UNIT HW ROTATION ANSWER KEY

Conceptual Questions

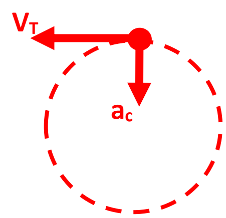
1) ____**D**_What type of linear acceleration does an object moving with constant linear speed (s_T) in a circular path experience?

A) free fall
C) linear acceleration
B) constant acceleration
D) centripetal acceleration

Expl. *The object tends to get pushed or pulled to the center by some force which could be the tension of the rope holding a spinning bucket or the gravity of the planet pulling a satellite to it, or the normal force of the walls of a washing machine pushing the clothes towards the center.*

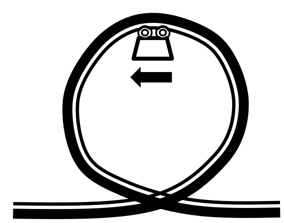
2) ____**C**__An object moves in a circular path at a constant linear speed. Compare the direction of the object's linear velocity (v_T) and centripetal acceleration (a_c) vectors.

A) Both vectors point in the same direction.
B) The vectors point in opposite directions.
C) The vectors are perpendicular.
D) The question is meaningless, since the acceleration is zero.



Expl. The velocity is tangential, around the circumference of the circle the object is rotating about. The acceleration always points towards the center of the circle.

3) ___**A**__A roller coaster car is on a track that forms a circular loop in the vertical plane. If the car is to just maintain contact with track at the top of the loop, what is the minimum value for its centripetal acceleration at this point?



A) g
B) 0.5g
C) 4g
D) 2g

Expl. The minimal value for $\boldsymbol{a_c = v^2/r}$ is $\boldsymbol{g}$, the **acceleration** due to gravity. If the car is moving faster than the minimum $\boldsymbol{v^2/r}$ (if $\mathbf{a_c > g}$) then there will also be an acceleration $\boldsymbol{a}$ downwards applied to the car due to the **normal force**, F_N, of the loop-the-loop pushing down on the car. If the car moves any slower than the minimum $\boldsymbol{v^2/r}$ (if $\mathbf{a_c < g}$), then the velocity will not be fast enough to overcome **g** and the car will fall downwards and crash.

4) ___**E**__ A cowboy spins a lasso. Which of the following quantities changes as the radius of the lasso rope gets longer?

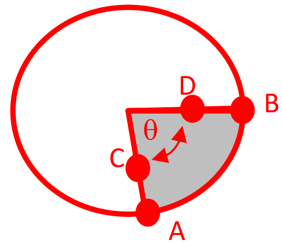
A) the angular velocity, ω, of the lasso
B) the tangential velocity, v_T, of the lasso
C) the centripetal acceleration, a_c, of the lasso
D) only A) and B)
E) only B) and C)

Expl. Only quantities affected by r, the **radius** of the lasso, will be affected. The **angular velocity** formula is $\omega = \frac{\Delta\theta}{\Delta t}$, so it will not be affected. The formula for **tangential velocity**, is $v_T = \omega \cdot r$, so it will be affected. The formula for centripetal acceleration is $a_c = \frac{v^2}{r}$, so it will be affected.

5) __**D.**__ Two flies land on a spinning lazy Susan on the table. One lands on the outer edge and the other one closer to the middle. Which one experiences a greater angular velocity, ω?

A) the one on the outer edge.
B) the one nearer the middle.
C) it depends on their mass.
D) their angular velocities are the same.

Expl. The **angular velocity** is $\omega = \frac{\Delta\theta}{\Delta t}$, so it only depends on the amount of **angular displacement** $\Delta\theta$ and the **amount of time** Δt passed, not the **radius**. If we look at the figure at right, we can see that the point C and A travel the same **angular distance** θ ending up at B and D respectively, irrespective of their distance (r) from the center of the circle.

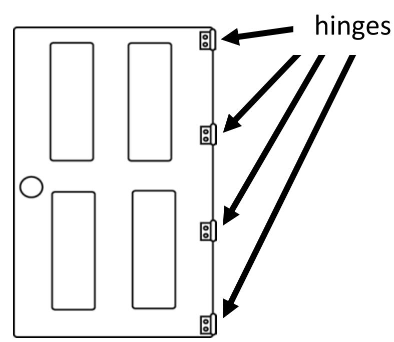


6) __**C.**__ The name of the quantity which is greater for a long 100 kg barbell than for a short 100 kg barbell, and makes the long barbell harder to twist is rotational

A) momentum.
B) kinetic energy.
C) inertia.
D) weight.

