Unit 8
Plane Geometry

Lesson Outline

*Note: This unit could stand alone and be placed anywhere in the course.

BIG PICTURE	

Students will:

- investigate properties of geometric objects using dynamic geometry software and manipulatives;
- illustrate and explain the relationship between angles formed by parallel lines cut by a transversal and interior and exterior angles of triangles and quadrilaterals;
- determine some properties of sides and diagonals of quadrilaterals.

Note: Students may have a very broad range of experience with using The Geometer's Sketchpad®4.
Skills can be taught as they are needed for each lesson, or alternatively, Introduction to The Geometer's Sketchpad®4 file (included in Day 2) could be used at the beginning of the unit.

Day	Lesson Title	Math Learning Goals	Expectations
1	It's a Parallel World	• Describe the properties and relationships of the angles formed by parallel lines cut by a transversal.	MG3.01, MG3.02 CGE 5a, 5c
2	Plane Geometry – Introduction (Part 1) *GSP®4 files:* **Plane Geometry, Introduction to Geometer's Sketchpad**	• Review angles, triangles, and parallel lines through exploration. • Build skills required for future use of The Geometer's Sketchpad®4 (GSP®4).	MG3.01, MG3.02 CGE 5a, 5e
3	Plane Geometry – Introduction (Part 2)	• Explore geometrical concepts (angles, triangles, parallel lines). • Build skills required for future use of GSP®4.	MG3.01, MG3.02 CGE 4b, 5b
4	What's So Special? (Part 1) GSP®4 file: **What's So Special?**	• Build investigation skills by exploring geometric concepts, using GSP®4. • Develop communication skills and geometric vocabulary.	MG3.01, MG3.02 CGE 2a, 5a
5	What's So Special? (Part 2)	• Build investigation skills by exploring geometric concepts, using GSP®4. • Develop communication skills and geometric vocabulary.	MG3.01, MG3.02 CGE 5a, 5b
6	Interior and Exterior Angles of Triangles and Quadrilaterals *GSP®4 files:* **Tutorial GSP, Sum of the Exterior Angles of a Triangle, Sum of the Interior Angles of a Polygon**	• Investigate the sum of the interior and exterior angles of triangles and quadrilaterals using GSP®4 and demonstration. • Develop skills with GSP®4 in preparation for summative assessments.	MG3.01, LR1.03, LR4.04 CGE 2a, 3c
7	Using the Properties: Connecting Algebra to Geometry	• Practise solving problems using the geometry explored in previous lessons. • Make connections to solving equations.	MG3.01, MG3.02, NA2.07, LR2.02, LR4.04 CGE 4b, 5a
8	Freaky Folds	• Use paper folding to illustrate geometric properties.	MG3.03 CGE 3c, 4b, 5g
9		Instructional Jazz	
10		Instructional Jazz	
11		Assessment presentation and/or test	

Math Learning Goals

- Describe the properties and relationships of the angles formed by parallel lines cut by a transversal.

75 min

Materials
- BLM 8.1.1, 8.1.2
- washable markers
- stickers
- butterfly fasteners
- protractors
- electronic presentation
- overheads or pictures of everyday examples of parallel lines
- scissors
- Acetate copies of BLM 8.1.1, 1 /pair
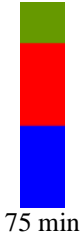
Assessment Opportunities

Minds On ... Small Groups → Brainstorm/Pass the Paper

In groups, students pass one piece of paper and write examples of everyday objects or situations that can be modelled using parallel lines. Examples could include fencing, ironing board top with the floor, any rectangular shape, railway tracks, etc. Record verbal responses from the groups, asking clarifying questions with respect to where the parallelism is. As a class, define parallel lines.

Have pictures, overheads, or an electronic presentation for followup to reinforce the possibilities.

Action! Pairs → Guided Exploration

Provide each pair with an acetate copy of BLM 8.1.1, washable markers, a protractor, and some stickers. Cut the acetate in half, and poke a hole through each piece on the dot. Fasten the pieces of acetate with butterfly fasteners through the dots. Explain that the images on the acetates should be superimposed and rotated to create parallel lines.

Students explore, record, and justify any angle relationships they observe on BLM 8.1.2. Students can use any of the tools (e.g., stickers, markers, protractors) to determine angle relationships.

Selecting Tools and Computation Strategies/Observation/Checklist: Circulate and observe the students as they choose their tools and strategies and as they investigate.

Teachers can differentiate for students by helping them select an appropriate tool.

Consolidate Debrief Whole Class → Summarizing

Record and consolidate the relationships established by student groups. Establish that the transversal is an essential condition for angles with parallel lines. Ensure that students recognize all the possible variations of each relationship, i.e., alternate (Z), corresponding (F), co-interior (C). Students make summarizing notes.

Students must understand that equal corresponding or alternate angles only occur with parallel lines.

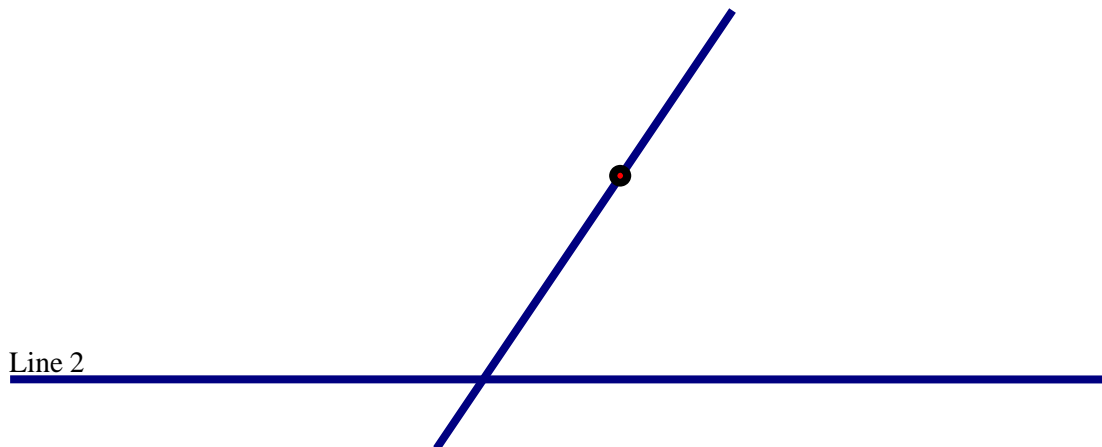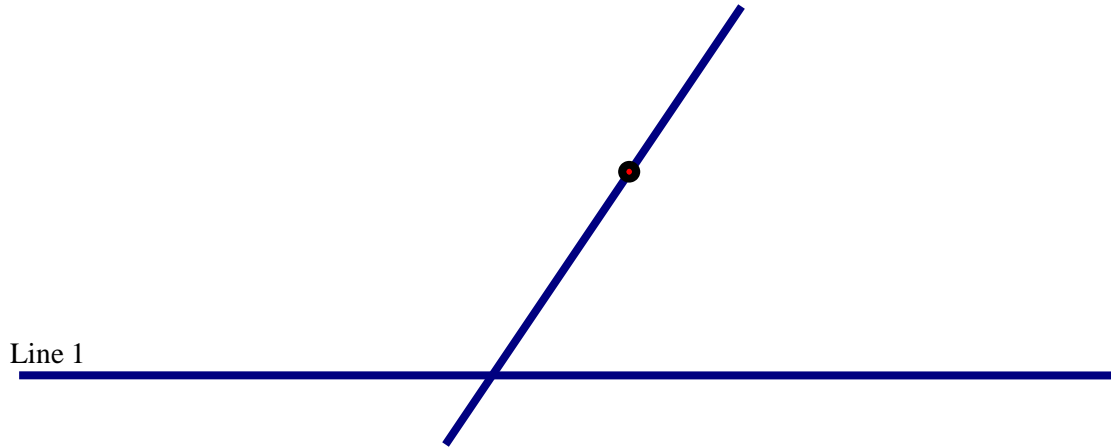### Home Activity or Further Classroom Consolidation

Application Concept Practice

Identify the types of angles found in the examples used in **Minds On**. Justify your answers.

Complete the practice questions.

Provide appropriate practice questions for students.

8.1.1: Parallel Lines Exploration

Line 1

Line 2

8.1.2: Parallel Lines Exploration

Explore and Reflect	Sketch
1. How did you know when Line 1 and Line 2 were parallel?	
Angle Relationships	
2. Find one pair of equal angles. Explain how you know they are equal.	
3. Find another pair of equal angles. Explain how you know they are equal.	
4. Find as many pairs of angles that are supplementary (add to 180°) as you can. Explain how you know.	
Summary (to be completed as a whole class)	

75 min
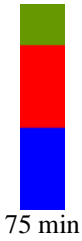
Math Learning Goals

- Review angles, triangles, and parallel lines through exploration.
- Build skills required for future use of The Geometer's Sketchpad®4 (GSP).

Materials

- BLM 8.2.1, 8.2.2, 8.2.3
- BLM 8.2.4 (Teacher)
- chart paper
- Computer with data projector
- Computer with Geometers Sketchpad 4, 1/ pair

Assessment Opportunities

Minds On ...

Small Groups → Brainstorm

Curriculum Expectations/Observation/Mental Note: Circulate while students are working to assess prior learning diagnostically.

For the Graffiti activity, post sheets of chart paper with the following topic titles: Triangles, Quadrilaterals, Polygons, Angles, Lines/Line Segments/Rays. Instruct groups to record prior learning on one of the concept sheets. Students might draw sketches, write definitions, state properties, etc. Groups rotate so that each group visits each topic sheet and adds information.

Whole Group → Demonstration

Demonstrate how to navigate through the GSP®4 file Plane Geometry using the How Do I...? feature and the GSP®4 Notes Booklet (BLM 8.2.2). Students may refer to these notes throughout the unit.

Intro GSP.gsp

Plane Geometry.gsp

As an introductory activity, ask students to search the web to discover the meaning of the word *geometry*.

Word Wall
See BLM 8.2.4

The GSP®4 file Plane Geometry also has a "How to" button on each page with information that students may copy onto BLM 8.2.2.

Action!

Pairs → Guided Exploration

Students use the GSP®4 file Plane Geometry and BLM 8.2.1 to review geometric concepts from Grades 7/8 and to build skills required for future activities with The Geometer's Sketchpad®4. Student roles: Driver (runs the mouse), Recorder (takes notes for the pair).

Students exchange roles part way through the activity. Provide feedback to student responses.

Learning Skills/Work Habits/Observation/Rating Scale: Circulate to assess how individual students stay on task and in role to complete the investigations.

Consolidate Debrief

Whole Class → Discussion

Discuss what has been accomplished to date.

Ask students to bring an optical illusion for Day 3.

Reflection Concept Practice

Home Activity or Further Classroom Consolidation

Make a title page for this unit.

Complete practice questions using the theorems.
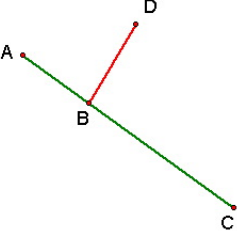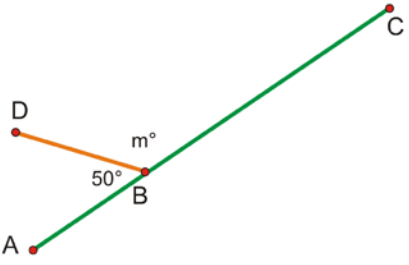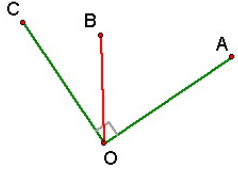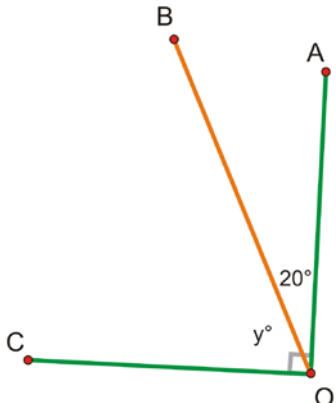
Complete worksheet 8.2.3.
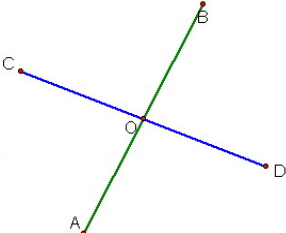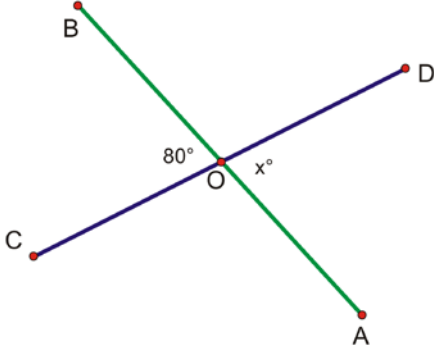
Provide appropriate practice questions.

