NAME ____________________

Date __________ Period__________

SYLLABUS

ALGEBRA 2 H

Unit 9: Rational Functions

DAY	TOPIC
1	Direct, Inverse and Combined Variation
2	Graphs of Inverse Variation Page 484 In Class
3	Rational Expressions Multiplying and Dividing
4	Adding and Subtracting Rational Expressions
5	Simplifying Complex Fractions with Addition and Subtraction
6	
7	Partial Fractions
8	Solving Rational Equations
9	Rational Equations Word Problems
10	Review

Direct variation: A linear function defined by an equation of the form $y = kx$. , where $k \neq 0$.

Constant of variation: When x and y are variables, you can write $k = \frac{y}{x}$, so the ratio y:x equates the constant k.

$$\frac{y_1}{x_1} = \frac{y_2}{x_2}$$

In general, an equation shows a direct variation if y is equal to the product of a nonzero constant and a function of *x*. The following examples illustrate direct variation

$$y = 5x^2$$
$$y = \frac{2}{5}x^3$$

Examples:

For each function, determine whether y varies directly with *x*. If so, find the constant of variation.

1.

x	*y*
2	8
3	12
5	20

2.

x	*y*
1	4
2	7
5	16

3.

x	*y*
-6	-2
3	1
12	4

4.

x	*y*
-1	-2
3	4
6	7

For each function, determine whether y varies directly with x. If so, find the constant of variation.

5. $3y = 2x$

6. $y = 2x + 3$

7. $y = \frac{x}{2}$

8. $2y - 1 = x$

<u>Inverse Variation</u>: A function of the form $y = \frac{k}{x}$ or $xy = k$ *where* $k \neq 0$

Examples

9. Suppose that *x* and *y* vary inversely, and *x* = 3 when *y* = -5. Write the function that models the inverse variation.

10. Suppose that *x* and *y* vary inversely, and *x* = 0.3 when *y* = 1.4. Write the function that models the inverse variation.

11. Is the relationship between the variables in each table a direct variation, an inverse variation, or neither? Write a function that models the direct and inverse variations.

a.

x	0.5	2	6
y	1.5	6	18

b.

x	0.2	0.6	1.2
y	12	4	2

c. If y varies directly as the square of *x*, and *y* = 50 when *x* = 2, find *y* when *x* = 6. (using proportions)

d. If *y* varies directly as *x* and *y* = 3 when *x* = 2, find *y* when *x* = 22.

e. The illumination, I, from a light source varies inversely as the square of the distance, d. If the illumination is 10 foot-candles when the distance is 2 m, find the illumination when the distance is 4m.

f. If y varies inversely as *x*, and *y* = 3 when *x* = 8, what is the value of *y* when *x* = 15? (using proportions)

$x_1 y_1 = x_2 y_2$

g. The amount of commission is directly proportional to the amount of sales. A realtor received a commission of $18,000 on the sale of a $225,000 house. How much would the commission be on a $130,000 house?

h. A train travels at a constant rate. If it travels 100 miles in 2 hours, how far will it travel in 7 hours?

<u>Combined variation</u>: combines direct and inverse variations in more complicated relationships

Examples:

Combined variation	Equation form
y varies directly with the square of *x*	
y varies inversely with the cube of *x*	
z varies jointly with *x* and *y*	
z varies jointly with *x* and *y* and inversely with *w*	
z varies directly with *x* and inversely with the product of *w* and *y*	

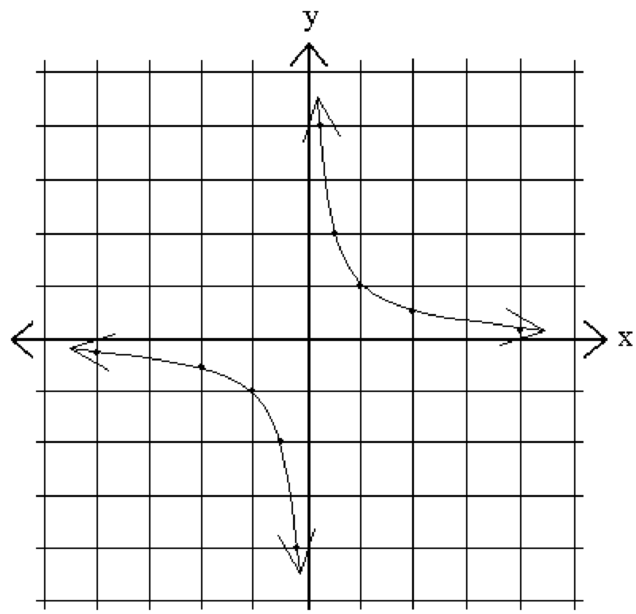
12. Newton's Law of Universal Gravitation is modeled by the formula $F = \frac{Gm_1m_2}{d^2}$. F is the gravitational force between two objects with masses m_1 *and* m_2, and d is the distance between the objects. G is the gravitational constant. Describe Newton's Law as a combined variation.

