Accelerated Mathematics III Frameworks Student Edition

Unit 5 Investigating Trigonometry Graphs

1st Edition
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Accelerated Mathematics III – Unit 5
Investigating Trigonometry Graphs
Student Edition

INTRODUCTION:

In this unit students develop an understanding of the graphs of the six trigonometric functions: sine, cosine, tangent, cotangent, cosecant, and secant. They will learn to recognize the basic characteristics of the trigonometric functions. Using these characteristics they will be able to write a function to match a given description or data set. Students will learn to apply the trigonometric functions in a variety of real-world settings. In the last task they will explore the inverses of the sine, cosine and tangent functions and will use these ideas to solve equations and real-world problems.

ENDURING UNDERSTANDINGS:

- There are many instances of periodic data in the world around us.
- Trigonometric functions can be used to model real world data that is periodic in nature.
- There is a direct relationship between right triangle trigonometry and trigonometric functions.
- The inverses of sine, cosine and tangent functions are not functions unless the domains are limited.

KEY STANDARDS ADDRESSED:

MA3A3. Students will investigate and use the graphs of the six trigonometric functions.

a. Understand and apply the six basic trigonometric functions as functions of real numbers.
b. Determine the characteristics of the graphs of the six basic trigonometric functions.
c. Graph transformations of trigonometric functions including changing period, amplitude, phase shift, and vertical shift.
d. Apply graphs of trigonometric functions in realistic contexts involving periodic phenomena.

MA3A8. Students will investigate and use inverse sine, inverse cosine, and inverse tangent functions.

a. Find values of the above functions using technology as appropriate.
b. Determine characteristics of the above functions and their graphs.

RELATED STANDARDS ADDRESSED:

MA3P1. Students will solve problems (using appropriate technology).

a. Build new mathematical knowledge through problem solving.
b. Solve problems that arise in mathematics and in other contexts.
c. Apply and adapt a variety of appropriate strategies to solve problems.
d. Monitor and reflect on the process of mathematical problem solving.

MA3P2. Students will reason and evaluate mathematical arguments.

a. Recognize reasoning and proof as fundamental aspects of mathematics.
b. Make and investigate mathematical conjectures.
c. Develop and evaluate mathematical arguments and proofs.
d. Select and use various types of reasoning and methods of proof.

MA3P3. Students will communicate mathematically.

a. Organize and consolidate their mathematical thinking through communication.
b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
c. Analyze and evaluate the mathematical thinking and strategies of others.
d. Use the language of mathematics to express mathematical ideas precisely.

MA3P4. Students will make connections among mathematical ideas and to other disciplines.

a. Recognize and use connections among mathematical ideas.
b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
c. Recognize and apply mathematics in contexts outside of mathematics.

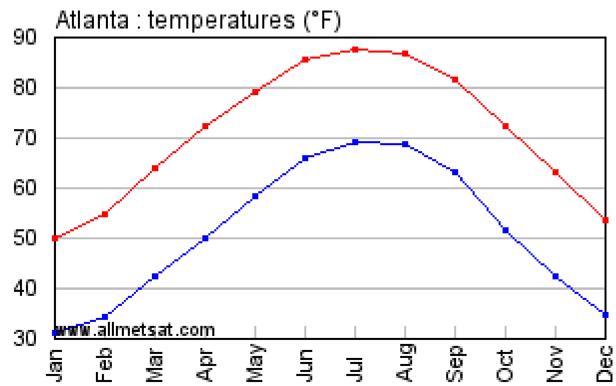
MA3P5. Students will represent mathematics in multiple ways.

a. Create and use representations to organize, record, and communicate mathematical ideas.
b. Select, apply, and translate among mathematical representations to solve problems.
c. Use representations to model and interpret physical, social, and mathematical phenomena.

What's Your Temperature? Learning Task:

Scientists are continually monitoring the average temperatures across the globe to determine if Earth is experiencing Climate Change. One statistic scientists use to describe the climate of an area is average temperature. The average temperature of a region is the mean of its average high and low temperatures.

1. The graph to the right shows the average high and low temperature in Atlanta from January to December. The average high temperatures are in red and the average low temperatures are in blue.



   a. How would you describe the climate of Atlanta, Georgia?

   b. If you wanted to visit Atlanta, and prefer average highs in the 70's, when would you go?

   c. Estimate the lowest and highest **average <u>high</u> temperature**. When did these values occur?

   d. What is the range of these temperatures?

   e. Estimate the lowest and highest **average low temperature**. When did these values occur?

   f. What is the range of these temperatures?

