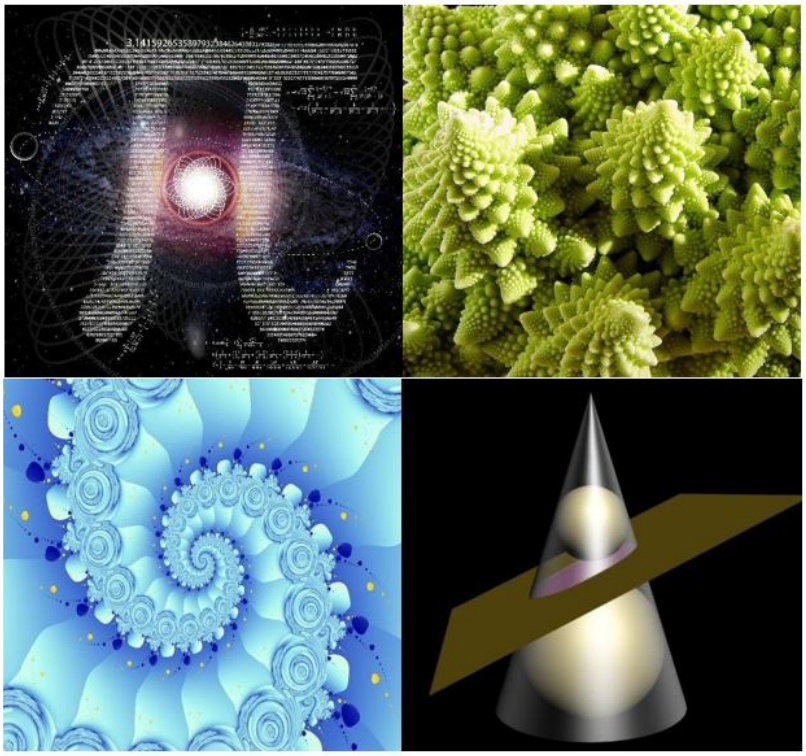
Washington State

Bridge to College Mathematics Course

Unit 6. Exponential Functions and an Introduction to Logarithms



Adapted from Math Ready

A Southern Regional Education Board Transition Course

Unit 6. Exponential Functions and an Introduction to Logarithms

Overview

PURPOSE

In this unit, students will experience exponential functions through a real-world lens of finance. Beginning with an overall look into financial decisions they will face as adults, students study the mathematics involved in purchasing a car, planning for retirement and even deciding on a job. They will extend their understanding of inverse functions as they explore logarithms as inverses to exponential functions and use logarithms to solve problems involving exponential growth and decay.

For Bridge to College course overview, refer to traditional Bridge to College course material.

Special Statement

The resources that follow were developed, adapted or compiled by more than 80 Washington Bridge to College Math teachers to support continuous learning during long term school closure determined necessary because of the COVID-19 pandemic. They are identified as Electronic or Printable resources to provide the most equitable access possible for students with varying levels of technology and internet capability. All possible steps were taken to compile meaningful and accurate resources and to attribute them appropriately, but if, in order to offer them in a timely fashion for immediate use, any errors exist, please notify the Mathematics Office at OSPI by email to mathematics@k12.wa.us.

We recognize that the collaborative problem solving structure developed in Bridge classrooms all year may not be possible at this time, but encourage teachers to seek ways to promote discussion and mutual supports between students using technology, email or phone calls.

This document currently contains Lesson 1, Unit 6 with a recommendation that this learning extends for approximately one week. Lesson 2 will be added within that week and the remaining Lessons 3 through 6 will follow shortly thereafter. Lesson 7, as a Capstone, will be left the discretion of Bridge to College teachers for the most suitable format and content for their students.