Expl. inertia depends not only on mass but where the mass is located. The further the **mass** is from the axis of rotation, the greater the **rotational inertia**. **Weight** depends on gravity which is the same for both barbells. **Momentum** and **kinetic energy** depend on the velocity of the spin but the assumption is that they are both spun at the same velocity.

7) __**D.**__ Why can't you open a door by pushing on its hinged side?



A) The distance r to the axis of rotation is zero.
B) the torque is zero
C) an infinite force would have to be applied
D) all of the above are true
E) only A) and B) are correct

Expl. In order to get some turning **force** (**torque**), you need a force applied at a certain distance from the **axis of rotation**. The hinges are located at the axis of rotation. If either *r* or *F* are zero, then the torque (τ) is zero.

8) __**C.**__ Rotational inertia

A) is more affected by the amount of mass than the distance that mass is from the axis of rotation.
B) is greatest for a solid sphere.
C) is the tendency for an object to keep rotating once set in motion.
D) is the tendency for an object to keep moving in a straight line once set in motion.

Expl. by definition, **rotational inertia** is the tendency of an object to keep rotating. Answer A is wrong because **radius**, *r*, affects *I* more than **mass**, *m*, since radius is squared (***I* = *mr*²** → $I = mr^2$). Answer B is wrong because I is greatest for a hollow hoop. Ans. D is wrong because it is the definition for linear inertia, not rotational.

9) __**C.**__ A solid sphere and a hoop roll down an incline. The hoop is slower than the sphere if

A) the mass of the hoop equals the mass of the solid sphere

B) the radius of the hoop equals the radius of the solid sphere

C) both masses and radii are equal

D) the hoop is always slower regardless of their masses and radii.

Expl. For a given **mass** and **radius**, the **hoop** will always be slower than the **sphere** because its rotational inertia will be larger ($I_{hoop} > I_{sphere}$). Compare the formulas $mr^2 > \frac{2}{5}mr^2$

10) __**A.**__ Suppose a solid sphere and a hoop are rolled up an incline plane with the same initial velocity. Which object will travel furthest up the plane?

A) The solid sphere

B) The hoop

C) maximum heights up the incline plane will be the same since their initial velocities are the same!

Expl. For a given **mass** and **radius**, the **hoop** will always require more energy than the **sphere** to overcome its rotational inertia ($I_{hoop} > I_{sphere}$). Compare the formulas $mr^2 > \frac{2}{5}mr^2$. So the initial kinetic energy given to each shape will be used more for rotation (spinning) by the **hoop** than for translation (moving in a straight line up the incline).

11) __**C.**__ An ice skater spins with her arms folded. When she extends her arms outward her angular momentum

A) increases B) decreases **C) remains the same**

Expl. **Angular momentum** (L) is a conserved quantity like energy. It is given by **$L = I\cdot\omega$.** It depends on the amount of **rot. inertia** and the **angular velocity**. With arms extended, the rot. inertia ***increases*** but the angular velocity ***decreases*** by the same amount. So angular momentum remains the same regardless of the placement of the mass.

12) __**B.**__ An ice skater spins with her arms folded. When she extends her arms outward her angular velocity

A) increases **B) decreases** C) remains the same

Expl. **Angular momentum** (L) is a conserved quantity like energy. It is given by **$L = I\cdot\omega$.** If the **rotational inertia**, **I,** increases by moving the **mass** away from the axis or rotation, then the **angular velocity**, ω, will decrease.

13) __**A.**__ An ice skater spins with her arms folded. When she extends her arms outward her rotational inertia

A) increases B) decreases C) remains the same

Expl. **Rotational Inertia** is given by $\boldsymbol{I = m\cdot r^2}$. If she extends her arms, they are further away from the axis of rotation and so r increases. If r increases, I increases.

14) __**B.**__ An ice skater spins with her arms folded. When she extends her arms outward her angular acceleration

A) increases **B) decreases** C) remains the same

Expl. angular acceleration is $\alpha = \frac{\Delta\omega}{\Delta t}$. If ω, the **angular velocity**, decreases when the ice skater's arms are extended, it means that the **angular acceleration**, α, will also decrease.