2. In mathematics, a function that repeats itself in regular intervals, or **periods**, is called **periodic**.
   a. If you were to continue the temperature graphs above, what would you consider its interval, or period, to be?

   b. Choose either the high or low average temperatures and sketch the graph for three intervals, or periods.

c. What function have you graphed that looks similar to this graph?

3. How do you think New York City's averages would compare to Atlanta's?

4. Use the data in the table below to create a graph for Sydney, Australia's average high and low temperatures.

	Jan	Feb	March	April	May	June	July	August	Sept	Oct	Nov	Dec
Avg high temp F°	78	79	77	73	67	64	62	65	69	72	74	77
Avg low temp F°	67	68	65	60	55	51	49	50	54	58	61	65

Use the graph to compare Sydney's climate to Atlanta's. What do you notice?

5. Sine and Cosine functions can be used to model average temperatures for cities. Based on what you know about these graphs from earlier units, why do you think these functions are more appropriate than a cubic function? Or an exponential function?

6. Using the data from #4, use your graphing calculator to find a sine function that models the data. Record your functions here.

a. Average high temp:

b. Average low temp:

The a, b, c and d values your calculator reported have specific effects upon the sin graph. In the next task you will investigate the characteristics of sine, cosine and tangent graphs. You will then revisit the functions in #6 to determine the effects of the values a – d.

Exploring Sine and Cosine Graphs Learning Task:

In the previous task you used your calculator to model periodic data using a sine graph. Now you will explore the sine, cosine, and tangent graphs to determine the specific characteristics of these graphs.

1. Using your knowledge of the unit circle, complete the following chart for $f(x) = \sin x$. (Use exact values.)

$x°$	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\sin x°$																	

   a. What do you notice about the values in the chart?

   b. When is $\sin x = 1$? 0?

   c. Does $\sin x$ appear to be a periodic function? If so, at what would you consider to be its period?

2. Use your graphing calculator to graph $\sin x$. Check your mode to make sure you are using radians and make sure you have an appropriate window for your data. Make a grid below and draw an accurate graph of $\sin x$. Make sure you draw a smooth curve.

3. Study the graph to answer the following questions:

   a. What is the period?

b. What is the domain and range?

c. What is the y-intercept?

d. Where do the x-intercepts occur?

e. What are the maximum values and where do they occur?

f. What is the minimum value and where does it occur?

g. How would your answers to questions d, e, and f change if your graph continued past 360°?

4. Using your knowledge of the unit circle and radians, complete the following chart for *f(x)=cos x*

x	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
cos x																	

5. Use your graphing calculator to graph *cos x*. Check your mode to make sure you are using radians and make sure you have an appropriate window for your data. Make a grid below and draw an accurate graph of *cos x*. Make sure you draw a smooth curve.

6. Study the graph to answer the following questions:

a. What is the period?

b. What is the domain and range?

c. What is the y-intercept?

d. Where do the x-intercepts occur?

e. What is the maximum value and where does it occur?

f. What is the minimum value and where does it occur?

g. Picture your graph continuing past 2π. How would your answers to questions d, e, and f change?

h. Re-write your answer to d, e and f and make sure to include all possible values.

i. Using your graph, find the value of *cos* $\frac{9\pi}{4}$.

j. Find all values of θ, such that $\cos\theta = -\frac{\sqrt{2}}{2}$

.

7. Using your calculator, graph *sin x* and *cos x* on the same axis. How are they alike? How are they different?

8. Using what you have learned about the graphs of the sine and cosine functions, practice graphing the functions by hand. You should be able to quickly sketch the graphs of these functions, making sure to include zero's, intercepts, maximums and minimums. Be sure to create a smooth curve!

The graphs of trigonometric functions can be transformed in ways similar to the function transformations you have studied earlier.

9. Using your knowledge of transformation of functions **make a conjecture** about the effect 2 will have on the following graphs of *cos x* and *sin x*. Then **graph it on a calculator** to determine if your conjecture is correct. Graph the parent function **each time** to compare the effect of 2 on the graph. (Make sure your mode is set on radians.)

a. $f(x) = 2\sin x$ $f(x) = 2\cos x$

b. $f(x) = \sin x + 2$ $f(x) = \cos x + 2$

c. $f(x) = \sin(x - 2)$ $f(x) = \cos(x - 2)$

d. *(x)= sin (2x)* *f(x)= cos (2x)*

Using your conjectures from above, **sketch** the graph of these functions. Use your calculator to check your graphs.

e. *f(x)=2 sin(x)+2* *f(x)=* 2*cos(2x)*

Consider the functions $f(\theta)=A\sin(k\theta+c)+h$ and $f(\theta)=A\cos(k\theta+c)+h$. A, k, c and h have specific effects on the graphs of the function. In trigonometry we also have special names for them based on their effect on the graph.

