ME2251 HEAT AND MASS TRANSFER L T P C
3 1 0 4

UNIT I CONDUCTION 11+3

Basic Concepts – Mechanism of Heat Transfer – Conduction, Convection and Radiation – Fourier Law of Conduction - General Differential equation of Heat Conduction –– Cartesian and Cylindrical Coordinates – One Dimensional Steady State Heat Conduction – Conduction through Plane Wall, Cylinders and Spherical systems – Composite Systems – Conduction with Internal Heat Generation – Extended Surfaces – Unsteady Heat Conduction – Lumped Analysis – Use of Heislers Chart.

UNIT II CONVECTION 10+3

Basic Concepts –Heat Transfer Coefficients – Boundary Layer Concept – Types of Convection – Forced Convection – Dimensional Analysis – External Flow – Flow over
Plates, Cylinders and Spheres – Internal Flow – Laminar and Turbulent Flow – Combined Laminar and Turbulent – Flow over Bank of tubes – Free Convection – Dimensional Analysis – Flow over Vertical Plate, Horizontal Plate, Inclined Plate, Cylinders and Spheres.

UNIT III PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGERS 9+3

Nusselts theory of condensation-pool boiling, flow boiling, correlations in boiling and condensation. Types of Heat Exchangers – Heat Exchanger Analysis – LMTD Method and NTU - Effectiveness – Overall Heat Transfer Coefficient – Fouling Factors.

UNIT IV RADIATION 8+3

Basic Concepts, Laws of Radiation – Stefan Boltzman Law, Kirchoffs Law –Black Body Radiation –Grey body radiation -Shape Factor Algebra – Electrical Analogy – Radiation Shields –Introduction to Gas Radiation

UNIT V MASS TRANSFER 7+3

Basic Concepts – Diffusion Mass Transfer – Fick's Law of Diffusion – Steady state Molecular Diffusion – Convective Mass Transfer – Momentum, Heat and Mass Transfer
Analogy – Convective Mass Transfer Correlations

L = 45 T = 15 TOTAL = 60 PERIODS

TEXT BOOKS

1. Sachdeva R C, "Fundamentals of Engineering Heat and Mass Transfer" New Age International, 1995.
2. Frank P. Incropera and David P. DeWitt, "Fundamentals of Heat and Mass Transfer", John Wiley and Sons, 1998.

REFERENCE BOOKS

1. Yadav R "Heat and Mass Transfer" Central Publishing House, 1995.

2. Ozisik M.N, “Heat Transfer”, McGraw-Hill Book Co., 1994.
3. Nag P.K, “ Heat Transfer”, Tata McGraw-Hill, New Delhi, 2002
4. Holman J.P “Heat and Mass Transfer” Tata McGraw-Hill, 2000.
5. Kothandaraman C.P “Fundamentals of Heat and Mass Transfer” New Age International, New Delhi, 1998.

UNIT I - CONDUCTION

INTRODUCTORY CONCEPTS AND BASIC LAWS OF HEAT TRANSFER

We recall from our knowledge of thermodynamics that heat is a form of energy transfer that takes place from a region of higher temperature to a region of lower temperature solely due to the temperature difference between the two regions. With the knowledge of thermodynamics we can determine the amount of heat transfer for any system undergoing any process from one equilibrium state to another. Thus the thermodynamics knowledge will tell us only how much heat must be transferred to achieve a specified change of state of the system. But in practice we are more interested in knowing the rate of heat transfer (i.e. heat transfer per unit time) rather than the amount. This knowledge of rate of heat transfer is necessary for a design engineer to design all types of heat transfer equipments like boilers, condensers, furnaces, cooling towers, dryers etc. The subject of heat transfer deals with the determination of the rate of heat transfer to or from a heat exchange equipment and also the temperature at any location in the device at any instant of time.

The basic requirement for heat transfer is the presence of a “temperature difference”. The temperature difference is the driving force for heat transfer, just as the voltage difference for electric current flow and pressure difference for fluid flow. One of the parameters , on which the rate of heat transfer in a certain direction depends, is the magnitude of the temperature gradient in that direction. The larger the gradient higher will be the rate of heat transfer.

Heat Transfer Mechanisms:-

There are three mechanisms by which heat transfer can take place. All the three modes require the existence of temperature difference. The three mechanisms are: (i) conduction,

(ii) convection and (iii) radiation

Conduction:-

It is the energy transfer that takes place at molecular levels. Conduction is the transfer of energy from the more energetic molecules of a substance to the adjacent less energetic molecules as a result of interaction between the molecules. In the case of liquids and gases conduction is due to collisions and diffusion of the molecules during their random motion. In solids, it is due to the vibrations of the molecules in a lattice and motion of free electrons.

