Unit 5 Guided Notes

Functions, Equations, and Graphs

Standards: **F.IF.7e, F.BF.1, F.BF.4a, F.BF.5, F.LE.4, F.IF.8**

Clio High School – Algebra 2A

Name: ____________________________________ Period: ________________

Need help? Support is available!

- Miss Seitz's tutoring:
  Tuesdays and Thursdays after school
- Website with all videos and resources
  www.msseitz.weebly.com

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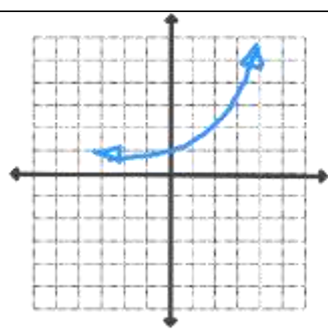
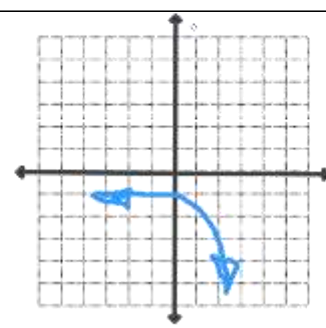
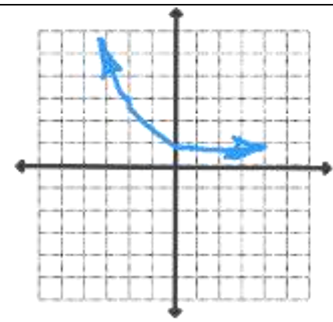
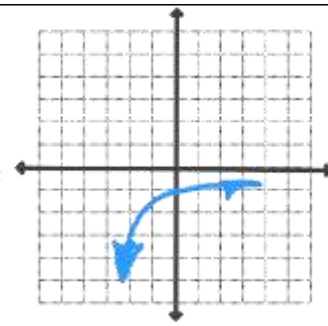
Concept #	What we will be learning…	Text
#1	**Graphing Exponential Growth and Decay** ☐ Substitute convenient values of x to generate a table and graph of an exponential function ☐ Classify exponential functions in function notation as growth or decay ☐ Determine the domain, range, and end behavior (horizontal asymptotes) of an exponential function when looking at a graph	7.1 7.2
#2	**Modeling Exponential Growth and Decay** ☐ Write an equation that describes how two things are related based on a real world context ☐ Distinguish between exponential functions that model exponential growth and exponential decay	7.1 7.2
#3	**Finding Linear Inverses** ☐ Write the inverse of a linear function in standard notation by replacing the x in my original equation with y and then solving for y	6.7
#4	**Translating Between Exponential and Logarithmic Functions** ☐ State that the inverse of an exponential function is a logarithmic function ☐ Explain the inverse relationship between exponents and logarithms ($y = b^x$) is equivalent to $\log_b y = x$	7.3
#5	**Evaluating a Logarithm** ☐ Solve for the values of logarithms by evaluating powers of the base (ex. $\text{Log}_5 25$ is 2 since $5^2 = 25$)	7.3
#6	**Solving Exponential and Logarithmic Equations** ☐ Solve problems with variables in an exponent or logarithm by applying the inverse relationship to logarithms and exponents	
#7	**Properties of Logarithms** ☐ Use product, quotient and power properties to rewrite logarithmic expressions	7.4
#8	**Solving by Taking Logs of Both Sides** ☐ Write an exponential equation $ab^{ct}=d$ in a logarithmic form $\log_b(d/a) = ct$ to solve it for t	7.5
#9	**Modeling With Exponential and Logarithmic Equations** ☐ Distinguish between exponential functions that model exponential growth and exponential decay	CB 7.5

#1	**Graphing Exponential Growth and Decay** **Text: 7.1, 7.2** ☐ Substitute convenient values of x to generate a table and graph of an exponential function ☐ Classify exponential functions in function notation as growth or decay ☐ Determine the domain, range, and end behavior (horizontal asymptotes) of an exponential function when looking at a graph Vocabulary: exponential function, initial value, y-intercept, domain, range, end behavior, exponential growth, exponential decay

Definitions
An **E**_______________ **F**________________ is of the form $y = ab^x$ where ***a*** is the **I**____________ **V**________ or also called the **Y- I**_____________.

