Unit 7

Chapter 5

Circular Motion; Gravitation

Units of Chapter 5

- Kinematics of Uniform Circular Motion
- Dynamics of Uniform Circular Motion

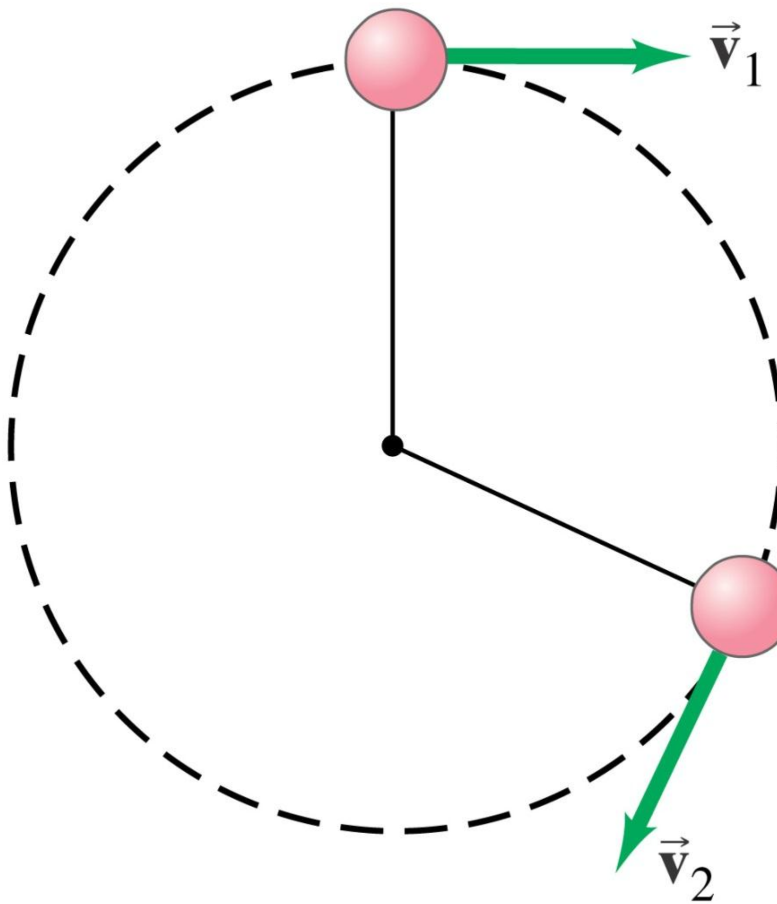
Newton’s Law of Universal Gravitation

- Gravity Near the Earth’s Surface; Geophysical Applications
- Satellites and “Weightlessness”
- Kepler’s Laws and Newton’s Synthesis
- Types of Forces in Nature

5-1 Kinematics of Uniform Circular Motion

Uniform circular motion: motion in a circle of constant radius at constant speed

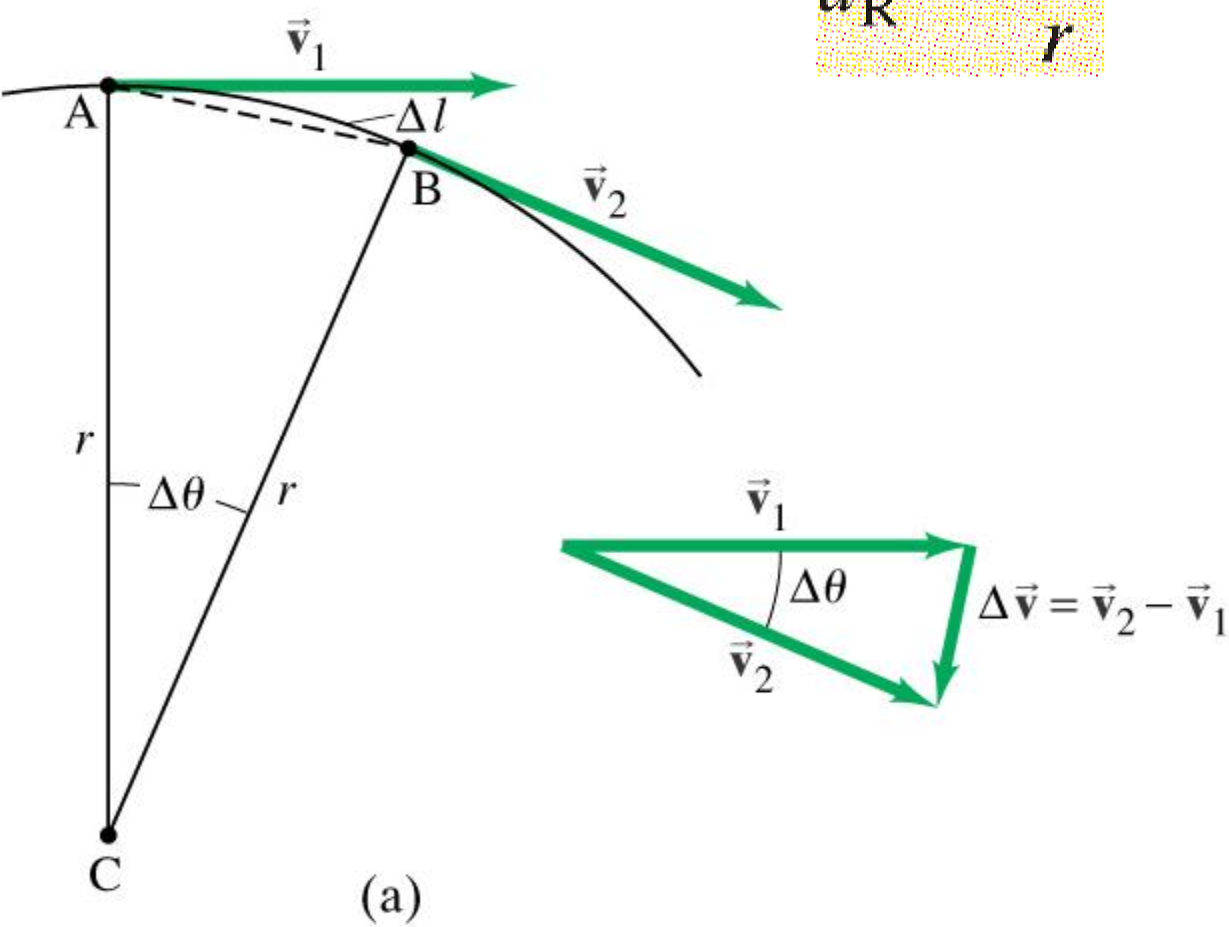
Instantaneous velocity is always tangent to circle.

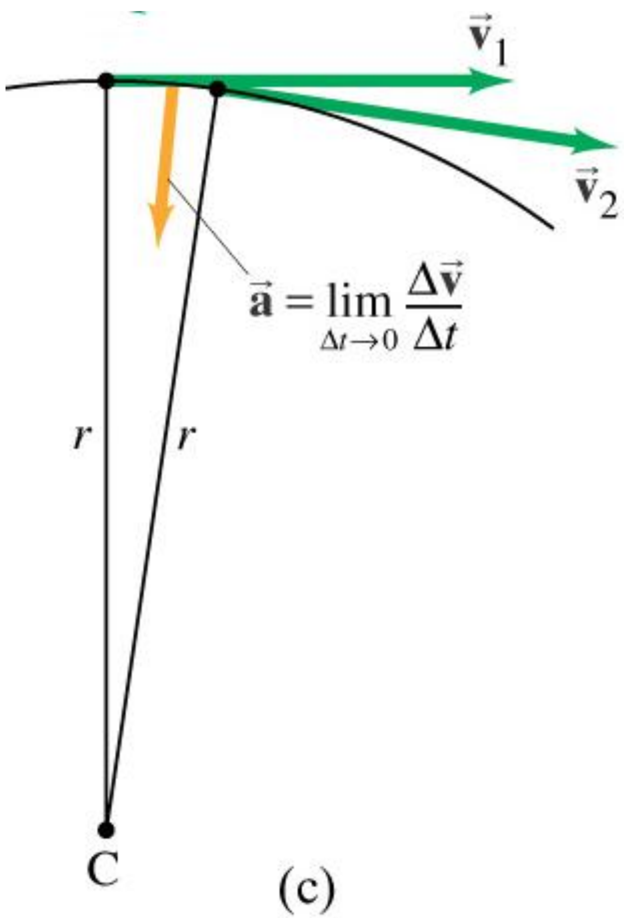


5-1 Kinematics of Uniform Circular Motion

Looking at the change in velocity in the limit that the time interval becomes infinitesimally small, we see that

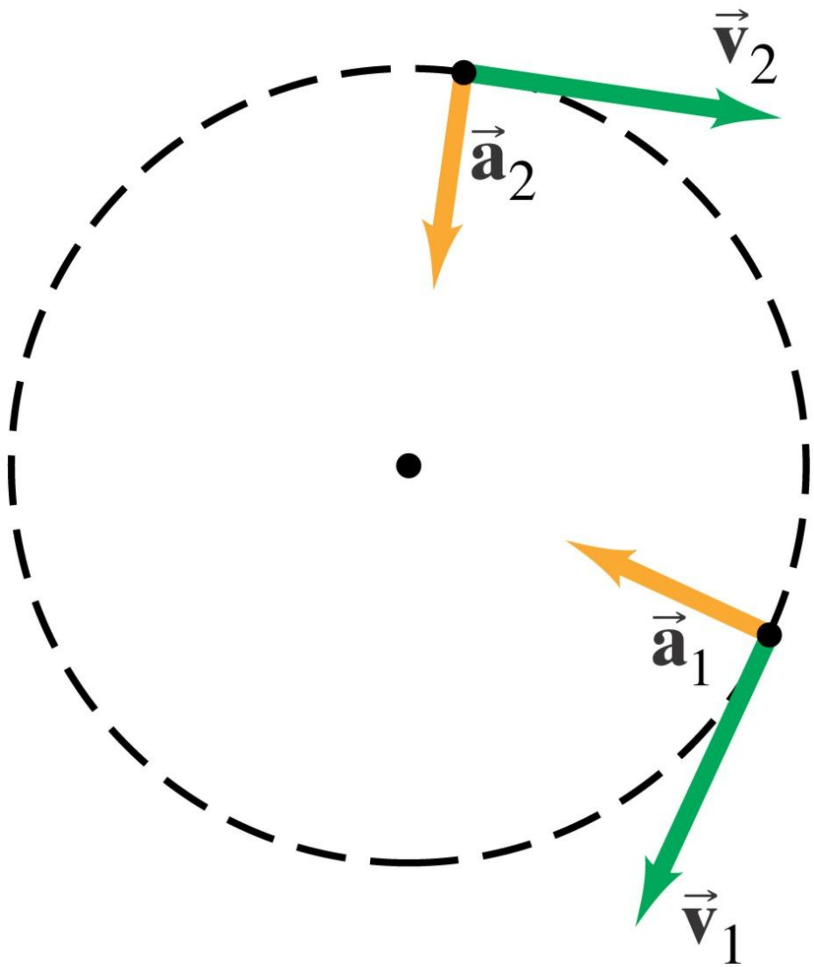
$$a_R = \frac{v^2}{r} \qquad (5\text{-}1)$$