8.2.1: Plane Geometry Record Sheet

Use this page to record your observations and conclusions from the Plane Geometry GSP®4 file.
Determine the unknown angle in the right column. Give reasons for your answer.

1) Supplementary Angles

Given: ∠ABC is a straight angle.

Explore: Measure ∠ABD and ∠DBC.

How do I ...?

Calculate the sum of these two angles.
Drag point D.

Conclude: What is the sum of the two angles?

2) Complementary Angles

Given: ∠AOC is a right angle.

Explore: Measure ∠AOB and ∠COB.

How do I ...?

Calculate the sum of these two angles.

Animate the diagram by pressing the button below. You can pause the animation by pressing the button again.

Animate

Conclude: What is the sum of the two angles?

3) Opposite Angle Theorem

Given: AB intersects CD at O.

Explore: Measure ∠AOC and ∠DOB.

How do I ...?

Animate the diagram by pressing the button below. You can pause the animation by pressing the button again.

Animate

Conclusion: What do you notice about ∠AOC and ∠DOB?

8.2.1: Plane Geometry Record Sheet (continued)

4) The Sum of the Interior Angles of a Triangle

Given: △ABC with the interior angles identified.

Explore: Click on the action button below.

| Show the Sum of the Angles | | Reset |
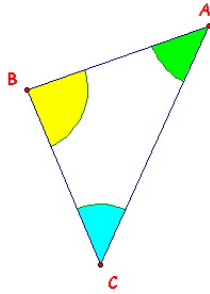
Drag each vertex of the triangle.

(Make sure the coloured sections marking *interior angles* are always inside the triangle.)

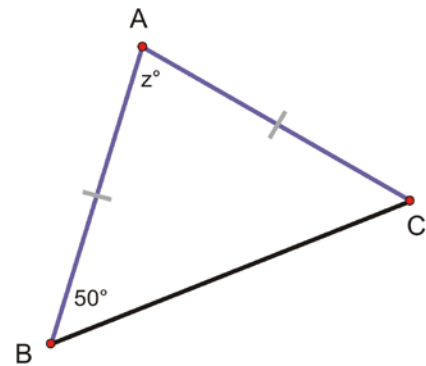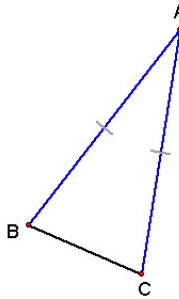**Conclude:** What is the sum of the interior angles of a triangle?

5) Isosceles Triangle Theorem

Given: Isosceles △**ABC**, where **AB** = **AC**

Explore: **Measure** the angles opposite the equal sides (∠**ABC** and ∠**ACB**)

| How do I ...? |
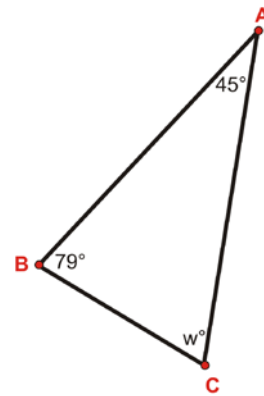
Drag each of the vertices.

Conclude: What do you notice about the angles opposite the equal sides?

6) Equilateral Triangles

Given: △**ABC**, where **AB** = **AC** = **BC**

Explore: **Measure** each of the interior angles then drag point C.

| How do I ...? |

Conclude: What do you notice about each of the interior angles?

7) Exterior Angle Theorem

Given: △**ABC** and exterior angle ∠**ABD**.

Explore: **Measure** the non-adjacent angles ∠**ACB** & ∠**CAB**.

| How do I ...? |
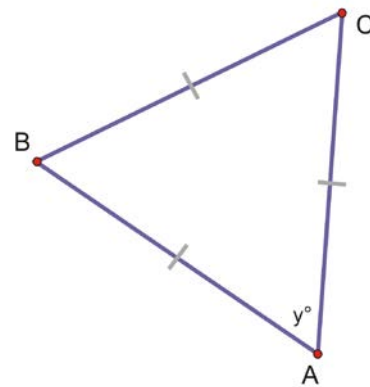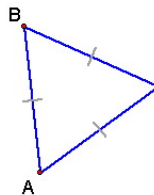
Calculate the sum of these two angles.
Measure the exterior angle ∠**ABC**.

Drag point A.

Conclude: What do you notice about the sum of the two non-adjacent angles, ∠**ACB** & ∠**CAB**, inside the triangle and the exterior angle, ∠**ABC**?

8) Parallel Line Theorem - Corresponding Angles

Given: **AB** is parallel to **CD** with transversal **PQ**.

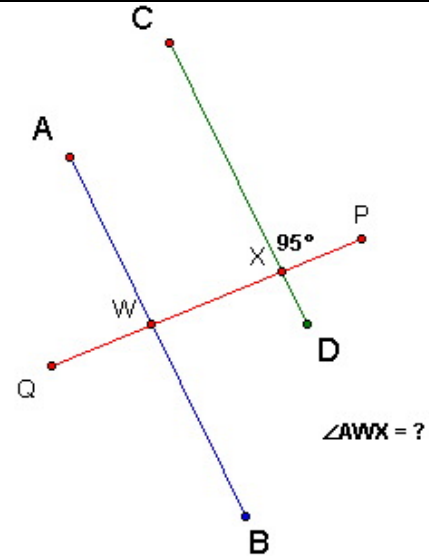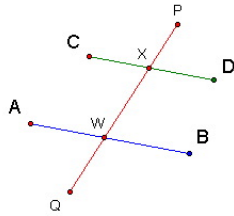**Explore:** **Measure** the *corresponding* angles ∠BWX and ∠DXQ.

How do I ...?

Drag points B and X.

Conclude: What do you notice about the *corresponding* angles ∠BWX and ∠DXQ?

More... Explain why this relationship is called the F-pattern.

There are four "F-patterns" in this diagram. Can you find them?

Help

∠AWX = ?

9) Parallel Line Theorem - Alternate Angles

Given: **AB** is parallel to **CD** with transversal **PQ**.

Explore: **Measure** the *alternate angles* ∠BWX and ∠WXC.

How do I ...?

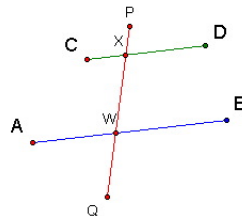**Drag points B and X.**

Conclude: What do you notice about the alternate angles ∠BWX and ∠WXC?

More... Explain why this relationship is called the Z-pattern.

There are two "Z-patterns" in this diagram. Can you find them?

Help

∠BWP = ?
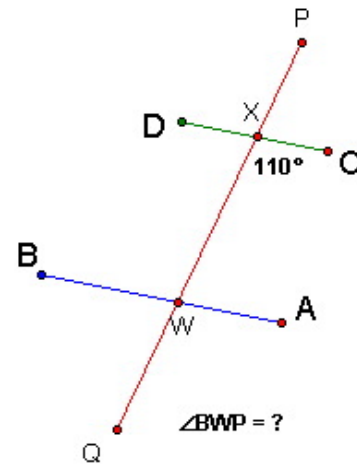
10) Parallel Line Theorem - Co-interior Angles

Given: **AB** is parallel to **CD** with transversal **PQ**.

Explore: **Measure** the co-interior angles ∠BWX and ∠WXD.
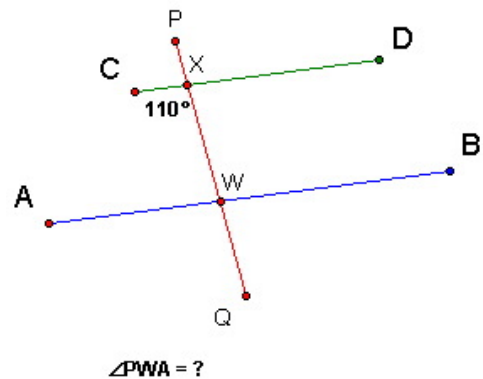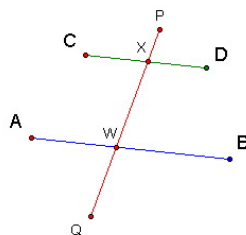
How do I ...?

Calculate the sum of these two angles.
Drag points B and X.

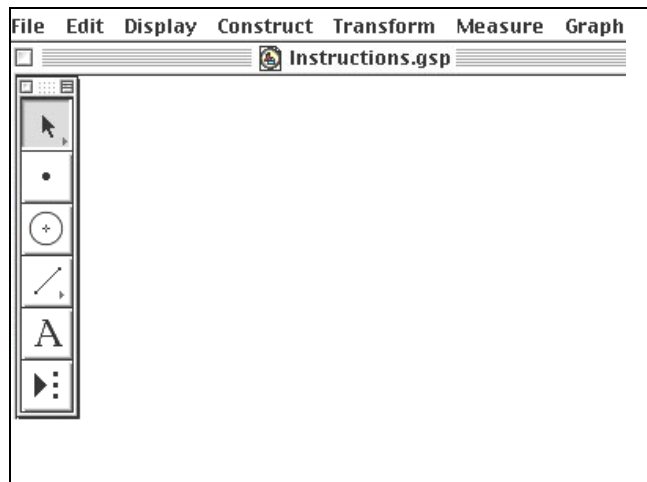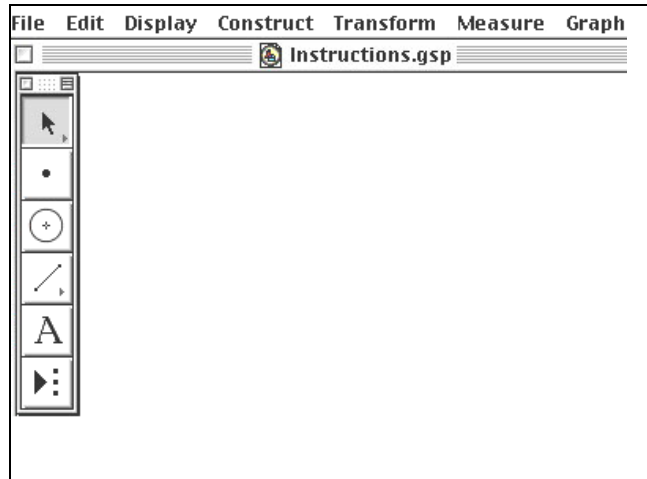**Conclude:** What do you notice about the sum of the co-interior angles ∠BWX and ∠WXD?
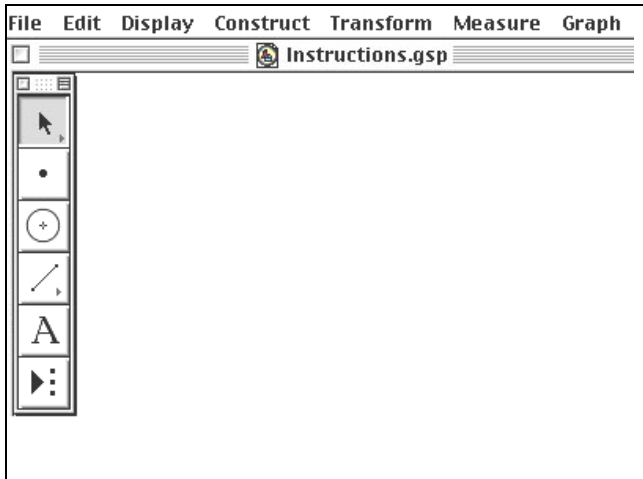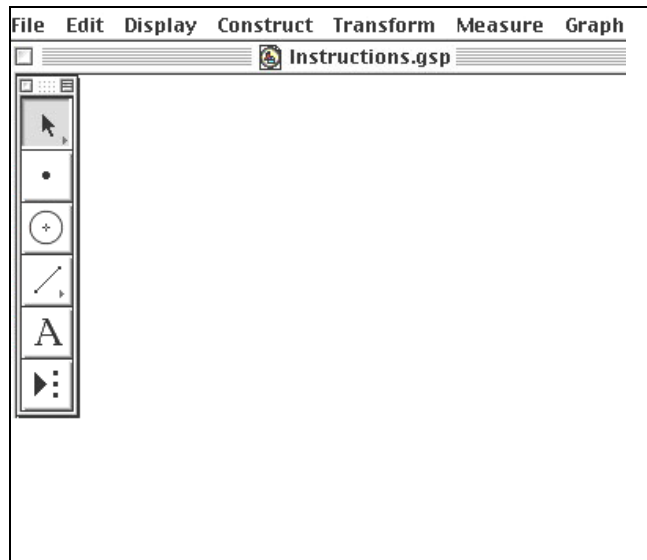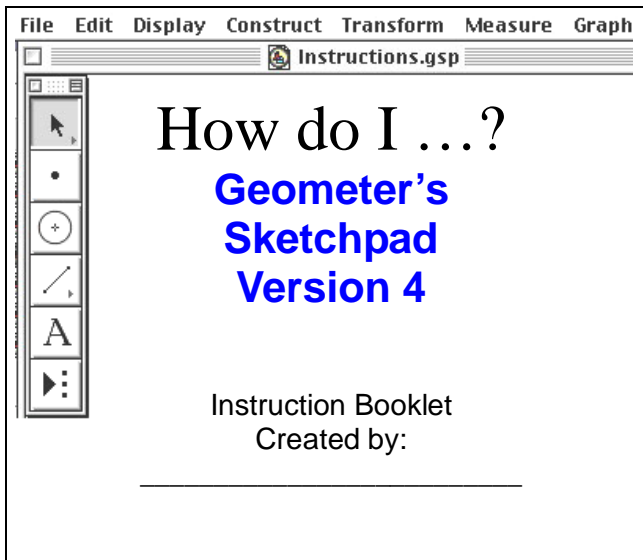
More... Explain why this relationship is called the C-pattern.

