Unit 7 : Chap. 1 Inductive and Deductive Reasoning

Inductive Reasoning: Involves making conjectures based on patterns or observations. Conjectures are much like hypothesis in science.

Ex.) Today it is below zero and cloudy , I expect that it will snow.

Deductive Reasoning: Deductions made based on factual information.

Ex.) Mike is older than Pete and Pete is older than Diane, therefore Mike is older than Diane.

We make conjectures using inductive reasoning.

We use deductive reasoning to Prove Conjectures.

Ex.)Conjecture: When you add two even numbers together,

you will always get an even number.

2 + 2 = 4 TRUE

2 + 4 = 6 TRUE

4 + 6 = 10 TRUE

PROOF:

2x + 2y = 2(x +y) This must be even. Therefore you have proved your conjecture.

Conjecture: When you add an odd number and an even number, you will always get an odd number.

1 + 2 = 3 True

2 + 3 = 5 True

5 + 6 = 11 True

PROOF:

ODD + Even

(2x +1) + 2y

2x + 2y + 1

2(x + y) + 1

Conjecture: When you square an odd number you will always get an odd number.

$1^2 = 1$ True

$3^2 = 9$ True

$7^2 = 49$ True

Proof:

$(2x + 1)^2$

(2x + 1)(2x +1) FOIL

$4x^2 + 4x +1$

$2(2x^2 + 2x) +1$ This MUST BE ODD. SINCE FIRST TERM MUST BE EVEN.

Testing for Validity: Some conjectures initially seem to be valid, but are shown not to be valid after more evidence is gathered.

Counter-example: If you can provide ONE example which is FALSE, you have proven the entire conjecture to be FALSE.

Ex.) The sum of the squares of two numbers is an odd number.

$1^2 + 2^2 = 1 + 4 = 5$ True

$3^2 + 4^2 = 9 + 16 = 25$ True

$1^2 + 3^2 = 1 + 9 = 10$ False

Since you have shown ONE example to be FALSE, the entire conjecture is FALSE.

Try to find a counterexample for each of the following:

i) If a number is divisible by 2, then it is divisible by 4.

ii) If $x + 4 > 0$, then x is positive.

What's Your Favorite Number?

Your Algebraic
Number Expressions

Choose your favorite number _____ _____

Add 3 _____ _____

Double it _____ _____

Subtract 5 _____ _____

Multiply by 5 _____ _____

Write down some of the answers given in the class with their number. Can you figure out the trick? See if you can use algebra to figure out why it works.

Answer Number Answer Number

_____ _____ _____ _____

_____ _____ _____ _____

_____ _____ _____ _____

What's Your Birthday? Your Algebraic Birthday Expressions

Choose the number of the month _____ _____

Double it _____ _____

Add 2 _____ _____

Multiply by 50 _____ _____

Add the day you were born _____ _____

Subtract 100 _____ _____

Answer Birthday Answer Birthday

_____ _____ _____ _____

_____ _____ _____ _____

Your Family Tree Your Algebraic Family Expressions

How many brothers do you have? _____ _____

Double it _____ _____

Add 3 _____ _____

Multiply by 5 _____ _____

Add the number of sisters you have _____ _____

Subtract 15 _____ _____

Answer Family Answer Family

_____ _____ _____ _____

_____ _____ _____ _____

Using **inductive reasoning** you can show number tricks for specific cases, using **deductive reasoning** you can shown it is true in general for all cases.

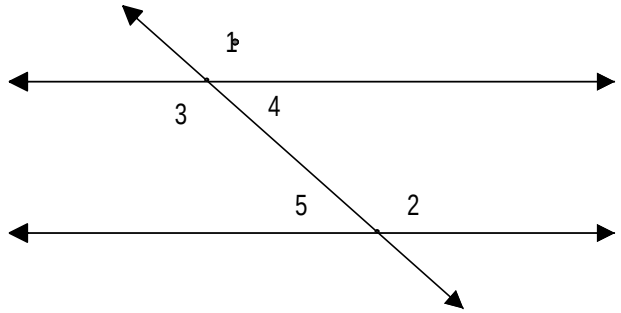
Using inductive and deductive reasoning to solve number tricks:

Ex #1.)	step 1 : choose a number	2
	Step 2 : triple it	6
	Step 3 : add 6	12
	Step 4 : subtract 3	9
	Step 5 : divide by 3	3
	Step 6 : subtract 1	2

Ex#2.)	step 1 : choose a number	6
	Step 2 : add 3	9
	Step 3 : double it	18
	Step 4 : add 4	22
	Step 5 : divide by 2	11

Reasoning about conjectures involving angles formed by transversals:

Conjecture: When a transversal intersects a pair of parallel lines, the **alternate interior angles** are equal.



