UNIT 5: Inequalities (Chapter 9 in textbook)

Lesson 1: Representing Inequalities (9.1 in book)

Inequality: a mathematical statement comparing expressions that may not be equal.

Boundary Point: separates the values less than from the values greater than a specified point. It may or may not be a possible solution.

Representing Inequalities:

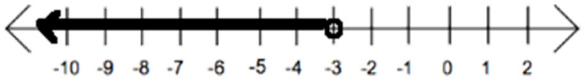
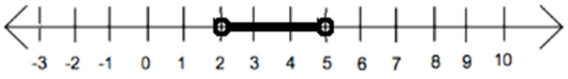
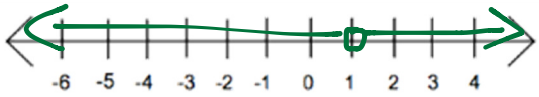
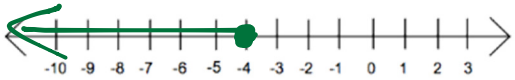
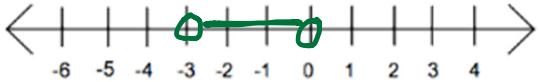
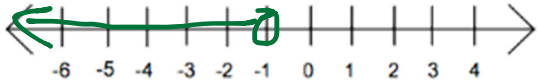
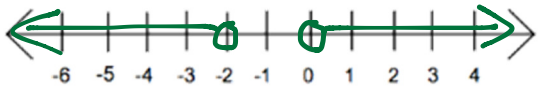
$a < b$	a is less than b
$a > b$	a is greater than b
$a \leq b$	a is less than or equal to b
$a \geq b$	a is greater than or equal to b
$a \neq b$	a is not equal to b

HINT: THE 'ALIGATOR' WANTS TO EAT THE BIGGER VALUE, SO ITS MOUTH OPENS TOWARDS IT!

Inequalities can be represented:

- Verbally
- Graphically
- Algebraically

Graphically, an **open circle** (not filled in) shows that the boundary point is NOT included; a **closed circle** shows that the boundary point IS included.

Verbally	Graphically	Algebraically
numbers greater than or equal to 2		$x \geq 2$ same as $2 \leq x$
y is less than -1		$y < -1$ same as $-1 > y$
numbers greater than 2 **and** less than 5		$a > 2$ ***and*** $a < 5$ same as $2 < a < 5$
b is less than -2 **or** greater than or equal to +2		$b < -2$ ***or*** $b \geq 2$
x is not equal to 1		$x \neq 1$
z is less than or equal to -4		$z \leq -4$
numbers greater than or equal to -5		$x \geq -5$
numbers greater than -3 and smaller than 0		$-3 < a < 0$
d is less than -1		$d < -1$
f is greater than 0 or smaller than -2		$f > 0$ OR $f < -2$

p. 346 #4, 5, 7, 9, 10, 11, 13, 15, 16 Challenge #23-25

UNIT 5 LESSON 2: Solving One-Step and Two-step Inequalities (9.2 in textbook)

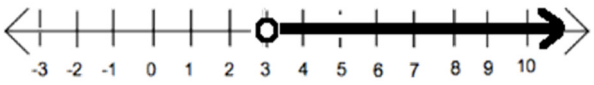
Just like with solving equations, we need to ISOLATE THE VARIABLE.

Example1:

$3x > 9$ 'x multiplied by 3 is greater than 9'

$\frac{3x}{3} > \frac{9}{3}$ To isolate x, both sides are divided by 3

$x > 3$



To check:

1. Test the boundary point.

At the boundary point, the LS = RS

LS = 3(3) = 9 RS = 9 Therefore, 3 is the b.p.

2. Check that the inequality sign is facing the correct way by testing a value for which the inequality should be true.

If x = 4, the inequality should be valid.

LS = 3(4) = 12 RS = 9 12 > 9 Therefore, it's valid.

Since the boundary point is valid and the inequality statement is true, then the solution x > 3 is correct.

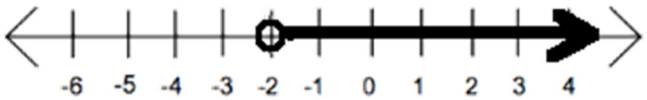
*IMPORTANT: WHEN YOU **MULTIPLY OR DIVIDE** BOTH SIDES BY A **NEGATIVE NUMBER**, YOU MUST **FLIP THE INEQUALITY SIGN**.*

Example 2:

$-5x < 10$ 'x multiplied by -5 is greater than 10'

$\frac{-5x}{-5} > \frac{10}{-5}$ To isolate x, both sides are divided by -5 and the sign is flipped

$x > -2$



Check:

1. Check the boundary point:
2. Check another value of x that makes the inequality true. A value for x > -2 is -1.

	LS		RS	
boundary	$-5\times(-2)=10$	$=$	10	✓
one other solution	$-5\times0=0$	$<$	10	✓

On your own:

$\frac{y}{4} < -1$

$\times 4 \quad \times 4$

$y < -4$

CHECK:

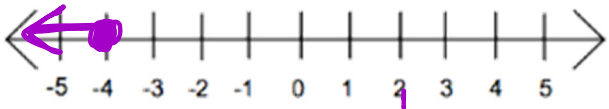
	LS		RS	
boundary	$-4\div4=-1$	$=$	-1	✓
one other	$-8\div4=-2$	$<$	-1	✓