Fourier's Law of Heat Conduction:-

The empirical law of conduction based on experimental results is named after the French Physicist Joseph Fourier. The law states that the rate of heat flow by conduction in any medium in any direction is proportional to the area normal to the direction of heat flow and also proportional to the temperature gradient in that direction. For example the rate of heat transfer in x-direction can be written according to Fourier's law as

$Qx \propto - A\,(dT / dx)$ ……………………(1.1)

Or

$Qx = - k\,A\,(dT / dx)$ W………………….. ..(1.2)

In equation (1.2), Qx is the rate of heat transfer in positive x-direction through area A of the medium normal to x-direction, (dT/dx) is the temperature gradient and k is the constant of proportionality and is a material property called "thermal conductivity". Since heat transfer has to take place in the direction of decreasing temperature, (dT/dx) has to be negative in the direction of heat transfer. Therefore negative sign has to be introduced in equation (1.2) to make Qx positive in the direction of decreasing temperature, thereby satisfying the second law of thermodynamics. If equation (1.2) is divided throughout by A we have qx is called the heat flux.

$q_x = (Q_x / A) = - k\ (dT / dx)$ W/m^2……….(1.3)

In the case of solids heat conduction is due to two effects: the vibration of lattice induced by the vibration of molecules positioned at relatively fixed positions, and energy transported due to the motion of free electrons. The relatively high thermal conductivities of pure metals are primarily due to the electronic component. The lattice component of thermal conductivity strongly depends on the way the molecules are arranged. For example, diamond, which is highly ordered crystalline solid, has the highest thermal conductivity at room temperature.

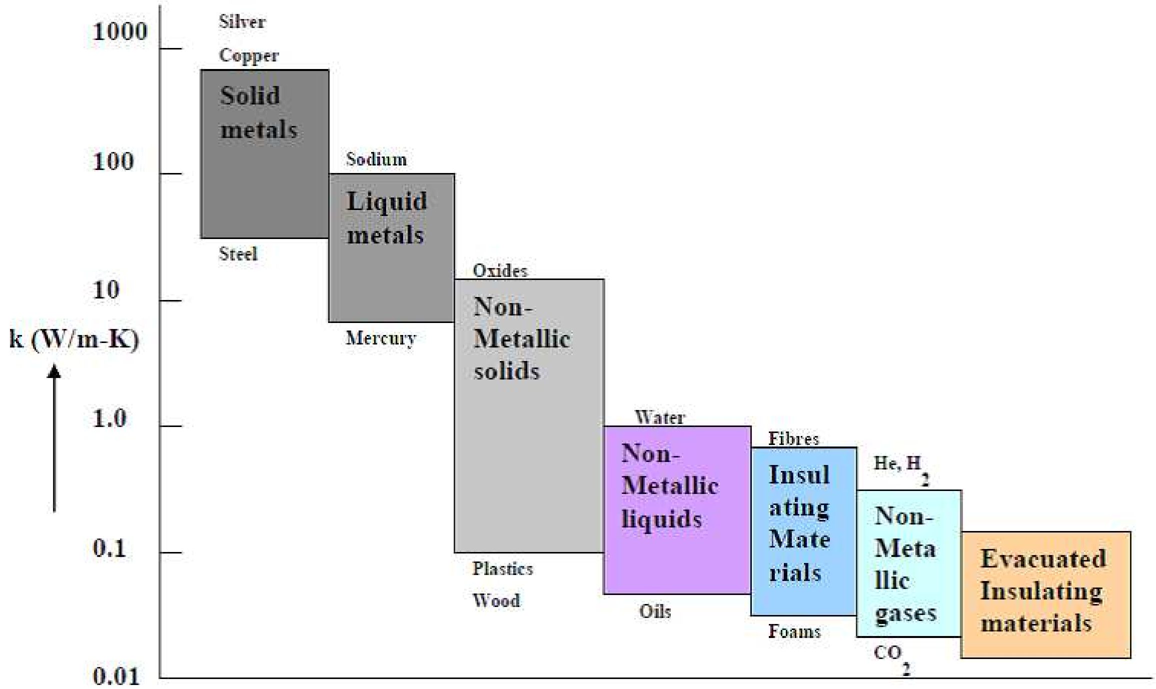


Fig. 1.1: Typical range of thermal conductivities of various materials

Unlike metals, which are good electrical and heat conductors, crystalline solids such as diamond and semiconductors such as silicon are good heat conductors but poor electrical conductors. Hence such materials find widespread use in electronic industry. Despite their high price, diamond heat sinks are used in the cooling of sensitive electronic components because of their excellent thermal conductivity. Silicon oils and gaskets are commonly used in the packaging of electronic components because they provide both good thermal contact and good electrical insulation.

One would expect that metal alloys will have high thermal conductivities, because pure metals have high thermal conductivities. For example one would expect that the value of the thermal conductivity k of a metal alloy made of two metals with thermal conductivities k1 and k2 would lie between k1 and k2.But this is not the case. In fact k of a metal alloy will be less than that of either metal.

The thermal conductivities of materials vary with temperature. But for some materials the variation is insignificant even for wide temperature range. At temperatures near absolute zero, the thermal conductivities of certain solids are extremely large. For example copper at 20 K will have a thermal conductivity of 20,000 W / (m-K), which is about 50 times the conductivity at room temperature. The temperature dependence of thermal conductivity makes the conduction heat transfer analysis more complex and involved. As a first approximation analysis for solids with variable conductivity is carried out assuming constant thermal conductivity which is an average value of the conductivity for the temperature range of interest.