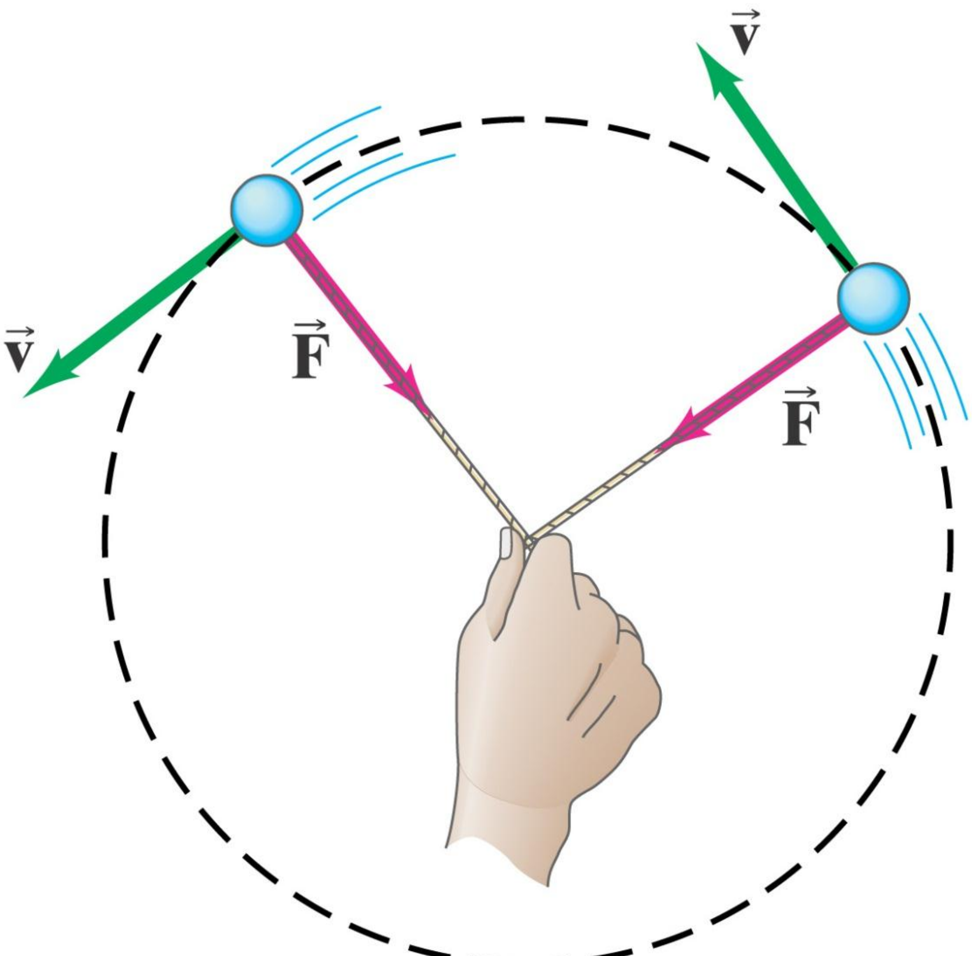
5-1 Kinematics of Uniform Circular Motion

This acceleration is called the centripetal, or radial, acceleration, and it points towards the center of the circle.



5-2 Dynamics of Uniform Circular Motion

For an object to be in uniform circular motion, there must be a net force acting on it.

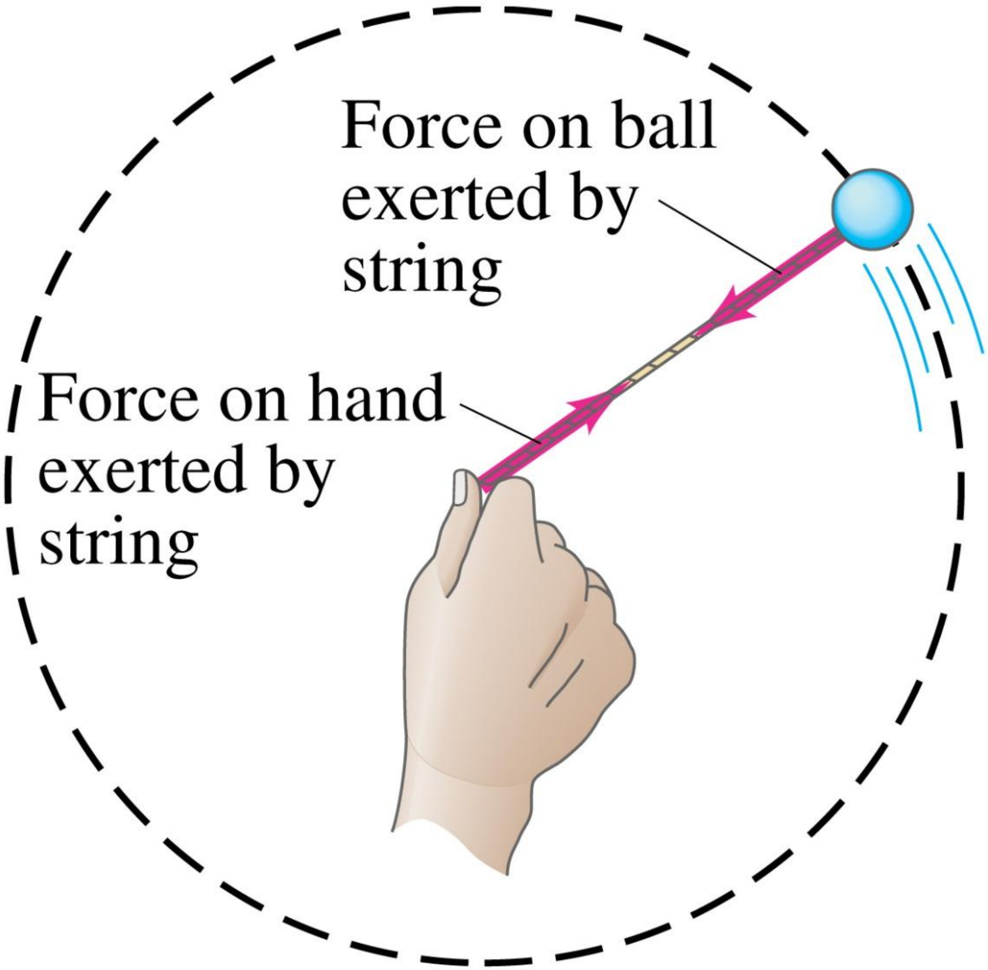


We already know the acceleration, so can immediately write the force:

$$\Sigma F_R = ma_R = m\frac{v^2}{r} \qquad (5\text{-}1)$$

5-2 Dynamics of Uniform Circular Motion

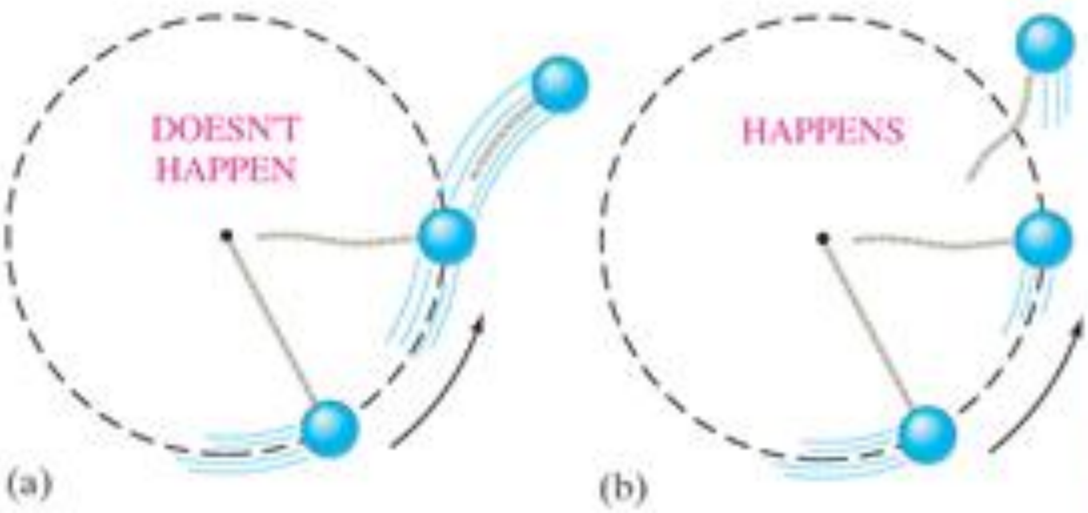
We can see that the force must be inward by thinking about a ball on a string:



5-2 Dynamics of Uniform Circular Motion

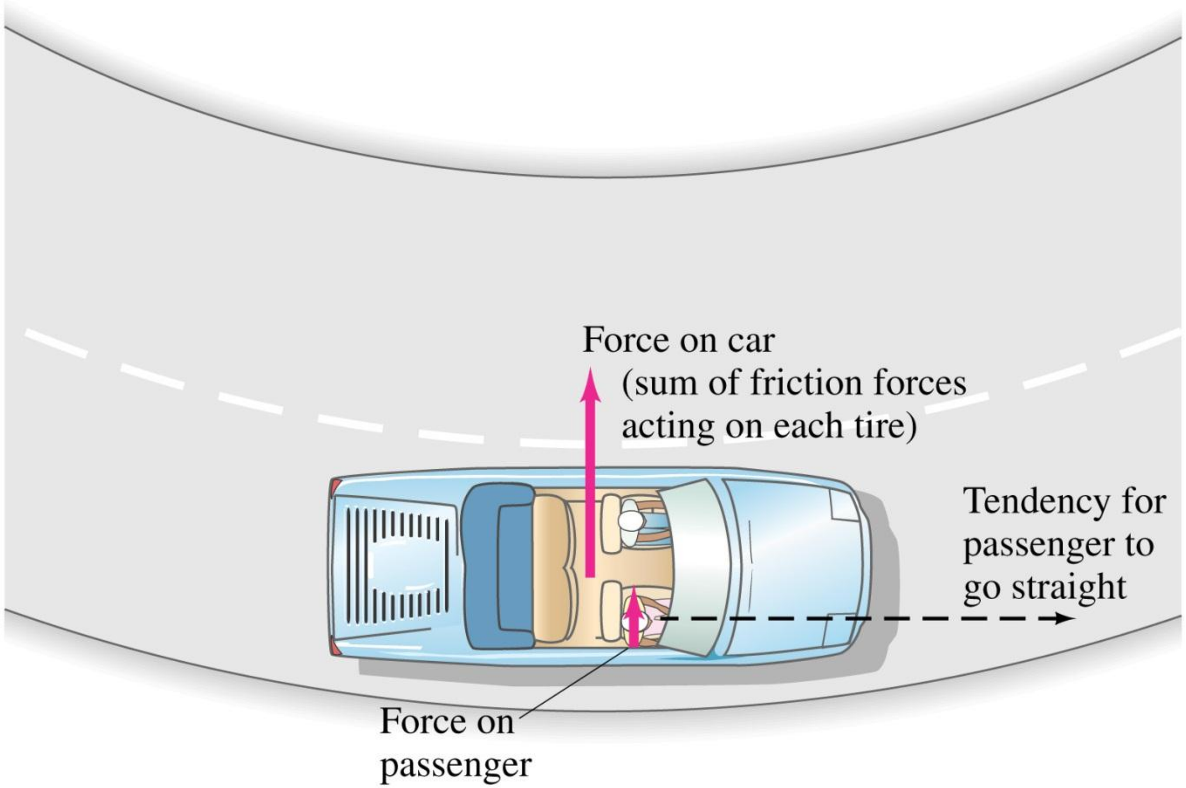
There is no **centrifugal** force pointing outward; what happens is that the natural tendency of the object to move in a straight line must be overcome.

If the centripetal force vanishes, the object flies off **tangent** to the circle.



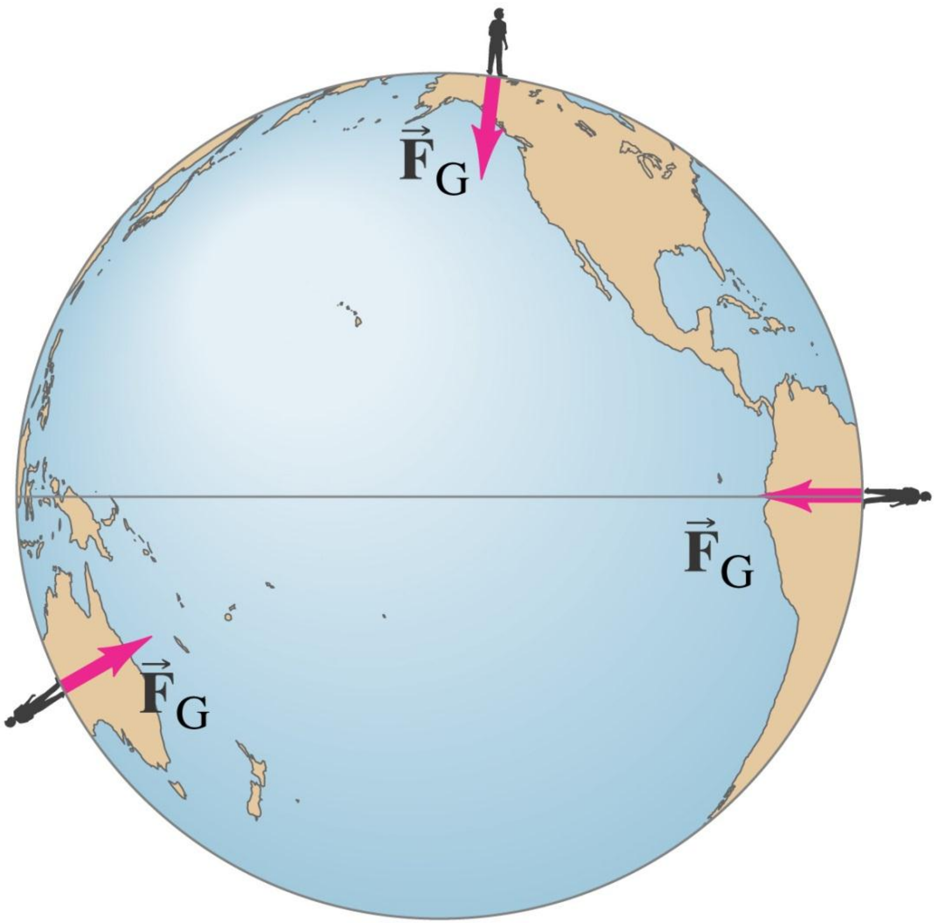
5-3 Highway Curves, Banked and Unbanked

When a car goes around a curve, there must be a net force towards the center of the circle of which the curve is an arc. If the road is flat, that force is supplied by friction.



5-6 Newton's Law of Universal Gravitation

If the force of gravity is being exerted on objects on Earth, what is the origin of that force?



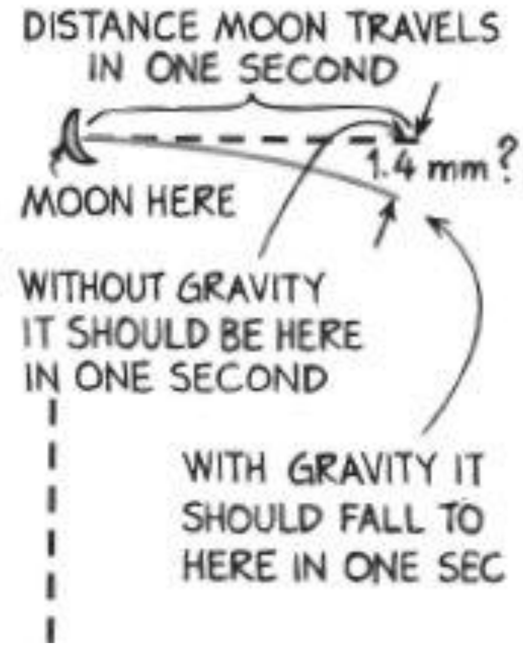
Newton's realization was that the force must come from the Earth.

He further realized that this force must be what keeps the Moon in its orbit.

Universal Gravitation and Orbits

- Newton reasoned that the **moon** is moving in a circular path around Earth (“falling toward Earth”) for the same reason an **apple** falls from a tree – they are both pulled by the Earth’s **gravity**.
- He hypothesized that the force of gravity was **universal**, and it was the key force for **planetary motion**.

