NAME ______________________

Date __________ Period__________

SYLLABUS
ALGEBRA 2 H

Unit 6: Polynomials

DAY	TOPIC
1	Polynomial Functions and End Behavior
2	Polynomials and Linear Factors
3	Dividing Polynomials
4	Synthetic Division and the Remainder Theorem
5	Solving Polynomial Equations
6	
7	Solving Polynomial Inequalities
8	Roots of Polynomial Equations
9	The Fundamental Theorem of Algebra
10	The Binomial Theorem
11	Review

U6 D1: Polynomial Functions & End Behavior

1. An expression that is a real number, a variable, or a product of a real number and a variable with whole-number exponents is known as ______________________________.
   a. A ______________________________ is a monomial or the sum of monomials. Standard form is written in descending order of exponents.
   b. The exponent of the variable in a term is the ________________________ of that term.

$$P(x) = \boxed{2}x^3 - 5x^2 - 2x + 5$$

constant

Leading coefficient   cubic term   quadratic term   linear term

Facts about polynomials:

1. classify by the number of terms it contains
2. A polynomial of more than three terms does not usually have a special name
3. Polynomials can also be classified by degree.
4. the **degree of a polynomial** is: ____
   ____

Degree	Name using degree	Polynomial example	Number of terms	Name using number of terms
0	constant			
1	linear			
2	quadratic			
3	cubic			
4	quartic			
5	quintic			

Examples:

1. Write each polynomial in standard form. Then classify it by degree and by the number of terms.

   a. $-7x+5x^4$

   b. $x^2-4x+3x^3+2x$

   c. $4x-6x+5$

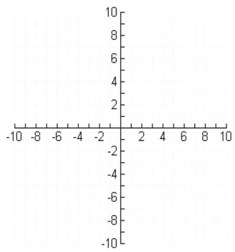
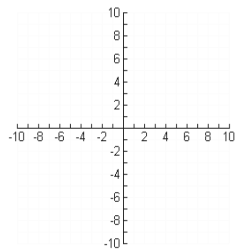
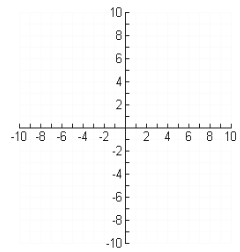
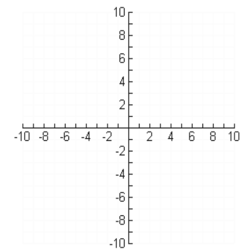
   d. $3x^3+x^2-4x+2x^3$

2. Use a graphing calculator to determine whether the data best fits a linear model, a quadratic model, or a cubic model. Predict y when $x=100$.

x	y
0	10.1
5	2.8
10	8.1
15	16
20	17.8

3. The table shows world gold production for several years. Find a quartic function that models the data. Use it to estimate production since 1988.

Year	**Production (millions of troy ounces)**
1975	38.5
1980	39.2
1985	49.3
1990	70.2
1995	71.8
2000	82.6

End Behavior: describes the far left and far right portions of a graph				
Behavior	Up and Up (↖,↗)	Down and Down (↙,↘)	Down and Up (↙,↗)	Up and Down (↖,↘)
Graph				
As x $\rightarrow \infty$ **y** $\rightarrow$ ______				
As x $\rightarrow -\infty$ **y** $\rightarrow$ ______				
Equation				

Examples: Determine the end behavior of the graphs of each function below

1) $y = 3x + 2$

2) $y = (x-2)^3$

3) $g(t) = -t^2 + t$

4) $h(x) = x^6$

Closure: Simplify the expression below (in standard form), and then provide all of the information …

$(-12x^3 + 5x - 23) - (4x^4 + 31 - 9x^3)$

a) Classify by # of terms

b) Find the degree

c) Find the end behavior

U6 D2: Polynomials and Linear Factors

Warm up: Write $(7x^2+8x-5)+(9x^2-9x)$ in standard form, and then determine the following:

Name of polynomial (degree) _______________, Name (# of terms) ______________

End behavior: ________________ Standard Form: _____________________

Factored form: $(x+1)(x+2)(x+3)$

Standard Form: _______________________

A polynomial can be written as a product of linear factors (degree ____). A linear factor is like a _______________ number, meaning it cannot be factored anymore. Factored form is extremely important because it helps us to find the ____________ of the polynomial functions. Remember, these are the ___ - intercepts!