Derive general heat conduction equation in Cartesian coordinates?

Consider a small rectangular element of sides dx, dy and dz as shown in figure. The energy balance of this rectangular element is obtained from first law of thermodynamics.

Net heat conducted into element from all the coordinate directions	+	**Heat generated within the element**	=	**Heat stored in the element -----1**

Net heat conducted into element from all the coordinate directions:

As per the Fourier law of heat conduction the rate of heat flow into the element in X, Y, Z directions through face ABCD, ABEF, ADHE are

$$Q_x = -k\,dy\,dz\,\frac{\partial T}{\partial x}dx \quad \}$$
$$Q_y = -k\,dx\,dz\,\frac{\partial T}{\partial y}dy \quad \} \text{-----------------}2$$
$$Q_z = -k\,dy\,dx\,\frac{\partial T}{\partial z}dz \quad \}$$

The rate of heat flow out of the element in X direction through the face EFGH is

$$Q_{x+dx} = Q_x + \frac{\partial}{\partial x}(Qx)dx \quad \text{------}3$$

Sub Equ 2 in Equ 3

$$Q_x - Q_{x+dx} = Qx - \{Q_x + \frac{\partial}{\partial x}(Qx)dx\}$$
$$Q_x - Q_{x+dx} = \frac{\partial}{\partial x}\{kdydzdx\ \frac{\partial T}{\partial x}\}$$
$$Q_x - Q_{x+dx} = kdxdydx\ \frac{\partial^2 T}{\partial x^2}$$
$$Q_x - Q_{x+dx} = kdxdydx\ \frac{\partial^2 T}{\partial x^2} \quad \text{------}4$$

Similarly

$$Q_y - Q_{y+dy} = kdxdydx\ \frac{\partial^2 T}{\partial y^2} \quad \text{------}5$$

$$\mathbf{Q_z - Q_{z+dz}} = k\,dx\,dy\,dx \frac{\partial^2 T}{\partial z^2} \text{ ------6}$$

$$\mathbf{Q = \{ (Q_x - Q_{x+dx}) + (Q_y - Q_{y+dy}) + (Q_z - Q_{z+dz}) \}} \text{ ---- 7}$$

Sub Equ 4 , 5, 6 in Equ 7

$$\mathbf{kdxdydz} \left\{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right\} \text{ ---8}$$

Heat generated internally : Qg = q dxdydz ------9

Heat stored in the element: $\mathbf{Q_{IE}} = \rho\,\mathbf{Cp}\,\frac{\partial T}{\partial t}\,dxdydz$ **-----10**

Sub Equ 8, 9, 10 in Equ 1

$$\mathbf{kdxdydz} \left\{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right\} + \mathbf{q\ dxdydz} = \rho\,\mathbf{Cp}\,\frac{\partial T}{\partial t}\,dxdydz \text{ ---11}$$

Take dxdydz commonly and divide by k

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{\rho \mathrm{Cp}}{\mathrm{k}} \frac{\partial T}{\partial t}$$ **---- Three dimensional heat conduction equation**

for Cartesian coordinates

Case 1: No heat generation $\frac{q}{k} = 0$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{\rho \mathrm{Cp}}{\mathrm{k}} \frac{\partial T}{\partial t}$$ **----- Fourier Equation**

Case 2: Steady state condition $\frac{\partial T}{\partial t} = 0$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = 0$$ **--- Poisson Equation**

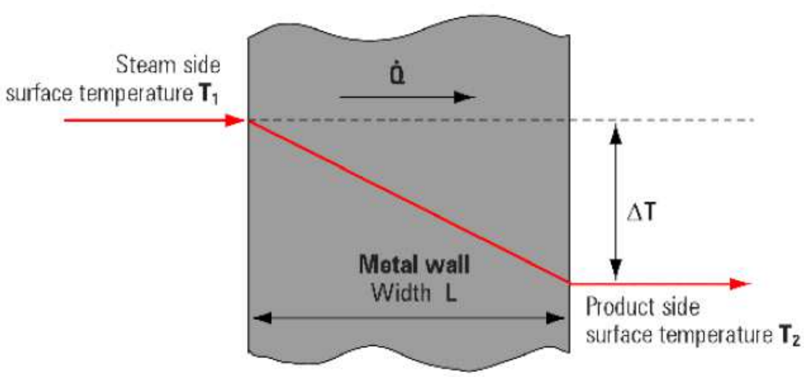
Case 3 : No heat generation and steady state : $\frac{q}{k} = 0 \quad \frac{\partial T}{\partial t} = 0$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$ **--- Laplace Equation**

Heat transfer by conduction through a simple plane wall

A good way to start is by looking at the simplest possible case, a metal wall with uniform thermal properties and specified surface temperatures.

T 1 and T 2 are the surface temperatures either side of the metal wall, of thickness L; and the temperature difference between the two surfaces is ΔT.



Ignoring the possible resistance to heat flow at the two surfaces, the process of heat flow through the wall can be derived from Fourier's law of conduction as shown in following equation. The term 'barrier' refers to a heat resistive film or the metal wall of a heat exchanger.

