Name: _____ Block: _____

Unit 7 – Central Tendency and Probability

7.1 Central Tendency
7.2 Introduction to Probability
7.3 Independent Events
7.4 Dependent Events

"I wish we hadn't learned probability
'cause I don't think our odds are good."

7.1 Central Tendency

A **central tendency** is a central or _____ value in a data set. We will look at three measures of central tendency:

- **Mean** (or _____), which is found by _____ all of the data values and dividing by _____

- **Median**, which is the _____ value when the data is arranged from _____ to _____. If there is an even amount of data values, then you must find the _____ of the two middle values.

- **Mode**, which is the value that occurs _____. It is possible to have more than one mode; if there are two then the data is _____ and if there are three then the data is _____. If no value repeats itself, then there is _____ mode.

The **range** of a set of data, the _____ between the highest and lowest data values, is also a useful measure because it can tell us how spread out the data is. Sometimes, however, there are **outliers**, values that are _____ the other data values, which can increase the range and make the data seem more spread out than it really is.

Ex 1. A teacher collects the following data from a small class of 9 students.

Student exam scores (%)	75	81	7	90	77	63	67	93	63

(a) From this data, find:

i. The **mean**

ii. The **median**

iii. The **mode**

iv. The **range**

(b) Are there any **outliers** in this set of data?

(c) Does the mean or median give a better indication of how well a typical student did on the exam? Why?

Ex 2. Six students were surveyed and asked how many text messages they had sent that day. The results were as follows: 42 12 12 19 42 22

(a) Find:

i. The **mean**

ii. The **median**

iii. The **mode**

iv. The **range**

7.1 Practice

1. Given this set of data:

9 4 7 4 5 8 6 4 3

a) Find the mean to 1 decimal place.

b) Find the median.

c) Find the mode.

d) Find the range.

2. The monthly rainfall for 1992 is recorded below.

Jan 10 mm	Feb 8 mm	Mar 18 mm	Apr 35 mm
May 26 mm	Jun 12 mm	July 8 mm	Aug 15mm
Sep 23 mm	Oct 20 mm	Nov 14 mm	Dec 16 mm

a) Find the mean rainfall to 1 decimal place.

b) Find the median.

c) Find the mode.

d) Find the range.

e) Which month has a rainfall closest to the **mean**?

f) Which months had a rainfall within 10 mm of the **median**?

g) If the **range** were a small number, what does this tell you about the rainfall for 1992?

3. During the 1992 Winter Olympics in France, a Canadian skater had the following scores:

10.0 9.8 8.4 9.2 8.9 9.5 8.4 8.7 9.2 9.8

a) Find the mean.

b) Find the median.

c) Find the mode.

d) Find the range.

e) If the **range** were a large number, what does this tell you about the judges?

f) In these competitions, the score from the judge that gave the lowest score and the score from the judge that gave the highest score are not counted in calculating the mean. What is the new **mean** if these scores are not included?

g) Does the value of the **median** change when dropping the lowest and highest scores? Explain.

4. Christian obtained these scores on his math tests:

65% 96% 72% 70% 65% 62% 75% 65%

b) Find the mean.

b) Find the median.

c) Find the mode.

d) Find the range

e) Does the mean, median, mode or range best describe his math achievement? Explain.

f) Does the mean, median, mode or range best describe his consistency? Explain.

g) If the **range** were a very large number such as 50, does this necessarily mean that he does poorly half of the time and does well the other half of the time? Explain.

5. The scores out of 100 for 30 students are shown below.

16	30	60	75	83	43	47	59	89	92	75	59	62	73	69
83	45	63	87	88	65	39	67	64	59	78	89	54	20	68

a) Find the mean to 2 decimal places.

b) Find the median.

c) Find the mode.

d) Find the range.

e) If the **median** score is over 50, does this mean that most of the students passed or that most of them failed?

f) Does the mean, median or mode best describe the achievement of the class overall? Explain.

7.2 Introduction to Probability

The possible outcomes of an experiment are called the **sample space**. For example, when you roll a regular die, the sample space is: _____.