13. The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$. Describe this relationship.

Closure: Write an equation that shows what is meant for a quantity w to vary jointly as x and y and inversely as z?

Unit 9 (Rational Functions), Day 2: Graphing inverse variation

Unlike the graph of direct variation, the graph of inverse variation is not linear. Rather, it is a hyperbola- there are always two parts to the graph

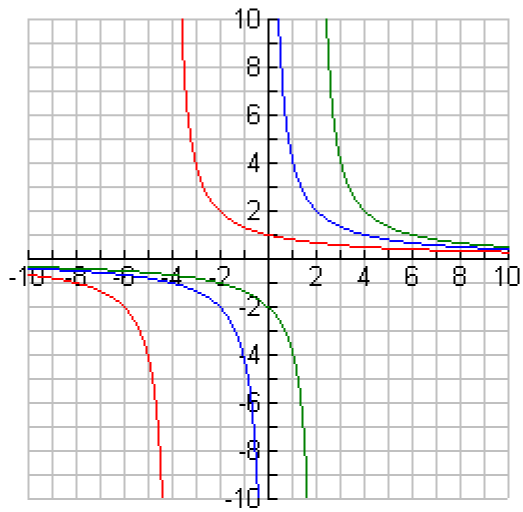


$$y = \frac{k}{x-b} + c$$

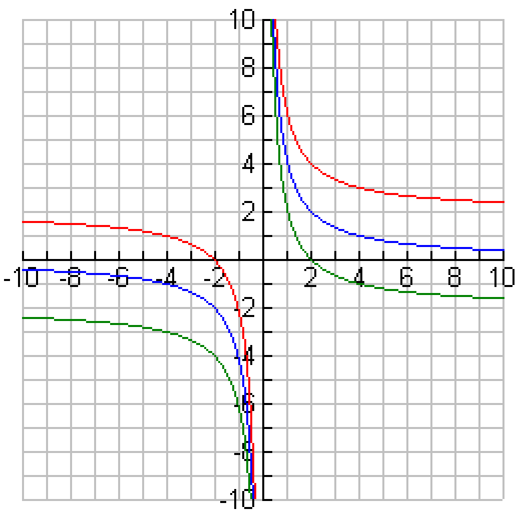
Note that the lines never cross the axes (unless it is a translation)-- they get closer and closer to $x = 0$ and $y = 0$, but x and y never equal zero.

To graph an inverse variation, make a data table and plot points. Then connect the points with a smooth (not straight) curve. There should be two curves -- one in the first quadrant (where both x and y are positive) and one in the third quadrant (where both x and y are negative).

Horizontal Translations



In your own words, describe the shift:



Vertical Translations

In your own words, describe the shift:

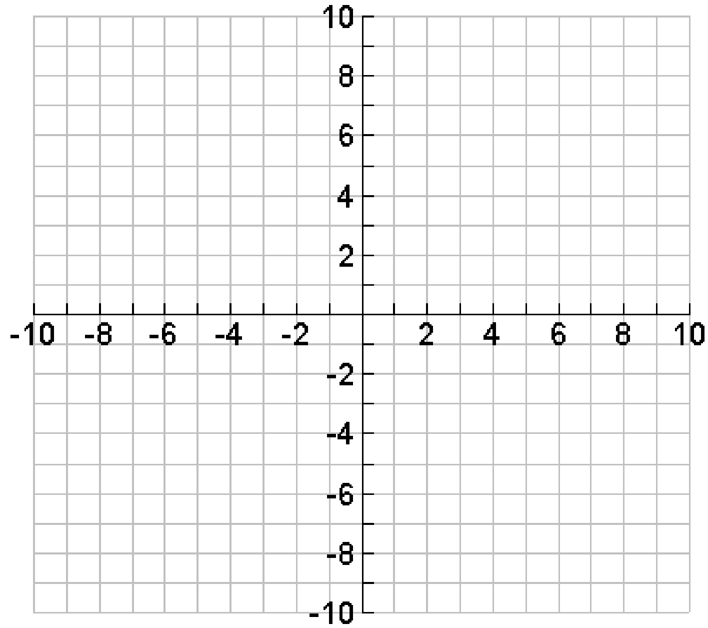
Sketch the graph of the following: $y = \frac{1}{x-2} - 3$ Remember $y = \frac{k}{x-b} + c$