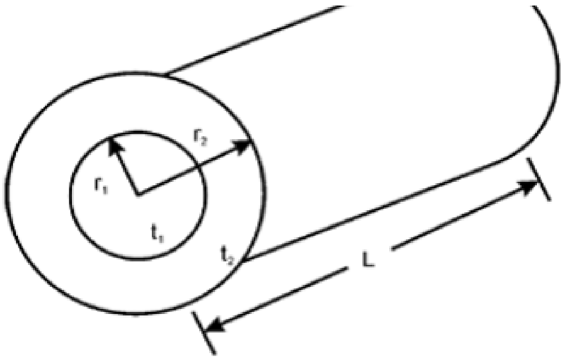
$$\dot{Q} = k A \frac{\Delta T}{x}$$

Where:

$\dot{Q}$ = Heat transferred per unit time (W)
A = Heat transfer area (m²)
k = Thermal conductivity of the barrier (W / m K or W / m °C)
ΔT = Temperature difference across the barrier (K or °C)
x = Barrier thickness (m)

$$\dot{Q} = A \frac{\Delta T}{x/k}$$

Heat Conduction through Hollow Cylinder



where,
r_1 = internal radius
r_2 = outer radius
t_1 = internal temperature
t_2 = external temperature

By Fourier's law Q = -KA.(dt/dx)

Q = -K.(A 2 λr L). dt/dr

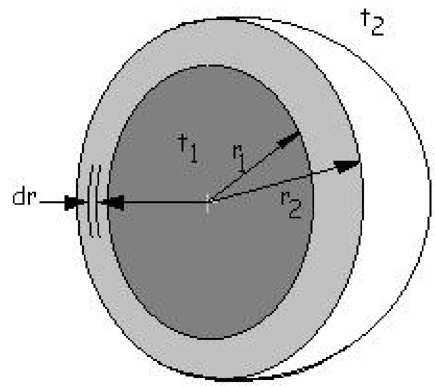
Or $\int_{t_1}^{t_2} dt = \int_{r_1}^{r_2} -(Q/\alpha.2.\Pi L).dr/r$

Or $t_1 - t_2$ = (Q/K.2.ΠL). log (r_2/r_1)

Or Q = 2Π K L ($t_1 - t_2$) / (log r_2/r_1)

Or Q = $t_1 - t_2$ / Rt , where Rt = log(r_2/r_1) / 2Π KL

Heat Conduction through Hollow Sphere



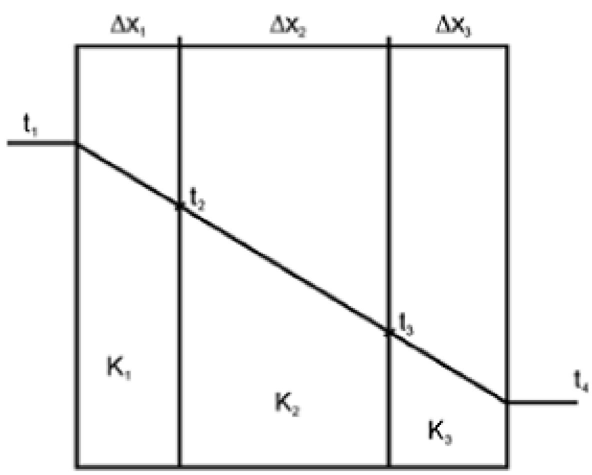
$$q_r = -kA\frac{dT}{dr} = -k\left(2\pi rL\right)\frac{dT}{dr}$$

$$q_r = \frac{4\pi\ k\left(T_{s,1} - T_{s,2}\right)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

Conductance through a Flat Composite Wall

Assume

1. Thickness of each of the three walls Δx_1, Δx_2, Δx_3
2. Thermal conductivity as K_1, K_2 and K_3
3. Surface temp. as t_1, t_2, t_3, t_4, where t_3 and t_4 are known as interfa temperatures
4. Q = heat flow through composite wall
5. A = Cross sectional area perpendicular to the path of heat flow



$$Q = -K_1 A (t_2 - t_1) / \Delta x_1 \quad = -K_2 A (t_3 - t_2) / \Delta x_2$$

$$= -K_3 A (t_4 - t_3) / \Delta X_3$$

$$\text{or} - (t2 - t1) = Q. \Delta x1 / K1\, A$$

$$\text{or } t_1 - t_2 = Q. \Delta x_1 / K_1 A$$

$$\text{Similarly } t_2 - t_3 = Q. \Delta x_2 / K_2 A$$

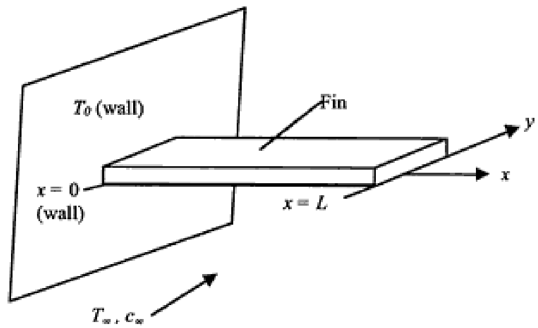
$$t_3 - t_4 = Q . \Delta x_3 / K_3 A$$

$$\text{or } (t_1 - t_2) + (t_2 - t_3) + (t_3 - t_4) = Q [(\Delta x_1/K_1A) + (\Delta x_2/K_2A) + (\Delta x_3/ K_3A)]$$

$$\text{or } Q = (t_1 - t_4) / Rt_1 + Rt_2 + Rt_3, \text{ where } Rt = \Delta x/ KA$$