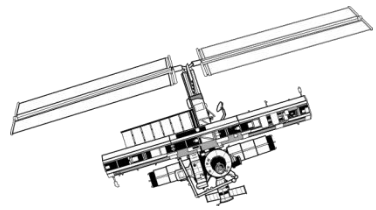
Quantitative Questions

1) ___**A)**___ What is the centripetal acceleration of a point on the perimeter of a bicycle wheel of diameter 0.70 m when the bike is moving 8.0 m/s?

A) 91 m/s^2 C) 2.1×10^2 m/s^2
B) 1.8×10^2 m/s^2 D) 2.7×10^2 m/s^2

Ans. $a_c = \frac{v^2}{r} = \frac{(8.0\frac{m}{s})^2}{0.70\ m} =$ **91 m/s^2**

2) ___**D)**___ The Gemini 11 mission attempted to produce artificial gravity by rotating the capsule around the Agena Target Vehicle which it was attached to by a tether. They were able to generate a small amount of artificial gravity, about 0.00015 g, by firing their side thrusters to slowly rotate the combined craft like a slow-motion pair of bolas. The resultant force was too small to be felt by either astronaut, but objects were observed moving towards the "floor" of the capsule. How many revolutions per minute must a circular, rotating space station of radius 1000 m rotate to produce an artificial gravity of 9.80 m/s^2?



A) 0.65 rpm C) 0.85 rpm
B) 0.75 rpm **D) 0.95 rpm**

Ans. $g = a_c = \omega^2 \cdot r \rightarrow 9.8\ m/s^2 = \omega^2 \cdot (1000\ m) \rightarrow \frac{9.8\ m/s^2}{1000\ m} = \omega^2 \rightarrow \sqrt{\frac{9.8\ m/s^2}{1000\ m}} = \omega \rightarrow 0.099\ rad/s = \omega$

In rotations per minute, this is: $\frac{0.099\ rad}{s} \times \frac{1\ rot}{2\pi\ rad} \times \frac{60\ s}{1\ min} =$ **0.95 rpm**

3) ___**D)**___ An F-16 figher jet can do 9g maneuvers and reach a maximum speed of Mach 2 (680 km/hr). If it is flying at a velocity of 600 m/s and experiences an acceleration of 4g when pulling out of the dive, what is the radius of curvature of the loop in which the F-16 is flying?

A) 640 m C) 7100 m
B) 1200 m **D) 9200 m**

Ans. $a_c = \frac{v^2}{r} \rightarrow 4g = \frac{(600\frac{m}{s})^2}{r} \rightarrow r = \frac{(600\frac{m}{s})^2}{4g} \rightarrow r = \frac{(600\frac{m}{s})^2}{4(9.8\frac{m}{s^2})} \rightarrow$ **$r = 9183.67\ m$**

4) ___**C)**___ The tallest Ferris wheel in the world is located in Singapore. Standing 42 stories high and holding as many as 780 passengers, the Ferris wheel has a diameter of 150 meters and takes approximately 30 minutes to make a full circle. Determine the angular velocity of riders on the Singapore Flyer.

A) 0.009 rads/s
B) 0.006 rads/s
C) 0.003 rads/s
D) 0.001 rads/s

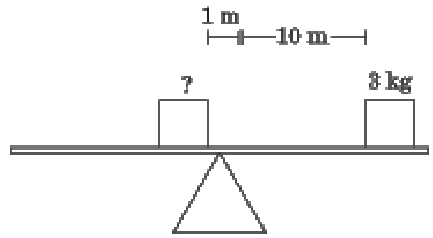
Ans. $\omega = \frac{\Delta\theta}{\Delta t} = \frac{1\ rot}{30\ min} = \frac{2\pi}{30\ min \times \frac{60\ s}{1\ min}} = = \frac{2\pi}{1800\ s} =$ **0.003 rad/s**

5) __D)__ A washing machine, starting from rest, accelerates within 3.14 s to a point where it is revolving at a frequency of 2.00 rev./sec. Its angular acceleration is most nearly:

(A) 0.100 rad/s^2
(B) 0.637 rad/s^2
(C)2.00 rad/s^2
(D) 4.00 rad/s^2
(E) 6.28 rad/s^2

Ans. $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{2.00\ rev/s}{3.14\ s} = \frac{2\frac{rev}{s}\times\frac{2\pi}{1\ rev}}{3.14\ s} = = \frac{4\pi}{3.14\ s} =$ **4.00 rad/s^2**

6) __D)__ Two objects rest on a seesaw. The first object has a mass of 3 kg and rests 10 m from the pivot. The other rests 1 m from the pivot. What is the mass of the second object if the seesaw is in equilibrium?