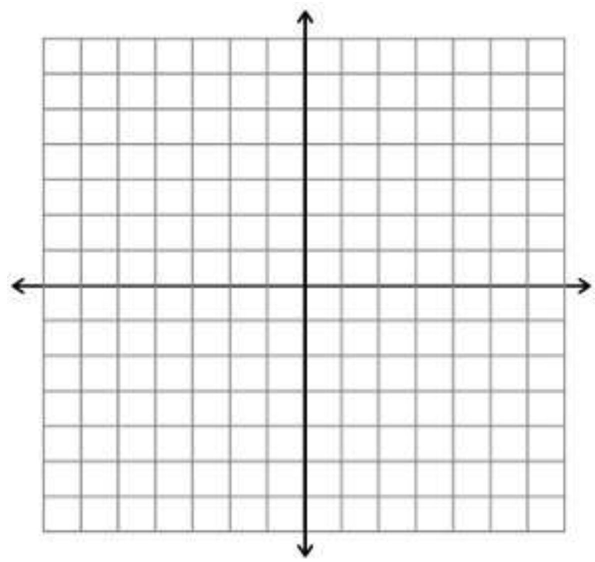
Domain, Range, End Behavior		
Domain	**Range**	**End Behavior**
All of the x's that will get us to the graph.	All of the y's that will get us to the graph.	What happens on the right and left ends of the graph.
Always the Same: All **R**_________ **N**_______________	**2 options:** • All **P**____________ real numbers • All **N**____________real numbers	**3 options:** • Approaching **Z**_______ • Approaching **P**_____________ ∞ • Approaching **N**_____________ ∞

$Y = a \cdot b^x$			
Exponential GROWTH			
	a > 0 b > 1 Example: $y=2(3^x)$		a < 0 b > 1 Example: $y= -2(3^x)$
Domain: _______________ Range: ________________ End Behavior:__________ & __________		Domain: _______________ Range: ________________ End Behavior:__________ & __________	

Exponential DECAY			
	a > 0 0 < b < 1 Example: $y = 3(\frac{1}{2})^x$		a < 0 0 < b < 1 Example: $y = -3(\frac{1}{2})^x$
Domain: ______________ Range: _______________ End Behavior:_________ & __________		Domain: ______________ Range: _______________ End Behavior:_________ & __________	

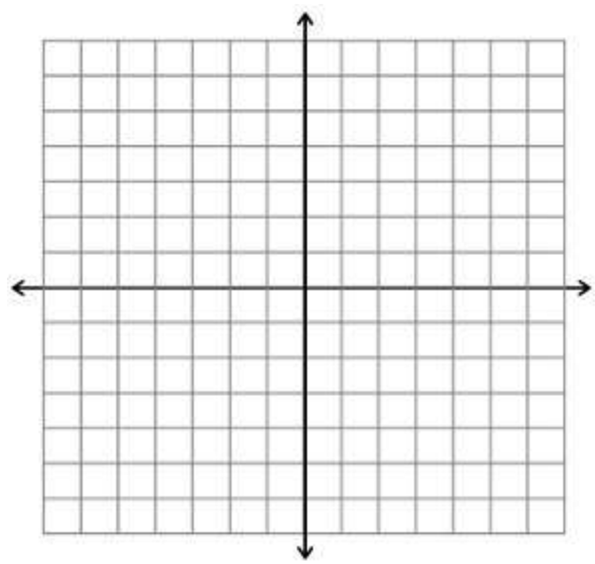
Graphing an Exponential Function

<u>Example 1:</u> Graph $y = 3(2)^x$



Domain:
Range:
End Behavior:

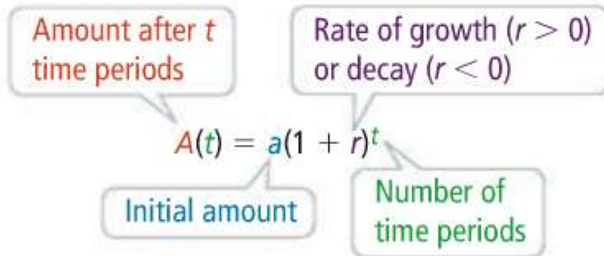
<u>Example 2:</u> Graph $y = -2(1/2)^x$



Domain:
Range:
End Behavior:

#2	**Modeling Exponential Growth and Decay** **Text: 7.1, 7.2** ☐ Write an equation that describes how two things are related based on a real world context ☐ Distinguish between exponential functions that model exponential growth and exponential decay Vocabulary: exponential growth, exponential decay, growth factor, decay factor

Exponential Growth and Decay



- Growth factor – **P**_______________
- Decay factor – **N**______________

Used when situations increase or decrease by a **F**__________ **A**___________

Annual Interest $A(t) = a(1 + r)^t$

<u>Example 1:</u> You invested $1000 in a savings account at the end of 6th grade. The account pays 5% ***annual interest***. How much money will be in the account after six years?

A) Determine if an exponential function is a reasonable model.

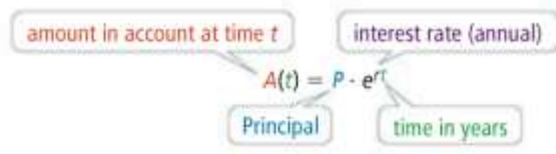
B) Define the variables and determine the model.

C) Use the models to solve the problem.

You Try It!

1.) The initial value of a car is $25,000. The value of the car decays by 15% each year. Estimate the value of the car after 5 years.

Continuous Interest $A(t) = Pe^{rt}$



There is a very special number we use a lot with exponential functions called

E__________

C____________.

e = 2.718281828...

<u>Example 2:</u> Find the amount in a continuously compounded account for the given conditions.

Principal: $5000
Annual interest rate: 3.5%
Time: 10 years

You Try It!

2.) Suppose you won a contest at the start of 5^{th} grade that deposited $3000 in an account that pays 5% ***annual interest compounded continuously***. How much will you have in the account when you enter high school 4 years later?

#3

Finding Linear Inverses **Text: 6.7**

☐ Write the inverse of a linear function in standard notation by replacing the x in my original equation

Vocabulary: inverse, relation

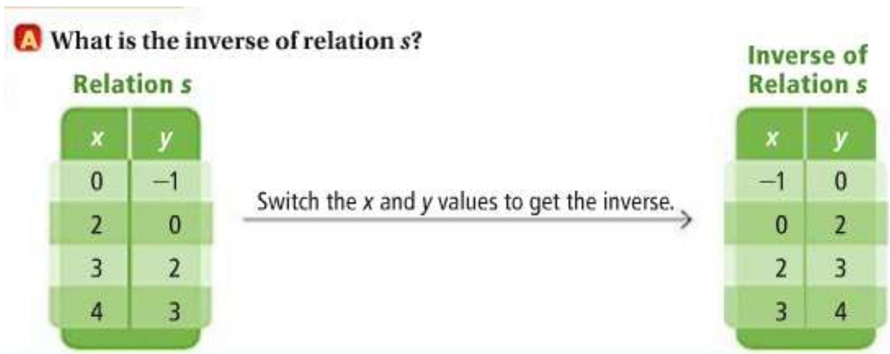
What is the inverse of the following:	
Addition:	**Subtraction:**
Multiplication:	**Division:**

Finding Inverses

A **relation** maps the **D**________ to the **R**_________

An **inverse relation** maps the **R**________ to the **D**_________

Example 1:



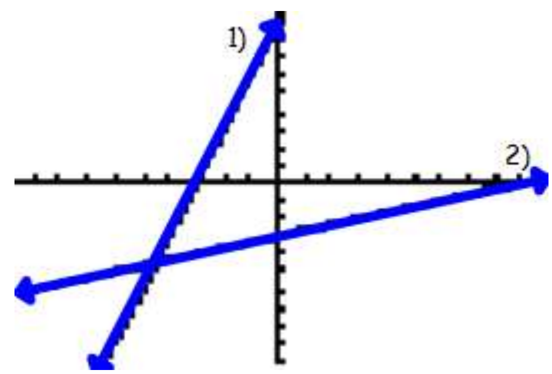
Example 2: Find the inverse of y = 2x + 3

STEP 1: Switch x and y	
STEP 2: Solve for y	
STEP 3: Write with **I**____________ **N**_____________ or $f^{-1}(x)$ for y.	

You Try It! Find the inverse of each.

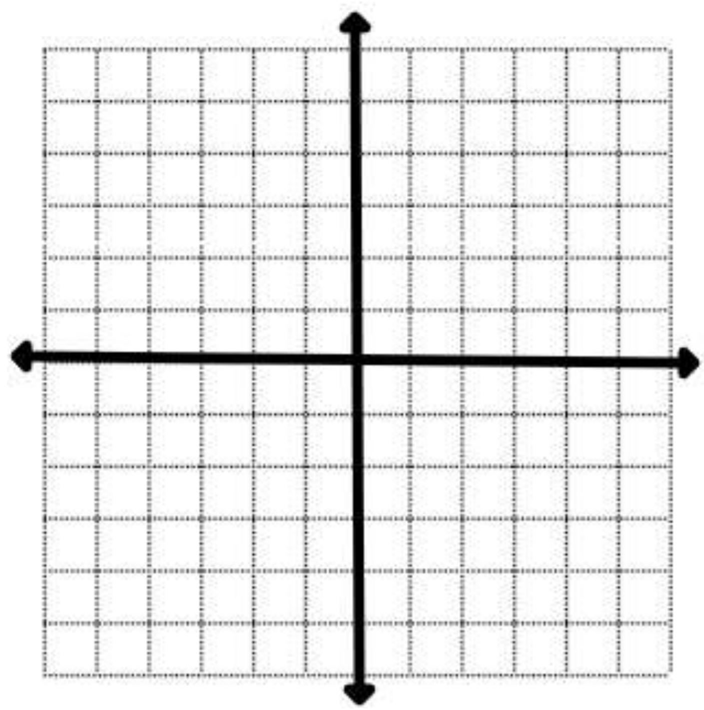
1.) $y = x^2 - 1$

2.) $f(x) = \sqrt{x-2}$

Graphs of Functions and their Inverses	
<u>Example 3:</u> 1.) y = 3x + 9 2.) y = (1/3)x – 3 Graphs of functions and their inverses are **S**________________ about the line ________.	

You Try It! Graph the function and its inverse.

3.) $y = x^2$



#4	**Translating Between Exponential and Logarithmic Functions** **Text: 7.3** ☐ State that the inverse of an exponential function is a logarithmic function ☐ Explain the inverse relationship between exponents and logarithms ($y = b^x$) is equivalent to $\log_b y = x$ Vocabulary: exponential form, logarithmic form, logarithm

Definitions
The inverse of an exponential function is a **L**_______________**F**_____________
take note **Key Concept** Logarithm A **logarithm** base *b* of a positive number *x* satisfies the following definition. For $b > 0, b \neq 1, \log_b x = y$ if and only if $b^y = x$. You can read $\log_b x$ as "log base *b* of *x*." In other words, the logarithm *y* is the exponent to which *b* must be raised to get *x*.

Translating Between Logarithmic and Exponential Forms	
Exponential Function $\text{base}^{\text{exponent}} = \text{number}$	**Logarithmic Function** $\log_{\text{base}} \text{number} = \text{exponent}$
$10^2 = 100$	
$3^4 = 81$	
	$\text{Log}_2 8 = 3$
	$\text{Log}_5 25 = 2$
Special Cases	
$y = 10^x$	**C**___________ **L**____________
$y = e^x$	**N**___________ **L**____________

You Try It!

