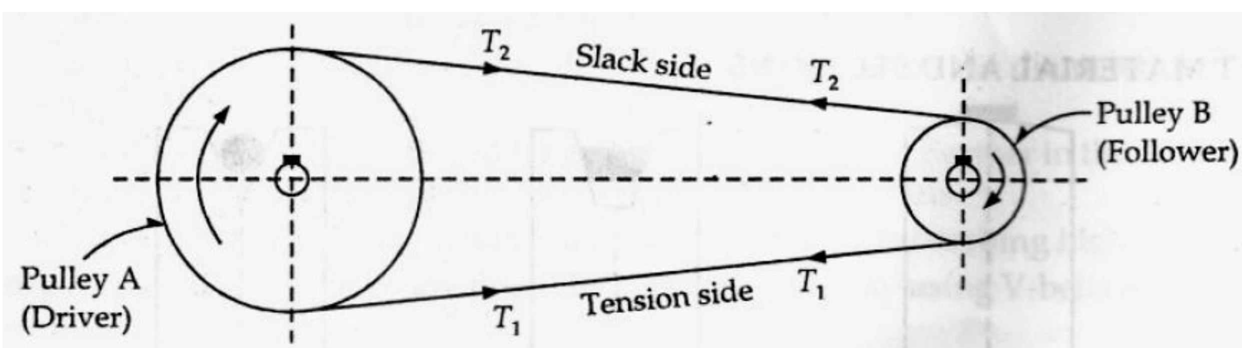
UNIT-5 (POWER TRANSMISSION Methods)

It is a method by which you can transfer cyclic motion from one place to another or one pulley to another pulley. The ways by which we can transfer cyclic motion are:-

1. **Belts and ropes** are used when the distance between the axes of the two shafts to be connected is considerable. Such connectors are non-rigid and undergo strain while in motion. These devices are called non-positive drive because of the possibility of slip occurring between the belt and pulley.
2. **Chain drive** is used when the distance between the shaft centers .is short and no slip is required. These connectors are referred to as a positive or non-slip drive.
3. **Gears** are used for transmitting motion and power when the distance between the driving & driven shafts is relatively small, and when a constant velocity ratio is desired.
4. **Clutches** are used for power transmission between co-axial shafts.

Belt & ropes drive

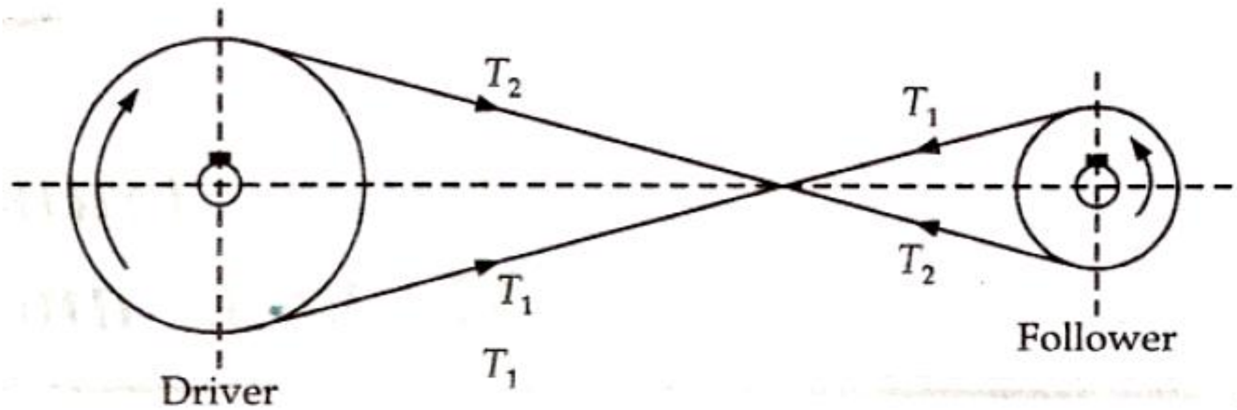
The transmission of power in factories from one rotating shaft to another that lies at a considerable distance is achieved through belts and ropes. The shafts are fitted with pulleys, the belt is wrapped round the pulleys and its ends are connected to form an endless connector. The belts and the pulley remain in contact by frictional grip.



Belt drive-open system

With reference of above figure, the pulley A which is connected to the rotating shaft is called the **driver**. The pulley B that needs to be driven is termed as **follower**. When the driver rotates, it **carries the belt because of friction** that exists between the pulley and the belt. The frictional resistance develops all along the contact surfaces. That makes the belt carry the follower which too starts rotating. The driving pulley pulls the belt from one side (called **tension or tight** side) and delivers it to the other side (called **slack side**). The tension T_1 in the belt on the tension side is more than tension T_2 on the slack side.

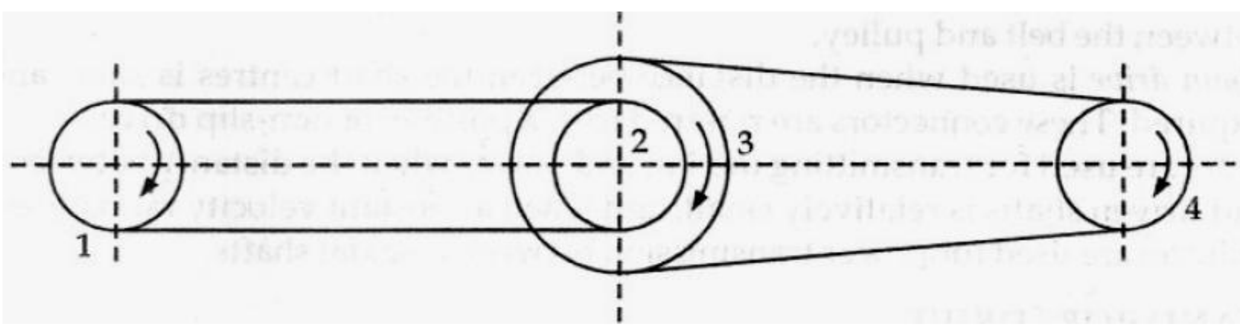
Two parallel shafts may be connected by open belt or by cross belt. In the open belt system, the **rotation** of both the pulleys is in the **same direction**. If a **crossed** belt system is used, the rotation of pulleys will be in the **opposite direction**.