To do this:

1. Draw asymptotes

2. Find your vertices: if k is positive, your graph is in the 1^{st} and 3^{rd} quadrant. If k is negative, then the graph is in the 2^{nd} and 4^{th} quadrant.

3. The graph intersects $y = x$ at $\left(\sqrt{k}, \sqrt{k}\right)$ and $\left(-\sqrt{k}, -\sqrt{k}\right)$, where k is the constant of variation. So, for this problem we have (1,1) and (-1,-1) as points. Plot these points starting at the NEW ORIGIN.



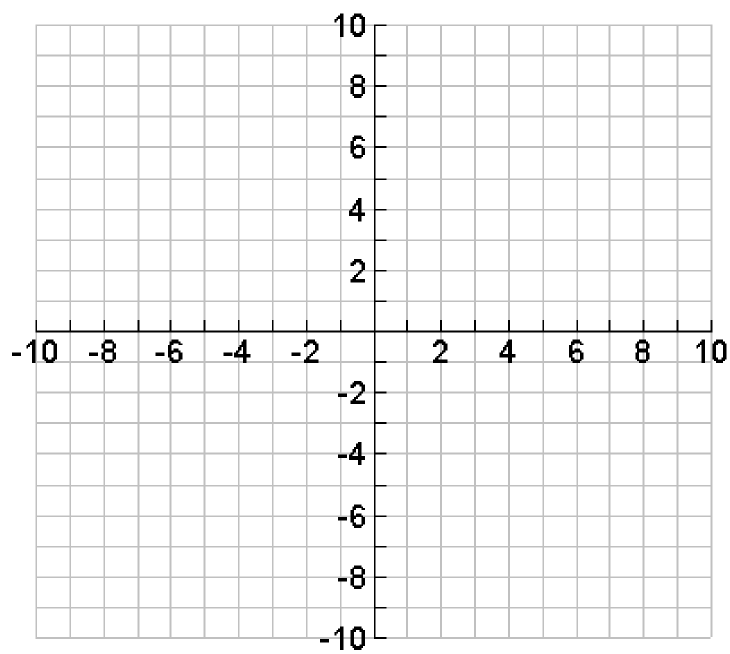
Sketch the graph of $y = \frac{-1}{x+2} - 3$

To do this:

1. draw the asymptotes

2. find your vertices, but remember, this graph is in the 2^{nd} and 4^{th} quadrant. So "ignore" the "-" and take $\sqrt{1}$, but your ordered pair is (-1,1) and (1, -1) because this graph <u>MUST BE IN THE 2^{nd} and 4^{th} QUADRANT.</u>

3. graph



Unit 9 Day 3: Multiplying and Dividing Rational Expressions

Factor the following:

1. $2x^2 - 3x + 1$ 2. $4x^2 - 9$ 3. $5x^2 + 6x + 1$

4. $10x^2 - 10$

A **rational expression** is in simplest form when its numerator and denominator are polynomials that have no common divisors.

Simplest form

$\frac{x}{x-1}$ $\frac{2}{x^2+3}$

Not in simplest form

$\frac{\frac{1}{x}}{x+1}$ $\frac{2(x-3)}{3(x-3}$

Examples:

1. Simplify and state any restrictions on the variable

a. $\frac{x^2+10x+25}{x^2+9x+20}$

b. $\frac{-27x^3y}{9x^4y}$

c. $\frac{6-3x}{x^2-6x+8}$

d. $\frac{x^2-6x-16}{x^2+5x+6}$

2. Multiply and state any restrictions on the variables

a. $\frac{2x^2+7x+3}{x-4} \cdot \frac{x^2-16}{x^2+8x+15}$

b. $\frac{a^2-4}{a^2-1} \cdot \frac{a+1}{a^2+2a}$

c. $\frac{3x^2+5x-2}{x-5}\cdot\frac{x^2-25}{3x^2-7x+2}$

3. Divide and state any restrictions on variables

a. $\frac{4-x}{(3x+2)(x-2)}\div\frac{5(x-4)}{(x-2)(7y-5)}$

b. $\frac{3-y}{(2x+1)(x-5)}\div\frac{6(y-3)}{(2x-1)(x-7)}$

Simplify each rational expression. Be sure to state restrictions.