Each of these outcomes has an equal chance of happening and is found by:

$$P(\text{outcome}) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

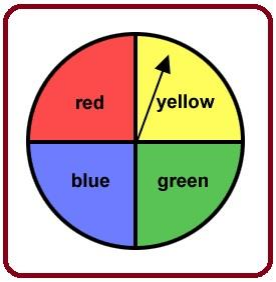
where $P(\text{outcome})$ is the **probability** of a particular outcome.

Ex 1. For a six-sided die, find each probability both as a fraction and as a percent:

a) $P(6)$ b) $P(\text{even number})$ c) $P(\text{number divisible by 3})$

d) $P(8)$ e) $P(4 \text{ or } 5)$ f) $P(\text{number from 1 to 6})$

The **theoretical probability** of an outcome is what we expect to happen, whereas the **experimental probability** is what actually happens when we try it out.



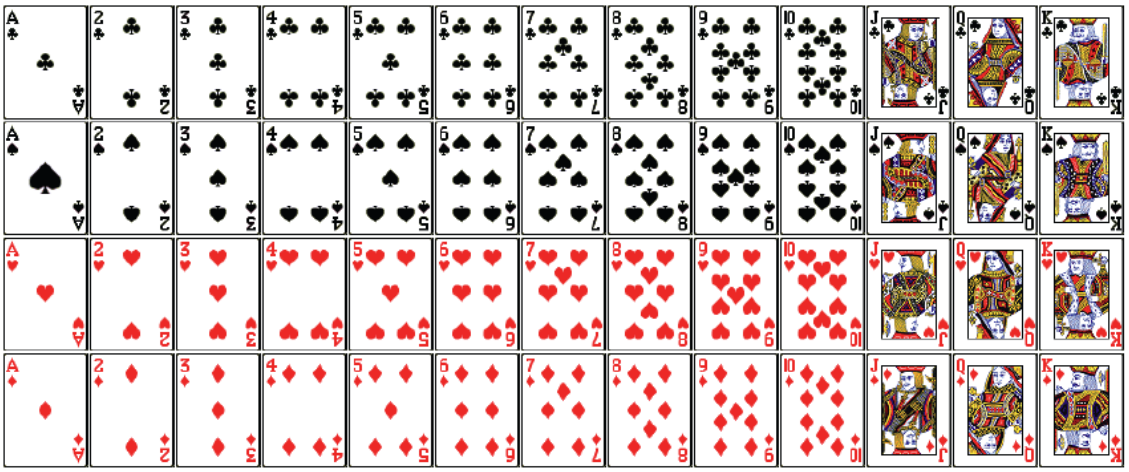
Ex 2. Consider the spinner pictured on the right.

a) What is $P(\text{green})$, the theoretical probability of landing on green?

b) If the spinner is spun 60 times, how many times would we expect it to land on green?

c) If the spinner lands on green 22 times in 60 spins, what is the experimental probability of landing on green?

Standard Deck of 52 cards



Ex 3. A card is drawn from a well-shuffled deck. Find:

a) $P(\heartsuit)$ b) $P(\text{ace})$ c) $P(\text{red})$ d) $P(7\clubsuit \text{ or } K\diamondsuit)$

7.2 Practice

1. A card is drawn from a well-shuffled deck. Find the probability, as a <u>percentage to 2 decimal places</u>, of drawing:

   a) a spade b) a jack c) a black

   d) a red jack e) a five f) a black 3, 6, or 9

2. If one letter is selected at random from the word "mathematics", what is the probability, as a fraction, that it will be:

   a) an "m" b) an "e" c) a vowel

   d) a consonant e) an "o" f) a "t" or an "h"

3. Natasha buys 3 tickets for a draw. What is the probability of her winning, as a percentage (to two decimal places where necessary), if the number of tickets sold is:

   a) 36 b) 600 c) 9450

4. A spinner has 8 equal sections, numbered from 1 to 8. Find each probability, as a reduced fraction:

   a) $P(4)$ b) P(a number greater than 5) c) P(an odd number)

   d) P(a two-digit number) e) P(a one-digit number) f) P(a number divisible by 3)

5. How many times should a die show a "1" if it is tossed:

   a) 60 times b) 450 times c) 12 000 times

6. A bag contains 40 marbles; 12 red, 10 yellow, and 18 blue. If one is taken out at random, what is each probability, as a percent:

   a) $P(\text{red})$ b) $P(\text{yellow})$ c) $P(\text{blue})$

   d) $P(\text{not blue})$ e) $P(\text{red or blue})$ f) $P(\text{green})$

7. Bag A contains 5 red and 7 green counters. Bag B contains 4 red and 6 green counters. Bag C contains 2 red and 2 green counters. From which bag would you stand the best chance of selecting a green counter in one draw?

