Unit 7 NOTES – Law of Sines and Law of Cosines

Essential Question:

1. *How can trigonometry be used in triangles that are "oblique" (not right triangles)?*
2. *How can the Law of Sines and Law of Cosines be applied to Aircraft Navigation?*

GPS Standard

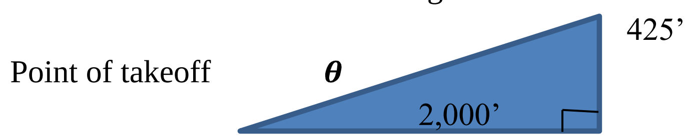
- MM4A6c – Law of Sines
- MM4A6c – Law of Cosines
- MM4A8a – Students will find the values of inverse sine and inverse cosine functions using technology.

Introduction

Anticipatory Set:

A plane takes off at an unknown angle of elevation. When the plane is directly over a landmark 2,000 feet from the point of takeoff, the plane is approximately 425 feet in the air.

Anticipated model resembles the following:



Anticipated equation is: $\tan\theta = \frac{425\prime}{2000\prime}$.

Remember that inverse trigonometric functions are used to isolate the variable, and if we take the tangent-inverse of both sides, we would have: $\theta = tan^{-1}\frac{425\prime}{2000\prime} \approx 12° \; angle\ of\ elevation$ (using a scientific calculator). NOTE: degree mode!!

There are two new formulas, Laws of Sines and Laws of Cosines, which allow us to utilize trigonometry in non-right, or "oblique" triangles.

Many (or most) real-life scenarios cannot be modeled exclusively with right-triangles; thus, Laws of Sines and Laws of Cosines are a way of finding missing angles or sides (distances) in many real-life situations.

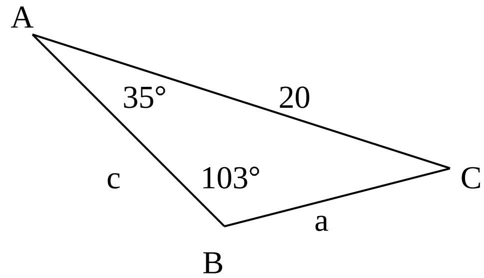
LESSON:

Draw and label ΔABC so that it is visibly NOT a right triangle. Label the sides a, b, and c as follows: side a opposite of angle A, side b opposite of angle B, and side c opposite of angle C.

The Law of Sines recognizes that there is a relationship, or common ratio, between the sine of an angle and the length of the side opposite of that angle.

Law of Sines: If a, b, and c represents the lengths of sides opposite of angles A, B, and C respectively, then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Example:



Using the Law of Sines, the following ratios must be equal:

$$\frac{\sin 35°}{a} = \frac{\sin 103°}{20} = \frac{\sin C}{c}$$

So, $\frac{\sin 103°}{20} \approx 0.0487$, then $\frac{\sin 35°}{a} \approx 0.0487$ and a = $\frac{\sin 35°}{0.0487}$ $or\ a \approx 11.77$

To find c, we need to first recall that the sum of the angles in a triangle is 180°, so $C\ = 180 - \ 35 + 103 \quad or\ C =\ 42°$. Then we plug C into the Law of Sines:

So, $\frac{\sin 42°}{c} \approx 0.0487$ and c = $\frac{\sin 42°}{0.0487}$ $or\ c \approx 13.74$

Law of Cosines recognizes that there is a relationship between the length of two sides and the cosine of the angle between them ("included angle").

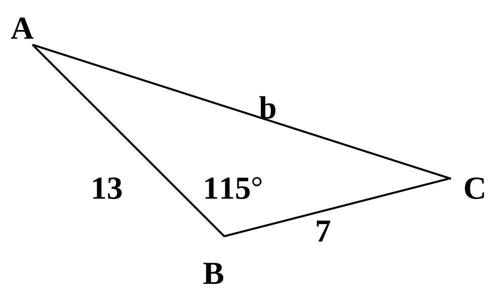
Law of Cosines: If a, b, and c represents the lengths of sides opposite of angles A, B, and C respectively, then the following are true:

$$\boldsymbol{a^2 = b^2 + c^2 - 2bc\cos A}$$

$$\boldsymbol{b^2 = a^2 + c^2 - 2ac\cos B}$$

$$\boldsymbol{c^2 = a^2 + b^2 - 2ab\cos C}$$

Example:



Using the Law of Cosines, $\boldsymbol{b^2 = 7^2 + 13^2 - 2(7)(13)\cos 115°}$

$$So,\ b^2 \approx 294.9165\ \ or\ \ b \approx 17.17$$

We could also determine A using:

$$7^2 = 13^2 + 17.17^2 - 2(13)(17.17)\cos A$$

$$\text{So, } -414.8089 = -2(13)(17.17)\cos A$$

$$0.9292 = \cos A$$

$$cos^{-1}0.9292 = A\ \ or\ A \approx 21.69°$$

…and C using:

$$13^2 = 7^2 + 17.17^2 - 2(7)(17.17)\cos C$$

$$\text{So, } -174.8089 = -2(7)(17.17)\cos C$$

$$0.7272 = \cos C$$

$$cos^{-1}0.7272 = C\ \ or\ C \approx 43.34°$$

SUGGESTION: Always verify that A+B+C = 180°

The Law of Sines and the Law of Cosines have many real-life applications in scenarios which are modeled with oblique triangles. Here is a video clip of these two Laws being used in Aircraft Navigation: http://player.discoveryeducation.com/index.cfm?guidAssetId=1899309E-1147-400E-8722-107A0B7E510E&blnFromSearch=1&productcode=US *NOTE: discoveryeducation.com requires a username and password, but many schools have a free code to subscribe.*

Check for Understanding (from the GaDOE Frameworks for Math IV)**:** Students will be asked to partner with the 1-2 other students. Each group will be given the following scenario, which will be shown on the Promethean ActivBoard, and asked to complete the items:

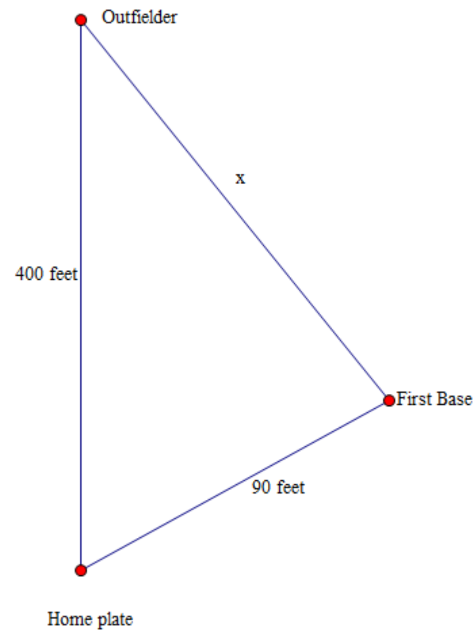
Scenario A: During a baseball game an outfielder caught a ball hit to dead center field, 400 feet from home plate. If the distance from home base to first base is 90 feet, how far does the outfielder have to throw the ball to get it to first base?

Items to complete:

1.) Utilize one individual white board for modeling the scenario above. *Hold up the whiteboard so that the instructor can verify drawing before proceeding.*
2.) Utilize a second individual white board for writing the Law of Cosines from the given information.
3.) Solve for c. *Hold up the whiteboard so that the instructor can confirm solution.*

SOLUTION:

In this problem a = 90, b= 400 and C=45°as shown to the right.



Using the Law of Cosines:

$c^2 = a^2 + b^2 - 2abcosC$

$c^2 = 90^2 + 400^2 - 2\ 90\ \ 400\ cos45$

$c \approx 342$

The distance is about 342 feet from dead centerfield to 1st base.

Guided Independent Practice (from the GaDOE Frameworks for Math IV)**:** Students will now work individually with the following scenario and assignment, which will be shown on the Promethean ActivBoard:

Scenario B: A surveyor is near a river (at point "A") and wants to calculate the distance across the river ("a"). He measures the angle between his observations of two points on the shore, one on his side (point "B") and one on the other side (point "C"), to be 28º (m$< BAC = 28°$). The distance between him and the point on his side of the river can be measured and is 300 feet. The angle formed by him, the point on his side of the river, and the point directly on the opposite side of the river is 128º ($m < ABC = 128$). What is the distance across the river?

Items to complete individually:

4.) Utilize your individual white board to modeling and label the above scenario. *Hold up the whiteboard so that the instructor can verify drawing before proceeding.*
5.) On the whiteboard, now write the three ratios found by using the Law of Sines.
6.) Solve for a. *Hold up the whiteboard so that the instructor can confirm solution.*

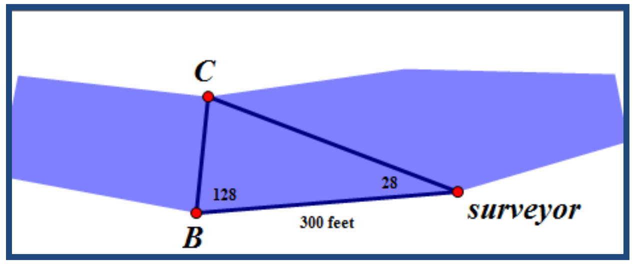
SOLUTION:

Angle C = 24°

$$\frac{300}{\sin 24} = \frac{a}{\sin 28}$$

$$\frac{300}{\sin 24} * \sin 28 = a$$

$$346.3\ feet = a$$



Independent Practice/Homework (from the textbook, pg. 293)**:** Students are expected to model the scenario with a picture, label known information, set up the appropriate equation using either Law of Sines or Law of Cosines, and solve for all missing sides and/or angles:

1.) An Earth-orbiting satellite is passing between the Oak Ridge Laboratory in Tennessee (A) and the Langley Research Center in Virginia (B), which are 46 miles apart. If the angles of elevation to the satellite from the Oak Ridge and Langley facilities are $58°\ and\ 72°$ respectively, how far is the satellite from each station?

2.) A triangular area of lawn has a sprinkler located at each vertex. If the sides of the lawn are a = 19', b = 24.3', and c = 21.8', what angle of sweep should each sprinkler be set to cover?

SOLUTIONS (pictures of scenarios will be placed on Teacher Page afterschool for students needing assistance):

1.) m<C = 50° ; $b \approx 57.11\ miles$; $a \approx 50.92\ miles$

2.) $m < A\ \approx\ 48.3°$; $m < B\ \approx\ 72.74°$; $m < C\ \approx\ 58.95°$

Closure: We often hear the shortest distance between two points is a straight line, but real-life often veers off path. Many of these situations involving distance can and have been modeled by triangles. Today we were reminded that sine and cosine, as well as sine-inverse and cosine-inverse are effective methods for finding unknown angle and side measures in triangles; however, we learned today that the Law of Sines and Law of Cosines allows us to apply these trigonometric functions to any triangle scenario, not just those directly modeled with a right triangle.

"Tomorrow we will revisit Area formulas for triangles ($A = \frac{1}{2}\ bh$) and learn how the Law of Cosines enables us to find areas of *non-right* triangles when the height is unknown—we will only need the lengths of all three sides."

Remediation:

For students who do not demonstrate sufficient understanding during the lesson and instead exhibit a need for remediation, the following sites may be helpful:

- Remediation for understanding Inverse Trigonometric Functions from a previous unit: http://www.intmath.com/analytic-trigonometry/7-inverse-trigo-functions.php
- A tutorial for determine when to use Law of Sines vs. Law of Cosines: http://www.mathwarehouse.com/trigonometry/law-of-sines-and-cosines.php

Enrichment:

For students who quickly master the concepts, an enrichment activity is provided here: http://www.regentsprep.org/Regents/math/algtrig/ATT12/derivelawofsines.htm. This site explains how the Law of Sines and the Law of Cosines were derived. Try to create an effective way for demonstrating same to the class tomorrow. NOTE: Additional information on the two Laws can be found in the textbook, pages 291-296.

****Handouts follow****

Independent Practice with Law of Sines and Law of Cosines

*****You will need to do the following on your own paper*****

i. Model with a picture and label all known information.
ii. Set up the appropriate equation using either Law of Sines or Law of Cosines
iii. Solve for all missing sides and/or angles

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<u>FFM 4/11/12:</u>

In $\Delta ABC, m\angle A = 22°, m\angle B = 113°, and\ a = 10$.
Use the Law of Sines to find all missing side lengths and angle measures.

(Assume a is the length of the side opposite $\angle A$, b is the length of the side opposite of $\angle B$, and c is the length of the side opposite of $\angle C$.)

Remember to keep 4 decimals in the middle of the problem,
but round your final answers to 2 decimal places.
Also, use appropriate symbols in your answers.

<u>FFM 4/12/12:</u>

During a baseball game an outfielder caught a ball hit to dead center field, 400 feet from home plate. If the distance from home base to first base is 90 feet, how far does the outfielder have to throw the ball to get it to first base?

1.) **First, work with your group to model the scenario on ONE whiteboard, and hold them up for me to verify BEFORE proceeding.**
2.) **Utilize a 2nd whiteboard to write the Law of Sines using the given information and a 3rd whiteboard to write the Law of Cosines using the given information.**
3.) **Determine which Law will help you solve for c.**
4.) **Solve for c, and show me your whiteboard when finished.** ***I will stamp each group member's FFM that understands and can explain. You may want to copy the model and solution on your FFM sheet, but you can just write "on whiteboard."***