UNIT-5

PART-A (2 MARKS)

THIN CYLINDERS, SPHERES AND THICK CYLINDERS

1) How does a thin cylinder fail due to internal fluid pressure? (May / June 2017)

Thin cylinder failure due to internal fluid pressure by the formation of circumferential stress and longitudinal stress.

2) Name the stress develops in the cylinder. [NOV/DEC 2016]

The stresses developed in the cylinders are:

1. Hoop or circumferential stresses.
2. Longitudinal stresses
3. Radial stresses

3) Define radial pressure in thin cylinder. [NOV/DEC 2016]

The internal pressure which is acting radially inside the thin cylinder is known as radial pressure in thin cylinder.

4)Differentiate between thin and thick cylinders [MAY/JUNE 2016] [APR/MAY 2015](Nov/Dec 2018) (Apr/May 2019)

S.No	Thin	Thick
1	Ratio of wall thickness to the diagram of cylinder is less than 1/20.	Ratio of wall thickness to the diagram of cylinder is more than 1/20
2	Hoop stress is assumed to be constant throughout the wall thickness.	Hoop stress varies from inner to outer wall thickness.

5) Describe the lame's theorem: [MAY/JUNE 2016][NOV/DEC 2014] [MAY/JUNE 2017] (Apr/May 2018)

(Apr/May 2019)

Ratio stress, $\sigma_r = b/r^2 - a$

Hoop stress, $\sigma_c = b/r^2 + a$

6) State the expression for max shear stress in a cylinder shell [NOV/DEC 2015]

In a cylindrical shell, at any point on it circumference there is a set of two mutually perpendicular stresses $\sigma_c \sigma_\gamma$ which are principal stresses and as such the planes in which these act are the principal planes.

$$\tau_{max} = \frac{\sigma_c - \sigma_\gamma}{2} = \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2} = \frac{pd}{8t}$$

$$\boxed{\tau_{max} = \frac{pd}{8t}}$$

7) Define-hoop stress & longitudinal stress **[NOV/DEC 2015](Apr/May 2018)**

(i) Hoop stress: (σ_c)

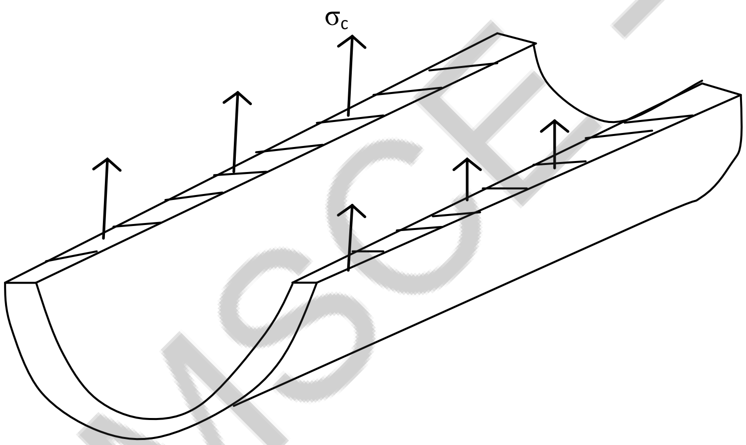
These act in a tangential dirn, to the circumference of the shell.

$$\sigma_c = \frac{pd}{2t}$$

(ii) Longitudinal stress: (σ_l)

The stress in the longitudinal direct due to tendency of busting the cylinder along the transverse place is called longitudinal stress

$$\sigma_l = \frac{pd}{4t}$$



8) State the assumption made in lame's theorem for thick cylinder analysis. [APR/MAY 2015] [NOV/DEC 2017] [NOV/DEC 2018]

1. The material is homogeneous and Isotropic.
2. The material is stressed within elastic limit.
3. All the fibers of the material are to expand (or) contract independently without being constrained by the adjacent fibers.
4. Plane section perpendicular to the longitudinal axis of the cylinder remain plane after the application of internal pressure.

9) What is meant by circumferential stress? **[NOV/DEC 2014]**

The stress in the circumferential direction in due to tendency of bursting the cylinder along the longitudinal axis is called circumferential stress (or) hoop stress.

$$\sigma_c = \frac{pd}{2t}$$

10) A storage tank of internal diameter 280 mm is subjected to an internal pressure of 2.56 MPa. Find the thickness of the tank. If the hoop & longtudital stress are 75 MPa and 45 MPa respectively

$\sigma_c = 75$ MPa, $\sigma_l = 45$ MPa, d = 280 mm, p = 2.5MPa

$\sigma_c > \sigma_l \Rightarrow$ use σ_c

$$\sigma_c = \frac{pd}{2t}$$

$$t = \frac{pd}{2\sigma_c} = \frac{2.5 \times 280}{2 \times 75}$$

$$\boxed{t = 4.66 \text{ mm}}$$

11) A spherical shell of 1m internal diameter undergoes a diameter strain of 10^{-4} due to internal pressure. What is the corresponding change in volume?

$$\delta V = e_v \times V$$

$$= 3 \times e \times V = 3 \times 10^{-4} \times \frac{\pi}{6} \times (1000)^3$$

$$\delta V = 157.079 \text{ mm}^3$$

12) A thin cylindrical closed at both ends is subjected to an internal pressure of 2 MPa. Internal diameter is 1m and the wall thickness is 10mm. What is the maximum shear stress in the cylinder material?

$$p = 2\,\text{mPa} = \frac{2N}{mm^2} \qquad d = 1m = 100mm \quad t = 10mm$$

$$\sigma_c = \frac{pd}{2t} = \frac{2 \times 1000}{2 \times 10} = 100\,N/mm^2$$

$$\sigma_l = \frac{pd}{4t} = \frac{2 \times 1000}{4 \times 10} = 50\,N/mm^2$$

$$\tau_{max} = \frac{\sigma_c - \sigma_l}{2} = \frac{100 - 50}{2} = \frac{50}{2}$$

$$\tau_{max} = 25\ N/mm^2$$

13) Find the thickness of the pipe due to an internal pressure of 10N/mm^2 if the permissible stress is 120 N/mm^2 and the diameter of the pipe is 750 mm

$$p = 10\ N/mm^2,\ \sigma_c = 120\,N/mm^2,\ d = 750\,mm$$

$$\sigma_c = \frac{pd}{2t}$$

$$t = \frac{pd}{2\sigma_c} = \frac{10 \times 750}{2 \times 120} = 31.25 \text{ mm}$$

14) A spherical shell of 1m diameter is subjected to an internal pressure 0.5 N/mm^2. Find the thickness if the allowable stress in the material of the shell is 75N/mm^2.

$$d = 1m = 1000 \text{ mm}, \qquad p = 0.5\,N/mm^2 \quad \sigma_c = 75N/mm^2$$

$$\sigma_c = \frac{pd}{4t}$$

$$t = \frac{pd}{4\sigma_c}$$

$$= \frac{0.5 \times 1000}{4 \times 75} = \boxed{1.67\,mm}$$

15) Define thick cylinder

When the ratio of thickness (t) to internal diameter of cylinder is more than 1/20 then the cylinder is known as thick cylinder

16) In a thick cylinder will the radial stress is vary over the thickness of wall?