10. **The amplitude of the function is |A|.** Look back at 9a: *f(x)=* 2*sin x and f(x)=* 2*cos x*

a. What is the value of A? How is it related to your graphs?

b. How is the amplitude related to the distance between the maximum and minimum values?

c. What effect does A have on the effect if A<0? Graph *y=-2sin x* to test your conjecture.

11. The **period** of the function is related to the value of **k**. Look at 9d: *f(x)=sin(2x) f(x)=cos(2x)*
The period of the sine and cosine function is defined as $\frac{2\pi}{k}$ **, where k>0.**

a. When x was multiplied by 2, the function repeated itself twice in the usual period of 2π. What is the period of the functions in 9d?

b. Using your calculator look at the graph of the functions *f(x)=sin x* and *f(x)=sin(4x)*. Notice the graph repeats 4 times in the length of time it took *sin x* to complete one period. So the period of *f(x)=sin(4x)* is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.

12. Look back at 9c. The 2 shifted the graph 2 units to the left. A horizontal translation of a trigonometric function is called a **phase shift**. When calculating the phase shift you have to consider the value of k, which determines the period of the function. **The phase shift =** $-\frac{c}{k}$ **.**

If c>0, the graph shifts to the left.
If c>0, the shift is to the right.

a. Graph *f(x) = sin x* and g*(x)=sin(x+2)*on the same axis using radians.

For what values of x is $f(x) = 0$? For what values of x is $g(x) = 0$?

b. What do you notice about your answers to II and III?

c. Use the table feature on your calculator to investigate this relationship for values other than $f(x)=0$.

13. Graph y=sin x and y= cos x on the same axes. How can you use the sine function to match the graph of the cosine function? How can you use the sine function to match the graph of the cosine function?

14. In trigonometric functions, **h translates** the graph h units vertically.
    a. What effect does 2 have on the graph of the function $y = f(x) + 2$?

    b. What effect did the 2 have on the graphs for 9b?

    c. What happens when h > 0? h < 0 ? h = 0?

15. Look at this equation that models the average monthly temperatures for Asheville, NC. (The average monthly temperature is an average of the daily highs and daily lows.)
    Model for Asheville, NC $f(t) = 18.5 \sin\left(\frac{\pi}{6}t - 4\right) + 54.5$ where t = 1 represents January

    a. Find the values of A, k, c, and h in the equation.

    b. Graph the equation on your calculator. The maximum and minimum values of a periodic function oscillate about a horizontal line called the **midline**. What is the midline of the equation modeling Asheville's temperature?

    c. How it the value of the amplitude related to this midline?

16. How is the amplitude related to the midline of $f(x)=\sin x$? $f(x)=3\sin x + 2$? $f(x)=-4\sin x -2$?

17. State the amplitude and period of the following functions describe the graph of the function.

    a. $f(\theta)=2\sin(6\theta)$ A = __ period = ___

b. $f(\theta) = -4\cos(1/2\ \theta)$ A = _______ period = __

The **frequency** of a *sine* or *cosine* function refers to the number of times it repeats compared to the parent function's period. Frequency is usually associated with the unit Hertz or oscillations/second and measures the number of repeated cycles per second. **The frequency and period are reciprocals of each other.**

Example: Determine the equation of a *cosine* function that has a frequency of 4.

The frequency and period are reciprocals of each other so the period $= \frac{1}{4}$.

Since $period = \frac{2\pi}{k}$, we can replace the period with $\frac{1}{4}$ and solve for k.

If $\frac{1}{4} = \frac{2\pi}{k}$, then $k = 8\pi$.

The *cosine* function with a frequency of 4 is $f(\theta)=\cos(8\pi\theta)$.

18. Find the equation of a sine graph with a frequency of 6 and amplitude of 4.

Applications of Sine and Cosine Graphs Learning Task:

Trigonometry functions are often used to model periodic data. Let's look at a few examples of real-world situations that can best be modeled using trigonometric functions.

1. The Flight of the Space Shuttle

As mathematicians in the US space program, you and your team have been assigned the task of determining the first orbit of the Space Shuttle on the next mission. Your project is to determine the orbit of the shuttle and any other information which might affect the remainder of the orbits during the remainder of this flight. (Will all orbits cross over the same initial points? Explain why or why not.)

Some points to remember are the shuttle may not cross land on the initial lift-off and the shuttle will launch from Kennedy Space Center in Florida.