Cross belt drive

The **angle of contact** in this system of drive is **more** and accordingly it can **transmit more power than** open belt drive system. However, the **belt wears out fast** at the places where crossing takes place in the crossed belt system. Further, for small centre distance, the belt is not fully utilized because of its larger slanted run off.

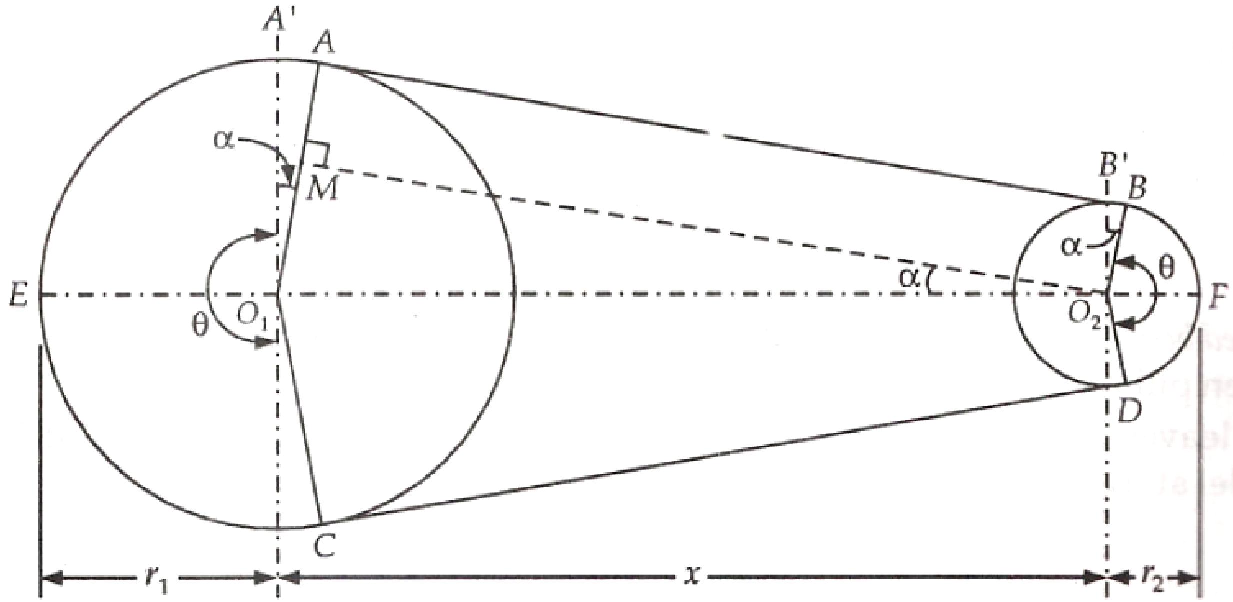
When a number of pulleys are used to transmit power from one shaft to another, then a ***compound*** **drive** is used.



Compound drive

Length of belt

(i) **Open belt system***:* the open belt system in which both the driving an driven pulleys rotate in the same direction.
Let $r_{1,}$ r_2= radius of the two pulleys
x = distai1ce between $O_1$and O_2; the centers of the two pulleys.
The belt leaves the bigger pulley at *A* and C, and the smaller pulley at Band D.
A line O_2M drawn parallel to *AB* will be perpendicular to O_1A also.



$$\angle A'O_1A = \angle B'O_2B = \angle O_1O_2M = \alpha$$

$$\sin\alpha = \frac{r_1 - r_2}{x}$$

Since α is very small,

$$\sin\alpha = \alpha = \frac{r_1 - r_2}{x}$$

Length of the belt, $l = 2(\text{arc } EA + AB + \text{arc } BF)$

$$\text{arc } EA = r_1 \times \left(\frac{\pi}{2} + \alpha\right) \quad \text{and} \quad \text{arc } BF = r_2 \times \left(\frac{\pi}{2} - \alpha\right)$$

$$AB = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2}$$

$$= \sqrt{x^2 - (r_1 - r_2)^2} = x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2} = x\left[1 - \left(\frac{r_1 - r_2}{x}\right)^2\right]^{1/2}$$

Since $\left(\frac{r_1 - r_2}{x}\right)^2$ is very small, Binomial expansion would give

$$AB = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2\right] = x\left[1 - \frac{(r_1 - r_2)^2}{2x^2}\right]$$

$$\therefore \quad l = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x\left\{1 - \frac{(r_1 - r_2)^2}{2x^2}\right\} + r_2\left(\frac{\pi}{2} - \alpha\right)\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 - r_2) + \left\{x - \frac{(r_1 - r_2)^2}{2x}\right\}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

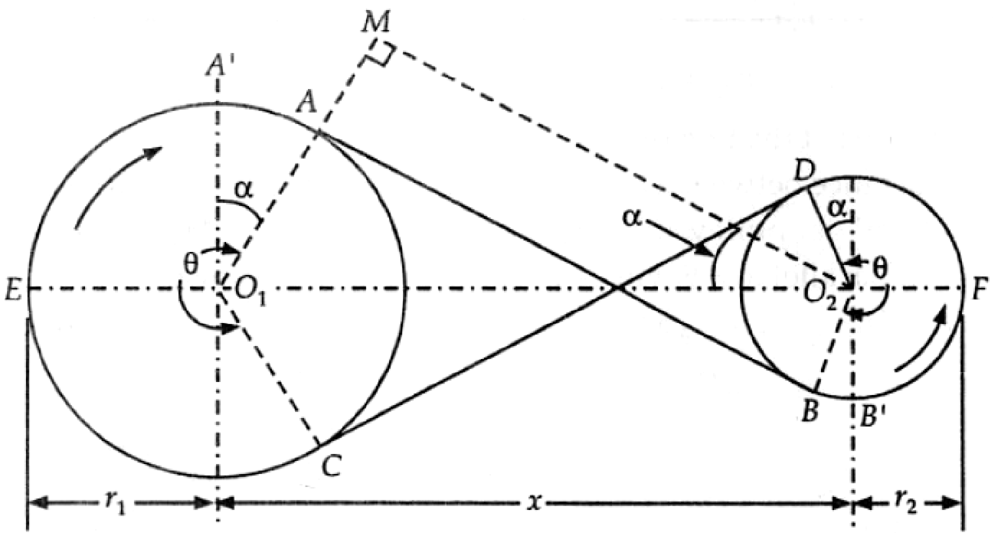
Substituting the value $\alpha = \frac{r_1 - r_2}{x}$, we get