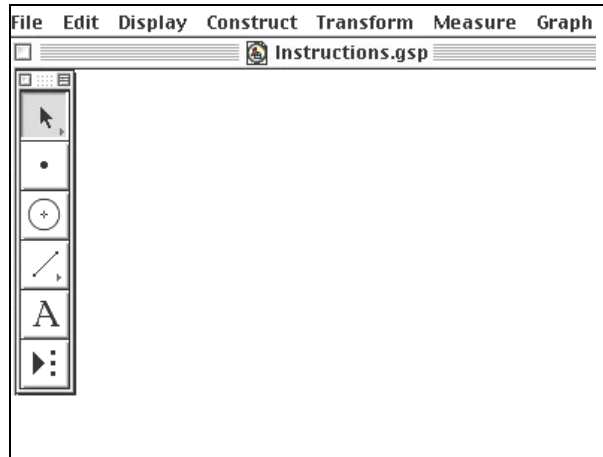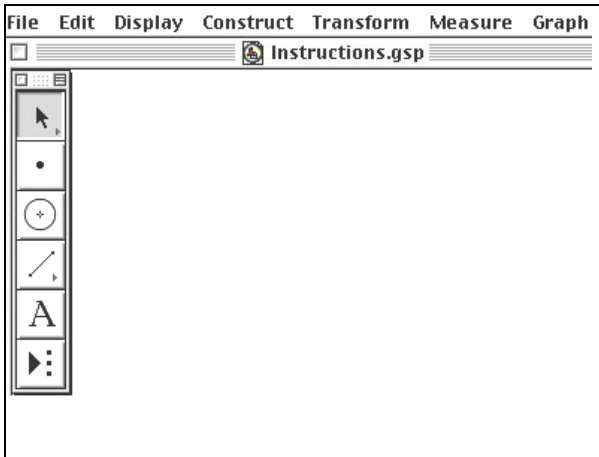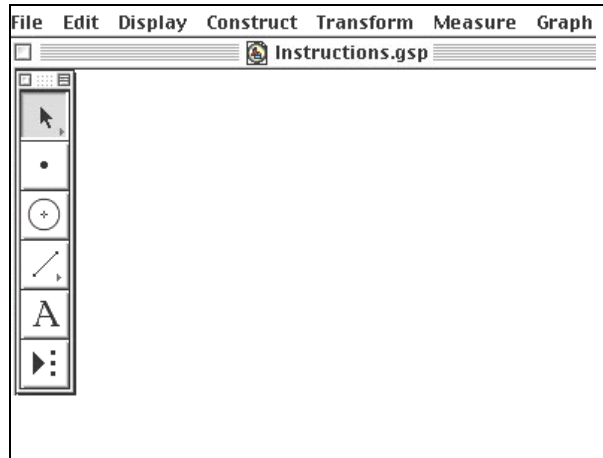There are two "C-patterns" in this diagram. Can you find them?

Help

∠PWA = ?

8.2.2: GSP® Notes Booklet

File Edit Display Construct Transform Measure Graph
Instructions.gsp

How do I …?
Geometer's Sketchpad Version 4

Instruction Booklet
Created by:
_____

File Edit Display Construct Transform Measure Graph
Instructions.gsp

File Edit Display Construct Transform Measure Graph
Instructions.gsp

File Edit Display Construct Transform Measure Graph
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File Edit Display Construct Transform Measure Graph
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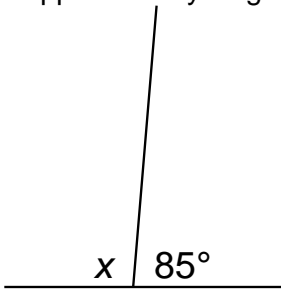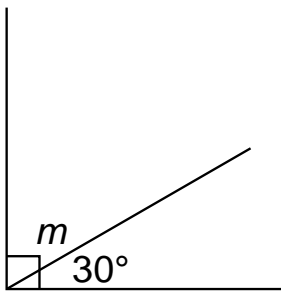
8.2.3: Theorems Practice Sheet

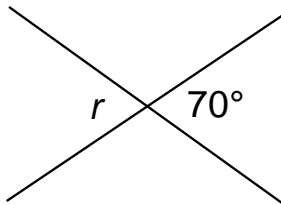Define each principle and determine the unknown angles.

1. Supplementary Angles

$x° =$

$=$

x | 85°

2. Complementary Angles

$m° =$

$=$

m

30°

3. Opposite Angle Theorem

$r° =$

$=$

r 70°

4. The Interior Angles of a Triangle

q

$q° =$

$=$

50°   55°
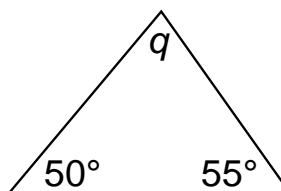
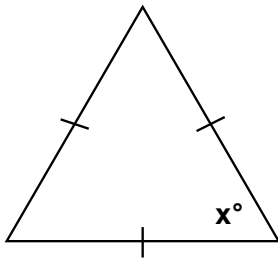# 8.2.3: Theorems Practice Sheet (continued)

5. Isosceles Triangle Theorem
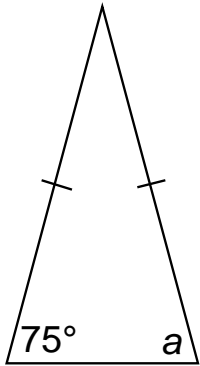
$a° =$

$=$

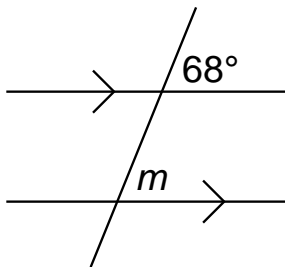6. Equilateral Triangles

$x° =$

$=$

7. Exterior Angle Theorem

$b° =$

$=$

8. Parallel Lines
    a) Corresponding Angles

    $m° =$

    b) Alternate Angles
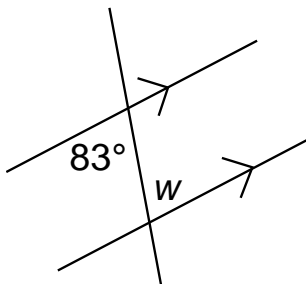
    $w° =$

    c) Co-interior Angles
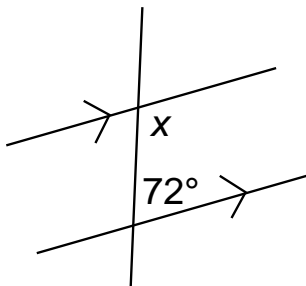
    $x° =$

    $=$

8.2.4: Word Wall List (Teacher)

acute angle	obtuse triangle
acute triangle	octagon
adjacent angles	opposite angle
alternate angles	parallelogram
bisector	pentagon
circle	perpendicular
co-interior angles	perpendicular bisector
complementary angles	point
congruent	polygon
corresponding angles	quadrilateral
diagonal	radius
diameter	ray
equilateral triangle	rectangle
exterior angle	reflex angle
hexagon	rhombus
interior angle	right angle
isosceles triangle	right angle triangle
kite	scalene triangle
line	side
line segment	similar figures
midpoint	supplementary angles
obtuse angle	transversal

Introduction to the Geometer's Sketchpad (GSP®4 file)

Intro GSP.gsp

Introduction to Geometer's Sketchpad.

Geometer's Sketchpad allows us to construct, measure, and animate and move geometric objects easily.

The four tools for constructing objects are at the left of the screen.
1) Point Tool (dot)
2) Compass Tool (circle)
3) Segment Tool (line segment)
4) Text Tool (letter A)

Try experimenting with these tools by constructing various objects on the right of the screen.

When you are finished, click on the *Link* button to go to *Page 2*.

Link to Page 2

Labeling Objects.

A point and a line segment are drawn at right. To label them click on the Text Tool (letter A) then click on the point or the line segment.

To hide a label, click on the object, choose the *Display* tab then choose *Hide Label*.

Another way to show or hide the label is to highlight the object, then from the *Display* tab, choose *Show/Hide Label*.

By highlighting an object you can also Hide the object from the *Display* tab. This does not delete it, but hides it from view.

Click on the *Link* button to go to *Page 3*
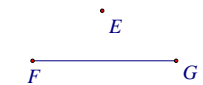
Link to Page 3

Constructions.

To construct a line segment between points A and B, click on the points then, from the *Construct* menu, choose *Segment*

To construct the midpoint of CD, click on the line segment then, from the *Construct* menu, choose *Midpoint*.

To construct a line parallel to FG highlight point E and segment FG Then, from the *Construct* menu, choose *Parallel Line*.
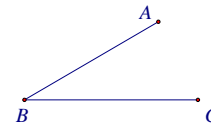
To construct a line perpendicular to FG, highlight point E and segment FG Then, from the *Construct* menu, choose *Perpendicular Line*.
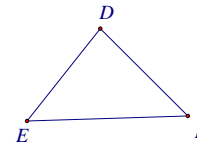
Click on the *Link* button to go to *Page 4*

Link to Page 4

Constructions 2

To construct the angle bisector of ABC, click on the points ABC (order is important) then, from the *Construct Menu*, choose *Angle Bisector*.

To construct the interior of △DEF , click on the points DEF. Then, from the *Construct Menu*, choose *Triangle Interior*.

You can change the color of the interior by highlighting it then, from the *Display* Menu, choose *Color*.

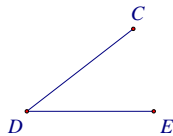Click on the *Link* button to go to *Page 5*

Link to Page 5

Measuring Objects (segments and angles).

A line segment is drawn at right. To measure the length of the line segment, click on it then go to the *Measure* menu above and choose *Length*.

Be sure that only the line segment is highlighted. To de-highlight an object click anywhere on the screen.

∠CDE is drawn at right. To measure the angle, click on points CDE (order is important) then go to the *Measure* menu and choose *Angle*.
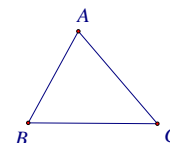
To change the precision of the measurements, go to the *Edit* tab and choose *Preferences*.

Click on the *Link* button to go to *Page 6*

Link to Page 6

Measuring Objects (perimeter and area)

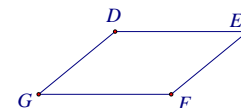Construct the interior of △ABC (click on points ABC then choose *Triangle Interior* from the *Construct* Menu).

Highlight the interior of the triangle then, from the *Measure* Menu, choose *Area*.