(A) 0.3 kg
(B) 3 kg
(C) 10 kg
(D) 30 kg
(E) 50 kg

Ans. the **clockwise torque** (τ_c) must equal the **counterclockwise torque** (τ_{cc}) for equilibrium to occur.

τ_c = F × r = 3 kg·9.8 m/s^2·10 m = 294 N·m

τ_{cc} = F × r = τ_c = 294 N·m

294.4 N·m = F × 1 m → **294.4** N = F → 294.4 N = **mg** → 294.4 N = m·(**9.8 m/s^s**) →**30.04 kg = m**

7) __E)__ What is the angular acceleration of a 0.1 kg record with a radius of 0.1 m to which a torque of 0.005 N· m is applied?

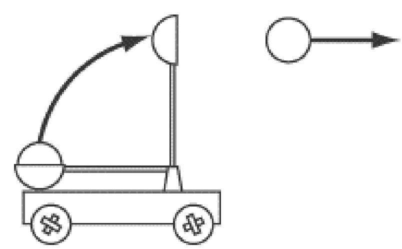
(A)0.1 rad/s^2 (B)0.5 rad/s^2 (C)1 rad/s^2 (D)5 rad/s^2 **(E)10 rad/s^2**

Ans. The other formula for torque is $\tau_c = I\cdot\alpha$

Solving for α→ $\frac{\tau_c}{I} = \alpha$

Substituting givens → $\frac{\mathbf{0.005\ N\cdot m}}{I} = \alpha \rightarrow \frac{0.005\ N\cdot m}{\frac{1}{2}Imr^2} = \alpha \rightarrow \frac{0.005\ N\cdot m}{\frac{1}{2}(0.1\ kg)(0.1)^2} = \alpha \rightarrow \mathbf{10\frac{rad}{s^2} = \alpha}$

8) __B)__ A catapult with a basket of mass 50 kg launches a 200 kg rock by swinging around from a horizontal to a vertical position with an angular velocity of 2.0 rad/s. Assuming the rest of the catapult is massless and the catapult arm is 10 m long, what is the velocity of the rock as it leaves the catapult?



(A)10 m/s **(B)20 m/s** (C)25 m/s (D)50 m/s (E)100 m/s

Ans. v = ωr = 2.0 rad/s· 10 m = **20 m/s**

Use the following information for Probls. 9 – 10.

Elmira, New York boasts of having the fastest carousel ride in the world. The merry-go-round at Eldridge Park takes riders on a spin at 18 mi/hr (8.0 m/s). The radius of the circle about which the outside riders move is approximately 7.4 m.

9) ___**E)**___ What is the rotational inertia of the 1500 kg carousel (assume it is disk shaped)?

(A) 9,840 kg·m^2
(B) 12,230 kg·m^2
(C) 16,260 kg·m^2
(D) 22,940 kg·m^2
(E) 41,070 kg·m^2

Ans. The **rotational inertia** of a disk shape is **$I = \frac{1}{2}mr^2$** = $\frac{1}{2}(1500\text{ kg})(7.4\text{ m})^2$ = **41,070 kg·m^2**

10) ___**A)**___ What is the angular momentum of the carousel?

(A)44,400 kg·m^2/s
(B) 22,200 kg·m^2/s
(C) 11,100 kg·m^2/s
(D) 5,550 kg·m^2/s
(E) 2,225 kg·m^2/s

Ans. The **angular momentum** is **$L = I\cdot\omega$** $=41{,}070\text{ kg}\cdot\text{m}^2\left(\frac{8\frac{m}{s}}{7.4\ m}\right)$= **44,400 kgm^2/s**

Remember that **$v = \omega\cdot r$.** Solving for **ω:** $\frac{v}{r} = \omega$