1. $\frac{20+40x}{20x}$

2. $\frac{4x+6}{2x+3}$

3. $\frac{3y^2-3}{y^2-1}$

4. $\frac{4x+20}{3x+15}$

5. $\frac{y^2-2y}{y^2+7y-18}\cdot\frac{y^2-81}{y^2-11y+18}$

6. $\frac{(y+6)^2}{y^2-36}\cdot\frac{3y-18}{2y+12}$

7. $\frac{y^2-49}{(y-7)^2}\div\frac{5y+35}{y^2-7y}$

8. $\frac{x^2-3x-10}{2x^2-11x+5}\div\frac{x^2-5x+6}{2x^2-7x+3}$

Unit 9 Day 4: Adding and Subtracting Rational Expressions

Add the following. Simplify where possible.

1. $\frac{2}{x}+\frac{3}{x}$

2. $\frac{4}{3x}+\frac{2}{3x}$

3. $\frac{3c}{2c-1}+\frac{5c+1}{2c-1}$

2. Explain the steps you used to simplify in question 1.

3. How is adding rational expressions (the examples in #1) similar to adding fractions?

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4. **Finding the LCM**

   a. Find the least common multiple of $4x^2-36 \ and \ 6x^2+36x+54$

   b. Find the LCM of $3x^2-9x-30 \ and \ 6x+30$

5. Adding/Subtracting Rational expressions.

   a. $\frac{1}{x^2+5x+4}+\frac{5x}{3x+3}$

   b. $\frac{1}{x^2-4x+12}+\frac{3x}{4x+8}$

   c. $\frac{7y}{5y^2-125}-\frac{4}{3y+15}$

Unit 9 Day 5: Simplifying Complex Fractions with + and -

Simplify the following. Be sure to state restrictions on the variables.

1. $\dfrac{2x^2 + 11x + 5}{3x^2 + 17x + 10}$

2. $\dfrac{7x - 28}{x^2 - 16}$

Multiply or divide. Write your answer in simplest form. Be sure to state restrictions on the variables.

3. $\dfrac{5a}{5a + 5} \cdot \dfrac{10a + 10}{a}$

4. $\dfrac{x^2 + 2x + 1}{x^2 - 1} \cdot \dfrac{x^2 + 3x + 2}{x^2 + 4x + 4}$

Complex Fractions: fractions that have a fraction in the numerator, denominator, or BOTH.

1. Simplify the following: $\dfrac{\frac{1}{x} + 3}{\frac{5}{y} + 4}$

Method 1

Method 2

2. $\dfrac{\frac{1}{x} + \frac{1}{y}}{\frac{2}{y} - \frac{1}{x}}$

3. $\dfrac{3}{1 - \frac{1}{2y}}$

Unit 9 Day 7: Partial Fractions

Resolve: $\frac{4x+41}{(x-2)(x+5)}$ into partial fractions.

$$\frac{4x+41}{(x-2)(x+5)} = \frac{A}{(x-2)} + \frac{B}{(x+5)}$$

$$\frac{4x+41}{(x-2)(x+5)} = \frac{A}{(x-2)} \bullet \frac{(x+5)}{(x+5)} + \frac{B}{(x+5)} \bullet \frac{(x-2)}{(x-2)}$$

$$\frac{4x+41}{(x-2)(x+5)} = \frac{A(x+5)}{(x-2)(x+5)} + \frac{B(x-2)}{(x+5)(x-2)}$$

$$\begin{aligned} 4x+41 &= A(x+5)+B(x-2) \\ &= Ax+5A+Bx-2B \\ &= Ax+Bx+5A-2B \\ &= x(A+B)+5A-2B \end{aligned}$$

$A+B=4$

$5A-2B=41$

solve using systems

A=7, B=-3

SO, $\frac{4x+41}{(x-2)(x+5)} = \frac{7}{(x-2)} - \frac{3}{(x+5)}$

Try: $\frac{2x}{x^2-1}$

(you should get $\frac{2x}{(x+1)(x-1)} = \frac{1}{(x+1)} + \frac{1}{(x-1)}$)

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WORKSHEET!
ALGEBRA 2

Unit 9, Day 7: Partial Fractions

Resolve each of the following rational expressions into partial fractions.