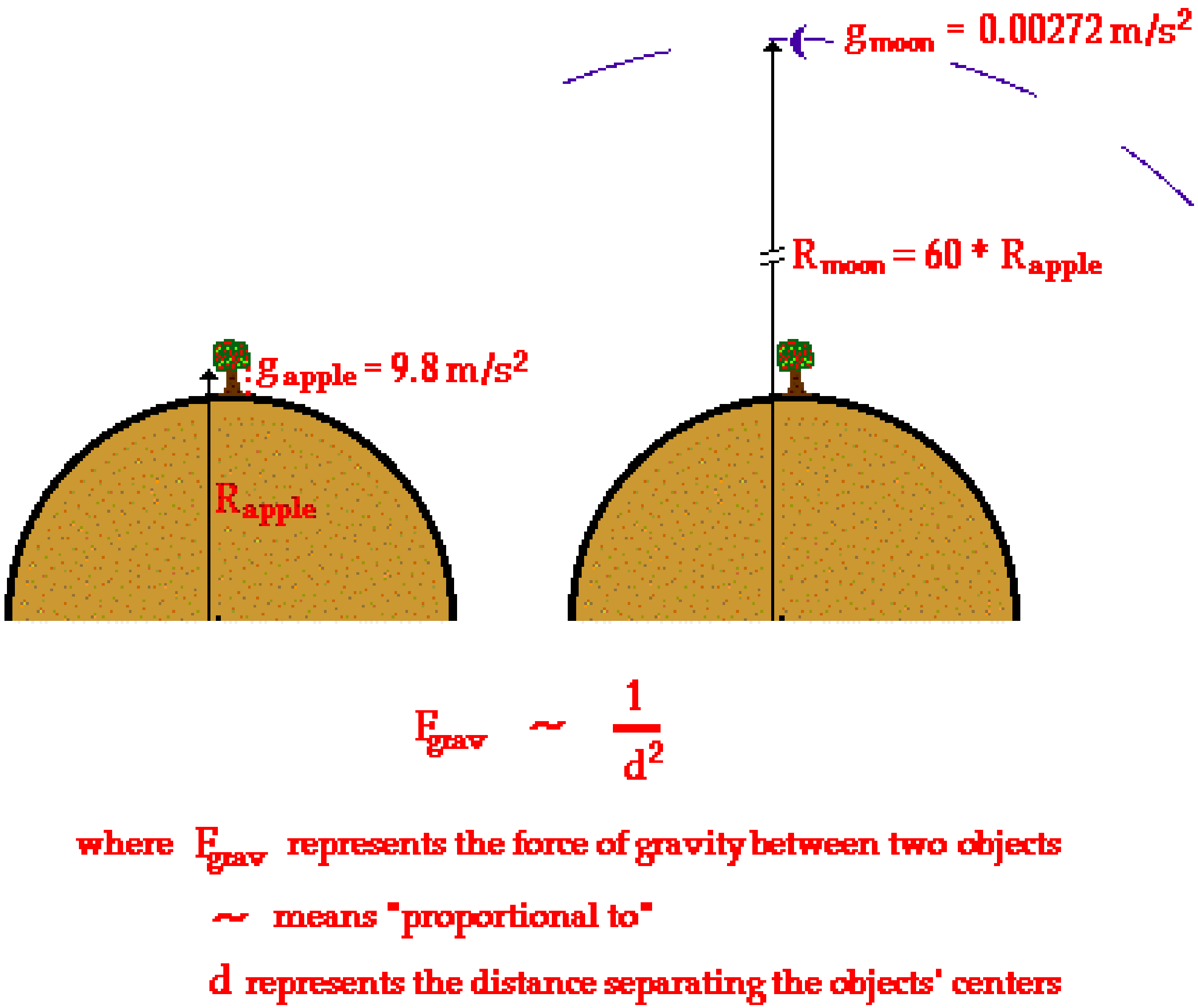
Gravitation and Distance



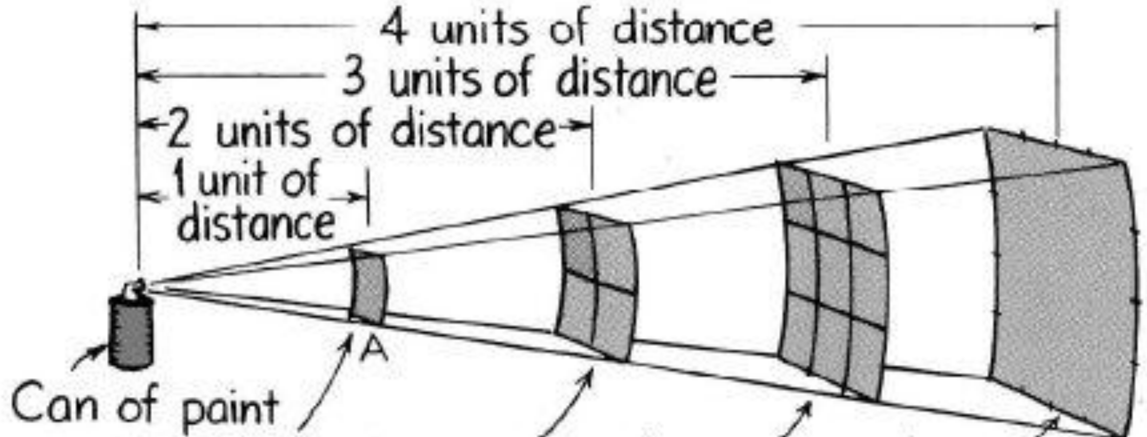
- Newton's goal was to show how the effect of gravity is **diluted with distance**.
- It was known at that time, that the force of gravity causes earthbound objects (such as falling apples) to accelerate towards the earth at a rate of **9.8 m/s^2.** And it was also known that the moon accelerated towards the earth at a rate of **0.00272 m/s^2**.

$$\frac{g_{moon}}{g_{apple}} = \frac{0.00272\ \mathrm{m/s^2}}{9.8\ \mathrm{m/s^2}} = \frac{1}{3600}$$

The Inverse Square Law



The moon, being 60 times further away than the apple, experiences a force of gravity that is $1/(60)^2$ times that of the apple. The force of gravity follows an **inverse square law**.

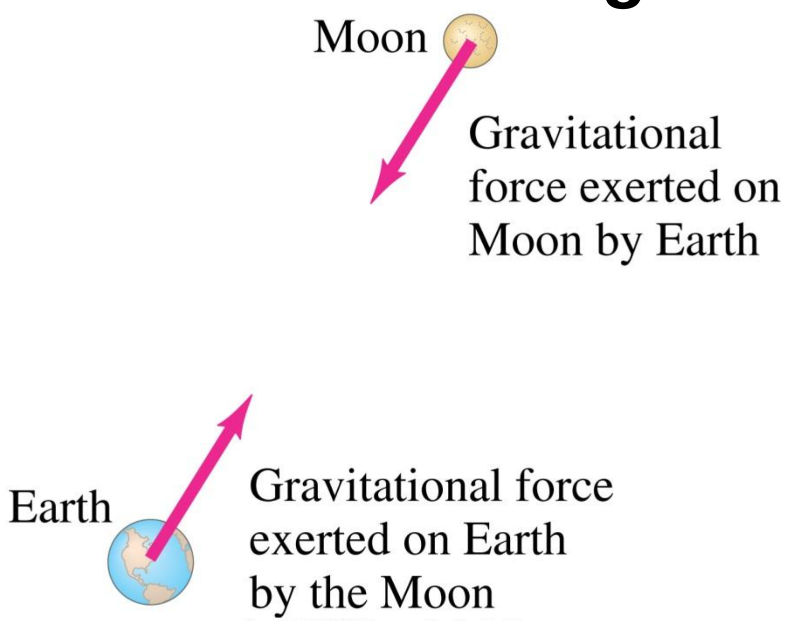


	1 area unit	4 area units	9 area units	16 area units
Paint spray	1 layer thick	¼ layer thick	1/9 layer thick	1/16 layer thick

5-6 Newton's Law of Universal Gravitation

The gravitational force on you is one-half of a Third Law pair: the **Earth** exerts a **downward** force on you, and you exert an **upward** force on the Earth.

When there is such a **disparity** in masses, the reaction force is undetectable, but for bodies more equal in mass it can be **significant**.



5-6 Newton's Law of Universal Gravitation

Therefore, the gravitational force must be proportional to **both** masses.

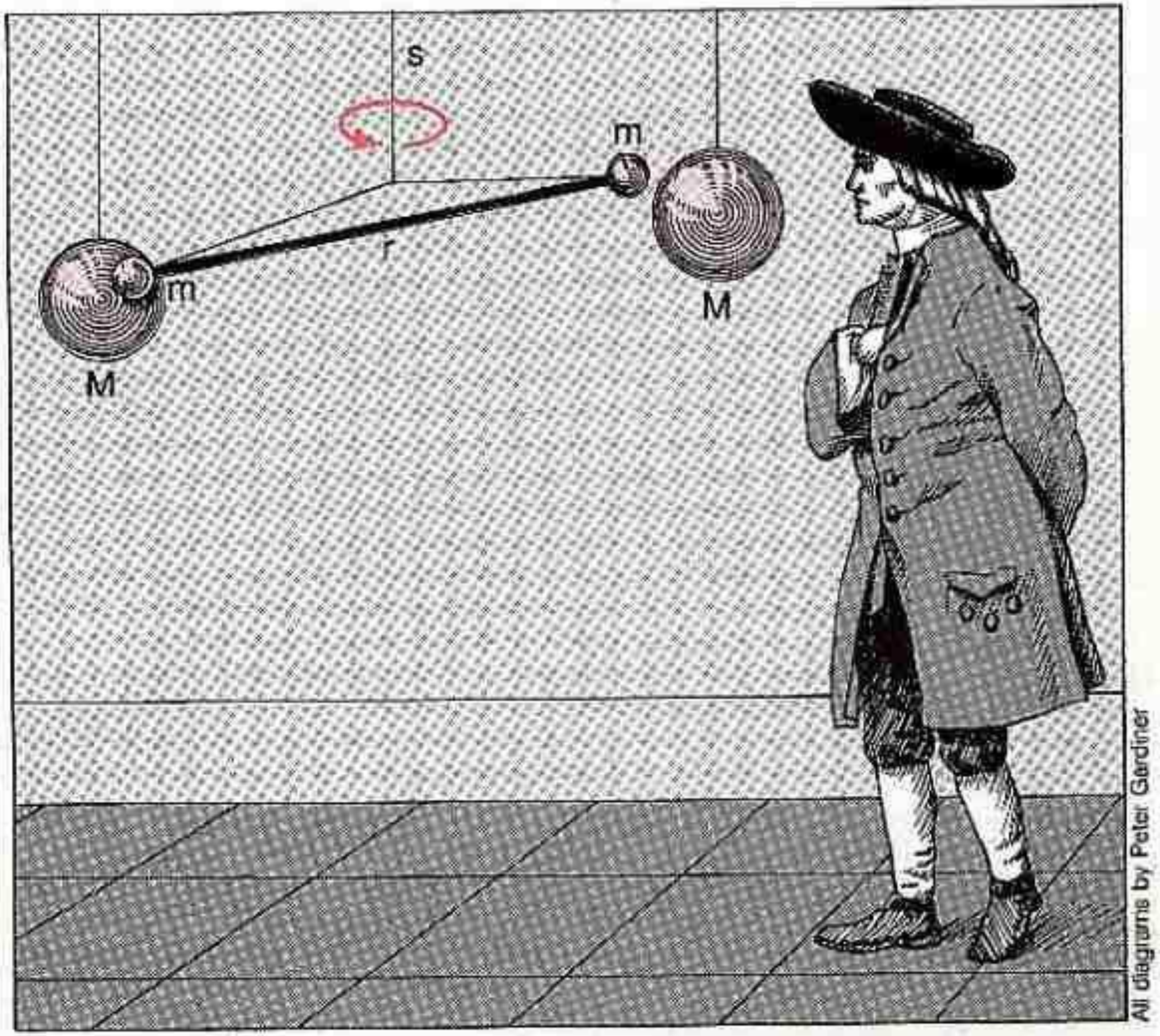
By observing planetary orbits, Newton also concluded that the gravitational force must decrease as the **inverse** of the **square** of the distance between the masses.