$-\frac{1}{2}x \geq 2$

$-0.5x \geq 2$

$\div(-0.5) \quad \div(-0.5)$

$x \leq -4$



CHECK:

	LS		RS	
boundary	$-\frac{1}{2}\times-4=2$	$=$	2	✓
one other	$-\frac{1}{2}\times-6=3$	$\geq$	2	✓

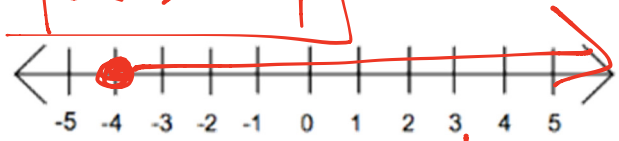
$-2(x+2) \leq 4$

$\div(-2) \quad \div(-2)$

$x+2 \geq -2$

$-2 \quad -2$

$x \geq -4$



CHECK:

	LS		RS	
boundary	$-2(-4+2)=4$	$=$	4	✓
one other	$-2(0+2)=-4$	$<$	4	✓

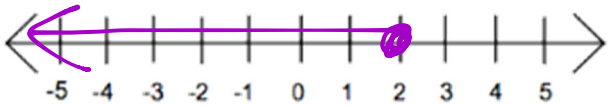
$-3x+1 \geq -5$

$-1 \quad -1$

$-3x \geq -6$

$\div(-3) \quad \div(-3)$

$x \leq 2$



CHECK:

	LS		RS	
boundary	$-3\times2+1=-5$	$=$	-5	✓
one other	$-3\times0+1=1$	$\geq$	-5	✓

Class work/HW: solving one-step and two-step inequalities worksheets;
challenge: p. 359 #23-28

UNIT 5 LESSON 3: Multi-Step Inequality Word Problems (9.3 in textbook)

To solve multi-step inequalities, use the same process as you would to solve a linear equation.

$-2x + 6 < x + 9$	We have x terms on both sides, so subtract an x term from both sides
$-2x + 6 - x < x + 9 - x$	
$-3x + 6 < 9$	x is multiplied by -3 and 6 is added to it
$-3x + 6 - 6 < 9 - 6$	Undo the addition by subtracting 6 from both sides.
$-3x < 3$	
$-3x \div -3 < 3 \div -3$	Undo the multiplication by dividing each side by -3. **Flip the inequality sign** since we are **dividing by a negative number**!!
$x > -1$	

Verify the solution:
Substitute the boundary point -1 to check that both sides are equal.

$-2(-1) + 6 = -1 + 9$
$+2 + 6 = 8$
$8 = 8$

Since the LS = RS, the boundary point is correct.

Since x is supposed to be greater than -1, we can check the inequality statement by substituting a value greater than -1 into the original inequality. An easy number to try greater than -1 is 0:

$-2(0) + 6 < 0 + 9$
$0 + 6 < 9$
$6 < 9$

Since 6 < 9, the inequality statement is correct.

SINCE THE BOUNDARY POINT IS VALID AND THE INEQUALITY STATEMENT IS TRUE, THEN THE SOLUTION x > -1 IS CORRECT

On your own: Solve then check $-3x - 10 > 5x + 38$

$+3x \quad +3x$

$-10 > 8x + 38$
$-38 \quad -38$

$-48 > 8x$
$\div 8 \quad \div 8$
$-6 > x$

$x < -6$

LS | RS

boundary

$-3x(-6) - 10$
$= 18 - 10$
$= 8$

$5x(-6) + 38$
$= -30 + 38$
$= 8$ ✓

$=$

one other

$-3x(-9) - 10$
$= 27 - 10$
$= 7$

$5x(-9) + 38$
$= -45 + 38$
$= -7$

$>$

Word Problem 1: **A student committee is planning a sports banquet. It costs $450 to rent the hall, and then $24 per person for the dinner. The committee needs to keep the total costs for the dinner under $2000. How many people can attend the dinner?**

< 2000

Step 1 (define variable): Let p represent the number of people attending the dinner.
Step 2 (write an inequality): The cost of the dinner is 450 + 24p and it must be less than $2000.

$450 + 24x < 2000$

Step 3 (solve):

$-450 \quad -450$

$24x < 1550$

$\div 24 \quad \div 24$

$x < 64.6$

OK $x \leq 64$

Step 4 (Check): check $x = 64$ & $x = 65$

$\text{cost} = 450 + 24 \times 64$

$= 1986$ ✓

$\text{cost} = 450 + 24 \times 65$

$= 2010$ ✓

Word problem 2: **A soccer club plans to buy shirts for its team and supporters. ProV Graphics charges a $75 setup fee plus $7 per shirt. BT Designs has no setup fee but charges $10.50 per shirt. How many shirts does the team need to order for ProV Graphics to be the better option?** (cost of ProV < cost of BT)

Let x represent the # of shirts.

inequality: cost of ProV < cost of BT

$75 + 7x < 10.50x$

solve:

$-7x \quad -7x$

$75 < 3.5x$

$\div 3.5 \quad \div 3.5$

$21.4 < x$

or $x > 21.4$

or $x \geq 22$

(minimum 22 shirts)

Class work/HW: p. 365 #3, 5, 6, 7, 8, 10, challenge #17-22
Start review questions: p. 368 #1-20; optional p. 370 #1-15