Yes, in thick cylinder radial stress is maximum at inner and minimum at the outer radius.

17) Define thin cylinder. (Nov/Dec 2017)

If the thickness of wall of the cylinder vessel is less than 1/15 to 1/20 of its internal diameter, the cylinder vessels is known as thin cylinder.

18) In a thin cylinder will the radial stress over the thickness of wall?

No, In the cylinder radial stress developed in its wall is assumed to be constant since the wall thickness is very small as compared to the diameter of cylinder

19) What is the ratio of circumference stress to longitudinal stress of a thin cylinder?

The ratio of circumferential stress to longitudinal stress of a thin cylinder is two.

20) Distinguish between cylinder shell and spherical shell.

S.No.	Cylindrical shell	Spherical shell
1.	Circumferencial stress is twice the longitudinal stress	Only hoop stress presents
2.	It withstands low pressure than spherical shell for the same diameter	It withstand more pressure than cylinder shell for the same diameter

21) What is the effect of riveting a thin cylinder shell?

Riveting reduce the area offering the resistance. Due to this, the circumferential and longitudinal stresses are more. It reduces the pressure carrying capacity of the shell.

PART-B

1) A cylindrical thin drum 80cm in diameter and 3m long has a shell thickness of 1cm. If the drum is subjected to an internal pressure of 2.5 N/mm^2, determine (i) change in diameter (ii) change in length and (iii) change in volume $E=2\times10^5 N/mm^2$ and poisons ratio=0.25 (Apr/May 2019)

d = 80cm

L = 3m = 300cm

t = 1cm

$p = 250 N/cm^2$

$E = 2 \times 10^7\ N/cm^2$

$\mu = 0.25$

Change in diameter (δd)

$$\delta d = \frac{pd^2}{2tE}\left[1-\frac{\mu}{2}\right]$$

$$= \frac{250\times80^2}{2\times1\times2\times10^7}\left[1-\frac{0.25}{2}\right]$$

$$\boxed{\delta d = 0.35cm}$$

Change in length (δℓ)

$$\delta\ell = \frac{pdL}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{250\times80\times300}{2\times1\times2\times10^7}\left[0.5-0.25\right]$$

$$\boxed{\delta\ell = 0.0375cm}$$

Change in volume (δv)

$$\frac{\delta V}{V} = 2\frac{\delta d}{d} + \frac{\delta l}{l}$$

$$\frac{\delta V}{V} = 2\frac{0.035}{80} + \frac{0.0375}{300} = 0.001$$

$$\text{original volume, } V = \frac{\pi}{4}d^2\times\ell = \frac{\pi}{4}\times80^2\times300$$

$$V = 1507964.473cm^3$$

$\delta V = 0.001 \times V = 0.001 \times 1507964.473 = 1507.96\ cm^3$

2) A spherical shell of internal diameter 0.9m and of thickness 10mm is subjected to an internal pressure of $1.4N/mm^2$. Determine the increase in diameter and increase in volume. $E=2\times10^5 N/mm^2$ and poissons ratio=1/3 (Apr/May 2019)

$d = 0.9\text{m} = 900\text{mm}$

$t = 10\text{mm}$

$p = 1.4\text{N/mm}^2$

$E = 2\text{x}10^5\text{N/mm}^2$

$\mu = \frac{1}{3}$

Change in diameter: (δd)

$$\delta d = \frac{pd^2}{4tE}\left[1-\frac{1}{m}\right]$$
$$= \frac{1.4\times900^2}{4\times10\times2\times10^5}\left[1-\frac{1}{3}\right]$$
$$\boxed{\delta d = 0.0945\text{mm}}$$

Change in volume (δv)

$$e_V = 3\text{x}\frac{\delta d}{d} = 3\text{x}\frac{0.0945}{900} = 315\text{x}10^{-6}$$
$$\frac{\delta V}{V} = 315\text{x}10^{-6}$$
$$V = \left(\frac{\pi}{6}\right)\text{x}d^3 = \left(\frac{\pi}{6}\right)\text{x}900^3$$
$$\delta V = 12028.5\text{mm}^3$$

3) A boiler shell is to be made of 15mm thick plate having tensile stress of 120 N/mm^2 If the efficiencies of the longitudinal and circumferential joints are 70% and 30%. Determine the maximum permissible diameter of the shell for an internal pressure of 2 N/mm^2 (Nov/Dec 2018)

Maximum diameter of circumference stress

$$\sigma_c = \frac{pd}{2t\eta_l}$$
$$120 = \frac{2\times d}{2\times15\times0.7}$$
$$d = \frac{120\times2\times15\times0.7}{2}$$
$$d = 1260\text{mm}$$

Maximum diameter for longitudinal stress

$$\sigma_l = \frac{pd}{4t\times\eta_c}$$
$$120 = \frac{2\times d}{4\times15\times0.3}$$
$$d = \frac{120\times4\times15\times0.3}{2}$$
$$d = 1080\,\text{mm}$$

4) A thin cylindrical shell with following dimensions is filled with a liquid at atmospheric pressure. Length=1.2m, external diameter=20cm, thickness of metal=8mm, Find the value of the pressure exerted by the liquid on the walls of the cylinder and the hoop stress induced if an additional volume of 25cm^3 of liquid is pumped into the cylinder. Take E=2.1×10^{5}N/mm^2 and poisons ratio=0.33 (Nov/Dec 2018)

$$L = 1.2m = 1200mm$$
$$D = 20cm = 200mm$$
$$t = 8mm$$
$$d = D - 2t = 184mm$$
$$\delta V = 25cm^3 = 25000mm^3$$
$$E = 2.1x10^5 N/mm^2$$
$$\mu = 0.33$$

$$\text{Volume, } V = \frac{\pi}{4} \times d^2 \times \ell$$
$$= \frac{3.14}{4} \times 184^2 \times 1200$$
$$= 31908528 \text{mm}^3$$
$$\delta V = V \times \frac{pd}{2tE}\left(\frac{5}{2} - \frac{2}{m}\right)$$
$$25000 = 31908528 \times \frac{p \times 184}{2 \times 8 \times 2.1 \times 10^5}\left[\frac{5}{2} - 2(0.33)\right]$$
$$p = 7.7 \text{N/mm}^2$$
$$\sigma_c = \frac{pd}{2t} = \frac{7.7x184}{2x8} = 89.42 \text{N/mm}^2$$

