Unit 5: Reasoning and Proof
Lesson 5.1: Patterns and Inductive Reasoning

Lesson 2.1 from textbook

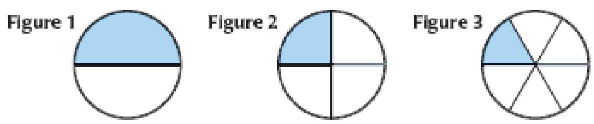
Objectives

- Find and describe patterns with pictures and number series.
- Use inductive reasoning to make real life conjectures.

FINDING PATTERNS

Example 1

Sketch the next figure in the pattern.



Example 2

Describe a pattern in the sequence of numbers. Predict the next number.

a. 1, 4, 16, 64 ……

b. -5, -2, 4, 13 …….

________________________ ________________________

USING INDUCTIVE REASONING

Much of the reasoning in Geometry consists of three stages

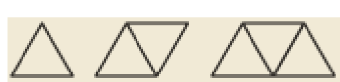
1. ____

2. ____

3. ____

Example 3

Given the pattern of triangles below, make a conjecture about the number of segments in a similar diagram with 5 triangles.



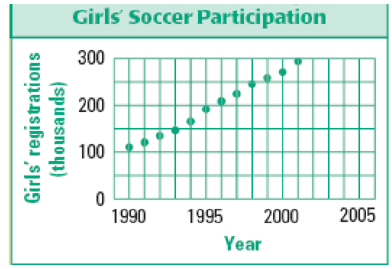
Number of triangles	1	2	3	4	5
Number of segments					

Conjecture: ____

Example 4

Make a conjecture that a high school athletic director could make based on the graph at right

Conjecture: ___________________________________



Example 5

Numbers 3, 4, and 5 are called consecutive integers. Make a test a conjecture about the sum of any three consecutive integers.

Pattern:

Conjecture: ______________________________________

Test:

DISPROVING CONJECTURES

Example 5

Show that the conjecture is false by finding a counterexample.

Conjecture: If the product of two numbers is positive, then those two numbers must be positive.

Counterexample: ____

Conjecture: In the series 1, 2, 4, ……, the next number is 8.

Counterexample: ____

Unit 5: Reasoning and Proof
Lesson 5.2 Analyze Conditional Statements

Lesson 2.2 from textbook

Objectives

- Recognize, analyze, and write a conditional statement and its counterexample, converse, inverse and contrapositive.
- Rewrite conditionals and converses of definitions
- Rewrite definitions as biconditional statements

RECOGNIZING CONDITIONAL STATEMENTS

If it is noon in Georgia, then **it is 9:00 A.M. in California**.

Hypothesis   **Conclusion**

Example 1

Rewrite the conditional statements in if-then form.

a. Two points are collinear if they line on the same line.

____

b. All sharks have a boneless skeleton.

____

NEGATION

Statement 1: The ball is red.   **Statement 2:** ______________________________

Statement 2: ______________________________   **Negation 2:** The cat is black.

VERIFYING STATEMENTS

In order for a conditional to be true, the conclusion ____

To show the conditional statement is false, you need a ______________________________.

Example 2

Write a counterexample to show that the following conditional statement is false.

a) If $x^2 = 16$, then $x = 4$.   Counterexample: __________________________

b) If a number is odd, then it is divisible by 3.   Counterexample: __________________________

RELATED CONDITIONALS

Example 3

Write the converse of the following conditional statement.

a) If two segments are congruent, then they have the same length.

Converse: ____

When you negate the hypothesis and conclusion of a **conditional statement**, you form the ___________.

When you negate the hypothesis and conclusion of a **converse of a conditional statement**, you form the ____________________.

Original	If m<A = 30°, then <A is acute.
Converse	If <A is acute, then m<A = 30°.
Inverse	If m<A ≠ 30°, then <A is not acute.
Contrapositive	If <A is not acute, then m<A ≠ 30°

Example 4

Write the inverse, converse, and contrapositive of the statement. Decide whether each statement is true or false.

If there is snow on the ground, then the flowers are not in bloom.

Inverse ____

Converse ____

Contrapositive ____

EQUIVALENT STATEMENTS

A conditional statement and the contrapositive are either ______________________________________.