Highlight the interior of the triangle then, from the *Measure* Menu, choose *Perimeter*

Practice: Determine the area and perimeter of parallelogram DEFG.
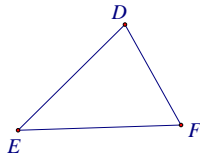
Click on the *Link* button to go to *Page 7*

Link to Page 7

Introduction to the Geometer's Sketchpad (GSP®4 file)
(continued)

Measuring Angles (tabulating)

ΔDEF is constructed at right.
All of the angles have been measured as well as
the sum of the angles.
Move point D to change the measures of the
angles, then double-click on the tabulation table.

What do you notice? _____

Move point F then double-click on
the tabulation table again.

On the next page you will learn
how to make a tabulation table.

m∠DFE = 63°
m∠DEF = 43°
m∠EDF = 74°
m∠DFE+m∠DEF+m∠EDF = 180°

m∠DFE	m∠DEF	m∠EDF	m∠DFE+m∠DEF+m∠EDF
63°	43°	74°	180°

Measuring (sum of angles)

Determine the measures of the three angles
of the triangle (see page 5).

When you have determined the three measures,
click on the *Measure* menu and choose
Calculate.

When you see the calculator, click on the first measure followed
by the plus sign (+) then click on the second measure followed
by the plus sign (+) then click on the third measure followed by
OK.

Highlight all three measures as well as the sum.
Then, from the *Graph* menu, choose *Tabulate.*

Change the size of the triangle then click the tabulate box.
What do you notice? Do this a few times.
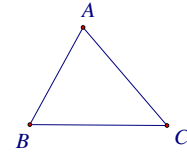
Practice

1- Construct any triangle and determine the
midpoints of the three sides.

2- Join the midpoints of the sides.

3- Determine the area of the small triangle in the middle
formed by joining the midpoints.

4- Determine the area of the large triangle.

5- Use the Calculator to divide the area of the large triangle
by the area of the smaller triangle.

6- What do you notice? _____

Practice

Construct line segments and triangles and
practice constructing the following:
1- parallel lines
2- perpendicular lines
3- angle bisectors
4- lengths of line segments
5- area and perimeters

Plane Geometry (GSP®4 file)

Plane Geometry.gsp

8.2.1 Plane Geometry

Explore each of the ten activities. (Start by clicking on number 1.)

Each activity has a [Main Menu] *button which will bring you back to this page.*

If you need help on how to use Geometer's Sketchpad go to the "How do I ...?" page by pressing the button: [How do I ...?]

Angles
[1] Supplementary Angles
[2] Complementary Angles
[3] Opposite Angles

Triangles
[4] Sum of the Interior Angles in a Triangle
[5] Isosceles Triangles
[6] Equilateral Triangles
[7] Exterior Angle of a Triangle

Parallel Lines
[8] Corresponding Angles (F-Pattern)
[9] Alternate Angles (Z-Pattern)
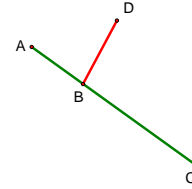[10] Co-interior Angles (C-Pattern)

1) Supplementary Angles

Given: ∠ABC is a straight angle.

Explore: **Measure** ∠ABD and ∠DBC.
[How do I ...?]

Calculate the sum of these two angles.
Drag point D.

Conclude: What is the sum of the two angles?

[Main Menu] [Next page]

2) Complementary Angles

Given: ∠AOC is a right angle.
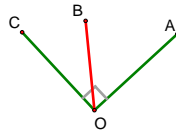
Explore: **Measure** ∠AOB and ∠COB.
[How do I ...?]
Calculate the sum of these two angles.

Animate the diagram by pressing the button below. You can pause the animation by pressing the button again.
[Animate]

Conclude: What is the sum of the two angles?

[Main Menu] [Next page]

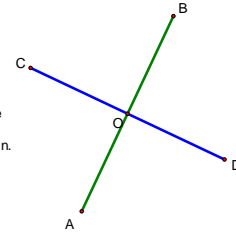## 3) Opposite Angle Theorem

Given: AB intersects CD at O.

Explore: **Measure** ∠AOC and ∠DOB.
[How do I ...?]
Animate the diagram by pressing the button below. You can pause the animation by pressing the button again.
[Animate]

Conclusion: What do you notice about ∠AOC and ∠DOB?

[Main Menu] [Next page]

4) The Sum of the Interior Angles of a Triangle

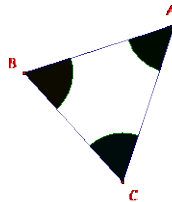**Given:**   △ABC with the interior angles identified.

Explore: Click on the action button below.
[Show the Sum of the Angles]   [Reset]

Drag each vertex of the triangle.

(Make sure the coloured sections marking *interior angles* are always inside the triangle.)

Conclude: What is the sum of the interior angles of a triangle?
[Main Menu] [Next page]

5) Isosceles Triangle Theorem

Given: Isosceles △ABC, where **AB** = **AC**

Explore: **Measure** the angles opposite the equal sides (∠ABC and ∠ACB)
[How do I ...?]
Drag each of the vertices.

Conclude: What do you notice about the angles opposite the equal sides?
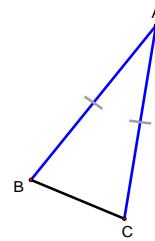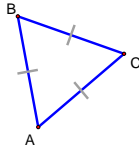
[Main Menu] [Next page]

Plane Geometry (GSP®4 file)
(continued)

6) Equilateral Triangles

Given: △**ABC**, where **AB** = **AC** = **BC**

Explore: **Measure** each of the interior angles then drag point C.
How do I ...?

Conclude: What do you notice about each of the interior angles?
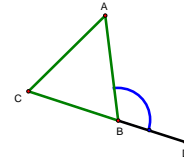
Main Menu   Next page

7) Exterior Angle Theorem

Given: △**ABC** and exterior angle ∠**ABD**.

Explore: **Measure** the non-adjacent angles∠**ACB** & ∠**CAB**.
How do I ...?

Calculate the sum of these two angles.
Measure the exterior angle ∠**ABD**.

Drag point A.

Conclude: What do you notice about the sum of the two non-adjacent angles, ∠**ACB** & ∠**CAB**, inside the triangle and the exterior angle, ∠**ABD**?

Main Menu   Next Page

8) Parallel Line Theorem - Corresponding Angles

Reset

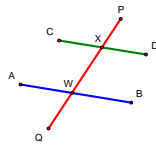**Given:** **AB** is parallel to **CD** with transversal **PQ**.

Explore: **Measure** the *corresponding* angles
∠BWX and ∠DXP.
How do I ...?

Drag points B and X.

Conclude: What do you notice about the *corresponding* angles ∠BWX and ∠DXP?

More... Explain why this relationship is called the F-pattern.

There are four "F-patterns" in this diagram. Can you find them? Help

Main Menu   Next page

9) Parallel Line Theorem - Alternate Angles

Reset Z Pattern

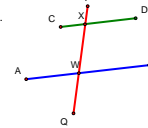**Given:** **AB** is parallel to **CD** with transversal **PQ**.

Explore: **Measure** the *alternate angles* ∠BWX and ∠WXC.
How do I ...?

Drag points B and X.

Conclude: What do you notice about the alternate angles ∠BWX and ∠WXC?

More... Explain why this relationship is called the Z-pattern.

There are two "Z-patterns" in this diagram. Can you find them? Help

Main Menu   Next page

10) Parallel Line Theorem - Co-interior Angles

Reset C Pattern

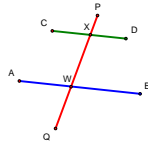**Given:** **AB** is parallel to **CD** with transversal **PQ**.

Explore: **Measure** the co-interior angles
∠BWX and ∠WXD.
How do I ...?

Calculate the sum of these two angles.
Drag points B and X.

Conclude: What do you notice about the sum of the co-interior angles ∠BWX and ∠WXD?

More... Explain why this relationship is called the C-pattern.

There are two "C-patterns" in this diagram. Can you find them? Help
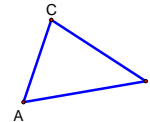
Main Menu

How do I ...?

...measure an angle or side?

... construct a polygon interior

... measure an area or perimeter?

... create a table?

... make a calculation?

Angle Measures in △ ABC

m∠CAB = 62°
m∠ABC = 43°
m∠BCA = 76°
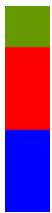
...measure an angle?

To measure ∠**ABC**:
1. Deselect.
2. Select point A, then B, then C. Order is important! The second point chosen is the vertex.
3. Under **Measure** choose **Angle.**.

...measure the length of a side?

To measure side **CB**:
1. Deselect.
2. Select the two endpoints (C and B).
3. Under **Measure** choose **Distance.**

Main Menu

Math Learning Goals
- Explore geometrical concepts (angles, triangles, parallel lines).
- Build skills required for future use of The Geometer's Sketchpad®4 (GSP).

Materials
- computer/pair
- BLM 8.2.1

75 min

Assessment Opportunities

Minds On ...

Pairs → Discussion
Students post their optical illusions and find concrete examples in the classroom of the geometric concepts that they explored yesterday. (For example, swing the door and ask what type of angles does the door create with the wall?)

Whole Group → Discussion
Review some of the concepts from the last lesson and ask students to share their concrete examples. Discuss some of the problems you observed with the students' use of GSP®4 and BLM 8.2.2 GSP®4 Notes Booklet (see Day 2).

As a class make up acronyms for some of the theorems e.g., students may suggest OAT for the Opposite Angles Theorem. In later lessons students may use the acronyms when asked to give reasons for their answers.

Discuss the idea of a theorem and the need to confirm your hypothesis with measurement.

Acronyms are good time savers when students are asked to give reasons for their answers. Students should use full terminology in work handed in. They could write acronyms on their practice sheet (BLM 8.2.3).
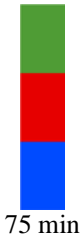
Examining the theorem with GSP®4 provides convincing evidence that the theorem always works.

Action!

Pairs → Guided Exploration

Learning Skills/Work Habits/Checklist/Rating Scale: Circulate to check homework and see unit title page while students are working.

Students use the GSP®4 file Plane Geometry and BLM 8.2.1 to complete the review of geometric concepts from Grades 7/8 and to build skills required for future activities with Geometer's Sketchpad. As well, they should work on completing BLM 8.2.2 GSP®4 Notes Booklet.

Student roles: Driver, Recorder. Students exchange roles part way through the activity. Provide feedback to student responses during circulation. (Students can check their answers by using the GSP®4 sketch.)

Encourage students to record GSP®4 instructions on BLM 8.2.2 GSP®4 Notes Booklet for future reference.

Consolidate Debrief

Whole Class → Discussion
Using the Graffiti sheets from Day 2, students determine if anything needs to be added/changed/deleted on the topic sheets, based on what they learned during the computer exploration.

Students provide more concrete illustrations of the concepts, using objects in the classroom, e.g., pages in a book, ceiling/floor tiles.

As a class, continue to make up acronyms for the theorems, e.g., students may suggest OAT for the Opposite Angles Theorem. Students record them on the Word Wall and/or in their notes. In later lessons students may use the acronyms when asked to give reasons for their answers.

Consider using the Word Wall list for a charade word game as a vocabulary review.

Home Activity or Further Classroom Consolidation
Ask students to bring an optical illusion to the next day's class if they haven't already.

Continue working on the title page for this unit, if necessary.

Complete practise questions using the theorems.

Reflection Concept Practice

Provide appropriate practice questions.

Unit 8: Day 4: What's So Special? (Part 1)

Grade 9 Applied

75 min

Math Learning Goals
- Build investigation skills by exploring geometric concepts, using GSP®4.
- Develop communication skills and geometric vocabulary.

Materials
- BLM 8.4.1, 8.4.2
- data projector

Assessment Opportunities

Minds On ...

Whole Class → Demonstration
Students post their optical illusions. Display the GSP®4 file What's So Special? using a data projector. After each demonstration, invite students to share their observations.
- Discuss the two optical illusions on the first sketch. Students will see different things when they look at geometric diagrams.
- Click the first two demonstration buttons. Emphasize the importance of discussing and recording observations so students can learn from each other. Use the posted optical illusions to reinforce messages.
- Click the Demonstration 3 button. Discuss the hidden geometry in the constructed equilateral triangle. (Students may respond that some sketches are special because some things always remain true in any drag test.)
- Click the Demonstration 4 button and drag the points to demonstrate how to collect evidence that shows that two quantities are proportional. Show students how to measure the areas of both circles and how to determine the ratio of the areas by using the Calculate command under the Measure menu.
- Use the first activity (Special Triangles) to demonstrate how to look for something special in a sketch. Explore the first two triangles. Demonstrate how to use "tabulate" to collect evidence that supports an hypothesis, e.g., for the second triangle collect at least three table entries that show that angle LKM has a measure of 90 degrees.
- Review the process by using BLM 8.4.1 and 8.4.2, and suggest that students use them as guides for GSP®4 explorations.