5) A cylindrical shell 3m long which is closed at the ends has an internal diameter of 1.5m and a wall thickness of 20mm. Calculate the circumferential and longitudinal stresses induced and also change in the dimensions of the steel. If it is subjected to an internal pressure of 1.5 N/mm^2 Take E=2×10^{5}N/mm^2 and poisons ratio=0.3 (Apr/May 2018)

$$l = 3m = 3000mm$$
$$t = 20mm$$
$$d = 1.5m = 1500mm$$
$$p = 1.5N/mm^2$$
$$E = 2x10^5 N/mm^2$$
$$\mu = 0.3$$

Hoop stress, $\sigma_c = \frac{pd}{2t} = \frac{1.5 \times 1500}{2 \times 20} = 56.25$

$$\sigma_c = 56.25 \text{N/mm}^2$$

Longitudinal stress, $\sigma_\ell = \frac{pd}{4t} = \frac{1.5\times1500}{4\times20} = 28.125$

$$\sigma_\ell = 28.125N/mm^2$$

Change in diameter (δd)

$$\delta d = \frac{pd^2}{2tE}\left[1-\frac{\mu}{2}\right]$$

$$= \frac{1.5\times1500^2}{2\times20\times200\times10^3}\left[1-\frac{0.3}{2}\right]$$

$$\boxed{\delta d = 0.7225mm}$$

Change in length (δℓ)

$$\delta\ell = \frac{pdL}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{1.5\times1500\times3000}{2\times20\times200\times10^3}\quad[0.5-0.3]$$

$$\boxed{\delta\ell = 0.16875mm}$$

Change in volume (δv)

$$\frac{\delta V}{V} = \frac{pd}{2tE}\left[\frac{5}{2}-\frac{2}{m}\right]$$

$$\text{original volume}, V = \frac{\pi}{4}d^2\times\ell = \frac{\pi}{4}\times1500^2\times3000$$

$$V = 5301437603mm^3$$

$$\delta V = \frac{1.5\times1500x5301437603}{2\times20\times200\times10^3}\left[\frac{5}{2}-2\times0.3\right]$$

$$\boxed{\delta V = 2832955.72mm^3}$$

6) A compound cylinder formed by shrinking one tube to another is subjected to an internal pressure of 90MN/m^2. Before the fluid is admitted, the internal and external diameter of the compound cylinders are 180mm and 300mm respectively and the diameter at the junction is 240mm. If after shrinking on, the radial pressure at the common surface is 12MN/m^2. Determine the final stresses developed in the compound cylinder (Apr/May 2018)

Solution. Internal pressure in the cylinder,

$$p_1 = 90 \text{ MN/m}^2$$

Internal radius of the cylinder, $r_1 = \frac{180}{2} = 90 \text{ mm} = 0{\cdot}09 \text{ m}$

External radius of the cylinder, $r_3 = \frac{300}{2} = 150 \text{ mm} = 0{\cdot}15 \text{ m}$

Radius at the junction, $r_2 = \frac{240}{2} = 120 \text{ mm} = 0{\cdot}12 \text{ m}$

Radial pressure at the common surface after shrinking on,

$$p = 12 \text{ MN/m}^2$$

Final stresses developed:

Let the *Lame's equations* be:

For inner tube: $\sigma_r = \frac{b}{r^2} - a$

and, $\sigma_c = \frac{b}{r^2} + a$

For outer tube: $\sigma_r = \frac{b'}{r^2} - a'$

and, $\sigma_c = \frac{b'}{r^2} + a'$

*(a) **Before the fluid is admitted:***

Inner tube:

At, $r = r_1 = 0{\cdot}09$ m, $\sigma_r = 0$

$$\frac{b}{0{\cdot}0081} - a = 0$$

$\therefore$ $123{\cdot}456\, b - a = 0$ ...(*i*)

or,

At, $r = r_2 = 0{\cdot}12$ m,

$\sigma_r = 12$ MN/m²

$\therefore$ $$\frac{b}{0{\cdot}0144} - a = 12$$

or, $69{\cdot}44\, b - a = 12$ ...(*ii*)

From eqns. (*i*) and (*ii*), we get

$$b = -\,0{\cdot}222 \text{ and } a = -\,27{\cdot}41$$

Hence circumferential stress at any point in the *inner tube* will be given by

$$\sigma_c = -\frac{0{\cdot}222}{r^2} - 27{\cdot}41$$

The *minus sign indicates that the stress will be wholly compressive.*

At, $r = r_1 = 0{\cdot}09$ m,

$$\sigma_{c(0\cdot09)} = -\frac{0{\cdot}222}{0{\cdot}09^2} - 27{\cdot}41 = 54{\cdot}82 \text{ MN/m}^2 \text{ (comp.)}$$

At, $r = 0{\cdot}12$ m,

$$\sigma_{c(0\cdot12)} = -\frac{0{\cdot}222}{0{\cdot}12^2} - 27{\cdot}41 = 42{\cdot}82 \text{ MN/m}^2 \text{ (comp.)}$$

Outer tube:

At, $r = 0{\cdot}15$ m, $\sigma_r = 0$

$\therefore$ $$\frac{b'}{0{\cdot}15^2} - a' = 0$$

or, $44{\cdot}44\, b' - a' = 0$ ...(*iii*)

At, $r = 0{\cdot}12$ m, $\sigma_r = 12$ MN/m²

$\therefore$ $$\frac{b'}{0{\cdot}12^2} - a' = 0$$

or, $69{\cdot}44\, b' - a' = 12$ ...(*iv*)