In its final form, the Law of Universal Gravitation reads:

$$F = G\frac{m_1 m_2}{r^2} \qquad (5\text{-}4)$$

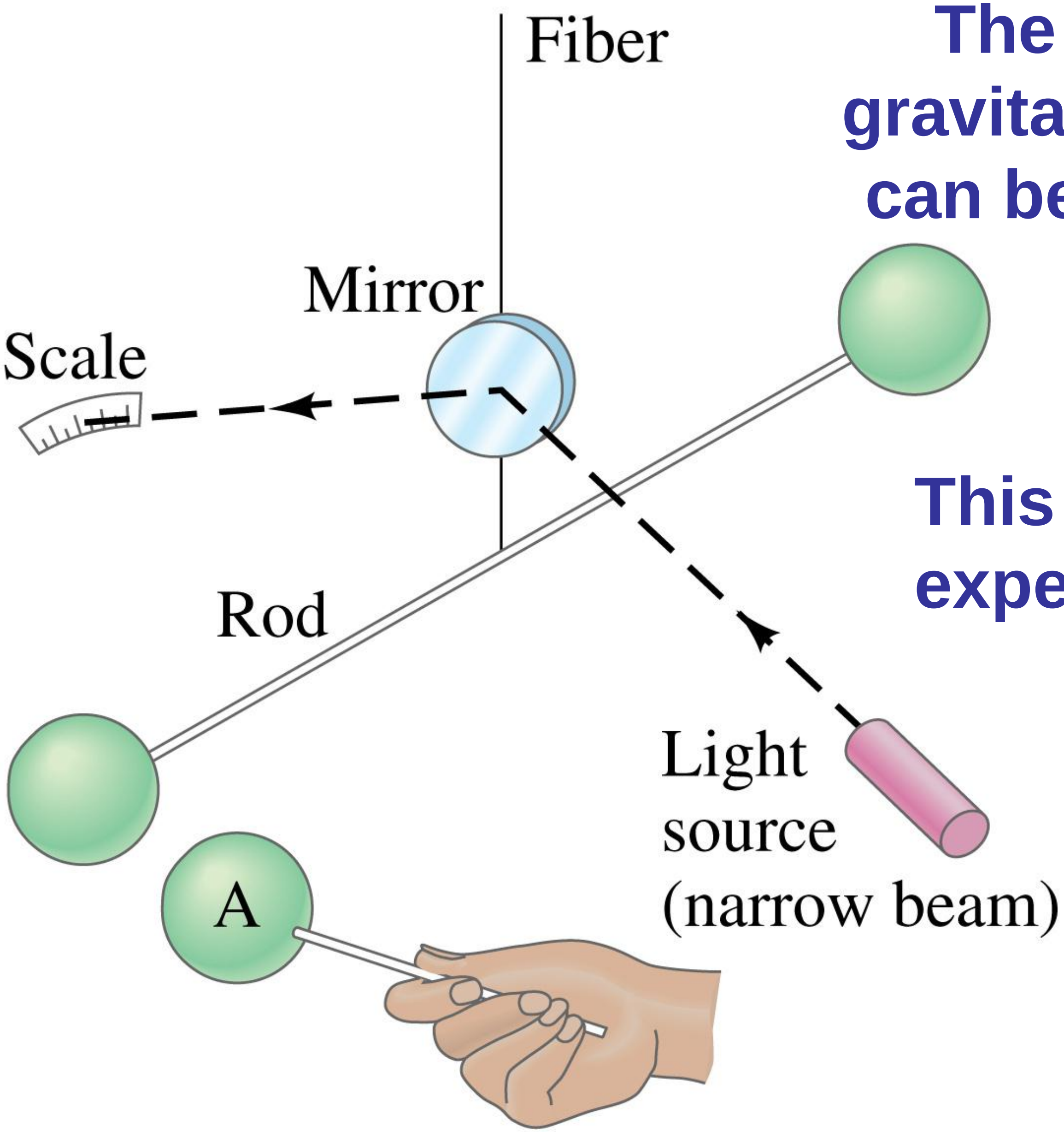
Where $G = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$

G – universal gravitation constant



Henry Cavendish with the famous torsion balance experiment that determined the gravitational constant G and demonstrated Newton's inverse-square law of gravitation. Large lead spheres placed close to small ones caused angular deflections

5-6 Newton's Law of Universal Gravitation



The **magnitude** of the gravitational constant *G* can be measured in the laboratory.

This is the **Cavendish** experiment.

5-7 Gravity Near the Earth's Surface; Geophysical Applications

Now we can relate the **gravitational constant** to the **local acceleration** of gravity. We know that, on the surface of the Earth:

$$mg = G\frac{mm_E}{r_E^2}$$

Solving for *g* gives:

$$g = G\frac{m_E}{r_E^2} \qquad \textbf{(5-5)}$$

Now, knowing *g* and the radius of the Earth, the mass of the Earth can be calculated:

$$m_E = \frac{gr_E^2}{G} = \frac{(9.80\ \mathrm{m/s^2})(6.38 \times 10^6\ \mathrm{m})^2}{6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}} = 5.98 \times 10^{24}$$

5-7 Gravity Near the Earth's Surface; Geophysical Applications

TABLE 5–1
Acceleration Due to Gravity at Various Locations on Earth

Location	Elevation (m)	g (m/s^2)
New York	0	9.803
San Francisco	0	9.800
Denver	1650	9.796
Pikes Peak	4300	9.789
Sydney, Australia	0	9.798
Equator	0	9.780
North Pole (calculated)	0	9.832

The acceleration due to gravity varies over the Earth's surface due to altitude, local geology, and the shape of the Earth, which is not quite spherical.

Exercise

Can you attract another person gravitationally?

A 50-kg person and 75-kg person are sitting on a bench Estimate the magnitude of the gravitational force each person exerts on the other. r=0.5m Let $G = 10^{-10}\, Nm^2 / kg^2$

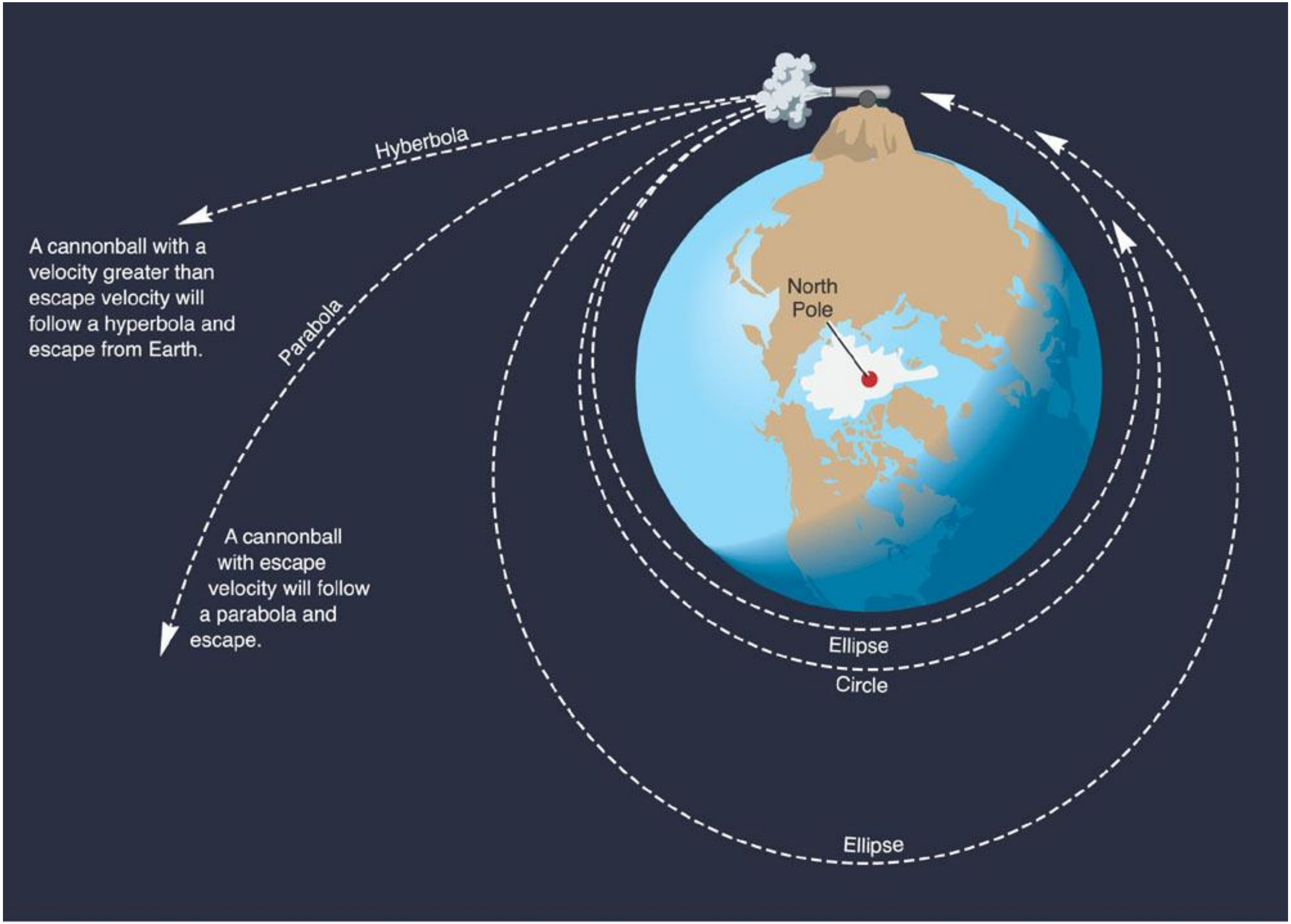
$$F = G\frac{m_1 m_2}{r^2} = \frac{10^{-10}(50)(75)}{(0.5)^2} = 10^{-6}\, N$$

Newton's Cannonball

- To explain orbits, Newton developed a "thought experiment". Newton visualizes a cannon on top of a very high mountain (above the atmosphere).
- If there were **no forces of gravitation** or **air resistance**, then the cannonball should follow **a straight line** away from Earth.
- If a **gravitational force acts** on the cannonball, it will follow a different path depending on its **initial velocity**.
- If the **speed is low**, it will simply **fall back** on Earth. (A and B)