8. Slips of paper, numbered from 1 to 30, are placed in a bowl. If one is selected at random, what is the probability, as a percent, that it bears a number with one or both digits a 2?

9. A card is drawn at random from a deck, replaced, and the deck shuffled. If this is done 1000 times, about how many times should the card drawn be:

   a) black

   b) a queen

   c) a diamond

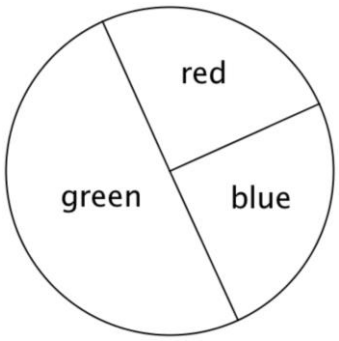
   d) the ace of spaces

10. For the spinner below, find each probability, as a <u>percent</u>:

   a) $P(\text{red})$

   b) $P(\text{green})$

   c) $P(\text{green or blue})$



7.3 Independent Events

Two events are said to be **independent** if the outcome of one has no effect on the outcome for the other. For example, rolling a die and tossing a coin are independent events.

Ex 1. A coin is tossed and a die is rolled at the same time. What is the probability of getting a head and a 6?

Method 1 – Make a **tree diagram** to show the sample space

Method 2 – Multiply the probabilities of each separate outcome: $P(\text{A and B}) = P(\text{A}) \times P(\text{B})$

Ex 2. Without looking, Trevor took one card from each of 3 decks. What is the probability that the 3 cards are the jack of clubs, the ace of spades and the 7 of diamonds (in that order)?

Ex 3. A bag contains 5 red balls, 3 green balls, and 4 yellow balls. Two draws are made. If the first ball is replaced before drawing the second, find:

a) $P(\text{red, red})$

b) $P(\text{green, yellow})$

7.3 Practice

1. What is the probability of tossing four coins and getting four tails? Express as a fraction and as a percent.

2. If it is equally likely that a child be born a girl or a boy, what is the probability that a family of six children will all be boys? Express your answer as a fraction.

3. A weather report gives the chance of rain on both days of the weekend as 80%. If this is correct, what is the probability, as a percent, that:

   a) there is rain on both days?

   b) it does not rain on either day?

4. A multiple-choice test has 4 questions. Each has 5 choices, only one of which is correct. If all the questions are attempted by guessing, what is the probability of getting all 4 right? Express as a fraction and as a percent.

5. A bag contains 3 red balls and 5 green balls. Find the probability, as a fraction and as a percent to one decimal place, of drawing two green balls if the first ball is replaced before drawing the second.

6. A meal at a fast-food outlet has the following choices:
   - A hamburger, cheeseburger, or hot dog
   - A soft drink or shake
   - A sundae, piece of pie, or cookies

   If a choice is made at random, what is the probability, as a fraction, that a meal will include a:

   a) hot dog and a shake

   b) cheeseburger, shake and cookies?

7. Two cards are drawn from a well-shuffled deck. If the first card is replaced before drawing the second, find the probability, as a percent to 2 decimal places, that they are:

   a) both spades

   b) both aces

   c) both face cards

   d) a heart, then the 3 of clubs

8. Five dice are tossed simultaneously. Find the probability, as a fraction, that:

   a) they all show 6

   b) no die shows 6

   c) no die shows 5 or 6

   d) challenge: they all show the same #

9. A bag contains 3 red and 2 blue cubes. Each cube is replaced after it is drawn. What is each probability, as a fraction?

   a) a red cube then a blue cube

   b) 2 red cubes

10. A red die, a blue die, and a white die are rolled. Find the probability, as a percent to 2 decimal places, of rolling a number greater than 3 on the red die, an even number on the blue die, and a 4 on the white die.