$$l = \pi(r_1 + r_2) + 2\frac{r_1 - r_2}{x} \times (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{(r_1 - r_2)^2}{x} + 2x \longrightarrow (1)$$

(ii) **Crossed belt system**: the crossed belt system in which the driving and the driven pulleys rotate in opposite directions.

The belt leaves the bigger pulley at A and C and the smaller pulley at Band D. A line 0_2M is drawn parallel at AB will be perpendicular to O_1A also.



let $r_1$and r_2 = radius of the two pulleys

x = distance between $O_1$and O_2; the centres of the two pulleys

angle $A'O_1A$ = *angle* $B'OB$ = *angle* $O_1O_2M = \alpha$

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1A + AM}{O_1O_2} = \frac{r_1 + r_2}{x}$$

Since α is very small,

$$\sin\alpha = \alpha = \frac{r_1 + r_2}{x}$$

length of belt,

$$l = 2(\text{arc } EA + AB + \text{arc } BF)$$

$$\text{arc } EA = r_1\left(\frac{\pi}{2} + \alpha\right) \quad \text{and} \quad \text{arc } BF = r_2\left(\frac{\pi}{2} + \alpha\right)$$

$$AB = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 + r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2} = x\left[1 - \left(\frac{r_1 + r_2}{x}\right)^2\right]^{1/2}$$

Through binomial expansion

$$AB = x\left[1 - \frac{1}{2}\left(\frac{r_1 + r_2}{x}\right)^2\right] = x\left[1 - \frac{(r_1 + r_2)^2}{2x^2}\right]$$

$$l = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x\left\{1 - \frac{(r_1 + r_2)^2}{2x^2}\right\} + r_2\left(\frac{\pi}{2} + \alpha\right)\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + r_2) + \left\{x - \frac{(r_1 + r_2)^2}{2x}\right\}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

Substituting value of α = ——— , we get

$$l = \pi(r_1 + r_2) + 2\frac{r_1 + r_2}{x} \times (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{(r_1 + r_2)^2}{x} + 2x \qquad \longrightarrow (2)$$

It may be noted from equations 1and 2 that:

(I) The length of a crossed belt is more than that of an open belt, other conditions remaining the same.

(ii) The total length of a crossed belt is a function of $(r_1 + r_2)$. If the sum of the radii of two pulleys be constant, the length of the cross belt required will be also remain constant.

Ratio of tension

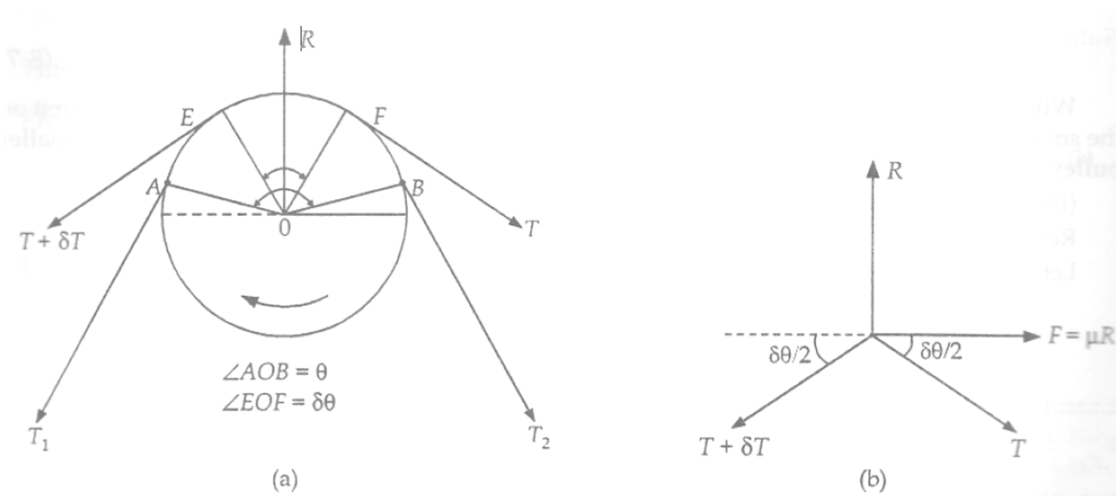
Figure shows a flexible belt resting over the flat rim of a stationary pulley. The tensions $T_1$and are such that the motion is impending Oust to take place) between the belt and the pulley. Considering the impending motion to be clockwise relative to the drum, the tension T_1 is more than T_2 is to be noted that only a part of the belt is in contact with the pulley. The angle subtended at the centre of the pulley by the position of belt in contact with it is called the *angle of contact* or the *angle of lap*

Angle of contact θ = *angle .AGB*

Let attention be focused on small element EF of the belt which subtends an angle δθ at the

centre. The segment EF is acted upon by the following set of forces:

- Tension *T* in the belt acting tangentially at S,
- Tension (*T* + 8T) in the belt acting tangentially at R
- Normal reaction R exerted by the pulley rim, and
- Friction force F = μR which acts against the tendency to slip and is perpendicular to normal reaction R.