LESSON PROGRESSION OVERVIEW

Lesson Big Idea	Lesson Details	Content Standards	Standards for Mathematical Practice
Lesson 1: Exponential Growth and Decay	Students are asked to compare and contrast linear and exponential functions as well as exponential growth and decay functions.	A-SSE.1 A-SSE.3 A-CED.2 F-IF.7 F-LE.1 F-LE.5	MP 4 MP 5 MP 6 MP 7 MP 8

Exponential Functions and an Introduction to Logarithms

Lesson 1 of 7

DESCRIPTION

In this lesson, students will explore two Big Ideas:

- What is the different between linear and exponential growth?
- How are exponential growth and decay related and used?

To explore and develop these ideas, students will:

- Write equations for linear and exponential functions
- Determine if a situation is linear or exponential
- Recognize linear and exponential situations using graphs, tables and scenarios

Lesson Key	
Blue Table	Electronic Resources
Orange Table	Printable (Paper) Resources

COMMON CORE STATE STANDARDS ADDRESSED

- A-SSE.1: Interpret expressions that represent a quantity in terms of its context.
- A-SSE.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- A-CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- F-IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- F-LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

- F-LE.5: Interpret the parameters in a linear or exponential function in terms of a context.
- 8-EE.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
- 8.F.1: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

MATHEMATICAL PRACTICE STANDARD(S) EMPHASIZED

- MP 4: Model with mathematics.
- MP 5: Use appropriate tools strategically.
- MP 6: Attend to precision.
- MP 7: Look for and make use of structure.
- MP 8: Look for and express regularity in repeated reasoning.

Sequence of Instruction	Activities Checklist

HOOK

Activity	Description	Link
Hook	Video that shows the effect of multiplying (or dividing) by powers of 10	Hook: Powers of 10 video

Activity	Description	Link
Hook	Students explore 3 possible salary schedules to see the effects linear v. exponential growth	Printable Resource: Exponential Finance

LESSON

Electronic Resources

Activity Description	Link
Students develop the idea of exponential growth	Printable Resource: Paper folding activity (tech)
Students identify graphs and varying coefficients and bases	Khan Academy – Exponential growth and decay
Students compare linear and exponential models	EngageNY – Algebra I, Module 3, Topic D Lesson 21
Writing Equations	Writing exponential equation practice Practice writing equations
Students explore the effect of changing values within an exponential function. Students vary a and b to draw conclusions about their effect	Identifying Exponential Function DESMOS LINK Task 2 Desmos Version
Students compare linear and exponential functions	Task 2 Desmos Version
Students explore to identify changes in exponential functions reflecting growth or decay	Marbleslides: Exponentials • Activity Builder by Desmos
Students develop understanding of the rate of change in exponential functions	Khan Academy – Modeling with Exponential Functions

Activity Description	Link
Students write exponential growth and decay equations using basic Desmos examples. Students practice how to write compound growth and decay equations	Desmos - Writing Exponential Equations
Students investigate the effects of changing a and b in exponential functions. The Desmos activity will investigate effects of changing a and b in exponential functions	Desmos – Exploring Graphs of Exponential Functions

Printable (Paper) Resources

Activity Description	Link
Students develop the idea of exponential growth	Printable Resource: Paper folding activity (no tech)
Students graph exponential functions to see the effects of changing the base	Printable Resource: Graphing Exponential Functions
Students compare linear and exponential models	EngageNY – Algebra I, Module 3, Topic D Lesson 21
Writing Equations and creating graphs	Printable Resource: Practice Writing Equations
Students compare linear and exponential functions and the effect of changing values within them	Printable Resource: Linear vs. Exponential Functions
Students determine whether scenarios can be represented by linear or exponential models	Printable Resource: Task 2 Part 1 with Supplemental Questions

Activity Description	Link
Students compare the growth of a linear function and an exponential one and the effect of changing initial amounts or growth rates	Printable Resource: Task 4 with Supplemental Questions
Students explore the effects of changing variables within exponential functions	Printable Resource: What happens if I change it?

ATTRIBUTION AND LICENSE INFORMATION

This work has been created in partnership with expert mathematics educators from across Washington state. We express our gratitude to all the contributors to this effort. Without their support and expertise, this resource would not be possible.