Special.gsp

When students identify a property they should consider: Is it always true? Have I provided evidence to support my answers?

Emphasize that students should look for special properties and measure to show the special feature.

Action!

Pairs → Investigation
Students work on triangle ABC and KLM of Investigation 1 in the GSP®4 file What's So Special? Circulate and assist groups who are experiencing difficulties.

Consolidate Debrief

Whole Class → Discussion
Using the demonstration, discuss what is so special about the triangle. Discuss what must be measured to show the special feature.

Curriculum Expectations/Quiz/Marking Key: Create a quiz that is based on material from Days 1–3.

Application

Home Activity or Further Classroom Consolidation
Find or create a logo that has a geometric design. Describe the design so someone else has a clear picture of it.

8.4.1: "What's So Special?" Guide Sheet

Explore!
Drag each vertex in the figure.

As you drag vertices, look for some of the following:
- measurements that always seem to be equal to each other
- measurements that never seem to change
- measurements that might have a constant ratio (proportional)
- lines that always seem to be parallel or perpendicular
- line segments that always seem to be bisected
- figures that always seem to be congruent
- objects that don't seem to be connected, yet they move together when something is dragged

Make an Hypothesis
Decide which measurements you need to test your hypothesis.
Drag each vertex again while you pay close attention to the way the object moves and to the way the measurements change.

Test Your Hypothesis
Collect and record evidence to test your hypothesis.

What can you measure?	What can you calculate?
• angles • lengths • areas • perimeters • slopes • •	• sums • ratios • formulas • • • •

8.4.2: Guiding Questions

<u>Examine the angles…</u>

- Are any of the angles equal?
- Do any of the angle measures always add to give the same total?
- Does the measure of any angle always stay the same?
- Are any of the angles cut in half (bisected)?

<u>Examine the line segments…</u>

- Are any of the lengths equal?
- Is any length proportional with any other length?
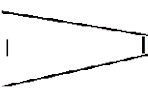- Are any of the line segments cut in half (bisected)?
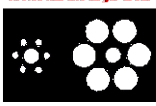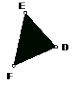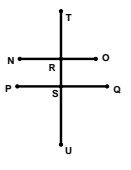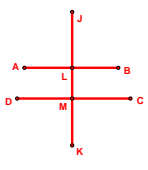
<u>Examine the lines…</u>

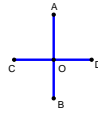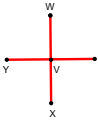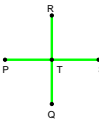- Are any of the lines parallel?
- Are any of the lines perpendicular?
- Are any of the slopes of the lines equal?

<u>Examine areas and perimeters…</u>

- Are any of the areas equal?
- Are any of the perimeters equal?
- Are any of the shapes congruent?
- Are any of the shapes similar?
- Are any of the areas proportional?
- Are any of the perimeters proportional?

What's So Special? (GSP®4 file)

Special.gsp

Main Menu

1) Special Triangles?
2) Parallel or Perpendicular?
3) Bisected?
4) Proportional?
5) Special Quadrilaterals?

How do I ...?

Which of the small center circles has the larger area?

Which line segment is longer?

What's so special?

Which shape was constructed to always be an equilateral triangle?

How does the area of the green circle compare to the area of the yellow circle?

Demonstration 1 - Circles
Demonstration 2 - Segments
Demonstration 3 - Triangles
Demonstration 4 - Circles

Investigation 1: Special Triangles?

What's a Special Triangle?

Hide Instructions
How do I ...?

Exploring:
Drag each vertex of every triangle.

Hypothesizing:
Make a hypothesis about each triangle. Are any of the triangles special?

Collecting Evidence:
What evidence can you find to support your hypothesis?
What are you going to measure? calculate? tabulate?

Making a Conclusion:
What kinds of triangles are shown? Were your hypotheses correct?
Justify your answers using the evidence you collected.

Main Menu
Next page

Investigation 2: Parallel or Perpendicular?

Hide Instructions
How do I ...?

Exploring:
Drag the endpoints of each line segment.

Hypothesizing:
Make a hypothesis based on your exploration.

Collecting Evidence:
What evidence can you find to support your hypothesis?
What are you going to measure? calculate? tabulate?

Making a Conclusion:
What is special about some of the line segments? Were your hypotheses correct?
Justify your answer using the evidence you collected.

Main Menu
Next page

Investigation 3: Bisected?

Hide Instructions
How do I ...?

Figure 1   Figure 2   Figure 3

Exploring:
Explore all three figures.
Do you notice anything special? Record your observations.

Hypothesizing:
Make a hypothesis

Collecting Evidence:
What evidence can you find to support your hypothesis?
What are you going to measure? calculate? tabulate?

Making a Conclusion:
What is special about the intersecting line segments? Were your hypotheses correct?
Justify your answer using the evidence you collected.

Main Menu
Next page
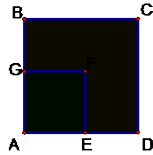
What's So Special? (GSP®4 file)
(continued)

Investigation 4: Proportional?

Show GSP Hint

Hide Instructions
How do I ...?

Exploring:
Drag each vertex. Do you notice anything special? Measure the area and the perimeter of figures AEFG and ABCD.

Hypothesizing:
Is there anything special in this diagram?

Collecting Evidence:
What evidence can you find to support your hypothesis?
What are you going to measure? calculate? tabulate?

Making a Conclusion:
Was your hypothesis correct?
Is anything in the diagram proportional?
Justify your answers using the evidence you collected.

Investigation 5: Special Quadrilaterals?

Hide Instructions
How do I ...?

Exploring:
Drag each vertex of every quadrilateral.

Hypothesizing:
Make a hypothesis about each quadrilateral. Are any of the quadrilaterals special?

Collecting Evidence:
What evidence can you find to support your hypothesis? What are you going to measure? calculate? tabulate?

Making a Conclusion:
What kinds of quadrilaterals are shown? Were your hypotheses correct?
(Hint: Match the blue names to the quadrilaterals so that each name is used exactly once.)
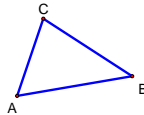Justify your answers using the evidence you collected.

Square

Trapezoid

Parallelogram

Rectangle

Rhombus

Kite

Quadrilateral

Rule: Do not drag any of the shapes into a "non-quadrilateral" shape like this one.

Main Menu

How do I ...?

...measure an angle or side?
... construct a polygon interior
... measure an area or perimeter?
... create a table?
... make a calculation?

Angle Measures in △ ABC

m∠CAB = 62°
m∠ABC = 43°
m∠BCA = 76°

...make a calculation?

To calculate the **sum of the interior angles**:
1. Deselect.
2. Under **Measure** choose **Calculate.**
3. Select the measurement m∠CAB (in the upper right corner) then
   the + sign (on the calculator) then
   the measurement m∠ABC, then the + sign, then
   the measurement m∠BCA, then OK

Summary:
Select measurements in the sketch and operations from the calculator and press OK at the end.

Main Menu

Defintions

Show | **Acute** △: A triangle in which **each** of the three interior angles is acute.

Show | **Obtuse** △: A triangle containing **one** obtuse angle.

Show | **Right** △: A triangle containing a 90° angle.

Show | **Equilateral** △: A triangle with all sides equal.

Show | **Isosceles** △: A triangle with two equal sides.

Show | **Scalene** △: A triangle with no equal sides.

Perpendicular : Intersecting at 90°.
Show

Parallel Lines: Lines in the same plane that do not intersect. They are always the same distance apart.
Show

Bisect: Means to cut in half. Angles or lengths can be bisected.
Show

Angle Bisector: A line that divides an angle into two equal parts.
Show

Right Bisector of a line segment: A line that is perpendicular to line segment and divides the line segment into two equal parts.
Show

Proportional: A statement that two ratios are equal. (Two objects have exactly the same shape but are different sizes)
Show

Unit 8: Day 5: What's So Special? (Part 2)

75 min

Math Learning Goals
- Build investigation skills by exploring geometric concepts, using GSP®4.
- Develop communication skills and geometric vocabulary.

Materials
- BLM 8.5.1
- Examples of logos with geometric designs
- Computer with Geometers Sketchpad 4, 1/ pair

Assessment Opportunities

Minds On ...

Small Groups → Brainstorm

Students form small groups and challenge each other to sketch the logo that they describe from the Home Activity assignment. Students brainstorm how to create an effective description.

Whole Class → Discussion

Demonstrate the two triangles from Day 4.

Emphasize that there is very little that is always special about triangle ABC. (It will always have three sides and three angles).

Emphasize that triangle KLM always has a right angle at K.

Explore the other triangles in the sketch to determine what is special.

Have available some examples of logos with geometric designs.

Action!

Pairs → Investigation

Students investigate geometric relationships using sketches in the GSP®4 file, What's So Special? (See Day 4.)

Assign one investigation to each pair for reporting purposes during **Consolidate/Debrief**.

Pairs may have time to do more than one of the investigations.

Learning Skills (Teamwork)/Observation/Rating Scale: Circulate to assess how students contribute to the group to complete the activity.

Consolidate Debrief

Pairs → Presentation

For each of the four activities choose pairs of students to present their work to the class.

Whole Class → Discussion

Debrief the presentations to help students understand how GSP®4 shows geometric principles.

Ask such questions as:
- How does The Geometer's Sketchpad®4 help to collect evidence?
- How much evidence is needed to convince you that something is always true?

Application
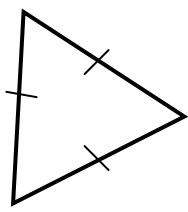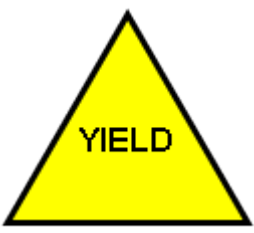
Home Activity or Further Classroom Consolidation

Complete worksheet 8.5.1 Learn the Lingo.

8.5.1: Learn the Lingo

1. Part a) shows an example of how to complete a word chart.
   Complete the remaining word charts.

a)

Term:	Visual Representation:
Equilateral Triangle	
Definition: An *equilateral triangle* is a triangle for which all sides have the same length.	**Association:** A Yield sign

b)

Term:	Visual Representation:
Triangle	
Definition:	**Association:**

c)

Term:	Visual Representation:
Exterior Angle	
Definition:	**Association:**

d)

Term:	Visual Representation:
Interior Angle	
Definition:	**Association:**

e)

Term:	Visual Representation:
Parallel Lines	
Definition:	**Association:**

f)

Term:	Visual Representation:
Transversal	
Definition:	**Association:**

g)
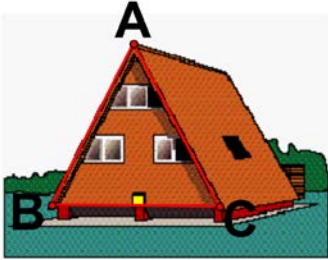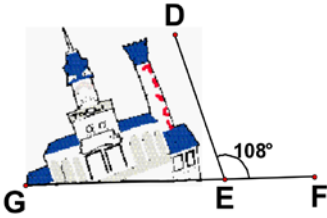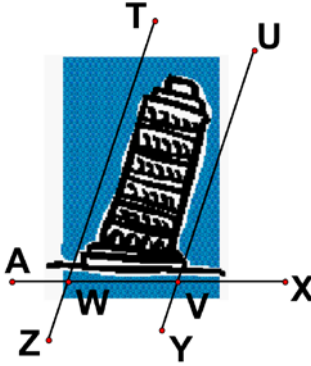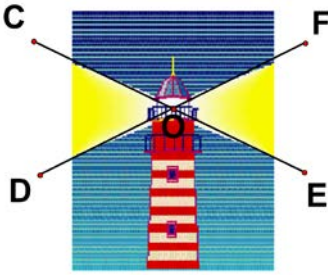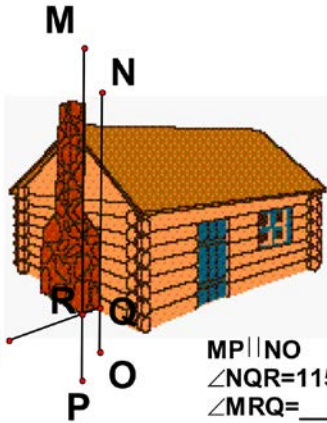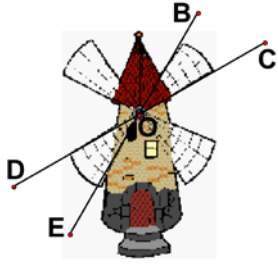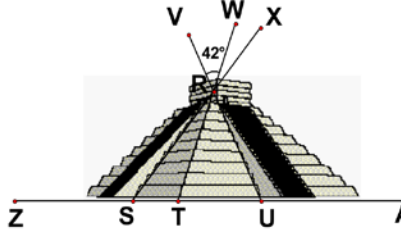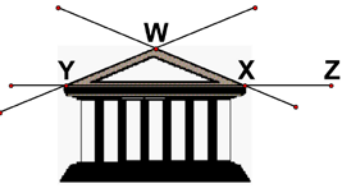
Term:	Visual Representation:
Perpendicular Bisector	
Definition:	**Association:**

h)