Materials which may be used to complete this project include:

- Globe
- Poster putty
- String
- Copy of a world map
- Colored pencils or pens
- Grid paper
- Ruler

To begin your project:

1. Use the string to measure the circumference of a "great circle" by measuring the circumference of the globe at the equator.
2. Beginning at Kennedy Space Center, use your string to mark a great circle which represents your proposed orbit. Your orbit should alternate north and south of the equator. Hold the string in place around the globe with the poster putty.
3. Plot the coordinates of the orbit as ordered pairs (longitude, latitude) on a flat map of the globe.
4. Plot the coordinates of your orbit on graph paper. Use the intersection of the equator and the prime meridian as your origin.

Find a sinusoidal (sine) equation to fit your data. Include your coordinates for the orbit and show how you determined the equation of the orbit.

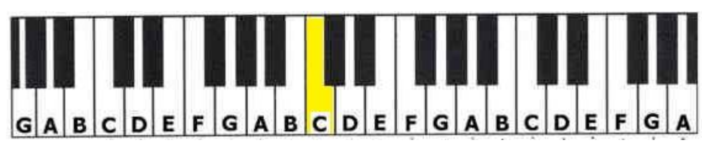
You may use a graphing calculator to check your work.

2. Musical Tones

There is a scientific difference between noise and pure musical tones.

A random jumble of sound waves is heard as noise.	
Regular, evenly spaced sound waves are heard as tones.	
The closer together the waves are the higher the tone that is heard.	
The greater the amplitude the louder the tone.	

Trigonometric equations can be used to describe the initial behavior of the vibrations that give us specific tones, or notes.



a. Write a *sine* equation that models the initial behavior of the vibrations of the note G above middle C given that it has amplitude 0.015 and a frequency of 392 hertz.

b. Write a *sine* equation that models the initial behavior of the vibrations of the note D above middle C given that it has amplitude 0.25 and a frequency of 294 hertz.

c. Based on your equations, which note is higher? Which note is louder? How do you know?

d. Middle C has a frequency of 262 hertz. The C found one octave above middle C has a frequency of 254 hertz. The C found one octave below middle C has a frequency of 131 hertz.

   i. Write a *sine* equation that models middle C if its amplitude is 0.4.

ii. Write a *sine* equation that models the C above middle C if its amplitude is one-half that of middle C.

iii. Write a *sine* equation that models the C below middle C if its amplitude is twice that of middle C.

3. The Ferris Wheel

There are many rides at the amusement park whose movement can be described using trigonometric functions. The Ferris Wheel is a good example of periodic movement.

Sydney wants to ride a Ferris wheel that has a radius of 60 feet and is suspended 10 feet above the ground. The wheel rotates at a rate of 2 revolutions every 6 minutes. (Don't worry about the distance the seat is hanging from the bar.) Let the center of the wheel represents the origin of the axes.

a. Write a function that describes a Sydney's height above the ground as a function of the number of seconds since she was ¼ of the way around the circle (at the 3 o'clock position).

b. How high is Sydney after 1.25 minutes?

c. Sydney's friend got on after Sydney had been on the Ferris wheel long enough to move a quarter of the way around the circle. How would a graph of her friend's ride compare to the graph of Sydney's ride? What would the equation for Sydney's friend be?

4. Weather Models

A city averages 14 hours of daylight in June, 10 in December, and 12 in both March and September. Assume that the number of hours of daylight varies periodically from January to December. Write a cosine function, in terms of t, that models the hours of daylight. Let **t** = 0 correspond to the month of January.

Graphing Other Trigonometric Functions Learning Task:

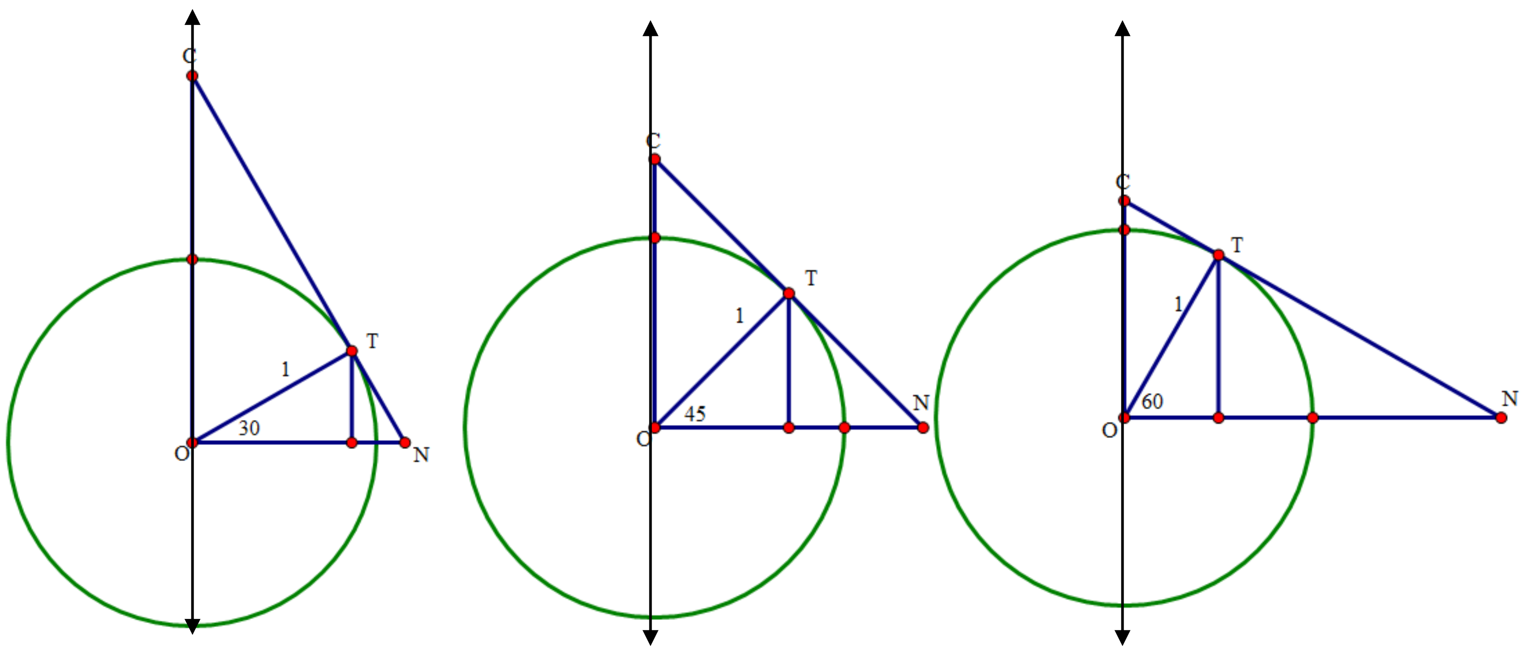
You have spent time working with Sine and Cosine graphs and equations. You will now have a chance to move beyond those functions into related functions.

1. Think back to right triangle trigonometry. How was the tangent function defined?

2. Solve the following problem. (Hint: Make sure to draw a picture.)
   A light-house keeper standing 50 feet above the ground sees a boat in distress at a 15° angle of depression. How far is the boat from the base of the lighthouse?

3. There is also a mathematical definition of tangent as related to circles. List everything you know about the tangent of a circle.

4. In the figures below segment TP is perpendicular to segment ON. Line CN is tangent to circle O at T. N is the point where the line intersects the x-axis and C is the where the line intersects the y-axis. (Only segment CN is shown.)
   Use your knowledge of sine and cosine to determine the length of segment TN. Use exact answers, no decimal approximations.



5. Using your understanding of the unit circle and $tangent\ \theta = \frac{sin\theta}{cos\theta}$, to complete the chart below for the indicated angles.

θ	Sin θ	Cos θ	Tan θ	Cotangent θ
30°				
45°				
60°				

How are these values of tangent related to the length you found in #4?

What would happen to the length of TN if the angle was changed to 0°?

What would happen to the length of TN if the angle was changed to 90°?

6. Complete the chart below using the unit circle. Remember, $tangent\ \theta = \frac{sin\theta}{cos\theta}$.

x	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	-π	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$
tan x													

How do you indicate an undefined solution for specific points in the domain of a graph?

Use your graph and chart to answer the following questions.

a. What is the period?

b. What is the domain and range?

c. What is the y-intercept?

d. Where do the x-intercepts occur?

e. Does the graph have any maximum or minimum values? If so, what are they?

f. Does the graph have any asymptotes? If so, where are they?

7. Use your graph to find the following values of tangent.

a. $\tan(-4\pi)=$	b. $\tan\frac{7\pi}{2}=$	c. $\text{Tan}\,\frac{-8\pi}{4}=$	d. $\tan\frac{-9\pi}{4}=$

8. Go back to the drawings in problem #4. Solve for the values of segment CT.

9. **The cotangent is the reciprocal of the tangent**. So cotangent $\theta = \frac{1}{tangent\ \theta}$. Using this definition, complete the chart in #5 by adding cotangent in the blank column beside tangent.

How are the values of cotangent in the chart related to the length of segment CT from problem #8?

What do you notice about the values of the tangent and cotangent values in the chart?

10. Complete the chart below and then graph the cotangent on the given grid.

x	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
Cotangent x									

How do you indicate an undefined solution for specific points in the domain of a graph?

Use your graph and chart to answer the following questions.

a. What is the period?

b. What is the domain and range?

c. What is the y-intercept?

d. Where do the x-intercepts occur?

g. Does the graph have any maximum or minimum values? If so, what are they?

h. Does the graph have any asymptotes? If so, where are they?