For example:

Statement: Guitar players are musicians.

Conditional: ____

Contrapositive: ____

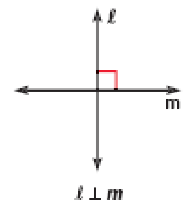
DEFINITIONS

A definition can be written as a ____.

In order to be a definition, ____.

Definition: Perpendicular Lines

If two lines intersect to form a 90^{o} angle, then they are perpendicular lines.



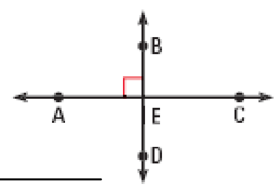
Converse:

____

Notation: instead of saying "line *l* is perpendicular to line *m*," this can be written as ____________________.

__Example 5__

Decide whether each statement about the diagram is true.



a. $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$ _________________________

b. $\angle AEB$ and $\angle CEB$ are a linear pair. _______________________

c. $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are opposite rays. ____________________________

BICONDITIONAL STATEMENTS

__Example 6__

Write the definition of perpendicular lines as a biconditional statement.

____

____

Unit 5: Reasoning and Proof
Lesson 5.3: Apply Deductive Reasoning

Lesson 2.3 from textbook

Objectives

- Use the Laws of Deductive Reasoning when problem solving.

USING THE LAWS OF LOGIC

Inductive reasoning uses previous examples and patterns to form a conjecture.

EX) Andrea knows that Robin is a sophomore and Todd is a junior. All the other juniors that Andrea knows are older than Robin. Therefore, Andrea, reasons ***inductively*** that Todd is older than Robin based on past observations.

Deductive reasoning uses facts, definitions, and accepted properties in a logical order to write a logical statement.

EX) Andrea knows that Todd is older than Chad. She also knows that Chad is older than Robin. Andrea reasons ***deductively*** that Todd is older than robin based on accepted statements.

Law of Detatchment:

____

Law of Syllogism

____

____

Example 1

Use the Law of Detachment to make a valid conclusion in the true situation.

a. If two segments have the same length, then they are congruent. You know that BC = XY.

b. Mary goes to the movies every Friday and Saturday night. Today is Friday

Example 2
If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

a. If Rick takes chemistry this year, then Jesse will be Rick's lab partner. If Jesse is Rick's lab partner, then Rick will get an A in chemistry.

b. If $x^2 > 25$, then $x^2 > 20$. If $x > 5$, then $x^2 > 25$.

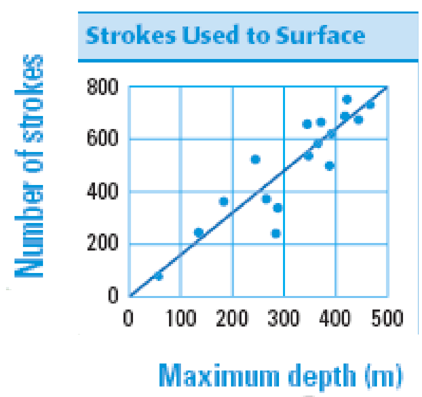
c. If a polygon is regular, then all angles in the interior of the polygon are congruent. If a polygon is regular, then all of its sides are congruent.

Example 3
What conclusion can you make about the product of an even integer and any other integer?

Example 4
Tell whether the statement is the result of *inductive reasoning* or *deductive reasoning*. Explain.

a. The northern elephant seal requires more strokes to surface the deeper it dives.

b. The northern elephant seal uses more strokes to surface from 250 meters than from 60 meters.



USING SYMBOLIC NOTATION

Conditional statements can be written using symbolic notation.

"p" represents the hypothesis "q" represents the conclusion "$\rightarrow$" means implies

EXAMPLE:

Conditional: If the sun is out, then the weather is good.

p: the sun is out q: the weather is good Symbolically: If p, then q or $p \rightarrow q$

Converse: ____

Symbolically: ____

Biconditional: ____

Symbolically: ____

<u>Example 1</u>

Let p be "the value of x is -5" and let q be "the absolute value of x is 5."

a) Write $p \rightarrow q$ in words. ____

b) Write $q \rightarrow p$ in words. ____

c) Decide whether the biconditional statement $p \leftrightarrow q$ is true. ______________________________

NOTE: In order to write the inverse and contrapositive, you need to be able to write the negation of a statement symbolically.