7.4 Dependent Events

Two events are said to be **dependent** if the outcome of one has an effect on the outcome for the other.

Ex 1. A bag contains 5 black balls and 5 red balls. Find the probability of drawing 2 red balls if the first ball is not replaced before drawing the second.

Ex 2. Three cards are chosen from a deck of cards. If a card is not replaced before the next is drawn, what is the probability of drawing a heart, then a black card, then the King of diamonds?

7.4 Practice

1. A bag contains 3 red balls and 5 green balls. Find the probability of drawing 2 green balls in succession if the first ball is not replaced before drawing the second. Express your answer as a reduced fraction.

2. Your sock drawer has four white socks, two polka dot socks and two striped socks. What is the probability, as a reduced fraction, of randomly picking out two socks and getting a matching pair of polka dot socks?

3. There are ten shirts in your closet. Four are blue and six are green. You randomly select one to wear on Monday and then a different one on Tuesday. What is the probability, as a percent to two decimal places, of wearing a blue shirt on both days?

4. A bag contains five red marbles, four blue marbles, and three yellow marbles. You randomly pick three marbles without replacement. What is the probability, as a percent to two decimal places, that the first marble is red, the second marble is blue, and the third marble is yellow?

5. Two cards are drawn from a well-shuffled deck. If the first card is NOT replaced before drawing the second, find the probability, as a percent to 2 decimal places, that they are:

   b) both spades

   b) both aces

   c) both face cards

   d) a heart, then the 3 of clubs

6. The word "mathematics" is spelled out with tiles and the tiles are put in a bag. What is the probability, as a reduced fraction, that two tiles drawn without replacement will be:

   a) both "m"?

   b) both vowels?

   c) both consonants?

ANSWERS

Section 7.1

1. a) 5.6
   b) 5
   c) 4
   d) 6
2. a) 17.1 mm
   b) 15.5 mm
   c) 8 mm
   d) 27 mm
   e) March
   f) All but April and May
   g) Rainfall was consistent each month
3. a) 9.19
   b) 9.2
   c) 8.4, 9.2 and 9.8 (trimodal)
   d) 1.6
   e) The judges had very different opinions of the performance (and some are maybe biased!)
   f) 9.1875
   g) Median won't change since middle will still be the same.
4. a) 71.25%
   b) 67.5%
   c) 65%
   d) 34%
   e) Median, since 96% is an outlier.
   f) Mode, since 3 of 8 scores were 65%.
   g) No, since there could be one really low or high outlier which increases the range.
5. a) 63.37
   b) 64.5
   c) 59
   d) 76
   e) Most passed, since the median is the value in the middle of the list.
   f) The median, since there were some really low outliers.

Section 7.2

1. a) 25%
   b) 7.69%
   c) 50%
   d) 3.85%
   e) 7.69%
   f) 11.54%
2. a) 2/11
   b) 1/11
   c) 4/11
   d) 7/11
   e) 0
   f) 3/11
3. a) 8.33%
   b) 0.5%
   c) 0.03%
4. a) 1/8
   b) 3/8
   c) 1/2
   d) 0
   e) 1
   f) 1/4
5. a) 10
   b) 75
   c) 2000
6. a) 30%
   b) 25%
   c) 45%
   d) 55%
   e) 75%
   f) 0%
7. Bag B
8. 40%
9. a) 500
   b) 77
   c) 250
   d) 19
10. a) 25%
    b) 50%
    c) 75%

Section 7.3

1. 1/16 or 6.25%
2. 1/64
3. a) 64%
   b) 4%
4. 1/625 or 0.16%
5. 25/64 or 39.1%
6. a) 1/6
   b) 1/18
7. a) 6.25%
   b) 0.59%
   c) 5.33%
   d) 0.48%
8. a) 1/7776
   b) 3125/7776
   c) 1024/7776 (or 32/243)
   d) 6/7776 (or 1/1296)
9. a) 6/25
   b) 9/25
10. 4.17%

Section 7.4

1. 5/14
2. 1/28
3. 13.33%
4. 4.55%
5. a) 5.88%
   b) 0.45%
   c) 4.98%
   d) 0.49%
6. a) 1/55
   b) 6/55
   c) 21/55