We also thank the following sources

Engage NY | Engage New York, New York State Education Department
Creative Commons Attribution 4.0 International license

Desmos | Desmos, Inc. *linked only*
Terms of Use: "(a) for your personal, non-commercial use or (b) as a teacher, for academic use by you and your students in individual classes."

PRINTABLE RESOURCES

Exponential Finance

MAIN TASK:

Jerry, David, and Sarah just graduated from Mount Tahoma High School and they each decided on different paths upon graduation.

- Jerry decided to enter the workforce. He found a job in Local community where he makes $29,000 per year.
- David decided to enroll in classes at Tacoma Community College to pursue an associate's degree. The cost of attending Tacoma Community College for David was $2550 per year and he finished his degree in two years. Upon completing his degree, he found a job where he makes $36,200 per year.
- Sarah decided to enroll in classes at Washington State University to pursue a bachelor's degree. The cost of attending Washington State University for Sarah was $13,950 per year and she finished her degree in four years. Upon completing her degree, she found a job where she makes $45,100 per year.

(Note: Since David and Sarah went to college directly after high school, they do not start earning an income until 2 and 4 years, respectively, after high school.)

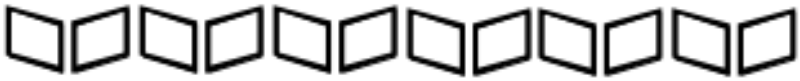
Due to inflation and pay raises, the annual income for Jerry, David, and Sarah, increases each year. Jerry's income increases at a rate of 2% each year, David's income increases at a rate of 2.5% each year, and Sarah's income increases at a rate of 4% each year.

Make a table for Jerry, David, and Sarah showing how much money they will earn the first 10 years out of high school. Count year as their first year out of school after graduation

Jerry's Table:

David's table:

Sarah's table:

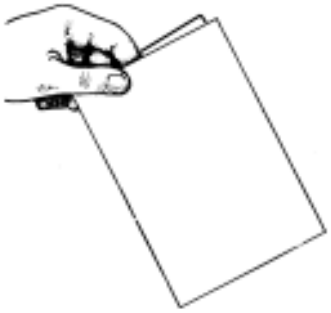


The Paper Folding Activity

Part I: Number of Sections

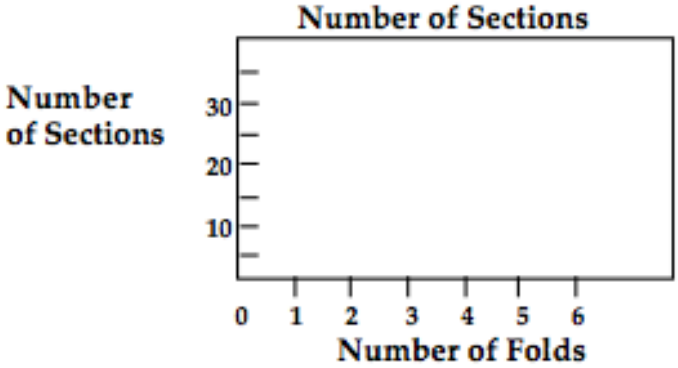
Number of Sections

Number of Folds	Number of Sections
0	
1	
2	
3	
4	
5	
6	



1. Fold an 8.5 x 11" sheet of paper in half and determine the number of sections the paper has after you have made the fold.
2. Record this data in the table and continue in the same manner until it becomes too hard to fold the paper.