STATEMENT	**SYMBOL**	**NEGATION**	**SYMBOL**
<3 measures 90°	p	<3 does not measure 90°	~ p
<3 is not acute	q	<3 is acute	~ q

The inverse and contrapositive of the statement $p \rightarrow q$ are as follows:

Inverse: If <3 does not measure 90°, then <3 is acute. Symbolically: ______________

Contrapositive: IF <3 is acute, then <3 does not measure 90°. Symbolically: ___________________

Example 2

Let p be "it is raining" and let q be "the soccer game is canceled."

a) Write the contrapositive of p → q.

____

b) Write the inverse of p → q.

____

Unit 5: Reasoning and Proof
Lesson 5.4: Use Postulates and Diagrams

Lesson 2.4 from textbook

Objectives

- Understand and apply the Point, Line, and Plane Postulates.

POINT, LINE, AND PLANE POSTULATES

5. ____

6. ____

7. ____

8. ____

9. ____

10. ____

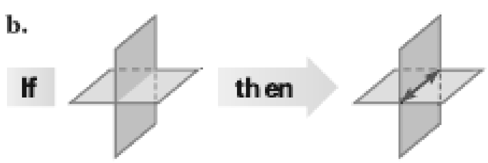
11. ____

Example 1

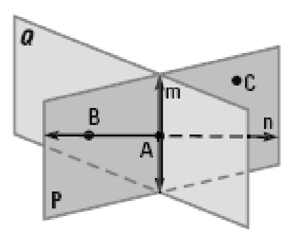
Identify the postulate illustrated by the diagram.

a.

b.



Example 2



Use the diagram to write examples of postulates 9 and 10.

Postulate 9 ____

Postulate 10 ____

Example 3

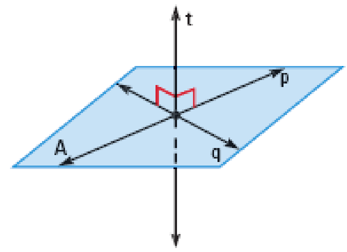
Sketch a diagram showing $\overleftrightarrow{TV}$ intersecting $\overline{PQ}$ at point W, so that $\overline{TW} \cong \overline{WV}$.

PERPENDICULAR FIGURES

A line is **perpendicular to a plane** ______________________________

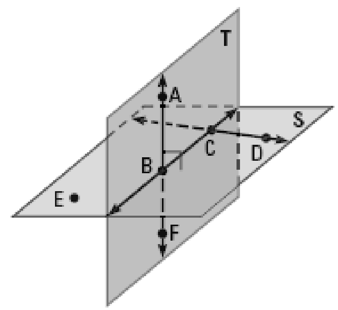
____

____.



Example 4

Which of the following statements cannot be assumed from the diagram? Explain.



a. A, B, and F are collinear

b. E, B, and D are collinear

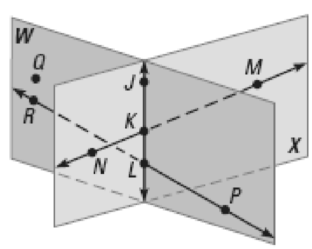
c. $\overline{AB} \perp$ plane S

d. $\overline{CD} \perp$ plane T

e. $\overleftrightarrow{AF}$ intersects $\overleftrightarrow{BC}$ at point B.

Example 5

Use the diagram to determine if the statement is true or false.



a. Planes W and X intersect at $\overleftrightarrow{KL}$.

b. Points K, L, M, and R are coplanar.

c. <PLK is a right angle.

d. <JKM and <KLP are congruent angles.

Unit 5: Reasoning and Proof
Lesson 5.5: Reasoning Using Properties from Algebra

Lesson 2.5 from textbook

Objectives

- Apply Algebraic Properties of Equality such as Addition, Subtraction, Multiplication, Division, Reflexive, Symmetric, Transitive, and Substitution.
- Use properties of length and measure to justify segment and angle relationships.

Properties of Equality for Real Numbers:
Let a, b, and c be real numbers.