Example #2: $2x(x-3)^2$

Standard form: _______________________

Work backwards: Factor! Remember, think GCF first.

a) $2x^3+10x^2+12x$

b) $3x^3-3x^2-36x$

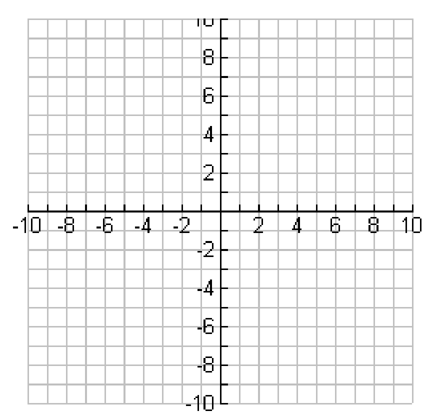
Standard form is helpful because it tells us the _____________ _____________________ easily.

Factored form is helpful because it tells us the _____________, and therefore helps us to graph.

Example: Find the zeros and graph!

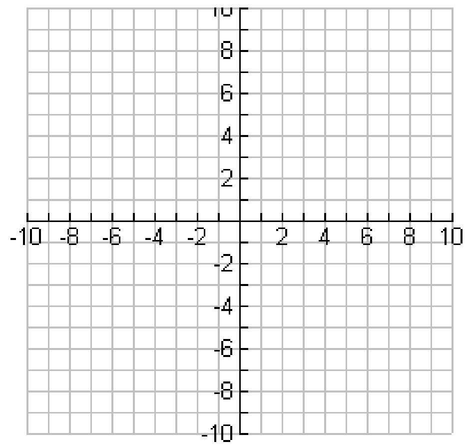
$y=x(x-3)(x+1)$

* This technique is known as the ____________ _________________ property



Example #2: Find the zeros and graph. Label zeros on the graph!

$f(x) = (x-2)(x-2)(x-2)$



When a zero is repeated, it is said to have a ____________________.
A multiple zero has a ______________ equal to the number of times the zero occurs. So for this example, 2 has a multiplicity of _______.

Directions: Find the zeros of each polynomial and state the *multiplicity* of any multiple zeros.

a) $y = x(x+3)(x+3)$

b) $f(x) = x^4 + 6x^3 + 8x^2$

c) $f(x) = (x-2)(x+1)(x+1)^2$

d) $x^3 + x^2 - 4x - 4$

Sometimes you might know the zeros of a function, but need to find the equation in standard form.
(Review: why do we need an equation in standard form sometimes?)

a) zeros are -2, 3, and 3

b) zeros are -4, 2, 1

Closure: What are "standard form" and "factored form" of a polynomial and why are they useful?

U6 D3: Dividing Polynomials (Long Division)

Question #1: Is 8 a factor of 76?

Question #2: Is $(x-4)$ a factor of x^2-3x+1

Important fact: To be a factor, division must yield a **remainder of ______!**

Directions: Determine whether each divisor is a factor of each dividend. (Do this by __________ division!)

a) $(2x^2-19x+24)\div(x-8)$

b) $(x^3-4x^2+3x+2)\div(x+2)$

Write your final answers in "quotient times divisor plus remainder" form:

a)

b)

HW Start! After the class activity, you can start on your homework. The first problems are listed below