3. Make a scatter plot of your data.



4. Determine a mathematical model that represents this data by examining the patterns in the table.

5. What might be different if you tried this experiment with an 8.5 x 11″ sheet of wax paper or tissue paper?

Part II: Area of Smallest Section

Area of Smallest Section

Number of Folds	Area of Smallest Section
0	1
1	
2	
3	
4	
5	
6	

6. Fold an 8.5 x 11″ sheet of paper in half and determine the area of the smallest section after you have made the fold.

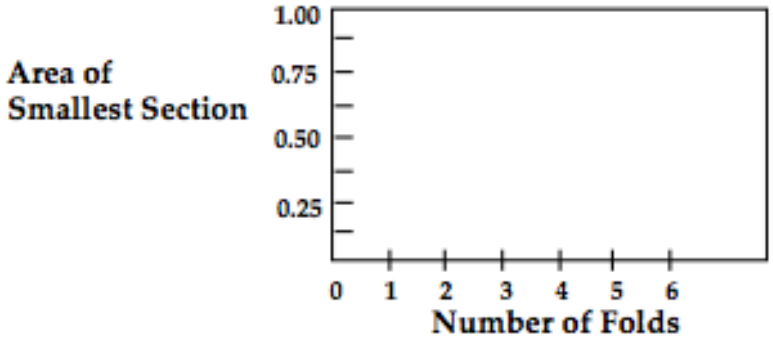
7. Record this data in the table and continue in the same manner until it becomes too hard to fold the paper.

8. Make a scatter plot of your data.

9. Determine a mathematical model that represents this data by examining the patterns in the table.

Did you know that paper was invented by the Chinese about 105 AD?

Area of Smallest Section



Now watch these two videos

https://www.youtube.com/watch?v=AmFMJC45f1Q

https://www.youtube.com/watch?v=kRAEBbotuIE

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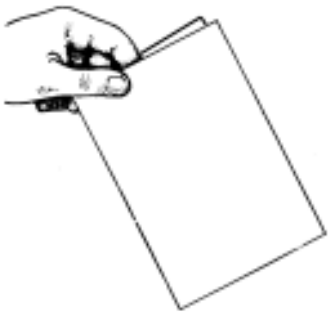
0	
1	
2	
3	
4	
5	
6	



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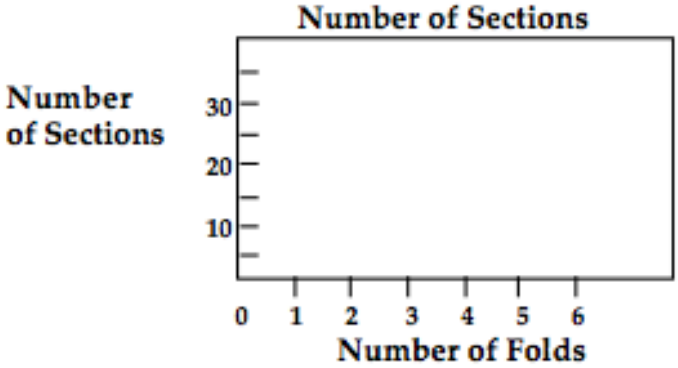

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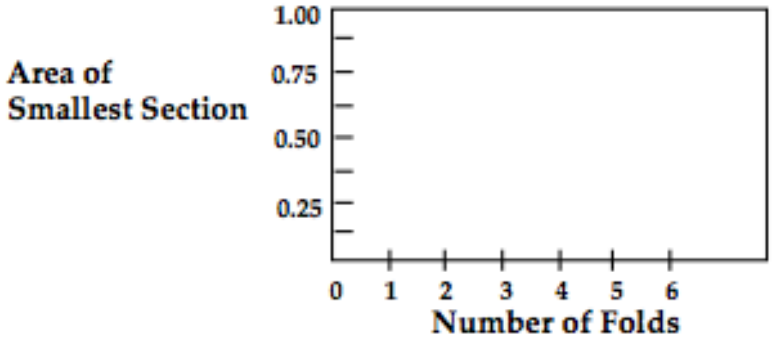
Area of Smallest Section



Part III: How many layers?

Now grab a bed sheet, the biggest flat sheet that you have. It might help to have someone else in your house give you a hand on this one!

1. Lay it out flat. If you don't have a space large enough, try draping it over the bed it could go on. You'll be folding it, so it won't be that big for very long.