Addition Property		If a = b, then a + c = ________________________________
Subtraction Property		If a = b, then a – c = ________________________________
Multiplication		If a = b, then a · c = ________________________________
Division Property	I	If a = b and c ≠ 0, then a ÷ c = ________________________
Substitution Property		If a = b, then ____

Example 1

Solve 2x + 5 = 20 – 3x . Write a reason for each step.

Explanation	Equation	Reason
1) Write original equation.	____________________	____________________
2) ____________________	____________________	____________________
3) ____________________	____________________	____________________
4) ____________________	____________________	____________________
5) ____________________	____________________	____________________

Distributive Property For all numbers a, b, and c, a(b + c) = ____________________________

Example 2

Solve -4(11x + 2) = 80. Write a reason for each step.

Explanation	Equation	Reason
1) Write original equation.	____________________	____________________
2) ____________________	____________________	____________________
3) ____________________	____________________	____________________
4) ____________________	____________________	____________________

Example 3

When you exercise, your target heart rate should be between 50% and 70% of your maximum heart rate. Your target heart rate r at 70% can be determined by the formula $r = 0.70(220 - a)$ where a represents your age in years. Solve the formula for a and write a reason for each step.

Explanation	Equation	Reason
1) ________________	____________________	____________________
2) ________________	____________________	____________________
3) ________________	____________________	____________________
4) ________________	____________________	____________________

Reflexive Property of Equality

- **Real Numbers** For any real number a, ____________________
- **Segment Length** For any segment AB, ____________________
- **Angle Measures** For any angle *A*, ____________________

Symmetric Property of Equality

- **Real Numbers** For any real numbers a and b, if a = b, then ____________________
- **Segment Length** For any segments AB and CD, if AB = CD, then ____________________
- **Angle Measures** For any angles *A* and *B*, if m<A = m<B, then ____________________

Transitive Property of Equality

- **Real Numbers** For any real numbers a, b and c, if a = b and b = c, then ____________________
- **Segment Length** For any segments AB, CD, and EF if AB = CD and CD = EF, then ____________________
- **Angle Measures** For any angles *A* , B and *C*, if m<A = m<B and m<B = m<C, then ____________________

Example 4

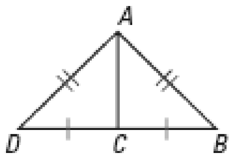
Use the property to copy and complete the statement.

If AB = 20, then AB + CD = ______________________________ (Substitution Property of Equality)

If AB = CD, then ______________ + EF = ______________ + EF (Addition Property of Equality)

Example 5

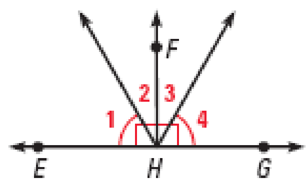
Show that the perimeter of triangle ABC is equal to the perimeter of triangle ADC.



Equation:	**Reason:**
1. ______________	______________
2. ______________	______________
3. ______________	______________
4. ______________	______________
5. ______________	______________

Example 6

Complete the table to show m<2 = m<3.



Equation	Explanation	Reason
$m\angle 1 = m\angle 4$, $m\angle EHF = 90°$, $m\angle GHF = 90°$	?	Given
$m\angle EHF = m\angle GHF$	?	Substitution Property of Equality
$m\angle EHF = m\angle 1 + m\angle 2$ $m\angle GHF = m\angle 3 + m\angle 4$	Add measures of adjacent angles.	?
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	Write expressions equal to the angle measures.	?
?	Substitute $m\angle 1$ for $m\angle 4$.	?
$m\angle 2 = m\angle 3$	?	Subtraction Property of Equality

Unit 5: Reasoning and Proof
Lesson 5.6 Proving Statements about Segments and Angles

Lesson 2.6 from textbook

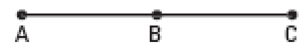
Objectives

- Justify statements about congruent segments and angles using definitions, properties, and postulates.
- Use two-column proofs to justify statements about congruence.

Example 1

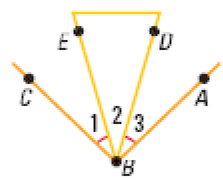
Four steps of the proof are given. Give the reasons for the last two steps.