Ratio of tight side & slack side

Considering equilibrium of forces in the radial (vertical) direction,

$$R = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2}$$

For small values of $\delta\theta$; $\sin\frac{\delta\theta}{2} \to \frac{\delta\theta}{2}$

$$R = (T + \delta T)\frac{\delta\theta}{2} + T\frac{\delta\theta}{2} = T\frac{\delta\theta}{2} + \delta T\frac{\delta\theta}{2} + T\frac{\delta\theta}{2}$$

The term $\delta T\frac{\delta\theta}{2}$ is small in magnitude and can be neglected

$$R = T\frac{\delta\theta}{2} + T\frac{\delta\theta}{2} = T\,\delta\theta$$

Considering equilibrium of forces in tangential (horizontal) direction,

$$\mu R = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2}$$

For small values of $\delta\theta$; $\cos\frac{\delta\theta}{2} \to 1$

$$\mu R = (T + \delta T) - T = \delta T$$

$$R = \frac{\delta T}{\mu}$$

From expressions (*i*) and (*ii*)

$$T\,\delta\theta = \frac{\delta T}{\mu}$$

Separating the variables and integrating between the limit $T = T_2$ at $\theta = 0$ and $T = T_1$ at $\theta = \theta$, we get;

$$\int_{T_2}^{T_1}\frac{\delta T}{T} = \mu\int_0^{\theta}\delta\theta$$

$$\log_e\frac{T_1}{T_2} = \mu\theta \quad \therefore \quad \frac{T_1}{T_2} = e^{\mu\theta}$$

When two pulley of unequal diameters are connected by open belt drive, the slip occurs first on the smaller pulley where the force of friction is less. Accordingly, the angle of contact on smaller pulley is taken into account while using the above equation.

ROPE DRIVE

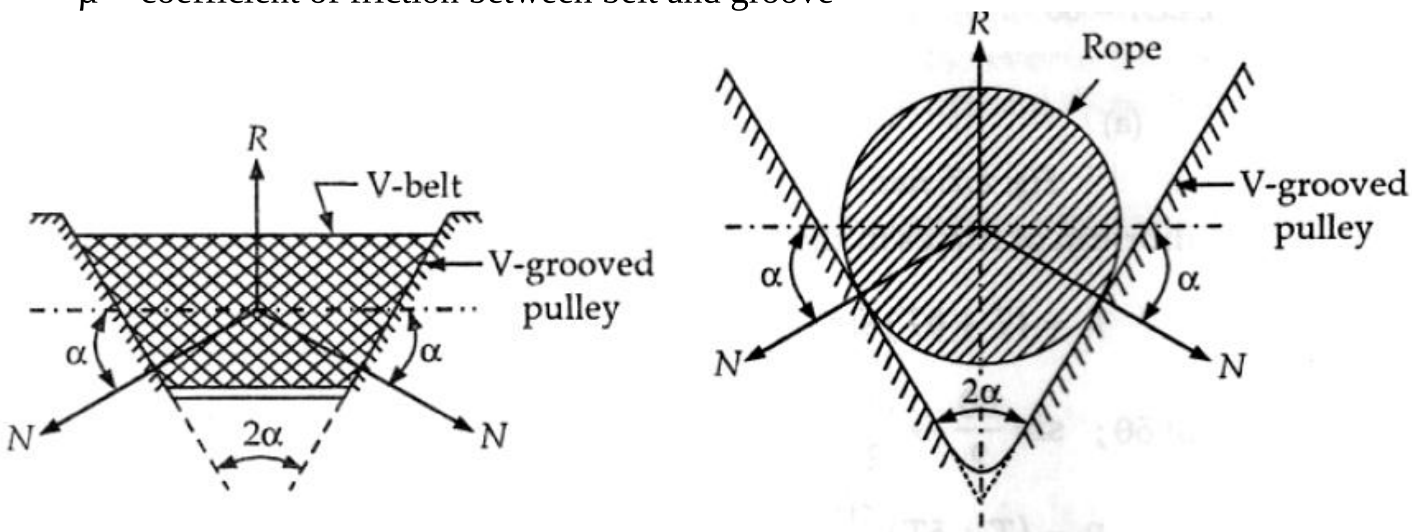
A V-belt or a rope runs into a grooved pulley. Figure which shows a V-belt of trapezoidal section resting in a grooved pulley.

Let 2α= angle of groove

N = normal reaction between belt and sides of the V-grooved pulley

R = total reaction in the plane of groove

μ = coefficient of friction between belt and groove



Considering equilibrium between R and N, we have

$$R = N \sin\alpha + N \sin\alpha = 2N \sin\alpha$$

$$\therefore \quad N = \frac{R}{2\sin\alpha} = \frac{R}{2}\operatorname{cosec}\alpha$$

$$\text{Friction resistance} = \mu N + \mu N = 2\,\mu N = 2\mu \times \frac{R}{2}\operatorname{cosec}\alpha = \mu R \operatorname{cosec}\alpha$$

Then with reference to Fig. 8.7

$$R = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2} = (T + \delta T)\frac{\delta\theta}{2} + T\frac{\delta\theta}{2} \qquad (\because \delta\theta \text{ is small})$$

$$= T\frac{\delta\theta}{2} + \frac{\delta T\,\delta\theta}{2} + T\frac{\delta\theta}{2}$$

The term $\frac{\delta T\,\delta\theta}{2}$ is the product of two small quantities and hence neglected. That gives

$$R = T\frac{\delta\theta}{2} + T\frac{\delta\theta}{2} = T\,\delta\theta \qquad \text{...(a}$$

Also : $\mu R \operatorname{cosec}\alpha = (T + \delta\theta)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} = (T + \delta T) - T = \delta T$ ...(b

This is because $\frac{\delta\theta}{2}$ is small and $\cos\frac{\delta\theta}{2} \to 1$

From expressions (a) and (b),

$$\mu\,(T\,\delta\theta)\operatorname{cosec}\alpha = \delta T$$

$$\text{or} \qquad (\mu \operatorname{cosec}\alpha)\,\delta\theta = \frac{\delta T}{T}$$

Upon integration with in appropriate limits, we have

$$\mu \operatorname{cosec}\alpha \int_{o}^{\theta} \delta\theta = \int_{T_2}^{T_1} \frac{\delta T}{T}$$

$$\text{or} \qquad (\mu \operatorname{cosec}\alpha) \times \theta = \log_e \frac{T_1}{T_2}$$

$\therefore$ Ratio of tensions for V-belt,

$$\frac{T_1}{T_2} = e^{(\mu \operatorname{cosec}\alpha)\,\theta}$$

Gear Drive

A gear is a wheel provided with teeth which mesh with the teeth on another wheel, or on to a rack, so as to give a positive transmission of motion from one component to another.