Divide using long division: Check your answers

1) $(x^2-3x-40)\div(x+5)$

2) $(3x^2+7x-20)\div(x+4)$

3) $(x^3+3x^2-x+2)\div(x-1)$

4) $(2x^3-3x^2-18x-8)\div(x-4)$

U6 D4: Synthetic Division & the Remainder Theorem

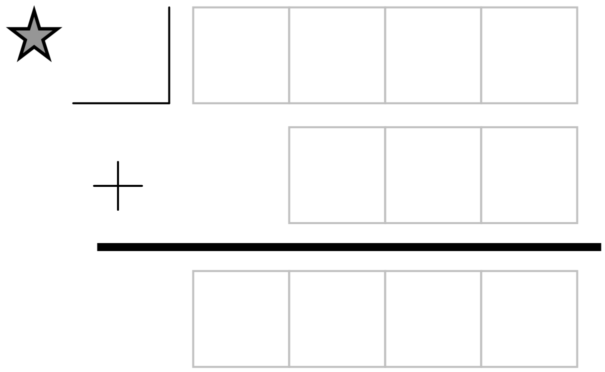
Today we will learn an alternative to long division: ____________________ division.

In this process, you omit all variables and exponents.

You must remember to _________________ the sign of the divisor, so you can add throughout the process

Example #1: $\left(x^3 - 4x^2 + 6x - 4\right) \div \left(x - 2\right)$

$$x - 2\overline{\big)\, x^3 - 4x^2 + 6x - 4}$$



Example #2: Divide $x^3 + 4x^2 + x - 6$ by $x + 1$

The Remainder Theorem: If a polynomial $P(x)$ of degree $n \geq 1$ is divided by $(x - a)$, where a is *a* constant, then the remainder is $P(a)$.

What the heck does that mean?

Use synthetic division to find $P(-1)$ for $P(x) = 2x^4 + 6x^3 - 5x^2 - 60$.

Practice:

1) Divide using synthetic division: $\left(6x^2-8x-2\right)\div\left(x-1\right)$

2) Use synthetic division and the remainder theorem to find $P(a)$.

a) $P\left(x\right)=x^3-7x^2+15x-9; a=3$

b) $P\left(x\right)=2x^3+4x^2-10x-9; a=3$

3) Extension: Divide using synthetic division, then check by using long division. Write your final answer in "quotient times divisor plus remainder" form.

$\left(x^4+3x^2+x+4\right)\div\left(x+3\right)$

U6 D5: Solving Polynomial Equations

Warm up questions:

a) Name 4 ways we have learned to solve quadratic equations.

b) How does the solution to a quadratic equation relate to the graph of a quadratic function?

c) What does it mean for $x = 5$ to be a solution to a higher degree polynomial (cubic, quartic, etc.)?

Today we are going to practice factoring & solving (or finding zeros) of higher degree polynomials.

From Day 2: $x^3 - 7x^2 - 18x$

Today: $x^3 + 8$

Some of our solving will require that we can factor the sum and difference of two cubes.

$a^3 + b^3$					

$a^3 - b^3$					

Perfect Cubes	x	1	2	3	4	5	6
	x^3						

Practice: Factor only (for now…)

1. $x^3 - 8$

2. $64x^3 + 125$

3. Solve $27x^3 + 1 = 0$

4. Solve $8x^3 - 1 = 0$

Now we need to be able factor quartic (degree 4) trinomials…

Let $a = x^2$, substitute, factor, then back substitute. Make sure that you find __________ factors (or roots)!

7. $x^4 - 2x^2 - 8$

8. $x^4 + 7x^2 + 6$

9. Solve $x^4 - x^2 = 12$

10. Solve $x^4 + 11x^2 + 18 = 0$

<u>Closure</u>: factor $x^3 + y^3$ and $x^3 - y^3$

<u>Extension</u>: How can we solve $x^3 - 2x^2 = -3$ on a graph?

U6 D7: Solving Polynomial Inequalities

1. $x^4 - 7x^2 - 30 > 0$

2. $-4x^2 + 4x > 0$

3. $x^3 - 3x^2 - x + 3 \le 0$

4. $8x^3 + 125 < 0$

U6 D7: Worksheet

Directions: Solve each polynomial inequality algebraically on the number line. Check your solution graphically. Remember you do not need to find imaginary roots because they are not on the number line. Real answers only!!