11. Use your graph to find the following values of cotangent.

a.	$\cot(-5\pi)=$	b.	$\text{Cot}\,\frac{9\pi}{2} =$	c.	$\text{Cot}\,\frac{-5\pi}{4}=$	d.	$\cot(0)=$

The cosecant and secant functions are reciprocals of the sine and cosine functions. The cosecant, csc, is the reciprocal of the sine function. The secant, sec, function is the reciprocal of the cosine function.

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

These graphs will be explored using a graphing calculator.

12. Graph the sine and cosecant function.
Enter the sine function in y1.
Check your window and mode to make sure there are at least two periods of the graph on your screen- one in the 1st quadrant and one in the 2nd quadrant.
Enter the cosecant function in y2.
The cosecant function may need to be entered as $\frac{1}{\sin\theta}$.

a. What are the values of sin x at x = -2π, $-\pi$, 0, π, and 2π?

b. What are the values of csc x at x = -2π, $-\pi$, 0, π, and 2π? Why?

c. What are the values of x where the graphs are tangent?

Answer the following questions for csc x.

d. What is the period?

e. What is the domain and range?

f. What is the y-intercept?

g. Where do the x-intercepts occur?

i. Does the graph have any maximum or minimum values? If so, what are they?

j. Does the graph have any asymptotes? If so, where are they?

13. Graph the cosine and secant function.
 Enter the cosine function in y1.
 Check your window and mode to make sure there are at least two periods of the graph on your screen- one in the 1st quadrant and one in the 2nd quadrant.
 Enter the secant function in y2.
 The secant function may need to be entered as $\frac{1}{\sin \theta}$.

a. What are the values of cos x at x = -3π/2, - π/2, π/2, and 3π/2?

b. What are the values of sec x at x = x = -3π/2, - π/2, π/2, and 3π/2? Why?

c. What are the values of x where the graphs are tangent?

Answer the following questions for sec x.

e. What is the period?

h. What is the domain and range?

i. What is the y-intercept?

j. Where do the x-intercepts occur?

k. Does the graph have any maximum or minimum values? If so, what are they?

l. Does the graph have any asymptotes? If so, where are they?

14. Which of the following functions have periods of 2π? Which have periods of π?
sin x, cos x, tan x, cot x, csc x, sec x

.

The transformations of the tangent, cotangent, secant, and cosecant functions work the same way as the transformations of the sine and cosine. The only difference is the period of the tangent and cotangent.

The **period** of functions $y = \sin k\theta$, $y = \cos k\theta$, $y = \csc k\theta$, and $y = \sec k\theta$ is $2\pi/k$.

The **period** of functions $y = \tan k\theta$, and $y = \cot k\theta$ is π/k.

15. Write an equation for the indicated function given the period, phase shift and vertical translation.
   a. Tangent function: period = 2π, phase shift = π, and vertical shift = -3

   b. Secant function: period = π, phase shift = $-\pi$, and vertical shift = 4

   c. Cosecant function: period = 4π, phase shift = $-\pi/2$, and vertical shift = -6

16. Graph the following functions.

a. $f(\theta) = \tan(\theta - \pi/4) + 2$

b. $f(\theta) = \cot(\theta - 3\pi/2) - 1$

c. $f(\theta) = \sec(2\theta + \pi/2) - 1$ (Hint: sketch $f(\theta) = \cos(2\theta + \pi/2) - 1$)

d. $f(\theta) = \csc(\frac{\theta}{2} - \pi) - 3$ (Hint: sketch $f(\theta) = \sin(\frac{\theta}{2} - \pi) - 3$)

17. Let's take one more look at your figures from problem 4. Find the lengths OC and ON.

a. What connection can you make between these values and the cosecant and secant values for the given angles?

b. CHALLENGE PROBLEM: Use similar triangles to show why these lengths are related to specific trigonometry functions.

Inverses of the Trigonometric Functions Learning Task:

In an earlier task you looked at this function that models the average monthly temperatures for Asheville, NC. (The average monthly temperature $\frac{\pi}{6}$ is an average of the daily highs and daily lows.)
Model for Asheville, NC *f(t)=18.5sin(t - 4)+ 54.5* where t=1 represents January.

How can you use this model to find the month that has a specific average temperature?

Recall your work with inverse relationships. The inverse of a function can be found by interchanging the coordinates of the ordered pairs of the function. In this case, the ordered pair (month, temperature) would become (temperature, month).

This task will allow you to explore the inverses the trigonometric functions from a geometric and algebraic perspective.