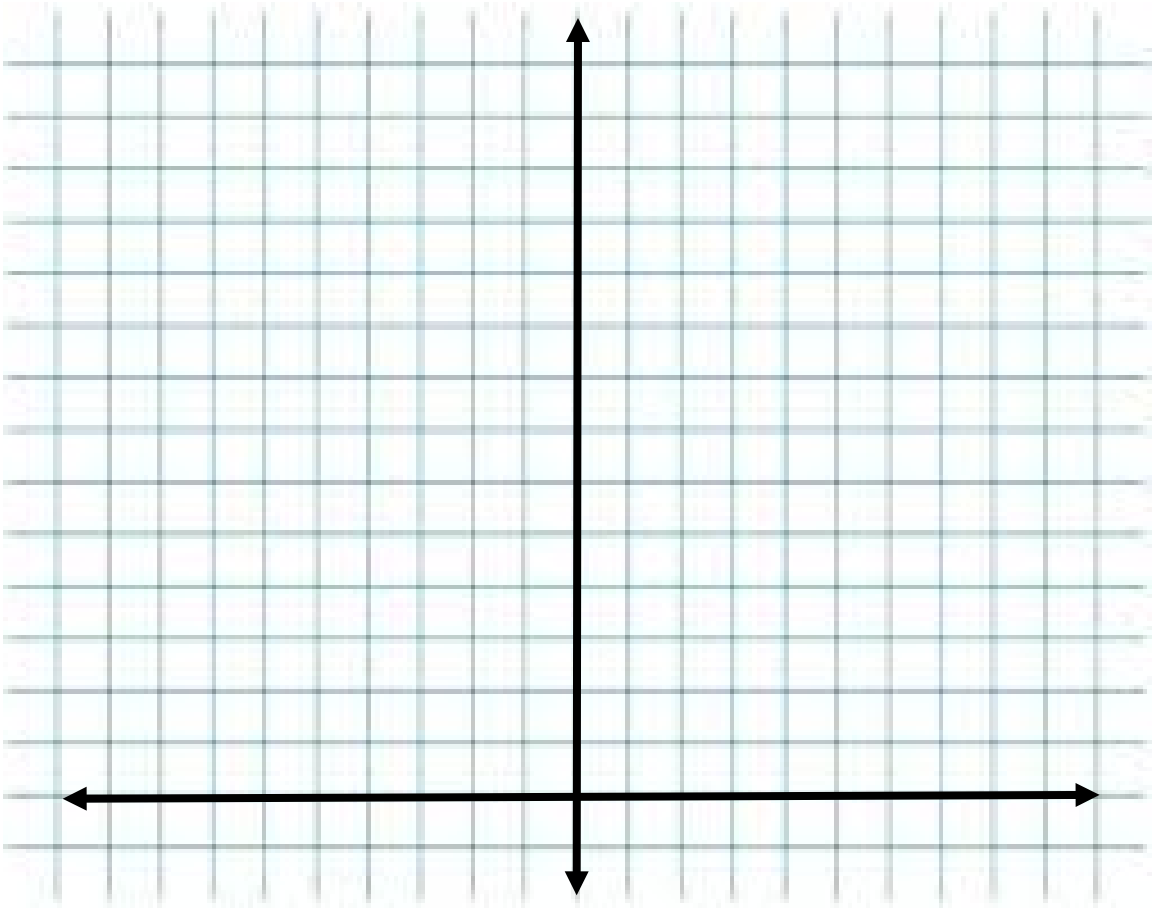
2. When it is spread out flat, it is one fabric layer thick. Fold it in half.
How many layers thick is it now? Start a table of folds and layers.

3. Fold it again and see how many layers you have.
Keep folding it, recording how many folds and layers you have each time.
Make folds until you can't really fold it again. Mythbusters did an episode about how many folds can be made. Check it out some time.

4. Look at your table. What pattern do you see? Can you write an function that describes the numbers of layers, f(x), in terms of the number of folds, x ?