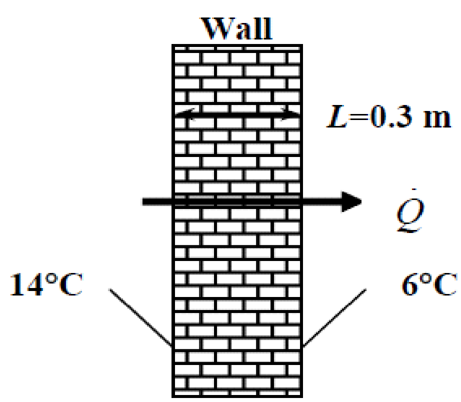
Heat Transfer From a Fin

Fins are used in a large number of applications to increase the heat transfer from surfaces. Typically, the fin material has a high thermal conductivity. The fin is exposed to a flowing fluid, which cools or heats it, with the high thermal conductivity allowing increased heat being conducted from the wall through the fin. The design of cooling fins is encountered in many situations and we thus examine heat transfer in a fin as a way of defining some criteria for design.



Solved Problems

1. Consider a 4-m-high, 6-m-wide, and 0.3-m-thick brick Wall whose thermal conductivity is k =0.8 W/m · °C. On a Certain day, the temperatures of the inner and the outer surfaces Of the wall are measured to be 14°C and 6°C, respectively. Determine the rate of heat loss through the wall on that day.



Assumptions 1 Heat transfer through the wall is steady since the surface temperatures remain constant at the specified values. 2 Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. 3. Thermal conductivity is constant. Properties The thermal conductivity is given to be k = 0.8 W/m°C

Analysis

The surface area of the wall and the rate of heat loss through the wall are

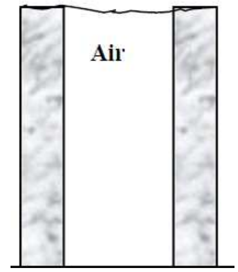
$$A = (4\text{ m}) \times (6\text{ m}) = 24\text{ m}^2$$

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.8\text{ W/m.°C})(24\text{ m}^2)\frac{(14-6)\text{°C}}{0.3\text{ m}} = \mathbf{512\ W}$$

2. Consider a 4-m-high, 6-m-wide, and 0.3-m-thick brick wall whose thermal conductivity is *k* =0.8 W/m · °C. On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be 14°C and 6°C, respectively. Determine the rate of heat loss through the wall on that day. assuming the space between the two glass layers is evacuated.

Assumptions 1 Heat transfer through the window is steady since the indoor and outdoor temperatures remain constant at the specified values. 2 Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. 3 Thermal conductivities of the glass and air are constant. 4 Heat transfer by radiation is negligible.

Properties The thermal conductivity of the glass and air are given to be kglass = 0.78 W/m.°C and kair = 0.026 W/m.°C.



Analysis The area of the window and the individual resistances are

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2.°\text{C})(2.4\ \text{m}^2)} = 0.0417\ °\text{C/W}$$

$$R_1 = R_3 = R_{\text{glass}} = \frac{L_1}{k_1 A} = \frac{0.003\ \text{m}}{(0.78\ \text{W/m.°C})(2.4\ \text{m}^2)} = 0.0016\ °\text{C/W}$$

$$R_2 = R_{air} = \frac{L_2}{k_2 A} = \frac{0.012\ \text{m}}{(0.026\ \text{W/m.°C})(2.4\ \text{m}^2)} = 0.1923\ °\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(25\ \text{W/m}^2.°\text{C})(2.4\ \text{m}^2)} = 0.0167\ °\text{C/W}$$

$$R_{total} = R_{conv,1} + 2R_1 + R_2 + R_{conv,2} = 0.0417 + 2(0.0016) + 0.1923 + 0.0167$$
$$= 0.2539\ °\text{C/W}$$

The steady rate of heat transfer through window glass then becomes

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{[24-(-5)]°\text{C}}{0.2539°\text{C/W}} = \mathbf{114\ W}$$

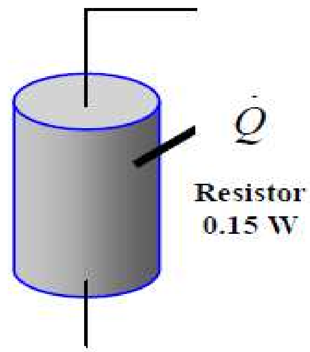
The inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty1} - T_1}{R_{conv,1}} \longrightarrow T_1 = T_{\infty1} - \dot{Q}R_{conv,1} = 24°\text{C} - (114\ \text{W})(0.0417°\text{C/W}) = \mathbf{19.2°C}$$

3. A cylindrical resistor element on a circuit board dissipates 0.15 W of power in an environment at 40°C. The resistor is 1.2 cm long, and has a diameter of 0.3 cm.