From eqns. (*iii*) and (*iv*), we get

$$b' = +\,0{\cdot}48, \text{ and } a' = +21{\cdot}33$$

Hence the circumferential stress at any point in the *outer tube* will be given by

$$\sigma_c = \frac{0{\cdot}48}{r^2} + 21{\cdot}33$$

At, $r = 0{\cdot}12$ m,

$$\sigma_{c(0\cdot12)} = \frac{0{\cdot}48}{0{\cdot}12^2} + 21{\cdot}33 = 54{\cdot}66 \text{ MN/m}^2 \text{ (tensile)}$$

At, $r = 0{\cdot}15$ m,

$$\sigma_{c(0\cdot15)} = \frac{0{\cdot}48}{0{\cdot}15^2} + 21{\cdot}33 = 42{\cdot}66 \text{ MN/m}^2 \text{ (tensile)}$$

(*b*) *After the fluid is admitted:*

Let the *Lame's equation* be:

$$\sigma_r = \frac{b}{r^2} - a$$

At, $r = 0{\cdot}09$ m, $\sigma_r = 90$ MN/m²

$$\therefore \quad 90 = \frac{b}{0{\cdot}09^2} - a$$

or, $90 = 123{\cdot}45\,b - a$ ...(*v*)

At, $r = 0{\cdot}15$ m, $\sigma_r = 0$

$$\therefore \quad 0 = \frac{b}{0{\cdot}15^2} - a$$

or $0 = 44{\cdot}44\,b - a$ ...(*vi*)

From eqns. (*v*) and (*vi*), we get

$$b = 1{\cdot}139 \text{ and } a = 50{\cdot}61$$

Hence, the circumferential stress at any point in the compound tube is given by,

$$\sigma_c = \frac{b}{r^2} + a$$

At, $r = 0{\cdot}09$ m, $\sigma_{c(0{\cdot}09)} = \frac{1{\cdot}139}{0{\cdot}09^2} + 50{\cdot}61 = 191{\cdot}23$ MN/m² (tensile)

$r = 0{\cdot}12$ m, $\sigma_{c(0{\cdot}12)} = \frac{1{\cdot}139}{0{\cdot}12^2} + 50{\cdot}61 = 129{\cdot}71$ MN/m² (tensile)

$r = 0{\cdot}15$ m, $\sigma_{c(0{\cdot}15)} = \frac{1{\cdot}139}{0{\cdot}15^2} + 50{\cdot}61 = 101{\cdot}23$ MN/m² (tensile)

The *final circumferential stresses* at different points are tabulated below:

Tensile stress........ +

Compressive stress..... –

Circumferential (or hoop) stress (MN/m²)	*Inner tube*		*Outer tube*	
	$r = 0{\cdot}09$ m	$r = 0{\cdot}12$ m	$r = 0{\cdot}12$ m	$r = 0{\cdot}15$ m
(*i*) Initially	– 54·82	– 42·82	+ 54·66	+ 42·66
(*ii*) Due to fluid pressure	+ 191·23	+ 129·71	+ 129·71	+ 101·23
Final	+ 136·41	+ 86·89	+ 184·31	+ 143·89

Hence the *final circumferential stresses are:*

Inner tube: $\sigma_{c\,(0{\cdot}09)}$ = **136·41 MN/m²** (*tensile*)

$\sigma_{c\,(0{\cdot}12)}$ = **86·89 MN/m²** (*tensile*) **(Ans.)**

Outer tube: $\sigma_{c\,(0.12)}$ = **184·31 MN/m²** (*tensile*)

$\sigma_{c\,(0{\cdot}15)}$ = **143·89 MN/m²** (*tensile*) **(Ans.)**

7) Determine the maximum and minimum hoop stress across the section of a pipe of 400 mm internal diameter and 100 mm thick, when the pipe contains a fluid at a pressure of 8 N/mm^2. Also sketch the radial pressure distribution and hoop stress distribution across the section. (May 2017) (Nov/Dec 2017)

Solution,

Given:

Internal dia = 400 mm

∴ Internal radius, $r_1 = \frac{400}{2} = 200\,\text{mm}$

Thickness = 100 mm

∴ External radius $r_2 = \frac{600}{2} = 300\,\text{mm}$

Fluid pressure, p_0 = 8N/mm^2

or at $x = r_1$, $p_x = p_0$ = 8N/mm^2

The radial pressure (p_x) is given by equation (18.1) as

$$p_x = \frac{b}{x^2} - a$$

Now apply the boundary conditions to the above equation. The boundary conditions are:

1. At $x = r_1$ = 200 mm, p_x = 8 N/mm^2

2. At $x = r_2$ = 300 mm, p_x = 0

Substituting these boundary conditions in equation(i), we get

$$8 = \frac{b}{200^2} - a = \frac{b}{40000} - a \qquad ...(ii)$$

and

$$0 = \frac{b}{300^2} - a = \frac{b}{90000} - a \qquad ...(iii)$$

subtracting equation (iii) from equation (ii), we get

$$8 = \frac{b}{40000} - \frac{b}{90000} = \frac{9b - 4b}{360000} = \frac{5b}{360000}$$

$$b = \frac{360000 \times 8}{5} = 5760000$$

Substituting this value in equation (iii), we get

$$0=\frac{5760000}{90000}-a \quad \text{or} \quad a=\frac{5760000}{90000}=6.4$$

The values of 'a' and 'b' are substituted in the hoop stress.

Now hoop stress at any radius x is given by equation (18.2) as

$$\sigma_x=\frac{b}{x^2}+a=\frac{576000}{x^2}+6.4$$

At x = 200 mm, $\sigma_{200}=\frac{576000}{200^2}+6.4=14.4+6.4=20.8\,\text{N/mm}^2$. Ans.

At x = 300 mm, $\sigma_{300}=\frac{576000}{300^2}+6.4=6.4+6.4=12.8\,\text{N/mm}^2$. Ans.

Fig.15 Shows the radial pressure distribution and hoop stress distribution across the section. AB is taken a horizontal line. AC = 8N/mm^2. The variation between B and C is parabolic. The curve BC shows the variation of radial pressure across AB.