5. How does this relate to the number of sections and the smallest area you looked at when folding paper?

Graphing Exponential Functions



Graph each equation, all on the same axes, using different colors. (Underline each equation to show the color)

a) $y = 2^x$ b) $y = 3^x$ c) $y = 5^x$ d) $y = 10^x$ e) $y = (\frac{1}{2})^x$ f) $y = (\frac{1}{5})^x$

Describe any patterns you see _____

How are the graphs of a) & e) or b) & f) related?_____

What generalizations can you make?_____

Test your ideas with other values for b where $y = b^x$

Bridge to College Math Name _____
Exponential Patterns
Practice Writing Equations Date _____ Period _____

For each problem below,

- Complete the table.
- Decide whether the table shows a linear or exponential pattern or neither.
- Decide whether the pattern represents growth or decay.
- Write "***y*** =" equations for the pattern.

1.

x	0	1	2	3	4	5	6	7	8
y				40	20	10			

Circle one: **Linear** or **Exponential** or **Neither**
Circle one: **Growth** or **Decay** or **Neither** ***y*** = _____

2.

x	0	1	2	3	4	5	6	7	8
y				25	30	35			

Circle one: **Linear** or **Exponential** or **Neither**
Circle one: **Growth** or **Decay** or **Neither** ***y*** = _____

3.

x	0	1	2	3	4	5	6	7	8
y				135	405	1215			

Circle one: **Linear** or **Exponential** or **Neither**
Circle one: **Growth** or **Decay** or **Neither** ***y*** = _____

4.

x	0	1	2	3	4	5	6	7	8
y				32	8	2			

Circle one: **Linear** or **Exponential** or **Neither**
Circle one: **Growth** or **Decay** or **Neither** ***y*** = _____

5.

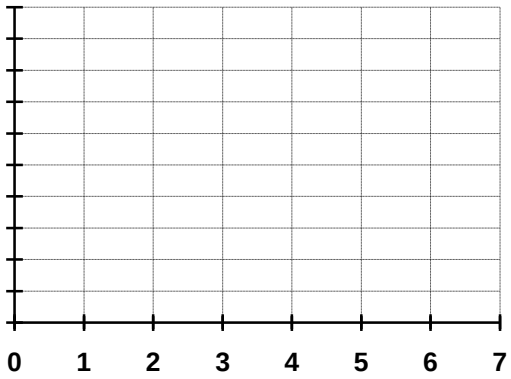
x	0	1	2	3	4	5	6	7	8
y				4	6	8			

Circle one: **Linear** or **Exponential** or **Neither**
Circle one: **Growth** or **Decay** or **Neither** ***y*** = _____

Complete the table, sketch a graph, and write an equation (linear or exponential) for each problem. Be careful when determining the scale of the y-axis.

6.

x	0	1	2	3	4	5	6	7
y				250	1250	6250		

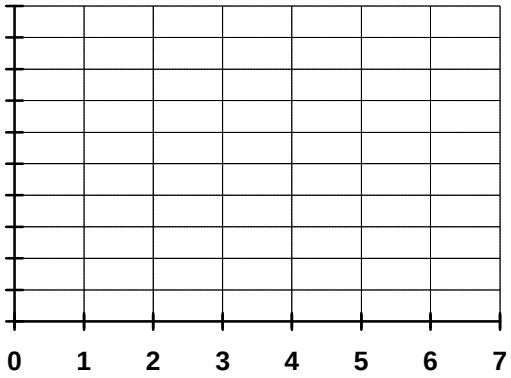


Circle one: Exponential Linear

Equation: _____

7.