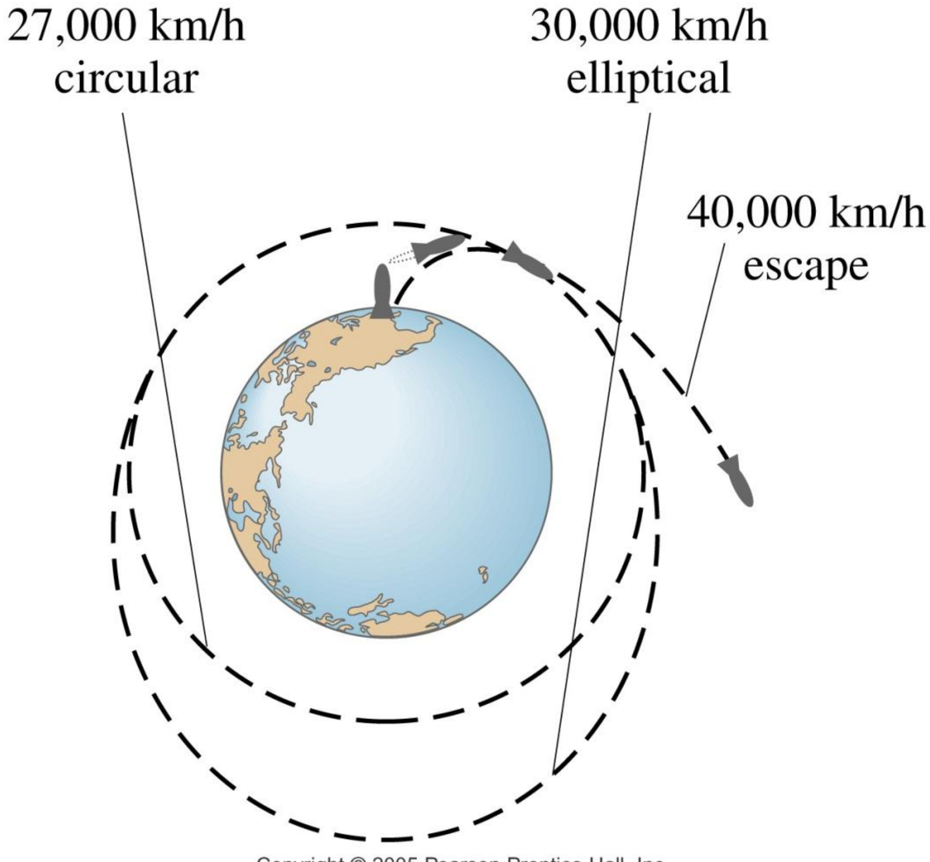
- If the speed is the **orbital velocity** at that altitude it will go on **circling around** the Earth along a fixed circular orbit just like the moon. (C)
- If the speed is **higher than the orbital velocity**, but not high enough to leave Earth altogether (**lower than the escape velocity**) it will continue revolving around Earth along an **elliptical orbit**.(D)
- If the speed is very high, it will indeed leave Earth.

- Newton's Cannonball

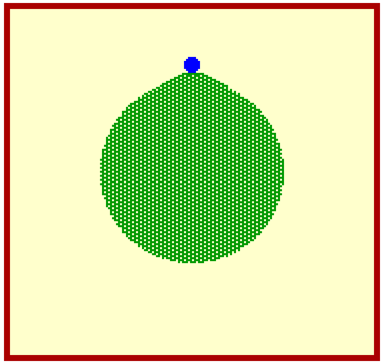

Hyberbola
A cannonball with a velocity greater than escape velocity will follow a hyperbola and escape from Earth.
Parabola
A cannonball with escape velocity will follow a parabola and escape.
North Pole
Ellipse
Circle
Ellipse

5-8 Satellites and "Weightlessness"

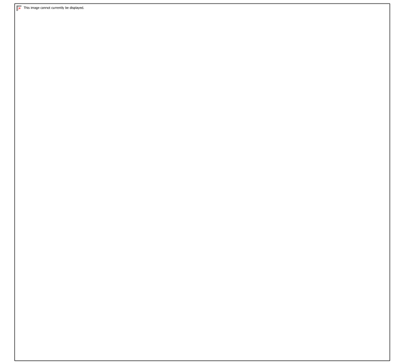
Satellites are routinely put into orbit around the Earth. The tangential speed must be high enough so that the satellite does not return to Earth, but not so high that it escapes Earth's gravity altogether.



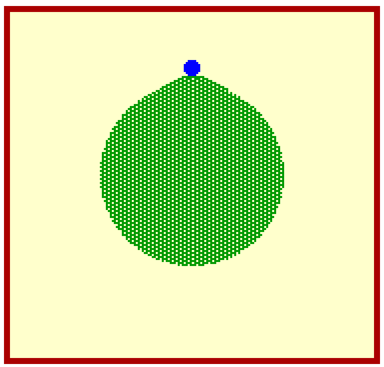
Satellite Motion



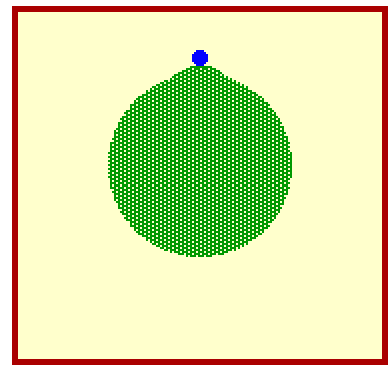
Launch Speed less than 8000 m/s
Projectile falls to Earth



Launch Speed less than 8000 m/s
Projectile falls to Earth



Launch Speed equal to 8000 m/s
Projectile orbits Earth - Circular Path

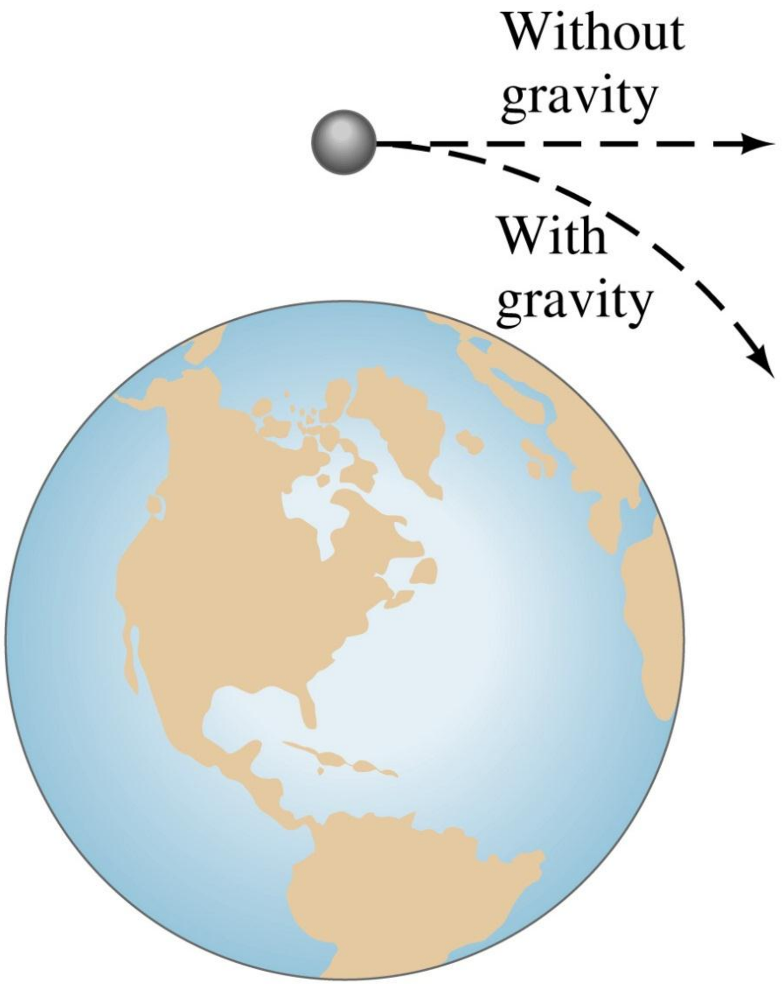


Launch Speed greater than 8000 m/s
Projectile orbits Earth - Elliptical Path

The same force that causes objects on Earth to fall to the earth also causes objects in the heavens to move along their circular and elliptical paths.

5-8 Satellites and "Weightlessness"

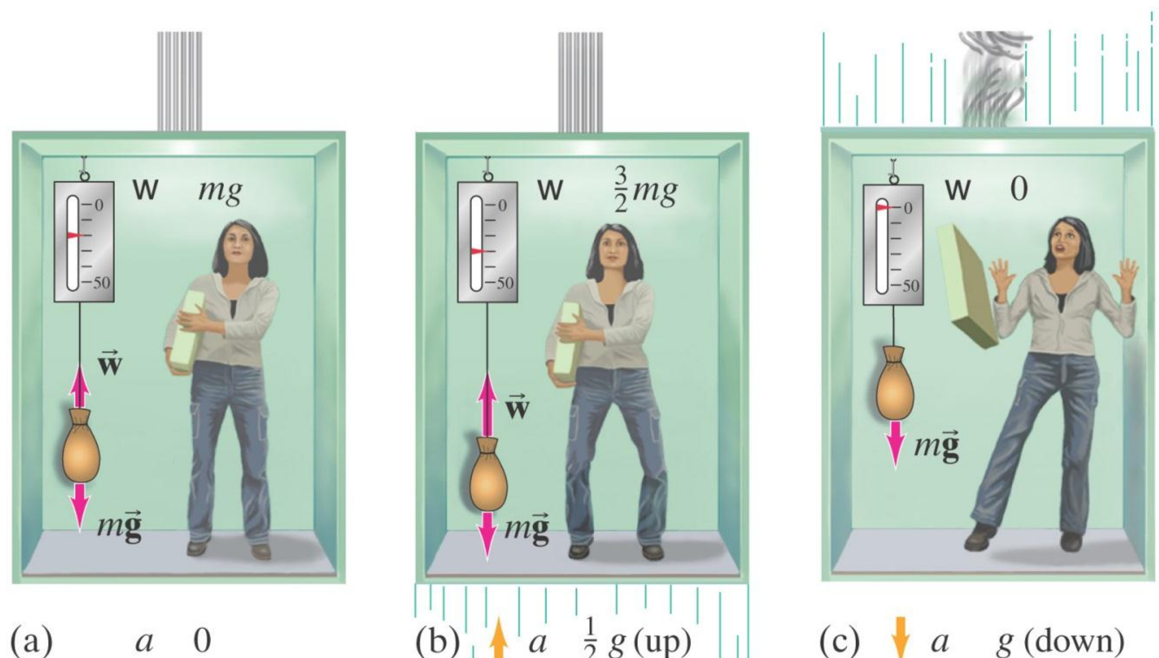
The satellite is kept in orbit by its speed – it is continually falling, but the Earth curves from underneath it.