I drew two parallel lines and a transversal as shown and I numbered the angles. I need to show that *L*3 = *L*2.

Statement	Justification
*L*1 = *L*2	Corresponding *L*'s
*L*1 = *L*3	Vertically opposite *L*'s
*L*3 = *L*2	Transitive Property

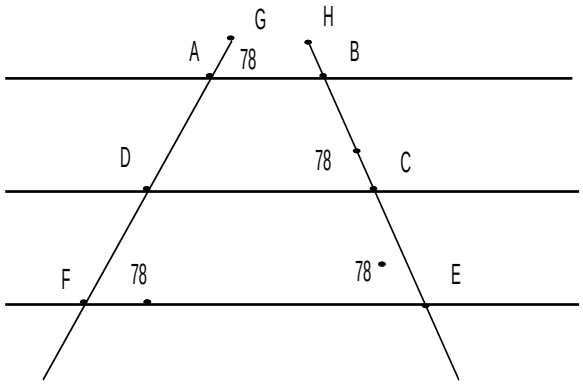
My conjecture is proved.

Ex.2) You can also use **Venn diagrams** to construct proofs involving logical reasoning.

Mammals have fur (or hair). Lions are classified as mammals.
What can be deduced about lions?

Example #3.

Using *L* properties to show that lines are parallel.



Possible Solution:

*L*GAB = *L*DFE = 78^0 Given

AB *II* FE When corresponding *L*'s are equal, the lines are parallel.

*L*BCD = *L*CEF = 78^0 Given

DC *II* FE When corresponding *L*'s are equal, the lines are parallel.

The three lines **MUST** be parallel.

<u>Proofs that are not valid.</u> A proof that contains an error in reasoning or that contains invalid assumptions.

Frank is trying to prove the following number trick.

Choose any number. Add 3. Double it. Add 4. Divide by 2. Take away the number you started with.

Each time Frank tries the trick, he ends up with 5. His proof however does not show the same result.

<u>Proof :</u>

n	**choose any number**
n + 3	**add 3**
2n + 6	**double it**
2n + 10	**add 4**
2n + 5	**divide by 2**
n + 5	**take away the number you started with**

Where is the error in Frank's proof ?

Ex. #2) Invalid Assumption.

Athletes do not compete in both Summer and Winter Olympics.

Sidney Crosby is an athlete and has competed in 2 Winter Olympics.

Therefore, Sidney Crosby has not participated in a Summer Olympics.

Where is the error ?

LOGIC PROBLEMS

Question 1

Jane, Bill and Kelly each have one pet. They all own different types of pet.

	Goldfish	Dog	Budgie
Jane			
Bill			
Kelly			

Use the logic table and the clues below to find out which pet each person owns.

Clue 1 : Kelly's pet does not have a beak.
Clue 2 : Bill's pet lives in a bowl.

Question 2

Karen, John and Jenny each play one sport: badminton, tennis or football.

	Badminton	Tennis	Football
Karen			
John			
Jenny			

Use the logic table and the clues below to find out who plays which sport.

Clue 1 : John hits a ball with a racket.
Clue 2 : Karen kicks a ball.

Question 3

Amanda, Jo, Alex and Zarah each have different coloured cars. One car is red, one blue, one white and the other is black.

	Red	Blue	White	Black
Amanda				
Jo				
Alex				
Zarah				

Use the logic table and the clues below to find out which person has which car

Clue 1 : Amanda's car is not red or white.
Clue 2 : Jo's car is not blue or white.
Clue 3 : Alex's car is not black or blue.
Clue 2 : Zarah's car is red.

Question 4

There are 4 children in a family aged 6, 8, 11 and 14 years old.

	6 years	8 years	11 years	14 years
Ali				
Mohammed				
Dipak				
Nesima				

Use the logic table and the clues below to find the age of each child.

Clue 1 : Dipak is three years older than Ali.

Clue 2 : Mohammed is older than Dipak.

Ex.) Sue signed up for games at her school's fun nights. Seven other people were assigned to her group, making up four pairs of partners.
The other members of her group were Dave, Angie, Josh, Tanya, Joy, Stu, and Linus.
When the games started, Dave and his partner were to the left of Stu. Across from Dave was Sue, who was to the right of Josh. Dave's brother's partner, Tanya, was across from Stu. Joy was not on Stu's right.

Name the four pairs of partners.

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Two girls, Arlene and Cathy, and two boys, Leander and Dean, are athletes. One is a
long distance runner, one is a softball player, one is a hockey player, and one is a golfer.
At lunchtime they sit around a square table, usually in the same places.

The runner sits on Arlene's left.
The hockey player sits across from Leander.
Cathy and Dean sit next to each other.
A girl sits on the softball player's left.

Who is the golfer?

	Runner	Softball	Hockey	Golf
Arlene				
Cathy				
Leander				
Dean				