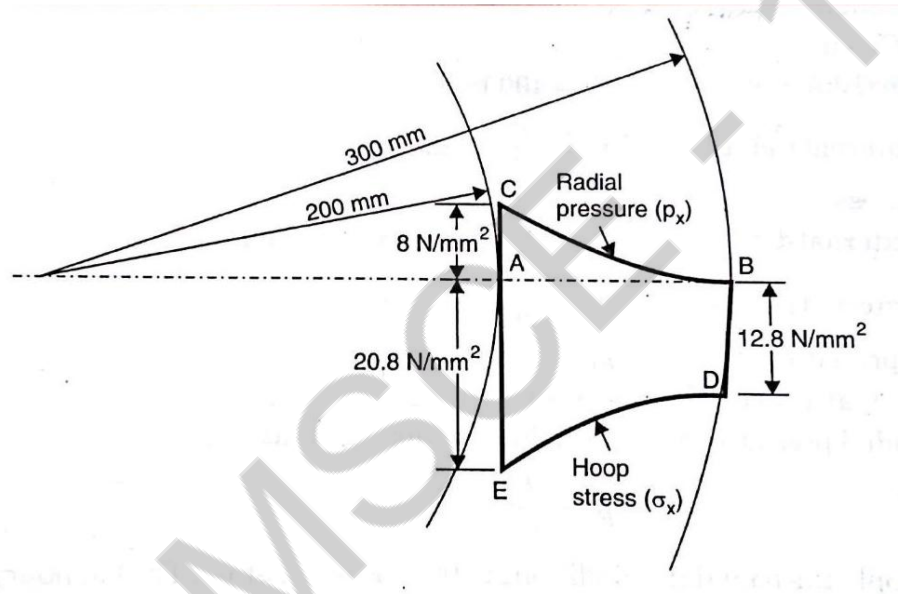


Fig. 15

The curve DE which is also parabolic, shows the variation of hoop stress across AB. Value BD = 12.8 N/mm^2 and AE = 20.8 N/mm^2 . The radial pressure is compressive whereas the hoop stress is tensile.

8) A cylindrical vessel is 2m diameter and 5m long is closed at ends by rigid plates. It is subjected to an internal pressure of 4N/mm^2 of the maximum principal stress is not to exceed 210N/mm^2. Find the thickness of the shell. Assume E=2×10^{5}N/mm^2 and poisons ratio=0.3, find the change in diameter, length and volume of the shell. [MAY/JUNE 2016-8 marks]

Given data:

Diameter, d=2m=2000mm

Length, l=5m=5000mm

Initial pressure, p=4N/mm^2

Maximum principal stress means the circumferential stress=σ_c=210N/mm^2

Young modulus =E=2×10^{5}N/mm^2

Poisons ratio=u=0.3

To find:

1.) Thickness of the shell (t)

2.) Change in diameter (∫d)

3.) Change in length and (∫ℓ)

4.) Change in volume (∫v)

Solution:

$$\sigma_c = \frac{pd}{zt}$$

$$t = \frac{pd}{2\times\sigma_c} = \frac{4\times 2000}{2\times 210} = 19.047\text{mm}$$

Change in diameter (∫d)

$$\int d = \frac{pd^2}{2t\,E}\left[1-\frac{1}{2}\times\mu\right]$$

$$= \frac{4\times 2000^2}{2\times 19.047\times 2\times 10^5}\left[1-0.5\times 0.3\right]$$

$$\int d = 1.785\text{mm}$$

Change in length (∫ℓ)

$$\int \ell = \frac{pd\ell}{2t\,E}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{4\times 2000\times 5000}{2\times 19.047\times 2\times 10^5}\left[\frac{1}{2}-0.3\right]$$

$$\int \ell = 1.050\text{mm}$$

Change in volume (∫v)

$$\frac{\int v}{v} = \frac{pd}{2tE}\left[\frac{5}{2}-2\times\mu\right] = \frac{4\times 2000}{2\times 19.047\times 2\times 10^5}\left[\frac{5}{2}-2\times 0.3\right]$$

$$\boxed{\int v / v = 1.995\times 10^{-3}\text{mm}^3} \qquad \left[V = \frac{\pi}{4}\times d^2\times L\right]$$

$$\int v = 1.995\times 10^{-3}\times\frac{\pi}{4}\times 2000^2\times 5000$$

$$\boxed{\int v = 313121500\text{mm}^3}$$

9) A spherical sheet of 1.50m internal diameter and 12mm shell thickness is subjected to pressure of 2N/mm^2. Determine the stress induced in the material of the shell [APR/-MAY/JUNE 2016-8marks]

Given data:

Internal diameter, d=1.5m=1500mm

Shell thickness, t=12mm

Pressure, P=2N/mm^2

To find:

(1) Stress induced in the material of shell

$$\sigma_1 = \frac{p}{4t}$$
$$= \frac{2\times1500}{4\times12}$$
$$= 62.5N/mm^2$$

10) A spherical shell of internal diameter 1.2m and of thickness 12mm is subjected to an internal pressure of 4N/mm^2. Determine the increase in diameter and increase in volume. Take E=2×10^{5}N/mm^2 and μ=0.33. [APR.MAY/JUNE 2016] 8marks

Given data:

Internal diameter of spherical shell, d=1.2m=1200mm

Thickness of spherical shell, t=12mm

Internal pressure, P=4N/mm^2

Young's modulus, E=2×10^{5}N/mm^2

Poisons ratio =μ= $\frac{1}{m} = 0.33$

To find:

(i) Increase in diameter, δd

(ii) Increase in volume, δv.

Change in diameter: (δd)

$$\delta d = \frac{pd^2}{4tE}\left[1-\frac{1}{m}\right]$$
$$= \frac{4\times1200^2}{4\times12\times2\times10^5}\left[1-0.33\right]$$
$$\boxed{\delta d = 0.402mm}$$

Change in volume (δv)

$$\delta v = v \times ev$$
$$= v \times \frac{3pd}{4tE}\left[1-\frac{1}{m}\right]$$
$$= \frac{\pi d^2}{6} \times \frac{3pd}{4tE}\left[1-\frac{1}{m}\right]$$
$$= \frac{\pi p d^4}{8tE}\left[1-0.33\right]$$
$$= \frac{3.14 \times 4 \times 1200^4}{8 \times 12 \times 2 \times 10^5}\left[1-0.33\right]$$
$$\boxed{\delta = 908,841.6mm^3}$$

Result:

1) Change in diameter =δd=0.402mm

2.) Change in volume =δv =908841.6mm

11) A steel cylinder of 300mm external diameter is to be shrunk to another steal cylinder of 150mm internal diameter. After shrinking the diameter at the function is 250mm and radial pressure at the common function is 28N/mm^2. Find the original difference in radial function. Take E=2×10^{5}N/mm^2 [Apr/May 2016-8 marks]

Given:

External diameter of outer cylinder =300mm

Radius of outer cylinder = r_2=150mm

Internal diameter of inner cylinder =150mm

Radius of inner cylinder =r_1=75mm

Diameter at the function =250mm

□radius at the function =r^*=125mm

Radial pressure at the function, P^*=N/mm^2

Young modulus =E=2×10^{5}N/mm^2

Original difference of radius at the function = $\frac{2r^*}{E}(a_1 - a_2) - - - (1)$

Find the values of a_1 and a_2 using the lame's equation.