Assuming heat to be transferred uniformly from all surfaces, determine (*a*) the amount of heat this resistor dissipates during a 24-h period, (*b*) the heat flux on the surface of the resistor, in W/m2, and (*c*) the surface temperature of the resistor for a combined convection and radiation heat transfer coefficient of 9 W/m2 · °C.

Assumptions **1** Steady operating conditions exist. **2** Heat is transferred uniformly from all surfaces of the resistor.



Analysis (*a*) The amount of heat this resistor dissipates during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.15\text{ W})(24\text{ h}) = \mathbf{3.6\ Wh}$$

(*b*) The heat flux on the surface of the resistor is

$$A_s = 2\frac{\pi D^2}{4} + \pi DL = 2\frac{\pi(0.003\text{ m})^2}{4} + \pi(0.003\text{ m})(0.012\text{ m}) = 0.000127\text{ m}^2$$

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{0.15\text{ W}}{0.000127\text{ m}^2} = \mathbf{1179\ W/m^2}$$

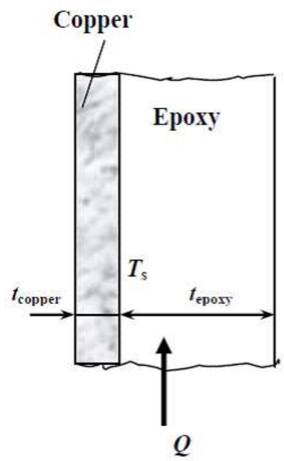
(*c*) The surface temperature of the resistor can be determined from

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = \frac{0.15\text{ W}}{(1179\text{ W/m}^2.°\text{C})(0.000127\text{ m}^2)} = \mathbf{171°C}$$

4. Heat is to be conducted along a circuit board that has a copper layer on one side. The circuit board is 15 cm long and 15 cm wide, and the thicknesses of the copper and epoxy layers are 0.1 mm and 1.2 mm, respectively. Disregarding heat transfer from side surfaces, determine the percentages of heat conduction along the copper (k =386 W/m · °C) and epoxy (k = 0.26 W/m · °C) layers. Also determine the effective thermal Conductivity of the board.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since heat transfer from the side surfaces is disregarded **3** Thermal conductivities are constant.

Properties The thermal conductivities are given to be k = 386 W/m□°C for copper and 0.26 W/m□°C for epoxy layers.



Analysis We take the length in the direction of heat transfer to be L and the width of the board to be w. Then heat conduction along this two-layer board can be expressed as

$$\dot{Q} = \dot{Q}_{copper} + \dot{Q}_{epoxy} = \left(kA\frac{\Delta T}{L}\right)_{copper} + \left(kA\frac{\Delta T}{L}\right)_{epoxy} = \left[(kt)_{copper} + (kt)_{epoxy}\right]w\frac{\Delta T}{L}$$

Heat conduction along an "equivalent" board of thickness $t = t_{copper} + t_{epoxy}$ and thermal conductivity k_{eff} can be expressed as

$$\dot{Q} = \left(kA \frac{\Delta T}{L} \right)_{\text{board}} = k_{\text{eff}} (t_{\text{copper}} + t_{\text{epoxy}}) w \frac{\Delta T}{L}$$

Setting the two relations above equal to each other and solving for the effective conductivity gives

$$k_{eff} (t_{\text{copper}} + t_{\text{epoxy}}) = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} \longrightarrow k_{eff} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}}$$

Note that heat conduction is proportional to *kt*. Substituting, the fractions of heat conducted along the copper and epoxy layers as well as the effective thermal conductivity of the board are determined to be

$$(kt)_{copper} = (386 \text{ W/m.°C})(0.0001 \text{ m}) = 0.0386 \text{ W/°C}$$

$$(kt)_{epoxy} = (0.26 \text{ W/m.°C})(0.0012 \text{ m}) = 0.000312 \text{ W/°C}$$

$$(kt)_{total} = (kt)_{copper} + (kt)_{epoxy} = 0.0386 + 0.000312 = 0.038912 \text{ W/°C}$$

$$f_{epoxy} = \frac{(kt)_{epoxy}}{(kt)_{total}} = \frac{0.000312}{0.038912} = 0.008 = \mathbf{0.8\%}$$

$$f_{copper} = \frac{(kt)_{copper}}{(kt)_{total}} = \frac{0.0386}{0.038912} = 0.992 = \mathbf{99.2\%}$$