x	0	1	2	3	4	5	6	7
y				81	27	9		

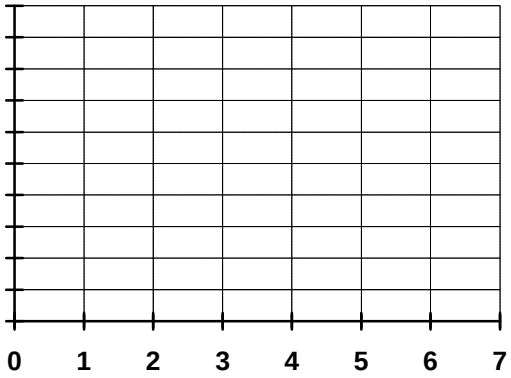


Circle one: Exponential Linear

Equation: _____

8.

x	0	1	2	3	4	5	6	7
y				13	16	19		



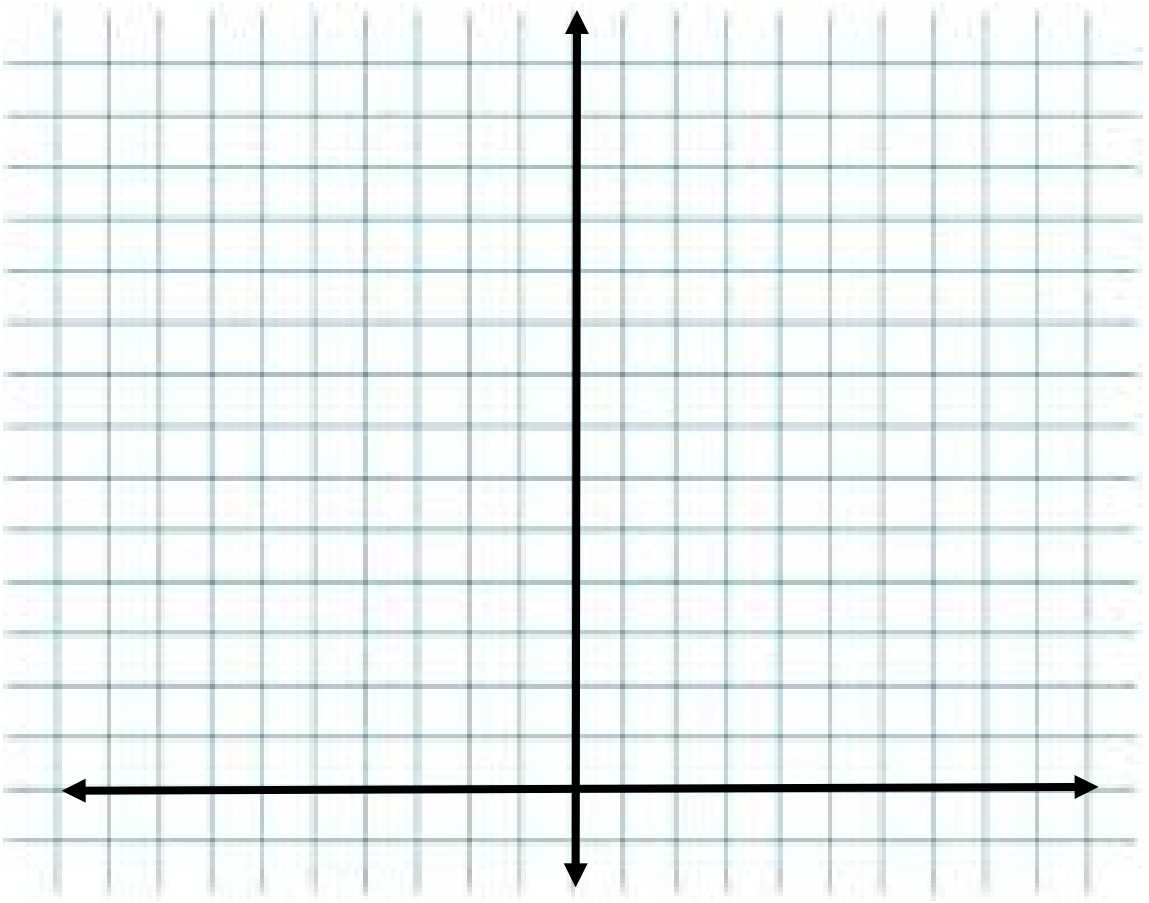
Circle one: Exponential Linear

Equation: _____

9. Which exponential growth equation has the fastest/highest growth rate? How do you know?

10. Which exponential growth equation has the highest y-intercept? How do you know?

Linear v. Exponential Functions



Graph each equation, all on the same axes, using different colors. (Underline each equation to show the color)

a) $y = 2x + 3$ b) $y = 3x - 2$ c) $y = 2^x + 3$ d) $y = 3^x - 2$ e) $y = (\frac{1}{2})^x$ f) $y = \frac{1}{2}x$