1.) Write in logarithmic form.

$6^2 = 36$

2.) Write in exponential form.

$\log_4 16 = 2$

#5	**Evaluating a Logarithm** **Text: 7.3** ☐ Solve for the values of logarithms by evaluating powers of the base (ex. $\log_5 25$ is 2 since $5^2 = 25$) Vocabulary: N/A

Evaluating a Logarithm To *EVALUATE* a logarithm means to write as an equation and solve (or translate into exponential).	
Example 1: Evaluate $\log_4 32$	
STEP 1: Write as an equation	
STEP 2: Rewrite as exponential equation	
STEP 3: Rewrite with the same base	
STEP 4: Set exponents equal and solve	

You Try It! Evaluate each logarithm.

1.) $\log_5 125$

2.) $\log_4 \frac{1}{32}$

Challenge! See if you can follow this example.
What is the value of $\log_8 32$?

$\log_8 32 = x$	Write a logarithmic equation.
$32 = 8^x$	Use the definition of a logarithm to write an exponential equation.
$2^5 = (2^3)^x$	Write each side using base 2.
$2^5 = 2^{3x}$	Power Property of Exponents
$5 = 3x$	Since the bases are the same, the exponents must be equal.
$\frac{5}{3} = x$	Solve for *x*.

Since $8^{\frac{5}{3}} = 32$, then $\log_8 32 = \frac{5}{3}$.

#6	**Solving Exponential and Logarithmic Equations** **Text:** ☐ Solve problems with variables in an exponent or logarithm by applying the inverse relationship to logarithms and exponents Vocabulary: N/A

Solving Exponential Equations	
Variable in the BASE *Write each side with the same* ***exponent***	Variable in the EXPONENT *Write each side with the same* ***base***
$x^3 = 1{,}000$	$25 = 5^x$
With a negative exponent $x^{-2} = \frac{1}{121}$	*Using a negative exponent* $\left(\frac{1}{3}\right)^x = \frac{1}{27}$
REMINDER	
$\sqrt[5]{x^4} = x^{\frac{4}{5}}$ $k^{\frac{3}{4}} = \sqrt[4]{k^3}$ $p^{\frac{1}{2}} = \sqrt{p}$	

You Try It!

1.) $81 = x^4$

2.) $4^x = 64$

3.) $\frac{1}{32} = x^{-5}$

4.) $2^x = \frac{1}{32}$

Solving **Logarithmic** Equations

Variable in the BASE *Rewrite as **Exponential** then… Write each side with the same **exponent***	
$2 = \log_x 9$	*Working with negative exponents…* $\log_x \frac{1}{169} = -2$

Variable in the NUMBER *Rewrite as **Exponential** then… just calculate!*	
$\log_{21} x = 2$	*Working with negative exponents…* $\log_7 x = -3$

You Try It!

5.) $\log_x 49 = 2$ **6.)** $\log x = \frac{1}{2}$ **7.)** $2 = \log_{25} x$

#7	**Properties of Logarithms** ☐ Use product, quotient and power properties to rewrite logarithmic expressions Vocabulary: N/A	**Text: 7.4**

Properties of Exponents		
• $a^0 = 1, a \neq 0$ • $\frac{a^m}{a^n} = a^{m-n}$	• $a^{-n} = \frac{1}{a^n}$ • $(ab)^n = a^n b^n$ • $(a^m)^n = a^{mn}$	• $a^m \cdot a^n = a^{m+n}$ • $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
Simplify $(x^4)^2$	Simplify $\frac{3b^2}{3b^4}$	Rewrite in radical form $m^{\frac{5}{4}}$

IMPORTANT NOTE: Remember that the Natural Logarithm ($\log_e$) is written as ln. The properties of logarithms work the EXACT same way for Natural Logarithms as they do for regular ones, but we still write them as ln instead of $\log_e$