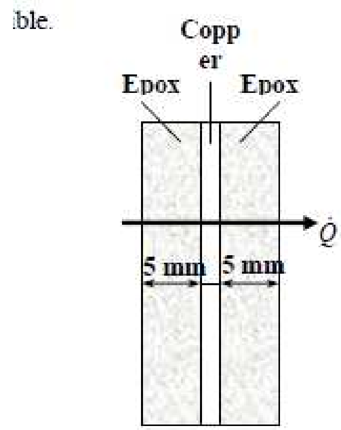
$$k_{eff} = \frac{(386 \times 0.0001 + 0.26 \times 0.0012) \text{ W/°C}}{(0.0001 + 0.0012) \text{ m}} = \mathbf{29.9 \text{ W/m.°C}}$$

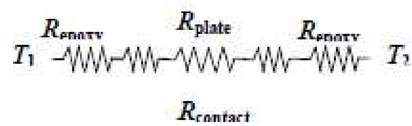
5. A1-mm-thick copper plate (k = 386 W/m · °C) is sandwiched between two 5-mm-thick epoxy boards (k = 0.26 W/m · °C) that are 15 cm 20 cm in size. If the thermal contact conductance on both sides of the copper plate is estimated to be 6000 W/m · °C, determine the error involved in the total thermal resistance of the plate if the thermal contact conductance are ignored.

Assumptions 1 Steady operating conditions exist. 2 Heat transfer is one-dimensional since the plate is large. 3 Thermal conductivities are constant.

Properties The thermal conductivities are given to be k = 386 W/m.°C for copper plates and k = 0.26 W/m.°C for epoxy boards. The contact conductance at the interface of copper-epoxy layers is given to be hc = 6000 W/m2. °C

Analysis The thermal resistances of different layers for unit surface area of 1 m2 are





$$R_{\text{contact}} = \frac{1}{h_c A_c} = \frac{1}{(6000\ \text{W/m}^2.°\text{C})(1\ \text{m}^2)} = 0.00017\ °\text{C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.001 \text{ m}}{(386 \text{ W/m.°C})(1 \text{ m}^2)} = 2.6 \times 10^{-6} \text{ °C/W}$$

$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.005 \text{ m}}{(0.26 \text{ W/m.°C})(1 \text{ m}^2)} = 0.01923 \text{ °C/W}$$

The total thermal resistance is

$$R_{\text{total}} = 2R_{\text{contact}} + R_{\text{plate}} + 2R_{\text{epoxy}}$$
$$= 2 \times 0.00017 + 2.6 \times 10^{-6} + 2 \times 0.01923 = 0.03914 \text{ °C/W}$$

Then the percent error involved in the total thermal resistance of the plate if the thermal contact resistances are ignored is determined to be

$$\%\text{Error} = \frac{2R_{\text{contact}}}{R_{\text{total}}} \times 100 = \frac{2 \times 0.00017}{0.03914} \times 100 = \mathbf{0.87\%}$$

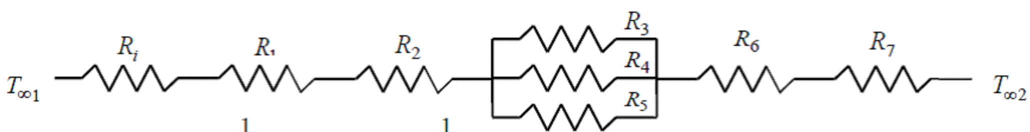
which is negligible.

6. A 4-m-high and 6-m-wide wall consists of a long 18-cm 30-cm cross section of horizontal bricks (k =0.72 W/m · °C) separated by 3-cm-thick plaster layers (k = 0.22 W/m · °C). There are also 2-cm-thick plaster layers on each side of the wall, and a 2-cm-thick rigid foam (k =0.026 W/m · °C) on the inner side of the wall. The indoor and the outdoor temperatures are 22°C and =4°C, and the convection heat transfer coefficients on the inner and the outer sides are $h1$ = 10 W/m2 · °C and $h2$ =20 W/m2 · °C, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be k = 0.72 W/m□°C for bricks, k = 0.22 W/m□°C for plaster layers, and k = 0.026 W/m□°C for the rigid foam.

Analysis We consider 1 m deep and 0.33 m high portion of wall which is representative of the entire wall. The thermal resistance network and individual resistances are



$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\ \mathrm{W/m^2.°C})(0.33 \times 1\ \mathrm{m^2})} = 0.303\ \mathrm{°C/W}$$

$$R_1 = R_{foam} = \frac{L}{kA} = \frac{0.02 \text{ m}}{(0.026 \text{ W/m.°C})(0.33 \times 1 \text{ m}^2)} = 2.33 \text{ °C/W}$$

$$R_2 = R_6 = R_{\substack{plaster\\side}} = \frac{L}{kA} = \frac{0.02 \text{ m}}{(0.22 \text{ W/m.°C})(0.30 \times 1 \text{ m}^2)} = 0.303 \text{ °C/W}$$

$$R_3 = R_5 = R_{\substack{plaster\\center}} = \frac{L}{h_o A} = \frac{0.18 \text{ m}}{(0.22 \text{ W/m.°C})(0.015 \times 1 \text{ m}^2)} = 54.55 \text{ °C/W}$$

$$R_4 = R_{brick} = \frac{L}{kA} = \frac{0.18 \text{ m}}{(0.72 \text{ W/m.°C})(0.30 \times 1 \text{ m}^2)} = 0.833 \text{ °C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(20 \text{ W/m.°C})(0.33 \times 1 \text{ m}^2)} = 0.152 \text{ °C/W}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{54.55} + \frac{1}{0.833} + \frac{1}{54.55} \longrightarrow R_{mid} = 0.81 \text{ °C/W}$$

$$R_{total} = R_i + R_1 + 2R_2 + R_{mid} + R_o = 0.303 + 2.33 + 2(0.303) + 0.81 + 0.152$$
$$= 4.201 \text{ °C/W}$$