5-8 Satellites and "Weightlessness"

Objects in orbit are said to experience **weightlessness**. They do have a gravitational force acting on them, though!

The satellite and all its contents are in **free fall**, so there is no **normal force**. This is what leads to the experience of weightlessness.



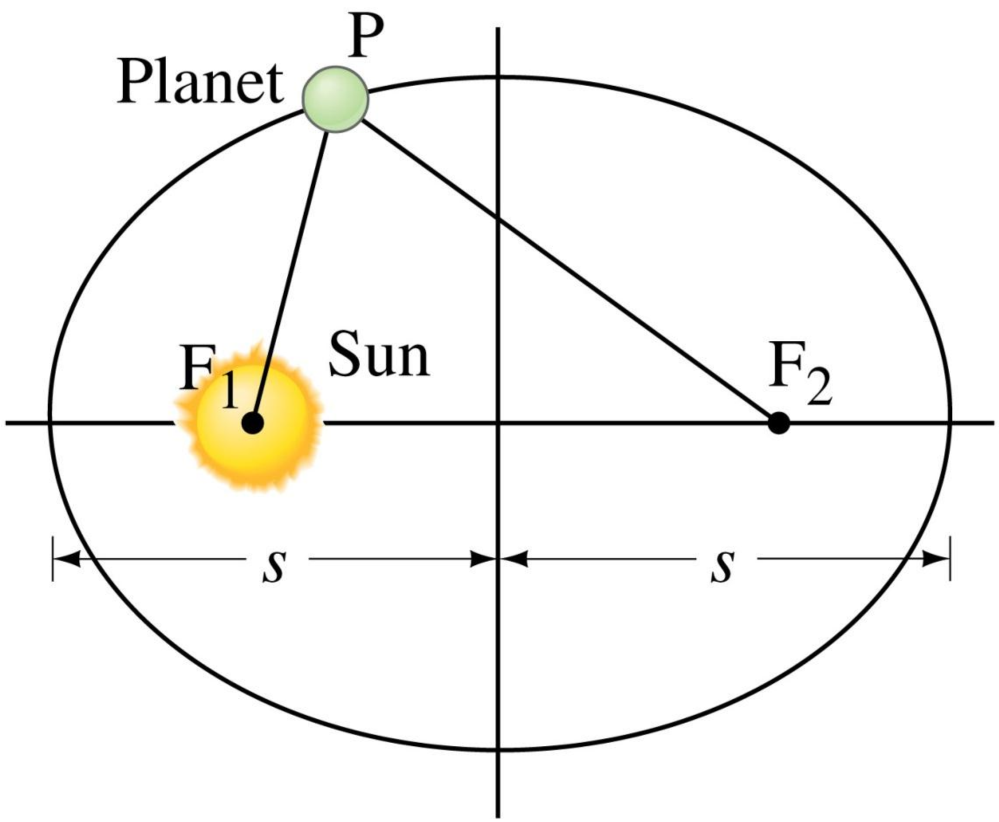
5-8 Satellites and "Weightlessness"

More properly, this effect is called **apparent weightlessness**, because the gravitational force still exists. It can be experienced on Earth as well, but only briefly:

5-9 Kepler's Laws and Newton's Synthesis

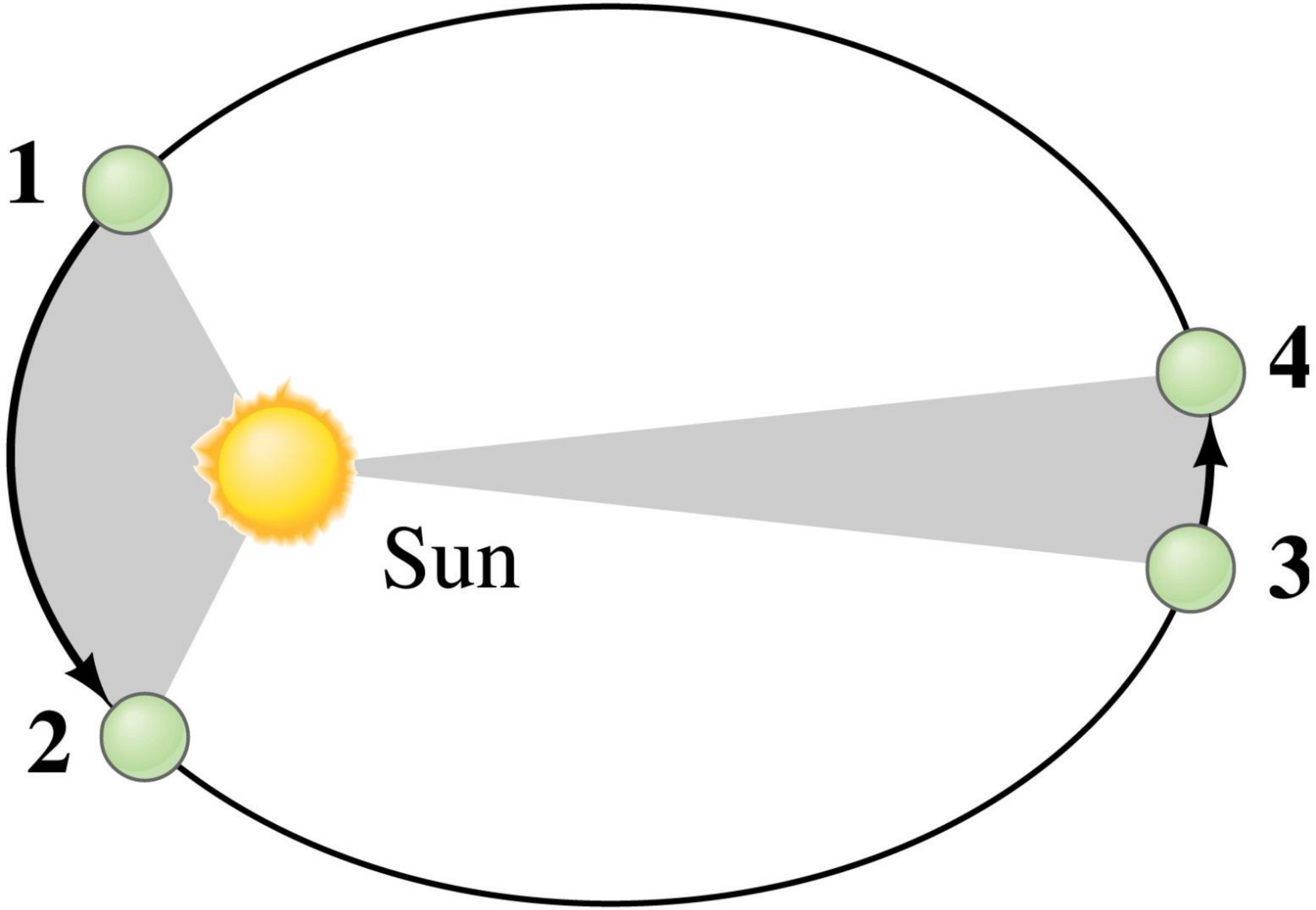
Kepler's laws describe planetary motion.

1. **The orbit of each planet is an ellipse, with the Sun at one focus.**



5-9 Kepler's Laws and Newton's Synthesis

2. An imaginary line drawn from each planet to the Sun sweeps out **equal areas** in **equal times**.



5-9 Kepler's Laws and Newton's Synthesis

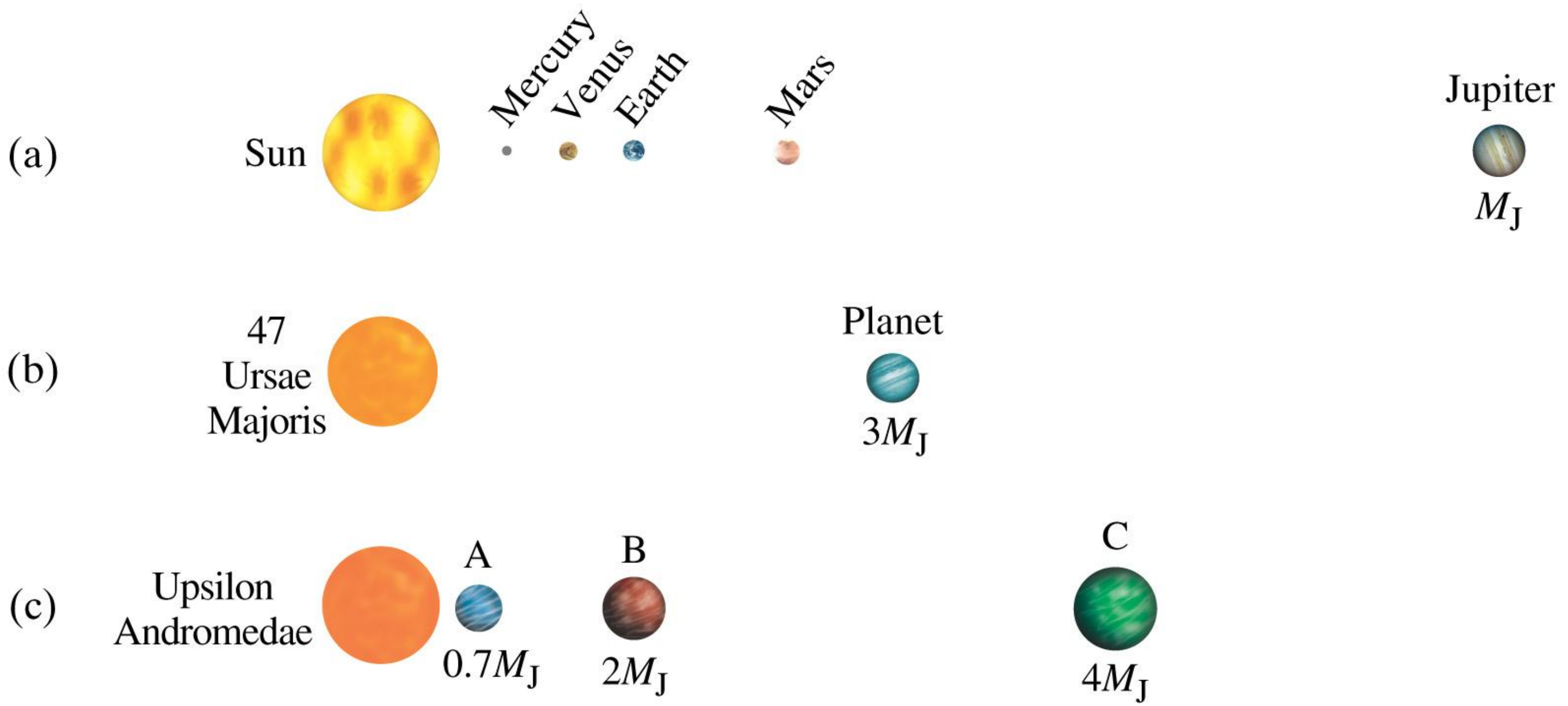
The ratio of the square of a planet's orbital period is proportional to the cube of its mean distance from the Sun.