1. $\dfrac{5x-1}{x^2-x-2}$

2. $\dfrac{5x-10}{x^2-x-6}$

3. $\dfrac{3x+18}{x^2+5x+4}$

4. $\dfrac{x-8}{x^2-5x+6}$

5. $\dfrac{2x+9}{x^2-x-6}$

6. $\dfrac{1}{x^2-x}$

Unit 9, Day 8: Solving Rational Equations

Just as extraneous solutions, solutions that are introduced into a problem that are NOT solutions to the original problem, can be introduced when raising both sides of an equation to a power (unit 7), they can be introduced when you multiply both sides of an equation by the same algebraic expression.

So what does this mean we have to do once we solve a rational equation?

Solve and remember to check for extraneous solution.

1. $\frac{5}{2x-2}=\frac{15}{x^2-1}$

2. $\frac{1}{x-3}=\frac{6x}{x^2-9}$

Since we have a sum/difference in these examples, we have to find the LCD first.

3. $\frac{1}{2x}-\frac{2}{5x}=\frac{1}{2}$

4. $\frac{4}{x}-\frac{3}{x+1}=1$

5. $\frac{5}{2x}-\frac{2}{3}=\frac{1}{x}+\frac{5}{6}$

Solve each equation. Check each solution.

1. $\frac{1}{x} = \frac{x}{9}$

2. $\frac{4}{x} = \frac{x}{4}$

3. $\frac{3x}{4} = \frac{5x + 1}{3}$

4. $-\frac{4}{x + 1} = \frac{5}{3x + 1}$

5. $\frac{3}{2x - 3} = \frac{1}{5 - 2x}$

6. $\frac{x - 4}{3} = \frac{x - 2}{2}$

7. $\frac{3}{1 - x} = \frac{2}{1 + x}$

8. $\frac{2x - 3}{4} = \frac{2x - 5}{6}$

9. $\frac{1}{x} = \frac{2}{x + 3}$

10. $\frac{x - 1}{6} = \frac{x}{4}$

11. $\frac{3 - x}{6} = \frac{6 - x}{12}$

12. $\frac{4}{x + 3} = \frac{10}{2x - 1}$

13. $\frac{x - 2}{10} = \frac{x - 7}{5}$

14. $\frac{3}{3 - x} = \frac{4}{2 - x}$

15. $\frac{1}{4 - 5x} = \frac{3}{x + 9}$

16. $\frac{4}{x - 3} = \frac{2}{x + 1} + \frac{16}{x^2 - 2x - 3}$

One thing vs the other

1. The aerodynamic covering on a bicycle increases a cyclist's average speed by 10mi/h. The time for a 75-mile trip is reduced by 2 hours. What is the average speed for the trip using the aerodynamic covering?

Remember: d=rt

Express the equation in words:

Define

	Distance (miles)	Rate (mile/hour)	Time (hours)
Without covering			
With covering			

Working Together

2. Mr. and Mrs. Smith have to paint 6000 square feet of walls in their house (yep, that's right… they live in a mansion!). Mr. Smith works twice as fast as Mrs. Smith (not really but for the sake of the problem we will go with it). Working together, they can complete the job in 15 hours. How long would it take each of them to work alone?
Express the equation in words:

Define:

	Time (hours)	Rate (square ft/hour)
Mr. Smith		
Mrs. Smith		
Together		

Unit 9, Day 9: Word Problems

EXAMPLE 2 The denominator of a fraction is 5 greater than the numerator. When 3 is added to both terms of the fraction, the new fraction is equal to $\frac{1}{2}$. Find the fraction.

SOLUTION $\frac{\text{numerator} + 3}{\text{denominator} + 3}$ is equal to $\frac{1}{2}$.

Use $\left[\begin{array}{ll} n & \text{for the numerator of the fraction.} \\ n+5 & \text{for the denominator of the fraction.}\end{array}\right.$

$$\frac{n+3}{(n+5)+3} = \frac{1}{2}$$

$$\frac{n+3}{n+8} = \frac{1}{2}$$

$$M_{2(n+8)} \qquad 2n + 6 = n + 8$$

$$n = 2$$

Also, $n + 5 = 7$.

CONCLUSION The fraction is $\frac{2}{7}$.