They are commonly used for power transmission or for changing power speed ratio in a power system but when they are not too far apart and when a constant velocity ratio is desired.

Advantages and Disadvantages of Gear Drive

The following are the advantages and disadvantages of the gear drive as compared to belt, rope and chain drives:

Advantages

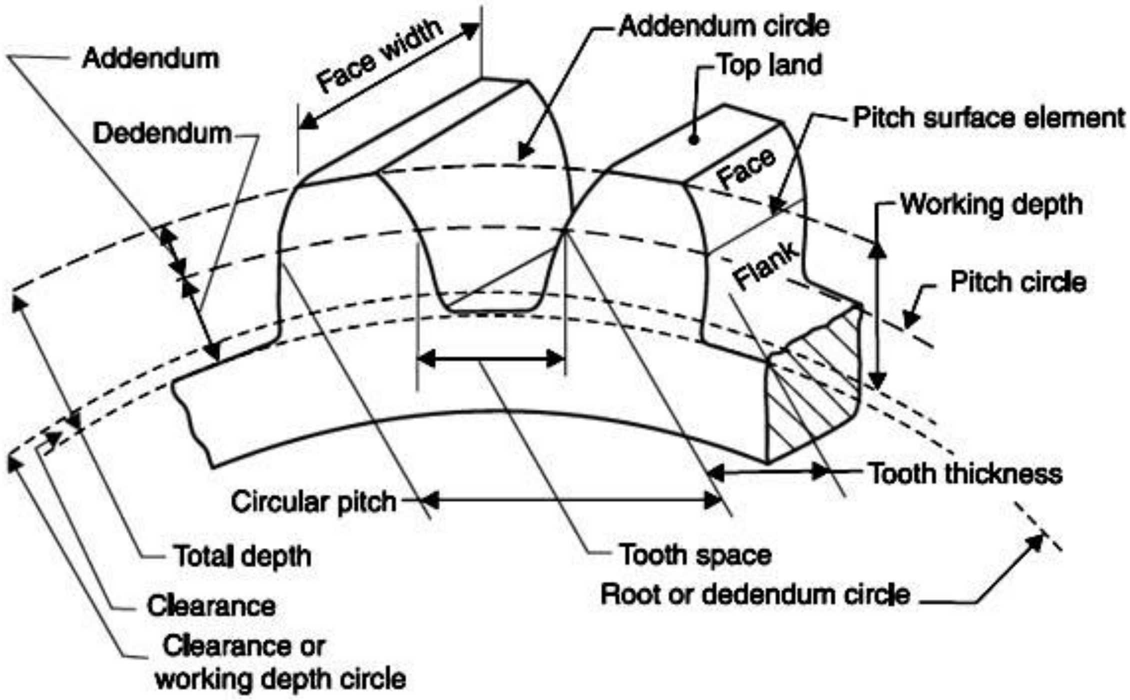
1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

Disadvantages

1. The manufacture of gears require special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation

Definitions

There are several notation which are given below:



1. Pitch circle. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

2. Pitch circle diameter. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as **pitch diameter**.

3. Pitch point. It is a common point of contact between two pitch circles.

4. Pitch surface. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. Pressure angle or angle of obliquity. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by φ. The standard pressure angles are $14\frac{1}{2}°$ and $20°$.

6. Addendum. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. Dedendum. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. Addendum circle. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. Dedendum circle. It is the circle drawn through the bottom of the teeth. It is also called root circle.

Note: Root circle diameter = Pitch circle diameter × cos φ, where φ is the pressure angle.

10. Circular pitch. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by pc. Mathematically,

Circular pitch, $P_c = \prod D/T$

where D = Diameter of the pitch circle, and
T = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

Note: If D1 and D2 are the diameters of the two meshing gears having the teeth T1 and T2 respectively, then for them to mesh correctly,

$\mathbf{P_C} = \prod D_1/T = \prod D_2/T$

11. Diametral pitch. It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by pd . Mathematically,

Diametral pitch, $P_d = T/D = \prod / \mathbf{P_C}$

where T = Number of teeth, and
D = Pitch circle diameter.

12. Module. It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m. mathematically,

Module, m = D /T

Note : The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, and 20. The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14 and 18 are of second choice.

13. Clearance. It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as **clearance circle**.

14. Total depth. It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.

15. Working depth. It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. Tooth thickness. It is the width of the tooth measured along the pitch circle.

17. Tooth space . It is the width of space between the two adjacent teeth measured along the pitch circle.

18. Backlash. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

19. Face of tooth. It is the surface of the gear tooth above the pitch surface.

20. Flank of tooth. It is the surface of the gear tooth below the pitch surface.

21. Top land. It is the surface of the top of the tooth.

22. Face width. It is the width of the gear tooth measured parallel to its axis.

23. Profile. It is the curve formed by the face and flank of the tooth.

24. Fillet radius. It is the radius that connects the root circle to the profile of the tooth.

25. Path of contact. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

26. ***Length of the path of contact**. It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

27. ** **Arc of contact**. It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, i.e.
(a) Arc of approach. It is the portion of the path of contact from the beginning of the engagement to the pitch point.
(b) Arc of recess. It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

Note : The ratio of the length of arc of contact to the circular pitch is known as **contact ratio** i.e. number of pairs of teeth in contact.