1. $x^3 + x^2 - 16x - 16 \geq 0$

2. $x^4 + 35x^2 - 36 \geq 0$

3. $x^4 - 6x^2 + 9 \leq 0$

4. $x^4 - 12x^2 + 11 < 0$

5. $x^3 - 216 \geq 0$

6. $x^4 - 13x^2 + 36 \leq 0$

7. $x^3 - 11x^2 - 8x + 88 \geq 0$

8. $x^3 + 1000 < 0$

9. $64x^4 - 81 > 0$

U6 D8: Roots of Polynomial Equations

So far we have looked at several ways to find the roots of an equation. Today, we'll learn another one…

The Rational Root Theorem

If $\frac{p}{q}$ is in simplest form and is a rational root of the polynomial equation $a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0 = 0$ with integer coefficients, then p must be a factor of a_0 and q must be a factor of a_n.

Example (this will be the easiest way to explain). Find the **rational roots** of $x^3 + x^2 - 3x - 3 = 0$

Possible rational roots are $\frac{\text{constant}}{\text{leading coef}}$

Find the leading coefficient: ______ → Factors: _________

Find the constant term: ______ → Factors: ____________

☆ Now, test all the potential points: Plug them back into the original equation and see if it's true!

Example #2: $x^3 - 4x^2 - 2x + 8 = 0$

You can use the **Rational Root Theorem** to find all the roots of a polynomial equation (Hooray!)

a) $2x^3 - x^2 + 2x - 1 = 0$ Step 1: Find all possible rational roots Step 2: Test each of the possible roots Step 3: Use synthetic division to find the quotient Step 4: Find the roots of the quotient	b) $3x^3 + x^2 - x + 1 = 0$

Irrational Root Theorem: If $a+\sqrt{b}$ is a root, then its conjugate is also a root. If $a-\sqrt{b}$ is a root, then so is its conjugate

6. A polynomial equation with integer coefficients has the roots $1+\sqrt{3}$ and $-\sqrt{11}$. Find two additional roots.	7. A polynomial equation with rational coefficients has the roots $2-\sqrt{7}$ *and* $\sqrt{5}$. Find two additional roots.

Imaginary Root Theorem: If the imaginary number $a+bi$ is a root of a polynomial with real coefficients, then the conjugate $a-bi$ is also a root.

8. If a polynomial equation with real coefficients has $3i$ and $-2+i$ among its roots, then what two other roots must it have?

9. A polynomial equation with integer coefficients has the roots $3-i$ and $2i$. Find two additional roots.

10. Find a third degree polynomial equation with rational coefficients that has roots 3 and $1+i$.

11. Find a cubic polynomial equation with rational coefficients that has roots -1 and $2-i$.

Summing it all up:

When/how do we use the following?

a. Remainder Theorem:

b. Long Division/Synthetic Division:

c. Rational Root Theorem:

d. Irrational Root Theorem:

e. Imaginary Root Theorem:

You have solved polynomial equations and found that their roots are included in the set of complex numbers. In other words, the roots have been integers, rational numbers, irrational numbers, and imaginary numbers. But, the question remains, can all polynomial equations be solved using complex numbers?

Carl Friedrich Gauss (1777-1855) proved that the answer to this question is yes! The roots of every polynomial equation, even those with imaginary coefficients, are complex numbers. The answer to this question was so important that it is now known as thee **Fundamental Theorem of Algebra.** More info about Gauss @ gradeamathhelp.com.

Fundamental Theorem of Algebra: If $P(x)$ is a polynomial of degree $n \geq 1$ with complex coefficients, then $P(x) = 0$ has at least one complex root.

Corollary to the Fundamental Theorem of Algebra: Including imaginary roots and multiple roots, an *n*th degree polynomial equation has exactly *n* roots; the related polynomial function has exactly *n* zeros.