★**Power Property** $\log_b m^n = n \cdot \log_b m$	
Expanding: Use when you have an exponent	**Condensing:** Use when you have a number out front
$\log_4 x^5$ $\log \sqrt[5]{x^3}$ $\log_2(2y - 4)^3$	**5logx**

Product Property **$\log_b m \cdot n = \log_b m + \log_b n$**	
Expanding: Use when you have multiplication	**Condensing:** Use when you have addition
$\log_4 xyz$ **$\log_5(5b^6c)$**	**$\log x + 3\log y$**

Quotient Property **$\log_b \frac{m}{n} = \log_b m - \log_b n$**	
Expanding: Use when you have division	**Condensing:** Use when you have subtraction
$\log_3 \frac{2a}{b}$	**$\log x - 3\log y$**

You Try It!

1.) Expand: **$\log_3 \frac{9v}{u^2}$**

2.) Write as a single logarithm: **$\log_2 8 + 3\log_2 X - \log_2 \sqrt{y}$**

#8	**Solve by Taking Logs of Both Sides** ☐ Write an exponential equation $ab^{ct}=d$ in a logarithmic form $\log_b(d/a) = ct$ to solve it for t Vocabulary: N/A	**Text: 7.5**

Solving Exponential Equations

Example 1: Solve $2^{8z-3} = 2^{5z}$

Example 2: Solve $10 \cdot 5^{2x} + 5 = 130$. Give the **exact** answer (with logs left in) and the **approximate** answer (decimal).

Steps:	
1. Get the number with the exponent by itself.	
2. Take the log of both sides	
3. Use the properties of logarithms	
4. Solve to get x by itself	

Exact Answer: **Approximate Answer:**

Solving a Logarithmic Equation With Logs on Each Side

Example 3: What is the solution of $\log_6 (3n+5) = \log_6 (2n+6)$

You Try It!

1.) $2^{2x+5} = 2^{4x}$

2.) $\log_8 (2n+5) = \log_8 (-3n+10)$

3.) $3^{3x} = 190$

#9	**Modeling with Exponential and Logarithmic Equations** **Text: Concept Byte 7.5** ☐ Distinguish between exponential functions that model exponential growth and exponential decay Vocabulary: N/A

Example 1

The pH of a solution measures its acidity on a scale from 1 to 14. It is given using the equation pH = $-\log H_0$ where H_0 is the concentration of the Hydrogen ions.

a.) If a solution has a hydrogen concentration of 0.00000001, what is its pH?

b.) If a solution has a pH of 4, what is the Hydrogen ion concentration?

Example 2

The population of Clio *t* years after 2000 is given by the equation $P = P_0(1.001)^t$ where P_0 is the population of Clio in 2000.

a.) If there were 19,500 people living in Clio in 2000, then how many were living there in 2008?

b.) If there were 19,500 people living in Clio in 2000, then in what year will its population be at least 20,000?

Example 3
The number of atoms of an isotope in a sample after *t* years is given by the equation $N = N_0e^{kt}$ where N_0 is the number of atoms of that isotope initially and k is the decay constant of that isotope. **a.)** Uranium – 235 has a decay constant of 9.72×10^{-10}. If a sample contains 200,000,000 atoms of Uranium, how many atoms of Uranium will it have after 1,000 years?   **b.)** A sample which originally contained 5,000,000 atoms of Potassium – 40 now contains 4,500,000 atoms. If the decay constant of Potassium – 40 is 5.34×10^{-10}, how many years old is this sample?

Example 4
The Richter Scale measures the magnitude of an earthquake. It is given by the equation $M = \log A - \log A_0$ where A is the maximum excursion of the seismograph measured in nanometers and A_0 is the epicentral distance of the station measured in kilometers. **a.)** If a seismograph station that is 100 km distant from the epicenter of an earthquake has a maximum excursion of 100,000 nanometers, then what is the magnitude of the earthquake on the Richter Scale?   **b.)** An earthquake with a magnitude of 6.9 on the Richter Scale is observed by a seismograph station that is 1,000 km distant. What should be the maximum excursion of the seismograph measure?