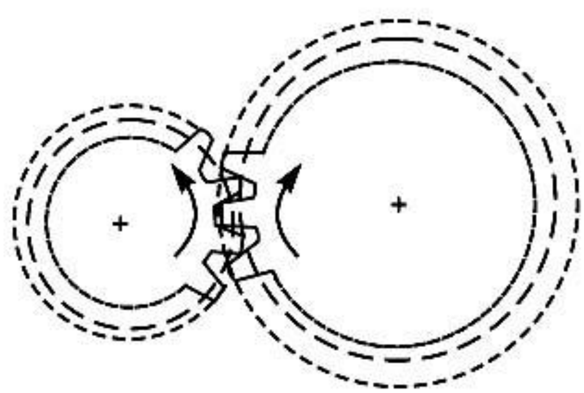
Types of gear

There are mainly five types of gear

1. Spur gear
2. Helical gear
3. Bevel gear
4. Worm gear
5. Rack & pinion gear

<u>Spur gear</u>

Spur gears or straight-cut gears are the simplest type of gear. They consist of a cylinder or disk, and with the teeth projecting radially, and although they are not straight-sided in form, the edge of each tooth thus is straight and aligned parallel to the axis of rotation. These gears can be meshed together correctly only if they are fitted to parallel axles. The arrangement is known as **spur gearing**.

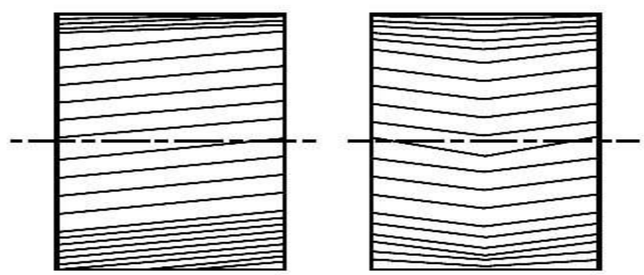


The efficiency of power transmission by these gears is very high & may be as much as 99% in case of high-speed gears with good material and workmanship of construction & good lubrication in operation. Under average condition efficiency of 96-98% are commonly attainable. The disadvantages are that they are liable to be more noisy in operation & may wear out & develop backlash more readily than the other types.

Helical gear

The teeth of helical gear are inclined to the axis In this type of gear. This ensures smooth action & more accurate maintenance of velocity ratio.

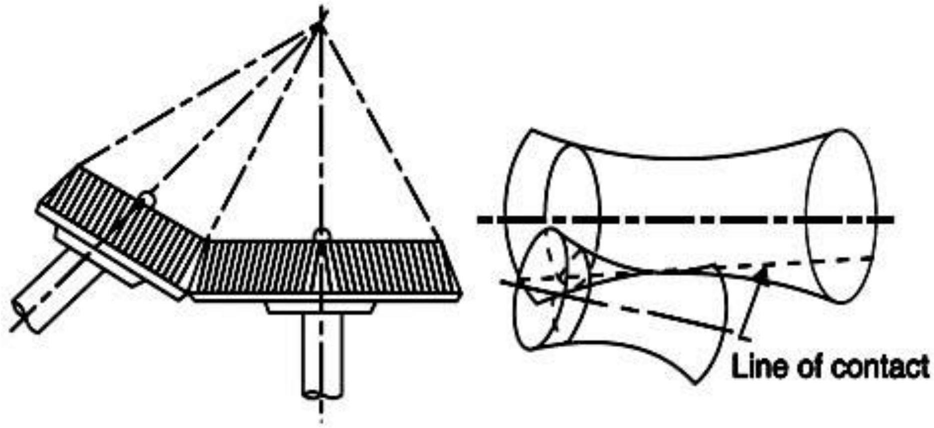
The disadvantage is that the inclination of the teeth sets up a lateral thrust. A method of neutralizing this lateral or axial thrust is to use double helical gears (also known as herring bone gear)



(a)Single helical gear (b) double helical gear

Bevel gear

The two non-parallel or intersecting, but coplanar shafts connected by gears are called **bevel gears** and the arrangement is known as **bevel gearing**. The bevel gears, like spur gears, may also have their teeth inclined to the face of the bevel, in which case they are known as **helical bevel gears**. The two non-intersecting and non-parallel i.e. non-coplanar shaft connected by gears are called **skew bevel gears** or **spiral gears** and the arrangement is known as **skew bevel gearing** or **spiral gearing**. This type of gearing also has a line contact, the rotation of which about the axes generates the two pitch surfaces known as **hyperboloids**.



(c) Bevel gear. (d) Spiral gear.

Notes:
(a) When equal bevel gears (having equal teeth) connect two shafts whose axes are mutually perpendicular, then the bevel gears are known as **mitres**.

(b) A hyperboloid is the solid formed by revolving a straight line about an axis (not in the same plane), such that every point on the line remains at a constant distance from the axis.

(c) The worm gearing is essentially a form of spiral gearing in which the shafts are usually at right angles

Worm gear

They connect non-parallel, non-intersecting shafts which are usually at right angles. One of the gear is called 'worm'. It is essential part of a screw, meshing with the teeth on a gear wheel, called the "worm wheel". The gear ratio is the ratio of number of teeth on the wheel to the number of thread on the worm. Its advantage is that it gives high gear ratio which are easily obtained & also smooth & quiet.