For outer cylinder

$$P_x = \frac{b_1}{x_1^{\,2}} - a_1$$

(i) At function x=r^* =125mm and P^*=28N/mm^2

(ii) At x=150mm, P_x=0

Substitute in above equation, we get

$$28 = \frac{b_1}{125^2} - a_1 = \frac{b_1}{15625} - a_1 ----(2)$$

$$0 = \frac{b_1}{150} - a_1 = \frac{b_1}{22500} - a_1 -----(3)$$

solving equation $(2) \times (3)$ we get

$$b_1 = 1432000 \qquad a_1 = 63.6$$

For inner cylinder

$$P_x = \frac{b_2}{x^2} - a_2$$

(i) At function $x=r^* = 125m$ $\qquad P_x = P^* = 28N/mm^2$

(ii) At x=75mm, $P_x = 0$

Substitute these two condition ion above equation

$$28 = \frac{62}{75^2} - a_2 = \frac{b_2}{15625} - a_2 -----(4)$$

$$0 = \frac{b_2}{75^2} - a_2 = \frac{b_2}{15625} - a_2 ------(5)$$

solving equation (4) & (3) we get

$$b_2 = -246100$$

$$a_2 = -43.75$$

substitute the valuies of a_2 & a_1 in equation

$$= \frac{2r^*}{E}(a_1 - a_2)$$

$$= \frac{2\times125}{2\times105} \qquad [63.6 - (-43.75)]$$

$$= \frac{125}{105}\times107.35$$

$$= 0.13mm$$

12) Calculate (i) the change in diameter (ii) Change in length and (iii) Change in volume of a thin cylindrical shell 100cm diameter, 1cm thick and 5m long, when subjected to internal pressure of $3N/mm^2$. Take the value of $E=2\times10^5 N/mm^2$ and poison's ratio, μ=0.3 (Nov/Dec 2017)[Nov/Dec 2016][13 marks] [Nov/Dec 2015]

Given data:

Diameter of cylindrical shell, (d) =100cm =1000mm

Thickness of shell (t) =1cm=10mm

Length of the shell (ℓ) =5m=5000mm

Internal pressure $=P=3N/mm^2$

Young modular$=E=2\times10^5 N/mm^2$

Poison's ratio =μ=0.3

Solution:

Longitudinal stress, $\sigma_1 = \frac{pd}{4t} = \frac{3\times1000}{4\times10} = 75$

$$\boxed{\sigma_1 = 75N/mm^2}$$

Hoop stress, $\sigma_c = \frac{pd}{2t} = \frac{3\times1000}{2\times10} = 150$

$$\boxed{\sigma_c = 150N/mm^2}$$

(i) Change in diameter

$$\delta d = \frac{pd^2}{2tE}(1-\frac{1}{2m})$$

$$= \frac{3\times1000^2}{2\times10\times2\times10^5}\left[1-\frac{1}{2}\times0.3\right]$$

$$\boxed{\delta d = 0.637mm}$$

(ii)Change in length (**δ**ℓ)

$$\delta\ell = \frac{pdL}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{3\times1000\times5000}{2\times10\times200\times10^3} \qquad [0.5-0.3]$$

$$\boxed{\delta\ell = 0.75mm}$$

(iii) Change in volume ,

$$\delta v = v\times\frac{pd}{2tE}\left(\frac{5}{2}\frac{-2}{m}\right)$$

$$\text{Volume}, v = \frac{\pi}{4}\times d^2\times\ell$$

$$= \frac{3.14}{4}\times1000^2\times5000$$

$$= 39.25\times10^8 mm^3$$

$$\delta v = 39.25\times10^8\times\frac{3\times1000}{2\times10\times2\times\times10^5}\left[\frac{5}{2}-2(0.3)\right]$$

$$\delta v = 5593125mm^3$$

Result:

(i) Change in diameter (δd) =0.637mm

(ii) Change in length (δℓ) =0.75mm

(iii) Change in length (δv) =5593125mm^3

13) Calculate the thickness of metal necessary for a cylindrical shell of internal diameter 16mm ton with slant of internal pressure of 25mN/m$_2$. If maximum permissible shell stress is 125MN/m$_2$. [NOV/DEC-2016]

Given data:

Internal diameter, d=160mm.

Internal pressure, P=25MN/m^2 =25N/Mm^2

Maximum permissible shell stress =125MN/m^2=125N/mm^2

To find:

Thickness (t)

Solution:

$$\sigma_{max} = \frac{pd}{8t}$$
$$125 = \frac{25\times160}{8\times t}$$
$$t = \frac{25\times160}{125\times8}$$
$$t = 4mm$$
Thickness of cylinderrical shell is 4mm

14) A boiler is subjected to an internal steam pressure of 2N/mm^2. The thickness of boiler plate is 2.6cm and permissible tensile stress is 120N/mm^2. Find the maximum diameter, when efficiency of longitudinal joint is 90% and that of circumference joint is 40%. [NOV/DEC 2015 , 16marks]

Given data:

Internal steam pressure, P=2N/mm^2

Thickness boiler plate, t=2.6cm & 26mm

Permissible tensile stress (σ) =120N/mm^2

Efficiency of longitudinal joint, η_l =90% =0.90

Efficiency of circumferences joint, η_c =40% =0.40

In case of joint the permissible stress may be longitudinal (or) circumferential stress.

To find:

Maximum diameter (d)

Solution:

Maximum diameter of circumference stress

$$\sigma_c = \frac{pd}{2t\eta_l}$$
$$120 = \frac{2\times d}{2\times0.90\times2.6}$$
$$d = \frac{120\times2\times0.90\times26}{2}$$
$$d = 2808mm$$

Maximum diameter for longitudinal stress

$$\sigma_2 = \frac{pd}{4t \times \eta_c}$$

$$120 = \frac{2 \times d}{4 \times 26 \times 0.40}$$

$$d = \frac{120 \times 4 \times 0.40 \times 26}{2}$$

$$d = 2496\,\text{mm}$$

The longitudinal (or) circumferential stresses induced in the material directly proportional to diameter (d). Hence the stress induced will be less if the value of ‘d’ is less. Hence take the minimum value of diameter.