Term:	Visual Representation:
Diagonal	
Definition:	**Association:**

2. Determine the unknown angle. Give reasons for your answer.

a)	b)	c)
AB = AC = BC ∠ACB = _____	∠DEG = _____	TZ‖UY ∠TWX = 75° ∠UVW = _____
d)	e)	f)
∠COD = 64° ∠FOE = ___ ∠COF = ___	MP‖NO ∠NQR=115° ∠MRQ=_____	∠BOC = 43° ∠COE = _____ ∠EOD = _____
g)	h)	i) Create your own question!
∠VRW = 42° RT = RU ∠SRT = 19° ∠RSZ =	WX = WY ∠YWX = 118° ∠WXZ = _____	

Unit 8: Day 6: Interior and Exterior Angles of Triangles and Quadrilaterals

Grade 9 Applied
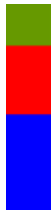
75 min

Math Learning Goals
- Investigate the sum of the interior and exterior angles of triangles and quadrilaterals using The Geometer's Sketchpad®4 and demonstration.
- Develop skills with The Geometer's Sketchpad®4 in preparation for summative assessments.

Materials
- data projector
- BLM 8.6.1, 8.6.2
- Computer with Geometers Sketchpad 4, 1 / pair

Assessment Opportunities

Minds On ...

Pairs → Tutorial

In pairs, students review how to use The Geometer's Sketchpad®4 tools for today's activity using the GSP®4 file, Quick Tutorial and BLM 8.2.2 GSP®4 Notes Booklet.

Whole Class → Demonstration

Demonstrate how to construct a triangle using rays.

Demonstrate how to measure the exterior angles. Have students make a prediction.

Demonstrate how to calculate the sum of the exterior angles and tabulate the sum. Drag each of the vertices of the triangle and tabulate the results.

Students make a prediction about the exterior angles of a quadrilateral.

Tutorial GSP.gsp

Angles Triangles. gsp

Angles Polygon.gsp

Make the rays in the same direction.

Action!

Pairs → Exploration

Students title a blank sketch "Exterior Angles of a Quadrilateral." They write their hypothesis about the sum of the exterior angles of the quadrilateral. They construct a quadrilateral, measure the angles, calculate the sum of the angles and tabulate their results, then write a conclusion.

Students create another sketch titled "Exterior Angles of a Polygon." They may choose a polygon with any number of sides and repeat the same process. Students can use BLM 8.6.1 to help guide their activity.

Reasoning and/or Proving/Observation/Mental Note: Observe students' facility with the inquiry process to determine if they need differentiated instruction to prepare for the assessment.

Whole Class → Demonstration

Consolidate what students should have learned from their investigation using the GSP®4 files, Sum of the Exterior Angles of a Triangle and Sum of the Interior Angles of a Polygon.

You may only want to present one of the three methods, Tearing Corners, Diagonal Divisions, or expand the idea of Tiny Triangles.

Consolidate Debrief

Pairs → Application

Students complete BLM 8.6.2.

Reflection

Home Activity or Further Classroom Consolidation

Write a letter to Abe, who missed Math class, explaining how he can determine the sum of the interior and exterior angles in a decagon (10-sided polygon).

Sample response:

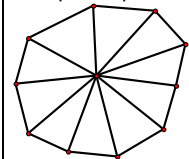$$S = 10 \times 180° - 360°$$
$$= 1440°$$

8.6.1: Exterior and Interior Angles of a Polygon

Part A – Exterior Angles of a Triangle
Follow the instructions carefully to make the diagram shown.

- Plot point A and draw a ray.
- Drag point B along the ray AB and then draw a ray from B.
- Select points C and A (order is important) then use the **construct** menu to construct ray CA.
- **Construct** points D, E, and F on rays AB, BC, and CA respectively.
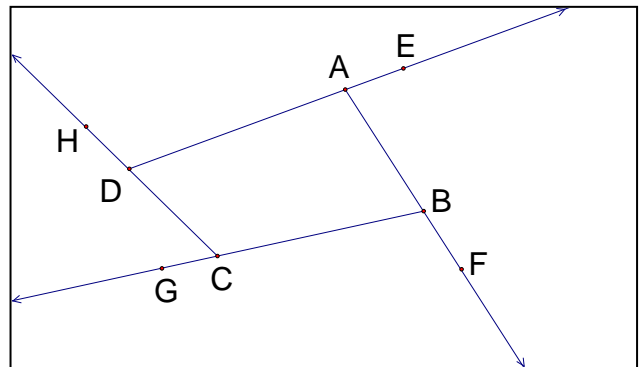- Drag test your construction.
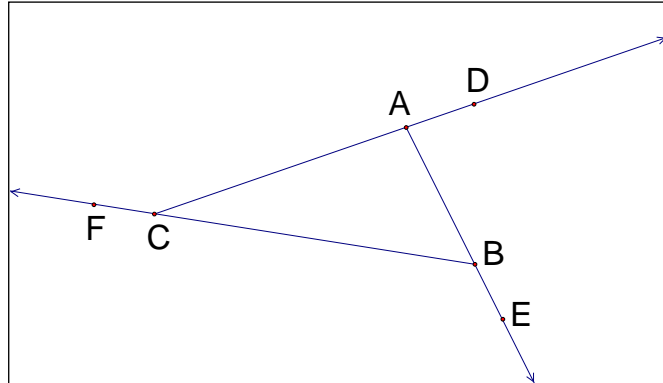
Hypothesis
I think that the sum of the exterior angles of a triangle is _____ because

Conclusion
Form a conclusion based on your evidence. Refer to your hypothesis.

Part B – Exterior Angles of a Quadrilateral
Use the steps from Part A to construct and explore the sum of the exterior angles of a quadrilateral.
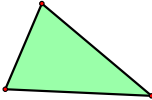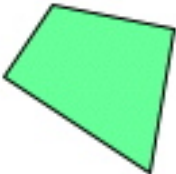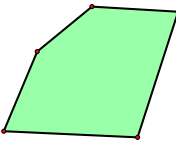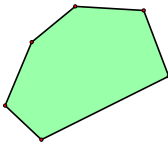
Part C – Exterior Angles of Any Polygon
Compare the conclusions you reached in Part A and Part B.
Test your hypothesis about the sum of the exterior angles of a polygon by constructing and measuring the angles of another polygon (pentagon, hexagon, etc.).
Record your information and write your final conclusion about the sum of the exterior angles of any polygon.

8.6.2: Interior Angle Sums

1. Complete the chart.

Diagram	Number of sides	Sum of interior angles	Understanding
	3	180°	The sum of the angles in any triangle is 180°.
	4		
	5		
	n		

2. a) Determine the sum of the interior angles in a polygon with 15 sides. Show your work.

   b) Determine the number of sides in a polygon if the sum of the interior angles is 5400°. Show your work.

8.6.2: Interior Angle Sums (continued)

3. Derek is building a deck for his summer job in the shape of a regular octagon.
   a) Define: regular octagon
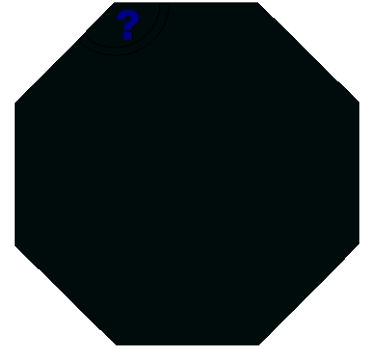
   b) Determine the measure of the interior angles of the deck.
      Show your work.

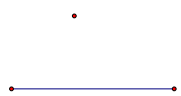4. A Canadian $1 coin, known as a loonie, is a regular polygon with 11 sides, called an *undecagon*.
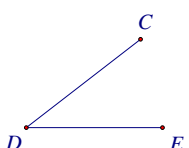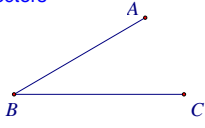
   a) Define a regular polygon with 11 sides.

   b) Determine the sum of the interior angles of the loonie.

   c) What is the size of one of the interior angles?

GSP® Quick Tutorial (GSP®4 file)

TutorialGSP.gsp

Quick Tutorial- Geometer's Sketchpad.

Geometer's Sketchpad allows us to construct, measure, and animate and move geometric objects easily.
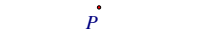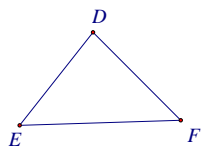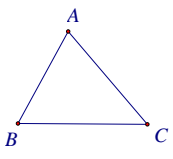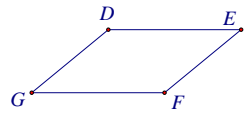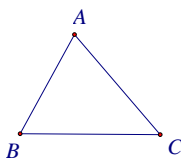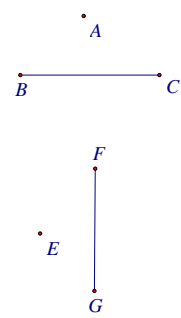
On the left side of your screen are your tools!

The **Selection Arrow** allows you to select and drag objects.

The **Point Tool** is used to construct points. It looks like a dot.

The **Compass Tool** is used to construct circles. It looks like a circle with cross hairs.

The **Straightedge Tool** is used to construct straight objects. If you hold the selection arrow over it it will change to allow you to select a *segment, ray* or *line* tool.

The **Text Tool** is used to create and edit text. It can be used to label objects. It looks like the letter A.

TRY IT-Try experimenting with these tools by constructing various objects on the right of the screen.

Name of Tools

- Selection Arrow
- Point Tool
- Compass Tool
- Straightedge Tools
- Text Tool
- Custom Tools

1) Labeling Objects.

A point and a line segment are drawn at right.

To label them click on the Text Tool (letter A) then click on the point or the line segment.

To hide a label, click on the *object*, choose the *Display* tab then choose *Hide Label*.

Another way to show or hide the label is to highlight the object, then from the *Display* menu, choose *Show/Hide Label*.

By highlighting an object you can also Hide the object from the *Display* tab. This does not delete it, but hides it from view.

2) Measuring Segments and Angles

Line segment AB is drawn at right.

One way to measure the length of the line segment, click on the line segment then go to the *Measure* menu above and choose *Length*.

Another way to measure the length of the line is to measure the distance between points. Select Point A then Point B. Go to the *Measure* menu above and choose *Distance*.

Which way do you like better?

∠CDE is drawn at right.
To measure the angle, click on points CDE (order is important) then go to the *Measure* menu and choose *Angle*.

HINT: To change the precision of the measurements, go to the *Edit* tab and choose *Preferences*.

3) Constructing Midpoints and Angle Bisectors

To construct the angle bisector of ABC, click on the points ABC (order is important) then, from the *Construct Menu*, choose *Angle Bisector*.

Did it really cut the angle in half? Measure it!

To construct the midpoint of DE, click on the line segment then, from the *Construct* menu, choose *Midpoint*.

Label the Midpoint M.

Is the distance between D and M the same as M and E?

Construct a ray through point P. Hold down the straightedge tool button and choose the middle option -ray tool. Click in the blank space then on point P.

4) Constructing Interiors

To construct the interior of △DEF , click on the **points** D, E, and F. Then, from the *Construct Menu*, choose *Triangle Interior*.

You can change the color of the interior by highlighting it then, from the *Display* Menu, choose *Color*.

TRY IT
Make a quadrilateral and shade the interior blue.