1. Graph $f(\theta) = \sin\theta$ and the line y = ½ .

   a. How many times do these functions intersect between -2π and 2π?

   b. How is this graph related to finding the solution to ½ = sin θ?

   c. If the domain is not limited, how many solutions exist to the equation ½ = sin θ?

   d. Would this be true for the other trigonometric functions? Explain.

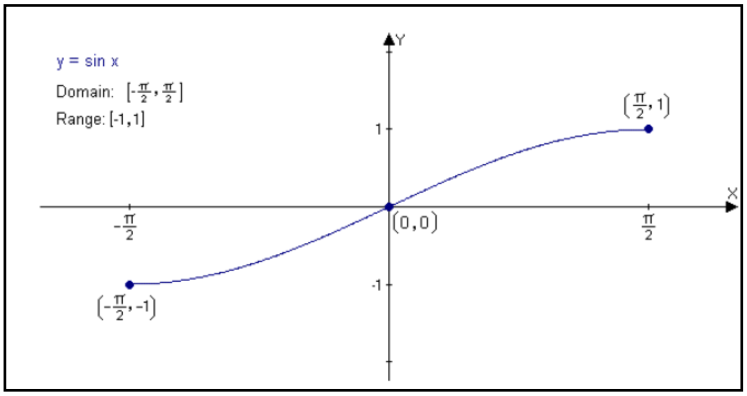
2. What happens to the axes and coordinates of a function when you reflect it over the line y = x?

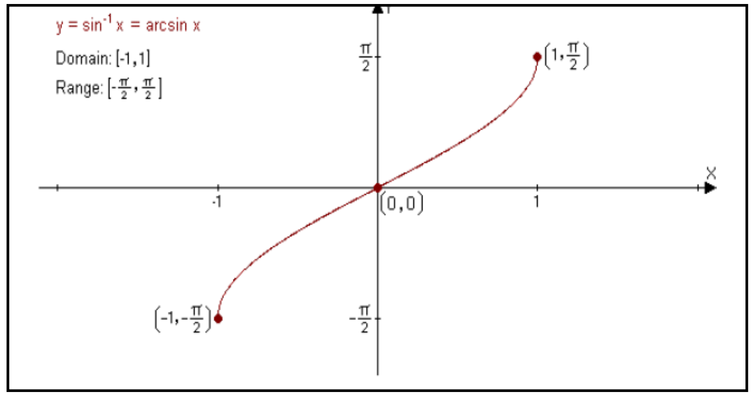
   a. Sketch a graph of $f(\theta) = \cos\theta$ and its reflection over the line y = x.

   b. How many times does the line x= ½ intersect the reflection of cos x?

3. Sketch the inverse of $f(\theta) = \tan\theta$ and determine if it is a function.

4. Look at this graph of $f(\theta) = \sin\theta$ with domain $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ and its inverse.

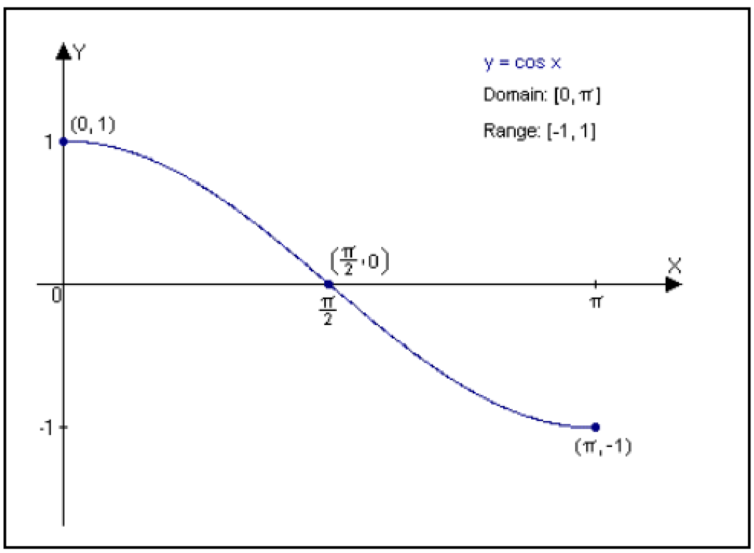


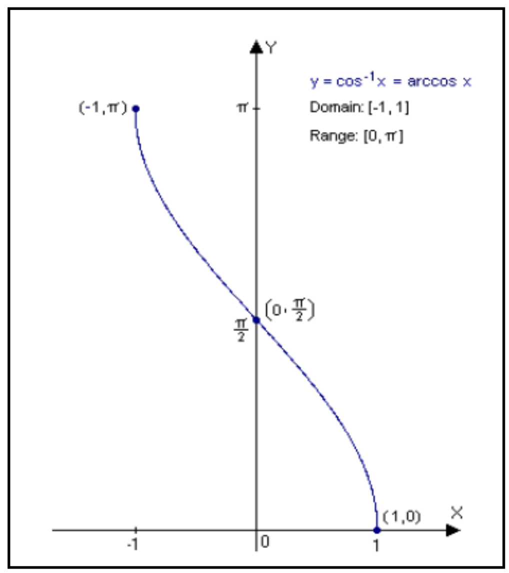


Is the inverse a function now?