Rack & pinion gear
A rack is a spur gear of infinite diameter, thus it assumes the shape of a straight gear. The rack is generally used with a pinion to convert rotary motion into rectilinear motion

Types of Gear Trains
Two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called **gear train** or **train of toothed wheels.** The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

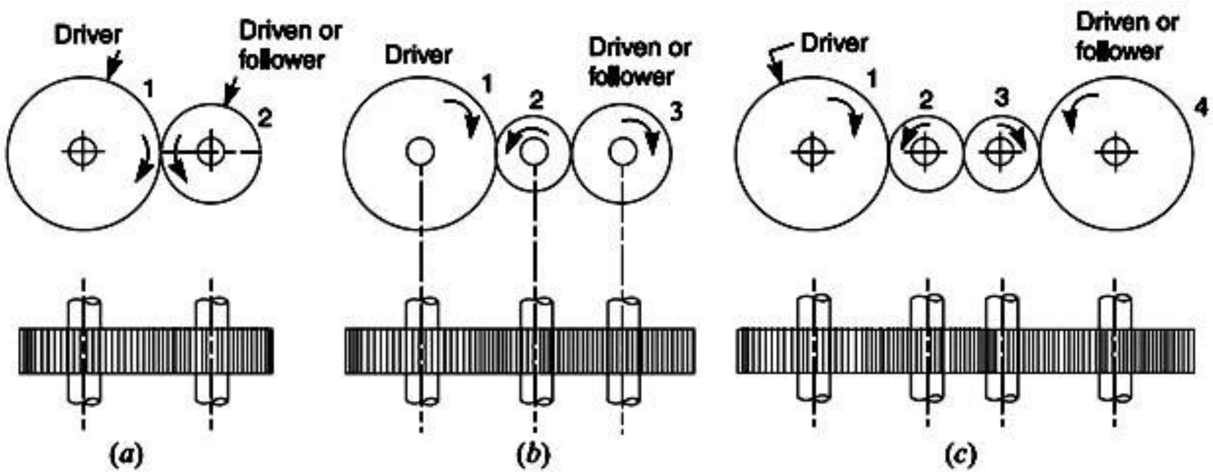
The different types of gear trains, depending upon the arrangement of wheels:

1. Simple gear train,
2. Compound gear train,
3. Reverted gear train, and
4. Epicyclic gear train.

In the first three types of gear trains, the axes of the shafts over which the gears are mounted are **fixed** relative to each other. But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may **move** relative to a fixed axis.

Simple gear train

When there is only one gear on each shaft, as shown in Figure, it is known as **simple gear train.** The gears are represented by their pitch circles. When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig.(a).Since the gear 1 drives the gear 2, therefore gear 1 is called the **driver** and the gear 2 is called the **driven** or **follower.** It may be noted that the motion of the driven gear is opposite to the motion of driving gear.



Simple gear train

Let
N1 = Speed of gear 1(or driver) in r.p.m.,
N2 = Speed of gear 2 (or driven or follower) in r.p.m.,
T1 = Number of teeth on gear 1, and
T2 = Number of teeth on gear 2.

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

Speed ratio = $N_1/N_2=T_2/T_1$

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as **train value** of the gear train. Mathematically,

Train value = $N_2/N_1=T_1/T_2$

From above, we see that the train value is the **reciprocal** of speed ratio.
Sometimes, the distance between the two gears is **large**. The motion from one gear to another, in such a case, may be transmitted by either of the following two methods:

1. By providing the large sized gear, or
2. By providing one or more intermediate gears.

A little consideration will show that the former method (i.e. providing large sized gears) is very inconvenient and uneconomical method ; whereas the latter method (i.e. providing one or more intermediate gear) is very convenient and economical. It may be noted that when the number of intermediate gears are **odd,** the motion of both the gears (i.e. driver and driven or follower) is **like** as shown in Fig. (b). But if the number of intermediate gears are **even,** the motion of the driven or follower will be in the opposite direction of the driver as shown in Fig. (c).

Now consider a simple train of gears with one intermediate gear as shown in Fig. (b).

Let
N1 = Speed of driver in r.p.m.,
N2 = Speed of intermediate gear in r.p.m.,
N3 = Speed of driven or follower in r.p.m.,
T1 = Number of teeth on driver,
T2 = Number of teeth on intermediate gear, and
T3 = Number of teeth on driven or follower.

Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is

$N_1/N_2=T_2/T_1$ ...**(i)**

Similarly, as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is

$$N_2/N_3=T_3/T_2 \text{...}\textbf{(ii)}$$

The speed ratio of the gear train as shown in Fig. 13.1 (b) is obtained by multiplying the equations **(i)** and **(ii).**

$$N_1/N_2 \times N_2/N_3=T_2/T_1 \times T_3/T_2$$

Or

$$\mathbf{N_1/N_3 = T_1/T_3}$$

i.e.

$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

and

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

Similarly, it can be proved that the above equation holds good even if there are any number of intermediate gears. From above, we see that the speed ratio and the train value, in a simple train of gears, is independent of the size and number of intermediate gears. These intermediate gears are called **idle gears,** as they do not affect the speed ratio or train value of the system. The idle gears are used for the following two purposes

1. To connect gears where a large centre distance is required, and
2. To obtain the desired direction of motion of the driven gear (i.e. clockwise or anticlockwise).

<u>Compound Gear Train</u>

When there are more than one gear on a shaft, as shown in Fig. 2, it is called a **compound train of gear .** The idle gears, in a simple train of gears do not effect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven.

But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts. In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in Fig. 2.

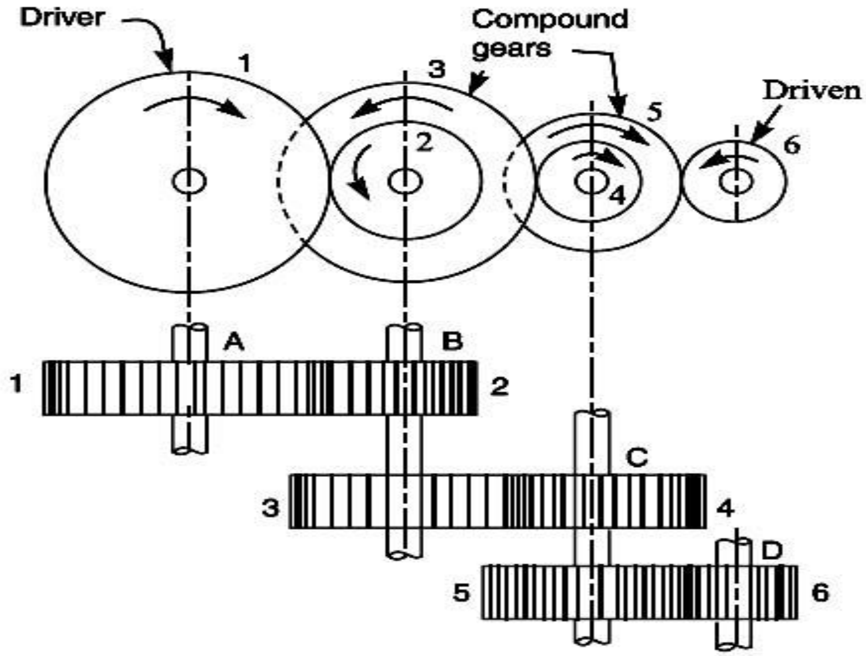


Fig 2 compound gear train

In a compound train of gears, as shown in Fig. 2, the gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let