Hence, diameter (d) =249.6cm

15) A thin cylindrical shell 2.5 long has 700 mm internal diameters and 8mm thickness, if the shell is subjects to an internal pressure of 1Mpa, find

(i) The hoop and longitudinal stresses developed

(ii) Maximum shell stress induced and

(iii) The change in diameter, length and volume. Take modulus of elasticity of the wall material as 200Gpa and poison’s ratio as 0.3 [AP/MAY 2015- 16 marks]

Given data:

Length of cylindrical shell, ℓ=2.5m=2500mm

Internal diameter ǂ d, =700mm

Thickness of shell, t=8mm

Internal pressure, P=1mpa=1N/mm^2

Modulus of elasticity = E=200Gpa=200×10^{3}N/mm^2

Poison’s ratio = μ=0.3

To find:

1.) Hoop stress and longitudinal stress

2.) Maximum shell stress induced.

3.) Change in diameter, (δd)

4.) Change in volume,(δv)

5.) Change in length (δ ℓ)

Solution:

Hoop stress, $\sigma_c = \frac{pd}{2t} = \frac{1 \times 700}{2 \times 8} = 43.75$

$$\sigma_c = 43.75\,\text{N/mm}^2$$

Longitudinal stress, $\sigma_\ell = \frac{pd}{ut} = \frac{1\times700}{4\times8} = 21.87$

$\sigma_\ell = 21.875 N/mm^2$

Change in diameter (δd)

$$\delta d = \frac{pd^2}{2tE}\left[1-\frac{\mu}{2}\right]$$

$$= \frac{1\times700^2}{2\times8\times200\times0^3}\left[1-\frac{0.3}{2}\right]$$

$$\boxed{\delta d = 0.130mm}$$

Change in length (δℓ)

$$\delta\ell = \frac{pdL}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{1\times700\times2500}{2\times8\times200\times10^3} \qquad [0.5-0.3]$$

$$\boxed{\delta\ell = 0.109mm}$$

Change in volume (δv)

$$\delta v = \frac{pdv}{2tE}\left[\frac{5}{2}-\frac{2}{m}\right]$$

$$\text{original volume, } V = \frac{\pi}{4}d^2\times\ell = \frac{\pi}{4}\times700^2\times2500$$

$$V = 961625000mm^3 = 96.16\times10^7 mm^3$$

$$\delta v = \frac{1\times700\times96.16\times10^7}{2\times8\times200\times10^3}\left[\frac{5}{2}-2\times0.3\right]$$

$$\boxed{\delta v = 399665mm^3}$$

Maximum shell stress induced (σmax)

$$\sigma_{max} = \frac{pd}{t} = \frac{1\times700}{8\times8} = 10.937 N/mm^2$$

$$\sigma_{max} = 10.937 N/mm^2$$

Result:

1.) Hoop stress σ_c =43.75N/mm^2

2.) Longitudinal stress, σ_ℓ=21.875N/mm^2

3.) Maximum shell stress, σ_{max} =10.937N/mm^2

4.) Change in diameter, δd=0.130mm

5.) Change in length, δℓ=0.109mm

6.) Change in length, δv=399665mm^3

16) A thick cylinder with external diameter 320mm and internal diameter 160mm is subjected to an internal pressure of 8N/mm^2. Draw the variation of radial and hoop stresses in the cylinder wall. Also determine the maximum shell stress in the cylinder wall. [APR/MAY- 2015 -16marks]

Given data:

Internal diameter, d_1 =160mm

External diameter, d_2 =320mm

Internal radius, r_1 =80mm

External radius, r_2 =160mm

Internal pressure, P_1 =[8N/mm^2

To find:

1.) To draw variation of radial and hoop stress.

2.) The maximum shell stress in the cylinder.

Solution: we know that by lame's equation

$$\sigma_r = \frac{b}{r^2} - a - - - - - (1)$$
$$\sigma_c = \frac{b}{r^2} + a - - - - - (2)$$

At, r=r_1 =80, and σ_r =P_1 =8N/mm^2

R=r_2 =160mm and σ_r =P_2 =0

Substitute in equation (1)

$$8 = \frac{b}{(80)^2} - a \Rightarrow 8 = 1.562\times10^{-4}\,b - a - - - - (3)$$
$$0 = \frac{b}{(160)^2} - a \Rightarrow 0 = 3.9\times10^{-5}\,b - a - - - - - (4)$$

Equation (3) and (4) becomes

$$a - 1.562\times10^{-4}\,b = -8 - - - - (5)$$
$$a - 3.9\times10^{-4}\,b = 0 - - - - - (6)$$

Solving equation (5) and (6)

A=13.34

B=34217.27

Substitute values of a and b in equation (2)

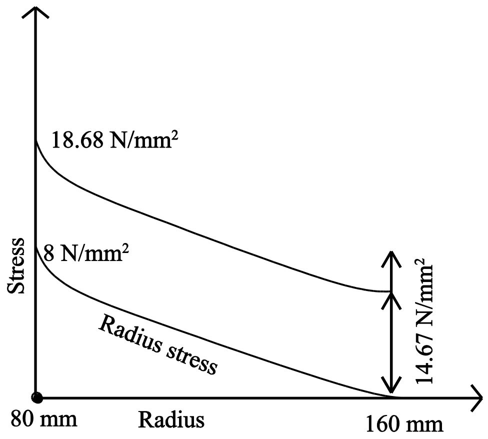
$$\sigma_c = \frac{b}{(80)^2} + a \Rightarrow = \frac{34217.27}{80^2} + 13.34$$

$$\sigma_c = 18.686 N/mm^2$$

$$\text{At } r = r_2 = 160mm$$

$$\sigma_c = \frac{b}{(160)^2} + a \Rightarrow \frac{34217.27}{(160)^2} + 13.34$$

$$\sigma_c = 14.67 N/mm^2$$



17) Desire relations for change in dimensions and change in volume of a thin cylinder subjected to internal pressure P. (May / June 2017) [NOV/DEC 2014]-16marks

Due to Internal pressure, the cylindrical shells are subjected to lateral and linear strain. Thus the change in dimensions such as length, diameter may increases.