5) Measuring Perimeter and Area of Objects

Construct the interior of △ABC.

Highlight the interior of the triangle then, from the *Measure* Menu, choose *Area*.

Highlight the interior of the triangle then, from the *Measure* Menu, choose *Perimeter*

TRY IT
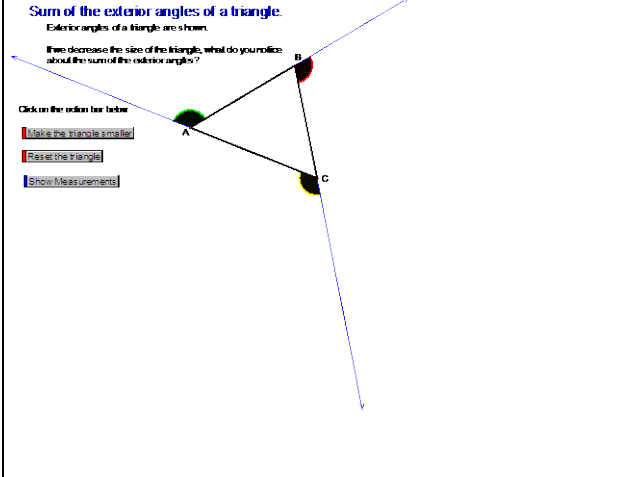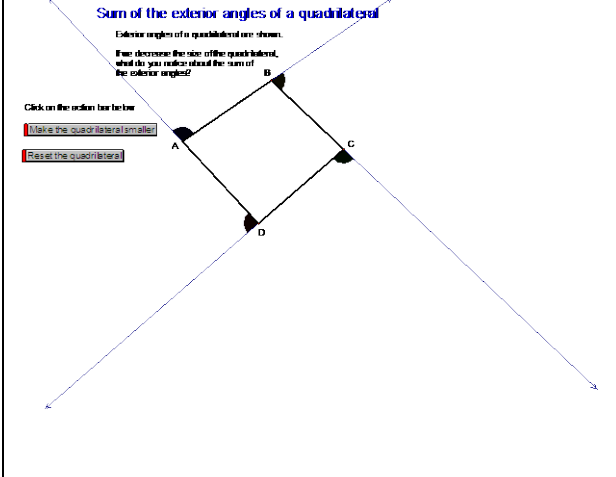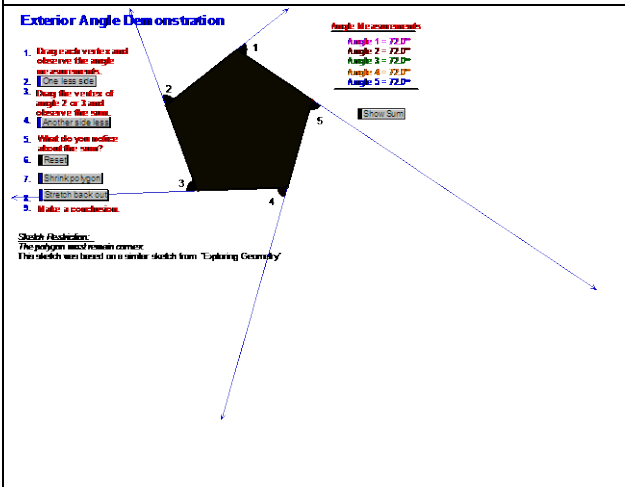Determine the area and perimeter of parallelogram DEFG.

GSP® Quick Tutorial (GSP®4 file)
(continued)

6) Measuring the sum of angles

Determine the measures of the three angles of the triangle ∠ABC, ∠ACB and ∠CBA

When you have determined the three measures, click on the *Measure* menu and choose *Calculate*.

When you see the calculator, click on the first measure (m∠ABC) followed by the plus sign (+) then click on the second measure followed by the plus sign (+) then click on the third measure followed by OK.

Highlight all three measures as well all three measures as the sum. Then, from the *Graph* menu, choose *Tabulate*.

Change the size of the triangle then doubleclick the tabulate box. What do you notice? Do this a few times.

7) Constructing Parallel and Perpendicular Lines

To construct a line parallel to BC through A. Highlight point A and segment BC Then, from the *Construct* menu, choose *Parallel Line*.

Test to see if they are parallel by dragging point C.

To construct a line perpendicular to FG through E. Highlight point E and segment FG then, from the *Construct* menu, choose *Perpendicular Line*.

Test to see if they are perpendicular.

8) Practice

1- Construct any triangle. Label it ABC.

2- Determine the midpoints of the three sides. Label them P, Q and R.

3- Join the midpoints of the sides.

4- Determine the area of the small triangle in the middle formed by joining the midpoints.

5- Determine the area of the large triangle.

6- Use the Calculator to divide the area of the large triangle by the area of the smaller triangle.

7- Tabulate your measurements.

8- What do you notice? _____

9) Picture Practice

Make a picture that includes construction of the following:

1- parallel lines

2- perpendicular lines

3- angle bisectors

4- lengths of the line segments

5- area and perimeters

Sum of the Exterior Angles of a Triangle (GSP®4 file)

Angles Triangle.gsp

Sum of the exterior angles of a triangle.

Exterior angles of a triangle are shown.

If we decrease the size of the triangle, what do you notice about the sum of the exterior angles?

Click on the action bar below

| Make the triangle smaller |

| Reset the triangle |

| Show Measurements |

Sum of the exterior angles of a quadrilateral

Exterior angles of a quadrilateral are shown.

If we decrease the size of the quadrilateral, what do you notice about the sum of the exterior angles?

Click on the action bar below

| Make the quadrilateral smaller |

| Reset the quadrilateral |

Exterior Angle Demonstration

1. **Drag each vertex and observe the angle measurements.**
2. | One less side |
3. **Drag the vertex of angle 2 or 3 and observe the sum.**
4. | Another side less |
5. **What do you notice about the sum?**
6. | Reset |
7. | Shrink polygon |
8. | Stretch back out |
9. **Make a conclusion.**

Angle Measurements
Angle 1 = 72.0°
Angle 2 = 72.0°
Angle 3 = 72.0°
Angle 4 = 72.0°
Angle 5 = 72.0°

| Show Sum |

Sketch Restriction:
The polygon must remain convex.
This sketch was based on a similar sketch from "Exploring Geometry"

Sum of the Exterior Angles of a Triangle (GSP®4 file)
(continued)

Sum of the Interior Angles of a Polygon

Tearing Corners

These two investigations simulate tearing off the corners of the polygon and joining them.

[1] Triangle

[2] Quadrilateral

Diagonal Divisions

These investigations use the fact that any polygon can be divided into triangles by drawing diagonals. It can also be used to explore the pattern of the sum of the angles and number of sides.

[3] Summary

[4] Quadrilateral

[5] Pentagon

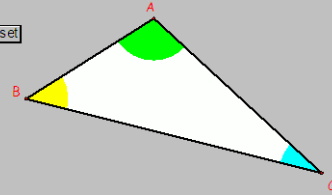[6] Hexagon vs. Regular Hexagon

Tiny Triangles

This investigation uses the fact that any polygon can be divided into triangles by connecting a center point to two of the vertices. It also uses the idea of subtracting one complete rotation.

[7] Quadrilateral

TRIANGLE -The Sum of the Interior Angles

Show the Sum of the Angles   Reset

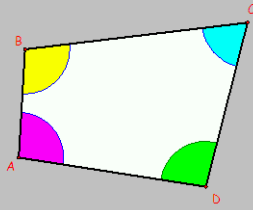If you move the vertices of the triangle does the sum of the angles remain the same?

Show SUM

QUADRILATERAL :Sum of the Interior Angles

Hypothesis: What is the sum of the interior angles of a quadrilateral?

Show The Sum of the Angles   Reset

If you move the vertices of the quadrilateral does the sum remain the same?

Show SUM

Interior Angles of Polygons-Looking for Patterns

Diagram	Number of Sides	Sum of Interior Angles	Understanding
	3	180°	FACT: The sum of the angles in any triangle is 180°.
	4	Show me!	The interior of a quadrilateral has two triangles, and sum sum of the angles in each triangle is 180°, so 2 x 180° = 360°
	5	Show me!	

Sum of the Interior Angles of a Polygon (GSP®4 file)

Angles Polygon.gsp

Examining the Interior Angles of a Quadrilateral

Return

Drag any point to form new quadrilateral.

Show Angle Measurements

Examining the Interior Angles of a Pentagon

Return

Drag any point to form new quadrilateral.

Number of Triangles = 3

Show Angle Measurements

Hexagon Drag any vertex

Show Angle Measurements
Show Triangle Sub-Divisions
Show Sum of Interior Angles

Regular Hexagon Drag point A

Show Angle Measurements
Show Triangle Sub-Divisions
Show Sum of Interior Angles

Sum of the Interior Angles of a Quadrilateral

Quadrilateral WXYZ consists of **4 triangles**. The 4 triangles share vertex O inside the quadrilateral. The **sum of the angles at point O is 360°** (since there are 360° in one complete rotation).

The rest of the angles in the triangles form the interior angles of the quadrilateral.
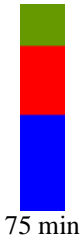
Therefore,
Sum of the interior angles in WXYZ
= (sum of interior angles in 4 triangles) - 360°
= 4 x 180° - 360°
= 360°

m∠XWZ = 82°
m∠WZY = 106°
m∠ZYX = 86°
m∠YXW = 87°
Sum of Interior Angles = 360°

Return

75 min

Math Learning Goals

- Practise solving problems using the geometry explored in previous lessons.
- Make connections to solving equations.

Assessment Opportunities

Minds On ...

Pairs → Equation Game

Use BLM 8.7.1 Equation Strips. Cut each solution into strips and place in an envelope.

Pairs of students solve the equations by correctly ordering the strips.

Whole Class → Discussion

Review the concept that solving an equation means finding the value of the variable that makes the statement true, i.e., so that the expression on the left side of the equal sign balances the expression on the right side of the equal sign.

Action!

Small Groups → Discussion

Students scan BLM 8.7.2 Connecting Algebra to Geometry and determine three clarifying questions to help them understand the task.

As a class, discuss the clarifying questions.

Pairs → Practice

Students complete BLM 8.7.2.

Learning Skills (Teamwork)/Observation/Mental Note: Continuously check as pairs work. Check one pair's work (this first pair becomes the expert pair on a question) and refer other pairs to the first pair for a correct solution.

Consolidate Debrief

Whole Class → Concept Map

Use Smart Ideas software to create a concept map with various titles, for example, Angle Theorems.

Brainstorm a list of theorems.

Review the list and categorize the theorems, e.g. triangle angle theorems.

Discuss the categories and display the concept map.

Extension: Some students are ready to solve equations with variables on both sides. Challenge these students to create new Equation Strips.

Some students will need more instruction on solving simple equations. This lesson provides an opportunity to differentiate instruction based on student readiness.

Application

Home Activity or Further Classroom Consolidation

Create five quiz questions, write solutions, and check them for accuracy.

These quiz questions could be exchanged with a partner for review or form part of a review package for the class.

8.7.1: Equation Strips

1.

$$2a + a = 90$$

$$3a = 90$$

$$\frac{3a}{3} = \frac{90}{3}$$

$$a = 30$$

2.

$$2b = 40 + 70$$

$$2b = 110$$

$$\frac{2b}{2} = \frac{110}{2}$$

$$b = 55$$

3.

$$2b + 3x = 100$$

$$5x = 100$$

$$\frac{5x}{5} = \frac{100}{5}$$

$$x = 20$$

4.

$$4c + 50 = 170$$

$$4c + 50 - 50 = 170 - 50$$

$$4c = 120$$

$$\frac{4c}{4} = \frac{120}{4}$$

$$c = 30$$

5.

$$5g - 32 = 108$$

$$5g - 32 + 32 = 108 + 32$$

$$5g = 140$$

$$\frac{5g}{5} = \frac{140}{5}$$

$$g = 28$$

8.7.2: Connecting Algebra to Geometry

1. a) The sum of the interior angles in a triangle is:
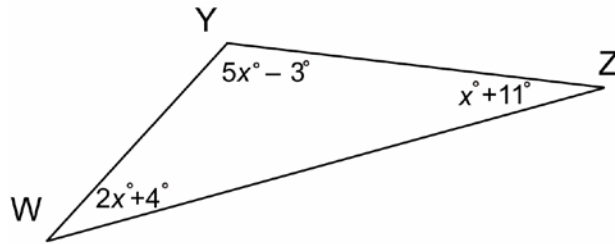
    b) An equation that models the sum of the interior angles in this triangle is:

    c) Solve the equation to determine the value of x.

    d) Use the value of x to calculate the size of:

    ∠W: ∠Y: ∠Z:

2. a) The sum of the angles in a right angle is:

    b) Write 2 equations to model the sums of the 2 sets of angles that add to 90°:

    (i)

    (ii)
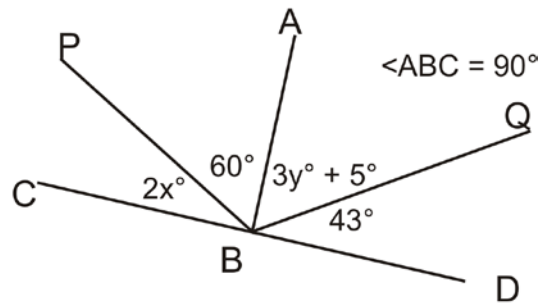
    c) Solve these equations to determine the values.