CHECK $\frac{2+3}{7+3} \stackrel{?}{=} \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$ ✓

CLASSROOM EXERCISES

Solve.

1. Tom can mow a lawn in 5 hours. His friend can mow the same lawn in 6 hours. How long does it take to mow the lawn when they work together?
2. The denominator of a fraction is one more than the numerator. When 4 is added to the numerator and 6 is added to the denominator the result equals $\frac{3}{4}$. Find the fraction.
3. Sue can build a table in 12 hours. When she works with Bob, it takes $6\frac{2}{3}$ hours to build a table. How long does it take Bob to build a table?
4. The denominator of a certain fraction is 8 less than the numerator. When 3 is subtracted from both the numerator and the denominator, the resulting fraction equals 3. Find the fraction.

EXERCISES

[A] Solve.

1. The numerator of a fraction is 4 less than the denominator. When both numerator and denominator are increased by 6, the resulting fraction equals $\frac{2}{3}$. Find the fraction.
2. The denominator of a fraction is 3 less than twice the numerator. When the

Unit 9 (Rational Functions), Day 10: Review

1. Write the function and find the value of z. z varies directly with the square of x and inversely with y. If z = 9 when x = 3 and y = 3, find z when x = 2 and y = 4.

2. Write an equation to model the data in the table:

x	-2	-1	4
y	-0.5	-1	0.25

Perform each operation and simplify. State any restrictions.

3. $\frac{12x^2-3x}{x^3-1}\cdot\frac{x^2+x-2}{4x^2+7x-2}$

4. $\frac{4x^2-64}{2x+8}\div\frac{(x-4)^2}{8x-32}$

5. $\frac{3}{y^2-1}+\frac{y}{7y-7}$

6. $\frac{3x}{x^2-4}-\frac{1}{x^2}$

7. $\frac{\frac{2x}{y^2}-1}{\frac{1}{y}+x}$

8. $\frac{2(x+2)^{-2}}{\frac{4}{x}\cdot(x+2)^{-1}}$

Resolve into partial fractions.

9. $\frac{9x-3}{2x^2+x-1}$

10. $\frac{2x}{x^2-25}$

Solve each equation. Check for extraneous solutions.

11. $\frac{2}{x}+\frac{6}{x-1}=\frac{6}{x^2-x}$

12. $\frac{6}{x-1}+\frac{2x}{x-2}=2$

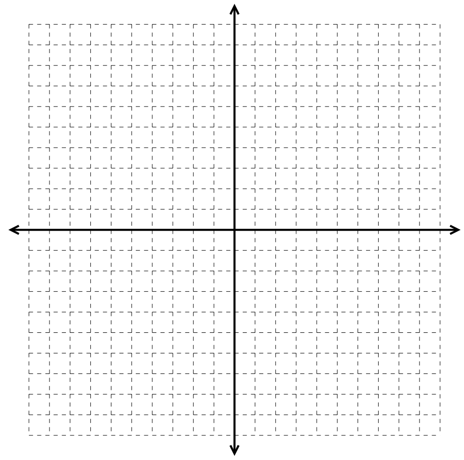
Word Problems – Write and equation and solve each.

13. It takes Patty one hour more to clean the house than Bill if they each work alone. If they work together, it takes them $1\frac{5}{7}$ hours to clean the house. How long does it take each of them working alone?

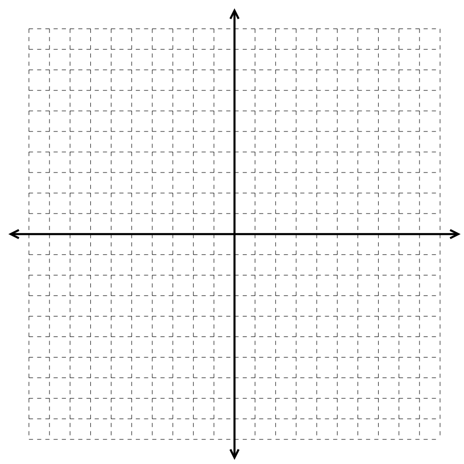
14. The denominator of a fraction is 2 more than the numerator. If the numerator is decreased by 5 and the denominator is decreased by 6, the result is $1\frac{1}{3}$. Find the fraction.

Graph each of the following and write the equation of the asymptotes. Show all work.

15. $y = 4 + \frac{1}{x-3}$



16. $y = \frac{5}{2x} + 1$



17. $y = \frac{-2}{x+5} - 3$