We know that

$$e_c = \frac{\delta d}{d} = \frac{\sigma_c}{E} - \frac{\sigma_a}{mE}$$

Where , δd – change in diameter

$\frac{1}{m}$ = poison's ratio

E – young's Modulus

Circumferential stress,

$$e_c = \frac{pd}{2tE} - \frac{pd}{\mu t\, mE}$$

$$\boxed{e_c = \frac{pd}{2tE}\left[1 - \frac{1}{2m}\right]}$$

Change in diameter,

$$\delta d = e_c \times d$$

$$\delta d = \frac{pd^2}{2tE}\left[1 - \frac{1}{2m}\right]$$

$$e_a = \frac{\delta\ell}{\ell} = \frac{\sigma_a}{E} - \frac{\sigma_c}{mE}$$

Longitudinal strain, $= \frac{pd}{4tE} - \frac{pd}{2tmE}$

$$\boxed{e_a = \frac{pd}{2tE}\left[\frac{1}{2} - \frac{1}{m}\right]}$$

Change in length,

$$\delta\ell = e_a \times \ell$$

$$\delta\ell = \frac{pd\ell}{2tE}\left[\frac{1}{2} - \frac{1}{m}\right]$$

Volume strain,

$$e_v = \frac{\text{final volume} - \text{initial volume}}{\text{initial volume}}$$

$$= \frac{\frac{\pi}{4}(d + \delta d^2)(\ell + \delta\ell) - \frac{\pi}{4}d^2\ell}{\frac{\pi}{4}d^2\ell}$$

By neglecting higher order terms of $\delta\ell$ and δd

$$e_v = \frac{2\delta d}{d} + \frac{\delta\ell}{\ell}$$

$$= 2e_c + e_a$$

$$= \frac{2pd}{2tE}\left(1 - \frac{1}{2m}\right) + \frac{pd}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

$$= \frac{pd}{2tE}\left[2 - \frac{2}{2m} + \frac{1}{2} - \frac{1}{m}\right]$$

$$= \frac{pd}{2tE}\left[2 + \frac{1}{2} - \frac{2}{m}\right]$$

$$e_v = \frac{pd}{2tE}\left[\frac{5}{2} - \frac{2}{m}\right]$$

Change in volume,

$$\delta v = e_v \times v$$

$$= \frac{pdv}{2tE}\left[\frac{5}{2} - \frac{2}{m}\right]$$

$$\delta v = v \times \frac{\sigma_c}{E}\left(\frac{5}{2} - \frac{2}{m}\right)$$

18) Find the thickness of metal necessary for a thick cylindrical shell of internal diameter 160mm to withstand an internal pressure on 8N/mm^2. The maximum hoop stress in section is not to exceed 35N/mm^2. [NOV/DEC- 2014 -] [16 marks]

Given data:

Internal diameter, d_1 =160mm

Internal radius = $r_1 = \frac{d_1}{2} = \frac{160}{2} = 80mm$

Internal pressure,=P_1 =8N/mm^2

Maximum hoop stress =σ_c =35N/mm^2

To find:

Thickness of metal (t)

Solution:

The lame's equation's are

$$\sigma_r = \frac{b}{r^2} - a - - - - - (1)$$

$$\sigma_c = \frac{b}{r^2} + a - - - - - (2)$$

At $r = r_i = 80mm$ and $\sigma_r = P_1 = 8N/mm^2$

$(\sigma_c)_{max} = 35N/mm^2$

substituting in equation (1) and (2), we get

$$8 = \frac{b}{(80)^2} - a \Rightarrow 8 = 1.56\times10^{-4}\, b - a - - - - (3)$$

$$35 = \frac{b}{(80)^2} + a \Rightarrow 35 = 1.56\times10^{-4}\, b + a - - - - (4)$$

Equation (3) and (4) becomes

$$a - 1.56\times10^{-4}\, b = -8 - - - - - (5)$$

$$-a - 1.56\times10^{-4}\, b = -35 - - - - (6)$$

Solving equation (5) and (6), we get

$(5)\times1 \qquad -a + 1.56\times10^{-4}\, b = -8$

$(6)\times1 \qquad -a - 1.56\times10^{-4}\, b = -35$

$-2a \qquad = -27$

$\boxed{a = 13.5}$

Substitute (a) value in equation (5)

$$13.5 - 1.56\times10^{-4}\,b = -8$$
$$-1.56\times10^{-4}\,b = -8 - 13.5$$
$$-1.56\times10^{-4}\,b = -21.5$$
$$b = \frac{21.5}{1.56\times10^{-4}}$$
$$\boxed{b = 137.82}$$

19) A cylindrical shell in diameter and 3m length is subjected to an internal pressure of 2MPa. Calculate the maximum thickness if the stress should not exceed 50MPa. Find the change in diameter and volume of shell. Assume poisson's ratio of 0.3 and young's modulus of 200kN/mm^2. [MAY/JUNE -2014- 16marks]

Given data:

Diameter of cylindrical shell, d=1m=1000mm

Length of cylindrical shell, ℓ=3, m=3000mm

Internal pressure, P=2Mpa = 2N/mm^2

Maximum stress,. σ_c =50Mpa =50N/mm^2

Young's modulus =E =200KN/mm^2 =2×10^{5}N/mm^2

Poison's ratio, $\frac{1}{m} = 0.3$

To find:

(i) Change in diameter, δd

(ii) Change in volume, δv.

Solution:

$$\sigma_c = \frac{pd}{2t} = \frac{2\times1000}{2\times t}$$

Hoop stress, $50 = \frac{2\times1000}{2\times t}$

$$t = 20\text{mm}$$

Change in diameter, δd

$$\delta d = \frac{Pd^2}{2tE}\left[1 - \frac{1}{2m}\right]$$
$$= \frac{2\times(1000)^2}{2\times20\times2\times10^5}\left[1 - \frac{1}{2}\times0.3\right]$$
$$\delta d = 0.2125\text{mm}$$

Change in volume,

$$\delta v = \frac{pdv}{2tE}\left[\frac{5}{2} - \frac{2}{m}\right]$$

Volume of cylinder, $V = \frac{\pi}{4} d^2 \times \ell$

$$= \frac{\pi}{4}(1000)^2 \times 3000$$

$$= 2.355 \times 10^9 \, mm^3$$

$$\delta v = \frac{Pdv}{2tE}\left[\frac{5}{2} - \frac{2}{m}\right]$$

$$= \frac{2 \times 1000 \times 2.35 \times 10^9}{2 \times 20 \times 2 \times 10^5} \quad [2.5 - 0.6]$$

$$\boxed{\delta v = 118625 mm^3}$$

Result:

(i) Thickness of cylinder. t=20mm

(ii) Change in diameter. δd=0.2125mm

(iii) Change in volume, δv=1118625mm^3.