    (i) solve for x^o (ii) solve for y^o

    d) Use the values of x and y to calculate the size of:

    ∠CBP: ∠ABQ:

8.7.2: Connecting Algebra to Geometry (continued)

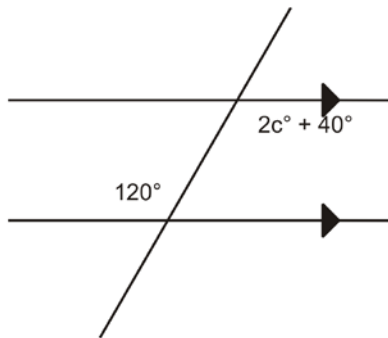3. Write an equation and solve for the unknown. State the theorem used to make the equation.
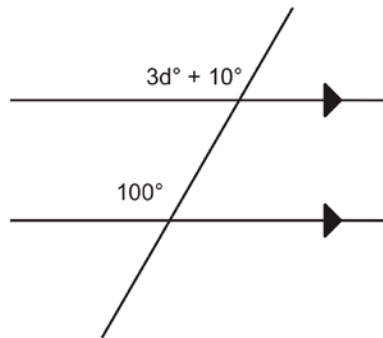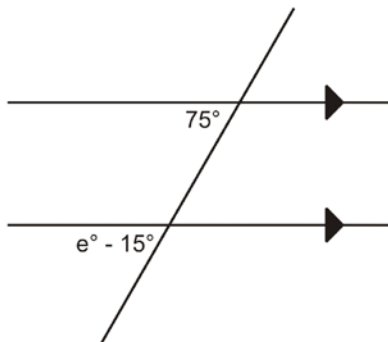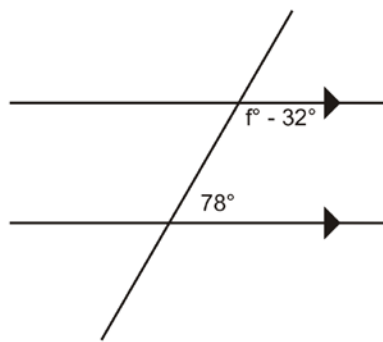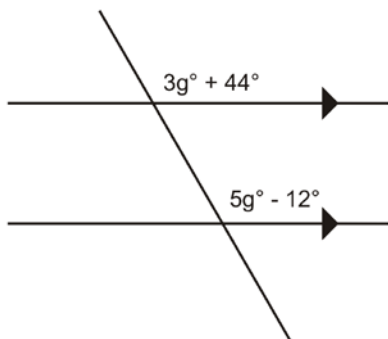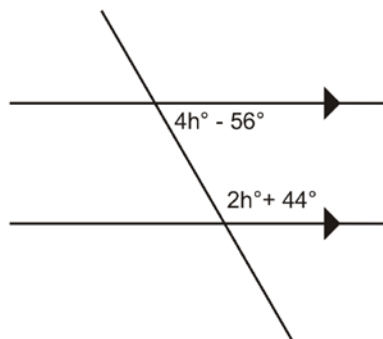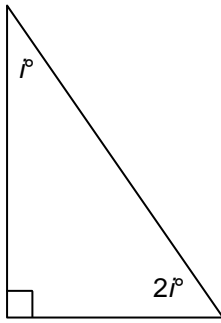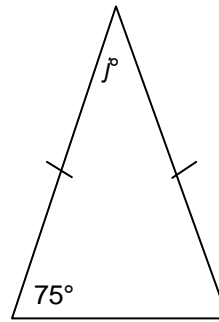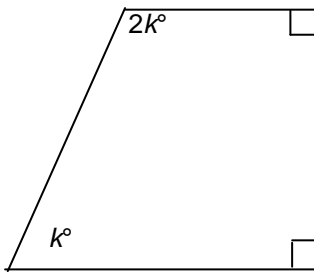
a)

$2a°$

$60°$

b)

$130°$

$b° + 20°$

c)

$2c° + 40°$

$120°$

d)

$3d° + 10°$

$100°$

e)

$75°$

$e° - 15°$

f)

$f° - 32°$

$78°$

g)

$3g° + 44°$

$5g° - 12°$

h)

$4h° - 56°$

$2h° + 44°$

i)

$i°$

$2i°$

j)

$j°$

75°

k)

$2k°$

$k°$

l)

$2l° + 20°$

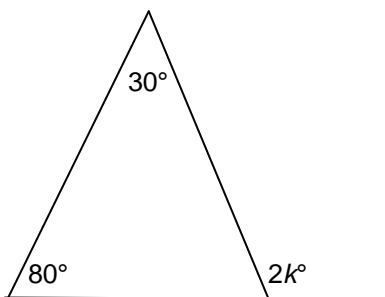$2l°$

$l°$

m)

30°
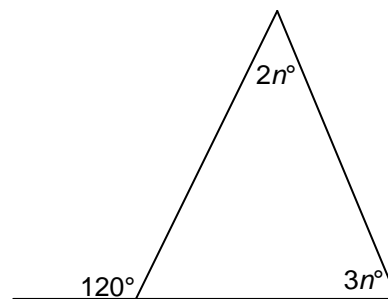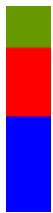
80°

$2k°$

n)

$2n°$

120°

$3n°$
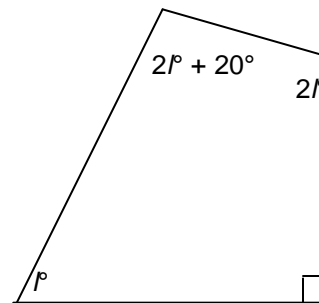
75 min

Math Learning Goals
- Use paper folding to illustrate geometric properties.

Materials
- origami paper
- scissors
- BLM 8.8.1 (Teacher)

Assessment Opportunities

Minds On ...

Whole Class → Demonstration

Introduce origami to the class as the art of paper folding. Give each student an origami or square piece of paper. Demonstrate how to fold the paper to form a paper drinking cup (use an overhead of BLM 8.8.1). Students perform the same folds.

Students then unfold their paper cup. Discuss the kinds of geometry they see in the folds.
For example:
- triangle EFB, triangle BFG, triangle DHI, triangle DIJ are congruent and isosceles
- triangle HCG and triangle JAE are right isosceles triangles
- EF is parallel to IH
- JI is parallel to FG
- JE ‖ DB ‖ GH

Visit:
www.tammyyee.com/origami.html.

This website is designed for young children but provides a good selection of appropriate origami patterns.
For example:
- fish
- iris (a bit tricky)
- cat (easy!)
- dragon fortune teller

Action!

Pairs → Investigation

Students make two of the same item – leave one folded and the second one unfolded. They tape their unfolded one onto another piece of paper and label all the geometry in preparation for their presentation. Ask students to make measurements to provide evidence that they have correctly identified the geometry.

Consolidate Debrief

Pairs → Presentation

Students present to the class their origami figure and explain the geometry they found in the unfolded shape.

Communicating/Presentation/Rubric: Assess students on their use of appropriate terminology and the clarity of their justifications as they make their presentations.

Differentiated
Reflection
Reasoning and
Proving

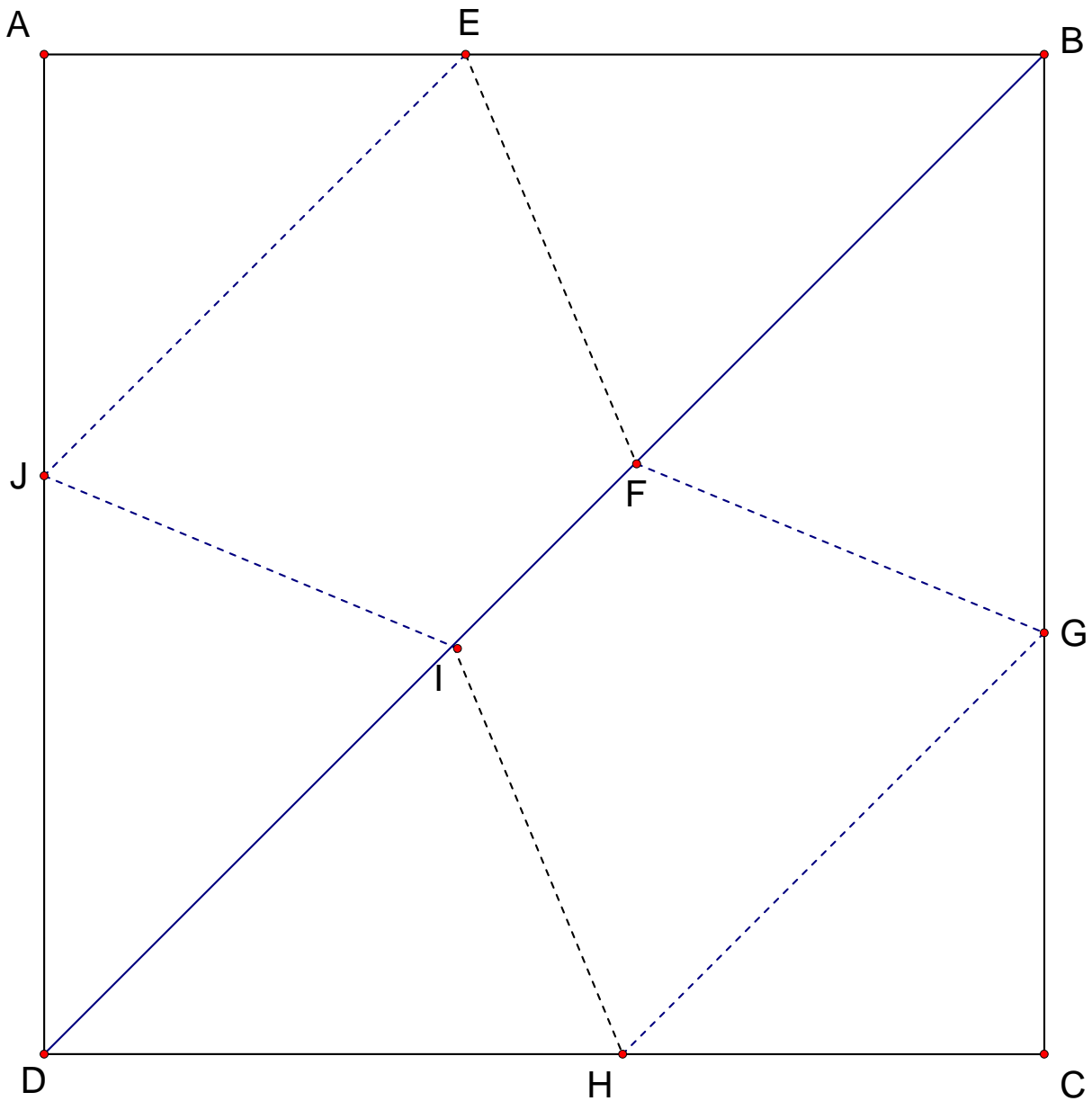### Home Activity or Further Classroom Consolidation

Look in newspapers, magazines, or on the Internet for a logo that has some geometric properties. Paste a picture of your logo on a piece of paper and identify and describe the geometry in the logo.

Submit your work for assessment.

8.8.1: Origami Paper Cup (Teacher)

1. Cut out the pattern and fold the square paper in half (lines facing out).

2. Fold right on the dotted line towards you, fold left outwards from you.

3. Tuck the top triangle inside the paper, one on each side paper.