TABLE 5–2 Planetary Data Applied to Kepler's Third Law

Planet	Mean Distance from Sun, s (10^6 km)	Period, T (Earth years)	s^3/T^2 (10^{24} km^3/y^2)
Mercury	57.9	0.241	3.34
Venus	108.2	0.615	3.35
Earth	149.6	1.0	3.35
Mars	227.9	1.88	3.35
Jupiter	778.3	11.86	3.35
Saturn	1427	29.5	3.34
Uranus	2870	84.0	3.35
Neptune	4497	165	3.34
Pluto	5900	248	3.34

5-9 Kepler's Laws and Newton's Synthesis

Kepler's laws can be derived from **Newton's laws**. Irregularities in planetary motion led to the discovery of **Neptune**, and irregularities in stellar motion have led to the discovery of many planets **outside** our Solar System.



5-10 Types of Forces in Nature

Modern physics now recognizes four fundamental forces:

1. **Gravity**
2. **Electromagnetism**
3. **Weak nuclear force** (responsible for some types of radioactive decay)
4. **Strong nuclear force** (binds protons and neutrons together in the nucleus)

5-10 Types of Forces in Nature

So, what about friction, the normal force, tension, and so on?

Except for gravity, the forces we experience every day are due to electromagnetic forces acting at the atomic level.

Summary of Chapter 5

• An object moving in a circle at constant speed is in uniform circular motion.

• It has a centripetal acceleration $$a_R = \frac{v^2}{r}$$

• There is a centripetal force given by

$$\Sigma F_R = ma_R = m\frac{v^2}{r}$$

•The centripetal force may be provided by friction, gravity, tension, the normal force, or others.

Summary of Chapter 5

- **Newton's law of universal gravitation:**

$$F = G\frac{m_1 m_2}{r^2}$$

- **Satellites are able to stay in Earth orbit because of their large tangential speed.**

Check Your Understanding:

- 1. Suppose that two objects attract each other with a gravitational force of 16 units. If the distance between the two objects is doubled, what is the new force of attraction between the two objects?
- Answer: If the distance is increased by a factor of 2, then force will be decreased by a factor of 4 (2^2). The new force is then 1/4 of the original 16 units.
- F = (16 N) / 4 = 4 units

- 2. Suppose that two objects attract each other with a gravitational force of 16 units. If the distance between the two objects is reduced in half, then what is the new force of attraction between the two objects?
- Answer: If the distance is decreased by a factor of 2, then force will be increased by a factor of 4 (2^2). The new force is then 4 times the original 16 units.
- F = (16 N) • 4 = 64 units

Exercise 1

Calculate the Earth's mass, if you know that the universal gravitational constant is

$$G = 6.67 \times 10^{-11} Nm^2 / kg^2$$

W=mg

Let the object on the Earth surface has a mass of 1 kg, then the weight of the object is

W = 1kg(9.8m/s/s)=9.8N

The gravitational force on the object exerted by Earth is

$$F_g = G\frac{m_1 m_2}{d^2}$$

Where m_2 = 1kg

m_1 is the mass of the Earth

$d = 6.4\times10^6 m$ radius of the Earth

W= F_g

mg= $G\frac{m_1 m_2}{d^2}$

$$9.8N = 6.67\times10^{-11}\,Nm^2/kg^2 \times \frac{1kg\times m_2}{\left(6.4\times10^6\,m\right)^2}$$

$$m_2 = 6\times10^{24}\,kg$$

Exercise 2

Can you attract another person gravitationally?

A 50-kg person and 75-kg person are sitting on a bench Estimate the magnitude of the gravitational force each person exerts on the other. r=0.5m Let $G = 10^{-10} Nm^2 / kg^2$

$$F = G\frac{m_1 m_2}{r^2} = \frac{10^{-10}(50)(75)}{(0.5)^2} = 10^{-6} N$$

Questions:

1. Why did Newton think that a force must act on the moon?
2. What did Newton conclude about the force that pulls apples to the ground and the force that pulls the moon in orbit?
3. If the moon falls, why does not if get closer to Earth?
4. How did Newton check his hypothesis that there is an attractive force between Earth and the moon?

5. What did Newton discover about gravity?

6. What does the small value of gravitational constant tell us about the strength of gravitational forces?

7. What are two masses and one distance that determine your weight?

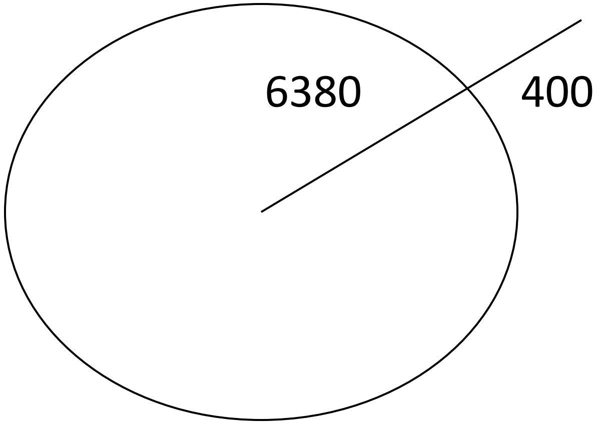
8. What be the difference in your weight if you were

a) five times father away from the Center of Earth than you are now?

b) ten times farther?

Do Now

- For a typical mission, the shuttle orbits Earth at altitude about of 400 km.
- The radius of Earth is 6380 km.
- Calculate the gravitational force on an astronaut in the shuttle, whose mass is 80 kg.



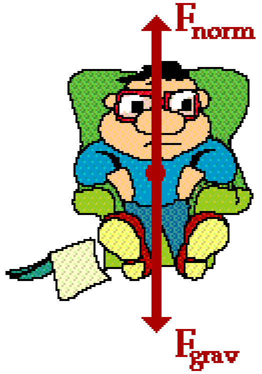
WEIGHTLESSNESS AND FREE FALL

Weightlessness

- Astronauts who are orbiting the Earth often experience sensations of weightlessness.
- These sensations experienced by orbiting astronauts are the same sensations experienced by anyone who has been temporarily suspended above the seat on an amusement park ride.

Forces

- There are two categories of forces - **contact forces** and **action-at-a-distance forces**.
- **Contact forces** can only result from the actual touching of the two interacting objects. As you sit in a chair, you experience two forces – the force of the Earth's gravitational field pulling you downward toward the Earth and the force of the chair pushing you upward.



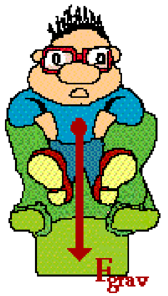
As you sit at rest in your chair, you feel the contact force (F_{norm}) balancing the non-contact force (F_{grav}).

Action-at-a-Distance Force

- Action-at-a-distance force .
  The force of gravity is the result of your center of mass and the Earth's center of mass exerting a mutual pull on each other; this force would even exist if you were not in contact with the Earth.
- The force of gravity can never be felt.

Meaning of Weightlessness

- **Weightlessness** is simply a sensation experienced by an individual when there are no external objects touching one's body and exerting a push or pull upon it.
- Weightless sensations exist when all contact forces are removed.
- Free fall gives you such a sensation.
- Weightlessness is only a sensation; it is not a reality corresponding to an individual who has lost weight.



A person in free fall does not experience any contact force and thus feels weightless.

Scale Reading and Weight

- While we use a scale to measure one's weight, the scale reading is actually a measure of the upward force applied by the scale to balance the downward force of gravity acting upon an object.
- The SCALE DOES NOT MEASURE YOUR WEIGHT. The scale is only measuring the external contact force that is being applied to your body.

Floating in Space

- So what does it mean to say that something is weightless in orbit?
- When you stand on a scale you are at rest and the net force on you is zero.
- The scale supports you and balances your weight by exerting an upward force.

Floating in Space

- The dial on the scale shows the upward force exerted by the scale, which is your weight.
- Now suppose you stand on the scale in an elevator that is falling.
- If you and the scale were in free fall, then you no longer would push down on the scale at all.
- The scale dial would say you have zero weight, even though the force of gravity on you hasn't changed.

Floating in Space

- A space shuttle in orbit is in free fall, but it is falling around Earth, rather than straight downward.
- Everything in the orbiting space shuttle is falling around Earth at the same rate, in the same way you and the scale were falling in the elevator.
- Objects in the shuttle seem to be floating because they are all falling with the same acceleration.

Do Now

- ***True or False?***

Astronauts on the orbiting space station are *weightless* because...

a)there is no gravity in space and they do not weigh anything.

b) space is a vacuum and there is no gravity in a vacuum.

c) the astronauts are far from Earth's surface at a location where gravitation has a minimal effect.

- Astronauts on the orbiting space station are *weightless* because...

Do Now

- What would be the force of attraction between the Earth and the moon if the distance between them were halved?