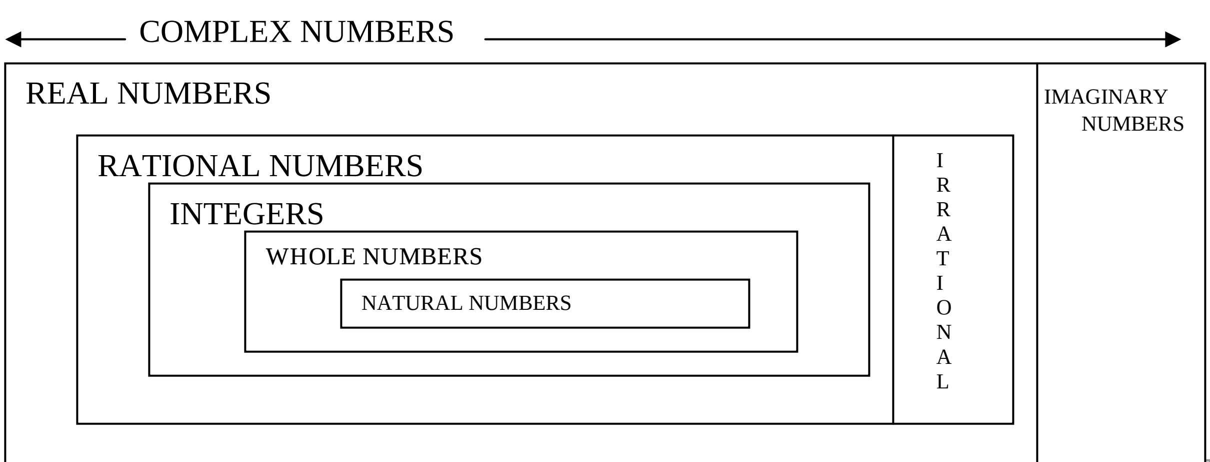
In other words, if the leading coefficient of a polynomial is x^8, then there are _____ complex roots!

Example #1: Find the number of complex roots of the equation below. Then, break up those roots into the number of possible real roots and the number of possible imaginary roots.

$x^3 + 2x^2 + 3x - 1$

Example #2: $x^4 + 5x^3 - x^2 + 6x - 4$

Review of our number system:



Let's put some of our theorems together:

<u>Directions</u>: For each equation, state the number of complex roots, the possible number of real roots, and the possible rational roots.

1) $x^4 + 4x^3 + 2$

2) $2x^5 + 3x^4 - 2x^3 + x^2 + 4x - 2$

3) Find the number of complex zeros of $f(x) = x^3 - 3x^2 + x - 3$. Find all the zeros.

Step #1: use synthetic division to find a rational zero

Step #2: Solve the remaining quadratic using one of our methods…

4) Find the number of complex zeros of $y = x^3 - 2x^2 + 4x - 8$. Find all the zeros.

<u>Closure</u>: How can you determine the number of complex roots of a polynomial? Number or real roots?

U6 D10: The Binomial Theorem (Combinations Review)

Warm up: Simplify the following. You may use your calculator where possible.

1) $\binom{6}{2}$ 2) ${}_5C_3$ 3) $\dfrac{8!}{3!5!}$ 4) $\dfrac{(n-2)!}{(n-4)!}$ 5) $\binom{4}{2}+\binom{7}{6}$

We will need to use combinations for the binomial theorem. If you did **not** take finite math and need extra help with this, please seek it out.