N1 = Speed of driving gear 1,
T1 = Number of teeth on driving gear 1,
N2 ,N3 ..., N6 = Speed of respective gears in r.p.m., and
T2 ,T3..., T6 = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$N_1/N_2=T_2/T_1 \qquad \text{...(i)}$$

Similarly, for gears 3 and 4, speed ratio is

$$N_3/N_4=T_4/T_3 \qquad \text{...(ii)}$$

and for gears 5 and 6, speed ratio is

$$N_5/N_6=T_6/T_5 \qquad \text{...(iii)}$$

The speed ratio of compound gear train is obtained by multiplying the equations **(i), (ii)** and **(iii),**

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \quad \text{or} \quad {}^*\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

i.e.

$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}} = \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

and

$$\text{Train value} = \frac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}} = \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}}$$

The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears. If a simple gear train is used to give a large speed reduction, the last gear has to be very large. Usually for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed.

Note: The gears which mesh must have the same circular pitch or module. Thus gears 1 and 2 must have the same module as they mesh together. Similarly gears 3 and 4, and gears 5 and 6 must have the same module.

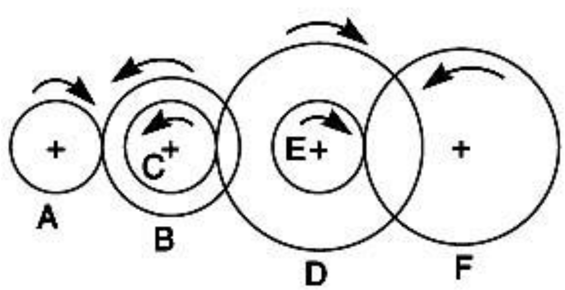


Fig 3

Reverted Gear Train

When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as **reverted gear train** as shown in Fig. a. We see that gear 1 (i.e. first driver) drives the gear 2 (i.e. first driven or follower) in the opposite direction.

Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (i.e. the last driven or follower) in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is **like.**

Let T1 = Number of teeth on gear 1,
r1 = Pitch circle radius of gear 1, and
N1 = Speed of gear 1 in r.p.m.

Similarly,

T2, T3, T4 = Number of teeth on respective gears,
r2, r3, r4 = Pitch circle radii of respective gears, and
N2, N3, N4 = Speed of respective gears in r.p.m.

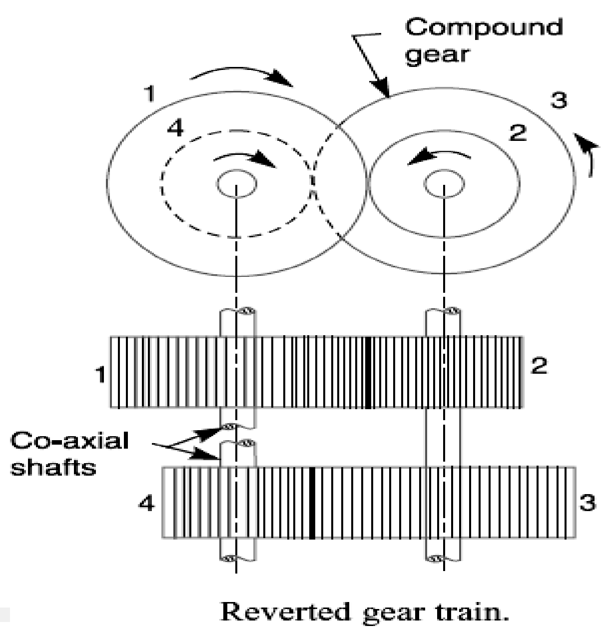
Since the distance between the centres of the shafts of gears 1 and 2 as well as gears 3 and 4 is same, therefore

$$r1 + r2 = r3 + r4 \quad ...\textbf{(i)}$$

Also, the circular pitch or module of all the gears is assumed to be same, therefore number of teeth on each gear is directly proportional to its circumference or radius.

$$T1 + T2 = T3 + T4 \quad ...\textbf{(ii)}$$

and Product of number of teeth on drivens



Reverted gear train.

Fig a

and $$\text{Speed ratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$

or $$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3} \qquad \text{... } (iii)$$

From equations **(i), (ii)** and **(iii),** we can determine the number of teeth on each gear for the given centre distance, speed ratio and module only when the number of teeth on one gear is chosen arbitrarily.

The reverted gear trains are used in automotive transmissions, lathe back gears, industrial speed reducers, and in clocks (where the minute and hour hand shafts are co-axial).

Epicyclic Gear Train

We have already discussed that in an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in Fig. b, where a gear A and the arm C have a common axis at O1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O2, about which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear B or **vice- versa**, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e. O1), then the gear B is forced to rotate **upon** and **around** gear A. Such a motion is called **epicyclic** and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as **epicyclic gear trains** (**epi.** means upon and **cyclic** means around). The epicyclic gear trains may be **simple** or **compound.**

The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

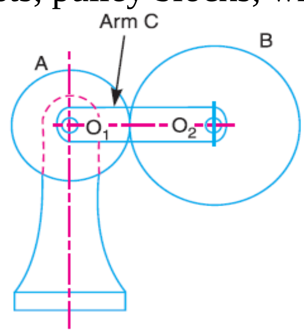


Fig b Epicyclic gear train.