Given: AC = AB + AB   Prove: AB = BC



STATEMENTS	REASONS
1. AC = AB + AB	1. Given
2. AB + BC = AC	2. Segment Addition Postulate
3. AB + AB = AB + BC	3.
4. AB = BC	4.

Example 2



Write a two-column proof.

Given: m<1 = m<3   Prove: m<EBA = m<DBC

STATEMENTS	REASONS
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

Theorem 2.1

Reflexive: For any segment AB, ________________________.

Symmetric: If $\overline{AB} \cong \overline{CD}$, then __________________________.

Transitive: If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then ____________________________.

Theorem 2.2

Reflexive: For any angle A, ________________________.

Symmetric: If <A ≅ <B, then __________________________.

Transitive: If <A ≅ <B and <B ≅ <C, then ____________________________.

Example 3

Name the property or definition illustrated by the statement.

a. If $\angle R \cong \angle T$ and $\angle T \cong \angle P$, then $\angle R \cong \angle P$. ______________________________________

b. If $\overline{NK} \cong \overline{BD}$, then $\overline{BD} \cong \overline{NK}$. ______________________________________

c. If $\overline{NK} \cong \overline{BD}$, then NK = BD. ______________________________________

d. If M is the midpoint of $\overline{CD}$, then CM = MD. ______________________________________

Example 4

Complete the two-column proof.

Given: M is the midpoint of $\overline{AB}$ Prove: a. $AB = 2 \cdot AM$ b. $AM = \frac{1}{2} \cdot AB$

STATEMENTS	REASONS
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

Unit 5: Reasoning and Proof
Lesson 5.7 Proving Angle Pair Relationships

Lesson 2.7 from textbook

<u>Objectives</u>

- Justify statements about supplementary, complementary, and vertical angle pairs.
- Develop mathematical arguments and proofs.

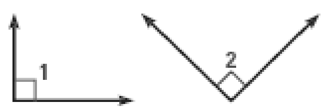
Right Angle Congruence Theorem

All right angles ____.

Proof:

GIVEN ▶ $\angle 1$ and $\angle 2$ are right angles.

PROVE ▶ $\angle 1 \cong \angle 2$

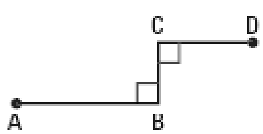


Statements	Reasons

<u>Example 1</u>

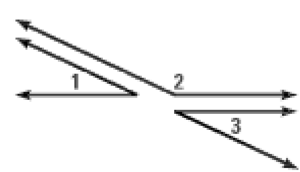
GIVEN ▶ $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$

PROVE ▶ $\angle B \cong \angle C$



Statements	Reasons

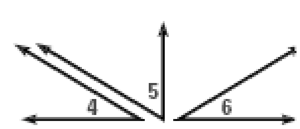
Congruent Supplements Theorem



If two angles are supplementary to the same angle (or two congruent angles),

then ____

Congruent Complements Theorem



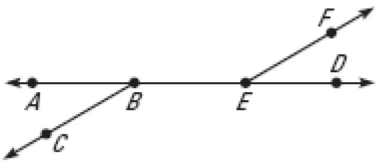
If two angles are supplementary to the same angle (or two congruent angles),

then ____

Example 2

Identify the pair(s) of congruent angles in the figures below. *Explain* how you know they are congruent.

$\angle ABC$ is supplementary to $\angle CBD$.
$\angle CBD$ is supplementary to $\angle DEF$.



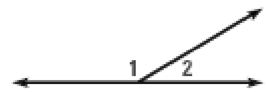
____

____

____

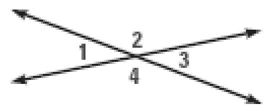
Linear Pair Postulate

If two angles form a linear pair, then ________________________________



Vertical Angles Congruence Theorem

Vertical angles ________________________________



Example 3

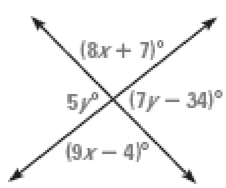
Use the diagram.

a) If m<1 = 112^{o}, then m<2 = ____________, m<3 = ____________, and m<4 = _____________.

b) If m<2 = 67^{o}, then m<1 = ____________, m<3 = ____________, and m<4 = _____________.

Example 4

Find the values of x and y.



x = __________ y = __________