Describe any patterns you see _____

Which are exponential functions? What do you notice about them compared to the linear functions?_____

When would linear growth be better? When would exponential growth be better?_____

Write a linear and an exponential function using the same numbers and graph them. Explain why the graphs look different.

Task #2: Linear or Exponential? Part 1.

1. In (a)–(e), say whether the quantity is changing in a linear or exponential fashion.

a) A savings account, which earns no interest, receives a deposit of $723 per month.
b) The value of a machine depreciates by 17% per year.
c) Every week, 9 of a radioactive substance remains from the beginning 10 of the week.
d) A liter of water evaporates from a swimming pool every day.
e) Every 124 minutes, half of a drug dosage remains in the body.

2. Think about the quantities you identified as linear. How are they the same? What makes them different from the ones you labeled exponential?

3. What are some things in your life that you can recognize as changing in a linear fashion? Name 3. How about exponential changes that you see in your life? Name 3 of those.

4. Choose one from each list and describe how it would be different if it changed the other way. What would happen to one of the linear relationships if it suddenly became exponential? What would be good or bad about one of the exponential quantities changing in a linear way?

5. Have a conversation with someone about your responses in #3 and #4. You could call a classmate or describe them for a sibling or parent. Share any highlights or surprises from that conversation.

Task #4: Population and Food Supply

The population of a country is initially two million people and is increasing at a rate of 4% per year. The country's annual food supply is initially adequate for four million people and is increasing at a constant rate adequate for an additional 0.5 million people per year.

1. Begin by writing an equation for the country's population:

   Is it growing linearly or exponentially?

   Keep it simple: since all of the populations are described in millions, we can use 2 in our equation to identify 2 million

   What are the initial population and the growth rate?

   Write your equation:

2. Now look at the food supply:

   Is it growing linearly or exponentially?

   What are the initial supply and constant rate of change?

   Write your equation:

3. Sketch a graph to see how these functions are related. Think about what the horizontal and vertical axes will represent and the scale to use. You may find creating an accurate graph for the following questions very helpful in understanding them.

4. Answer the following. Be sure to show your math and reasoning.

a) Based on these assumptions, in approximately what year will this country first experience shortages of food?

b) If the country doubled its initial food supply and maintained a constant rate of increase in the supply adequate for an additional 0.5 million people per year, would shortages still occur? In approximately which year?

c) If the country doubled the rate at which its food supply increases, in addition to doubling its initial food supply, would shortages still occur?

What happens if I change it?

1. For each of the following simple exponential functions $y = b^x$, draw the graph.

2. A general form for exponential functions can be written as $y = a \cdot b^x$, so these equations have **a**=1.

   What happens when **a** changes? See what affect **a** has on function (1) by selecting several different values, making a table of values and sketching the graph on the same axes. Use the same **a** values for function (2). How does the value of **a** effect an exponential function? Can you predict what the graph of (3) would look like for those values of **a** without making a table of values?

3. Consider **a, b** and **x** in exponential functions. What does each represent in them? Which has the greatest effect on the graph?

   Example: Which has the most impact, doubling **a**, doubling **b** or doubling **x**?

4. Think about the relationships you have seen that are exponential in nature. Reflect on what the effect of changing **a, b** or **x** might have on each.

Exponential Functions to graph:

(1) $y = 4^x$ (2) $y = 3^x$ (3) $y = 5^x$