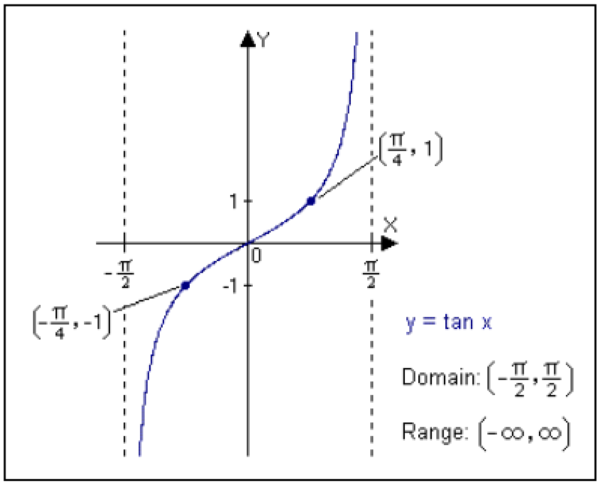
5. Use the following graphs to determine the limited domains on the cosine function used to insure the inverse is a function.

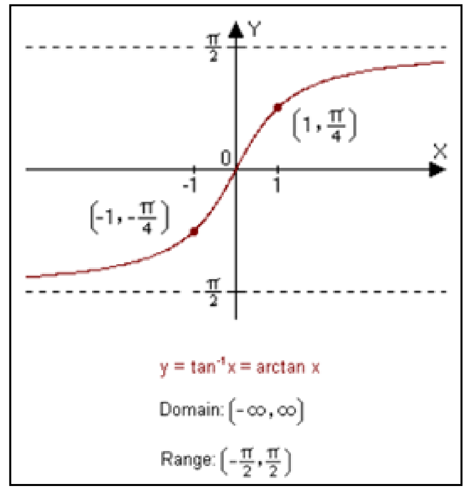
   Highlight the axes that represent the angle measure. (on both graphs)





6. Use the following graphs to determine the limited domains on the tangent function used to insure the inverse is a function. Mark the axes that represent the angle measure.





7. We use the names sin^{-1}, cos $^{-1}$, and tan^{-1} or arcsin, arccos, and arctan to represent the inverse of these functions on the limited domains you explored above. The values in the limited domains of sine, cosine and tangent are called **principal values**. (Similar to the principal values of the square root function.) Calculators give principal values when reporting sin^{-1}, cos $^{-1}$, and tan^{-1}.

   Complete the chart below indicating the domain and range of the given functions.

Function	Domain	Range
$f(\theta) = \sin^{-1}\theta$		
$f(\theta) = \cos^{-1}\theta$		
$f(\theta) = \tan^{-1}\theta$		

The inverse functions do not have ranges that include all 4 quadrants. Add a column to your chart that indicates the quadrants included in the range of the function. This will be important to remember when you are determining values of the inverse functions.

8. Use what you know about trigonometric functions and their inverses to evaluate the following expressions. Two examples are included for you. (Unit circles can also be useful.)

Example 1:	Example 2:
ArcCos $\left(\frac{\sqrt{3}}{2}\right)$ *The answers will be an angle.* Let θ = Arccos $\left(\frac{\sqrt{3}}{2}\right)$ *Ask yourself, what angle has a cos value of* $\frac{\sqrt{3}}{2}$. Cos θ = $\left(\frac{\sqrt{3}}{2}\right)$ *Using the definition of Arccos.* θ = $\left(\frac{\pi}{6}\right)$ *Why isn't* $\left(\frac{5\pi}{6}\right)$ *included*? So, ArcCos $\left(\frac{\sqrt{3}}{2}\right)$ = $\left(\frac{\pi}{6}\right)$	sin $(\cos^{-1} 1 + \tan^{-1} 1)$ *The answer will be a number, not an angle. Simplify parentheses first.* θ = $\cos^{-1} 1$ θ = $\tan^{-1} 1$ θ = 0^o θ = 45° sin $(0^o + 45^o)$ *Substitution* $\sin(45^o) = \frac{\sqrt{2}}{2}$ So, sin $(\cos^{-1} 1 + \tan^{-1} 1) = \frac{\sqrt{2}}{2}$