${}_nC_r$ or $\binom{n}{r}=\dfrac{n!}{r!(n-r)!}$ and $n!=$

The binomial theorem is used to expand (reverse or factor) binomials…

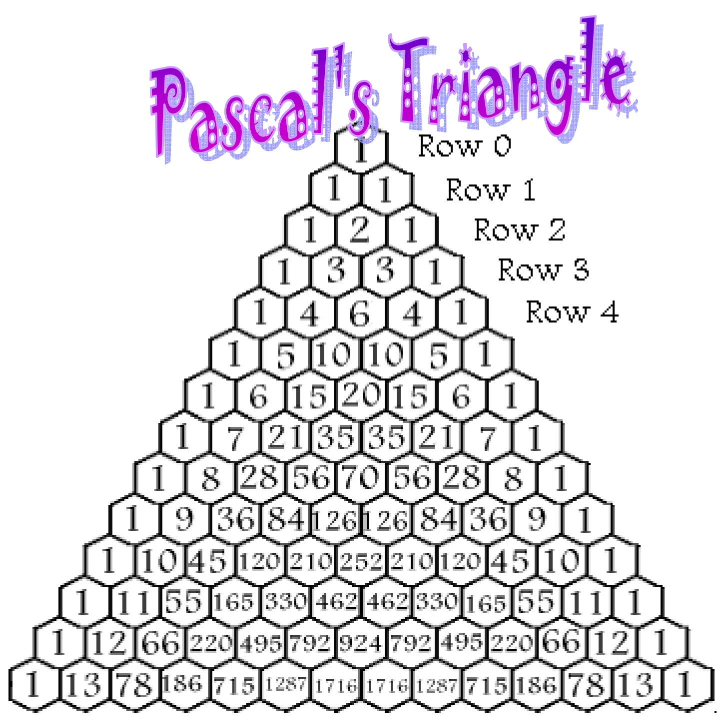
"Long way" $(x+3)^3$

"Short cut" $(x+3)^3$

Example #1: Use Pascal's triangle to expand $(a+b)^5$

Example #2: Use Pascal's triangle to expand $(x-3y)^4$

- The exponents of each term **add up to** ______.
- Do not forget parentheses when you have $(2x+y)$



Sometimes you might not want to write out all of the rows of Pascal's triangle, but you still need the coefficients. We can use combinations to "skip to" any row of Pascal's triangle.

Find $\binom{6}{0}$ $\binom{6}{1}$ $\binom{6}{2}$ $\binom{6}{3}$ $\binom{6}{4}$ $\binom{6}{5}$ $\binom{6}{6}$. Look back @ row 6

This is particular useful if we only need to find one term: For example, find the 5th term of $(3x-2y)^6$.

Let's find the pattern (it's easier to look at examples than to give a "formula" with all variables)

a) Third term of $(x+3)^{12}$

b) Second term of $(x+3)^9$

c) Eighth term of $(x-2y)^{15}$

d) Seventh term of $(x^2-2y)^{11}$

<u>Wrap up</u>: One term of a binomial expression is $\binom{7}{2}x^5y^2$. What is the term just before that term?

<u>Extension</u>: Example $(2x^2-5y)^7$

U6 D11: Review

1. Directions: Give the end behavior of each of the functions below.
   Write your answer as one of the following: $(\nwarrow, \searrow)$, $(\nwarrow, \nearrow)$, $(\swarrow, \searrow)$, $(\swarrow, \nearrow)$

a) $y = 5x^4 - 2x^3 + 5$

b) $y = -4x(x-2)^2$

c) $y = (x-1)(2x+4)^3(x+8)$

2. Directions: Simplify each of the following

a) $\binom{7}{2}$

b) ${}_9C_5$

c) ${}_xC_y$

3) Find the zeros of $x^3 - 64 = 0$

4) Classify with degree and # of terms:

a) $y = x^4 - 2x^2$

b) $y = x^2 + 2x + 10$

5) Which of the following is a rational zero of y = $x^3 - 10x^2 + 26x - 35$?

a. x = 35
b. x = -35
c. x = -7
d. x =7

6) State the zeros and the multiplicity for each for the function $f(x) = 2x(x-5)^2(x+3)^3$

7) Give the number of possible imaginary roots for each polynomial below

a) $y = 4x^6 + 3x^3 - 2x + 5$

b) $y = x^5 - 2x^4 + x^2 - 7x$

8) If $5 - 4i$ is a root of a polynomial, name another root of that same polynomial

9) The roots of a cubic polynomial are 1 and $-5i$. Write the polynomial in standard form. Show all work.

10) Use cubic regression to find the cubic model for the following data.

x	0	2	4	6	8	10
y	3.2	4.1	4.2	4.4	5.1	6.8

a. Write the equation of the cubic model. Round each number to three decimal places.

$y =$ ____

b. Use your model to predict the value of *y* when $x = 12$.

$y =$ ______________

11) Expand $(x-3y)^6$

12) Find the 3rd term of $(x-1)^7$

13) Find allllll zeros algebraicially.

$$x^3 + 4x^2 - x - 10 = 0$$