The steady rate of heat transfer through the wall per is 0.33 m^2

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[(22 - (-4)]\text{°C}}{4.201\text{°C/W}} = 6.19 \text{ W}$$

Then steady rate of heat transfer through the entire wall becomes

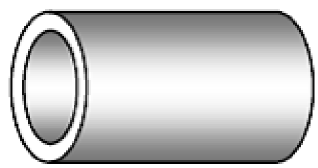
$$\dot{Q}_{total} = (6.19 \text{ W}) \frac{(4 \times 6)\text{m}^2}{0.33 \text{ m}^2} = \mathbf{450 \text{ W}}$$

7. A50-m-long section of a steam pipe whose outer diameter is 10 cm passes through an open space at 15°C. The average temperature of the outer surface of the pipe is measured to be 150°C. If the combined heat transfer coefficient on the outer surface of the pipe is 20 W/m2.°C, determine (*a*) the rate of heat loss from the steam pipe, (*c*) the thickness of fiberglass insulation (*k* =0.035 W/m · °C) needed in order to save 90 percent of the heat lost. Assume the pipe

Temperature to remain constant at 150°C.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivity is constant. **4** The thermal contact resistance at the interface is negligible. **5** The pipe temperature remains constant at about 150 □C with or without insulation. **6** The combined heat transfer coefficient on the outer surface remains constant even after the pipe is insulated.

Properties The thermal conductivity of fiberglass insulation is given to be k = 0.035 W/m.°C.



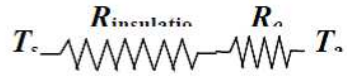
Analysis (*a*) The rate of heat loss from the steam pipe is

$$A_o = \pi DL = \pi(0.1\,\text{m})(50\,\text{m}) = 15.71\,\text{m}^2$$

$$\dot{Q}_{bare} = h_o A(T_s - T_{air}) = (20\,\text{W/m}^2.\text{°C})(15.71\,\text{m}^2)(150-15)\text{°C} = \mathbf{42{,}412\ W}$$

(*c*) In order to save 90% of the heat loss and thus to reduce it to 0.1□42,412 = 4241 W, the thickness of insulation needed is determined from

$$\dot{Q}_{insulated} = \frac{T_s - T_{air}}{R_o + R_{insulation}} = \frac{T_s - T_{air}}{\dfrac{1}{h_o A_o} + \dfrac{\ln(r_2 / r_1)}{2\pi kL}}$$



Substituting and solving for r_2, we get

$$4241\ \text{W} = \frac{(150 - 15)°\text{C}}{\dfrac{1}{(20\ \text{W/m}^2.°\text{C})[(2\pi r_2(50\ \text{m})]} + \dfrac{\ln(r_2 / 0.05)}{2\pi(0.035\ \text{W/m}.°\text{C})(50\ \text{m})}} \longrightarrow r_2 = 0.0692\ \text{m}$$

Then the thickness of insulation becomes

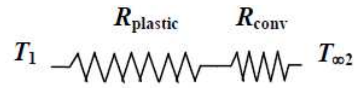
$$t_{insulation} = r_2 - r_1 = 6.92 - 5 = \mathbf{1.92\ cm}$$

8. Consider a 2-m-high electric hot water heater that has a diameter of 40 cm and maintains the hot water at 55°C. The tank is located in a small room whose average temperature i 27°C, and the heat transfer coefficients on the inner and outer surfaces of the heater are 50 and 12 W/m2 · °C, respectively. The tank is placed in another 46-cm-diameter sheet metal tank of negligible thickness, and the space between the two tanks is filled with foam insulation (*k* =0.03 W/m · °C). The thermal resistances of the water tank and the outer thin sheet metal shell are very small and can be neglected. The price of electricity is \$0.08/kWh, and the home owner pays \$280 a year for water heating. Determine the fraction of the hot water energy cost of this household that is due to the heat loss from the tank. Hot water tank insulation kits consisting of 3-cm-thick fiberglass insulation (*k* = 0.035 W/m · °C) large enough to wrap the entire tank are available in the market for about \$30. If such an insulation is installed on this water tank by the home owner himself, how long will it take for this additional insulation to pay for itself?

Assumptions 1 Heat transfer is steady since there is no indication of any change with time. 2 Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. 3 Thermal properties are constant. 4 The thermal contact resistance at the interface is negligible. 5 Heat transfer coefficient accounts for the radiation effects, if any.

Properties The thermal conductivity of plastic cover is given to be k = 0.15 W/m.°C. *Analysis* In steady operation, the rate of heat transfer from the wire is equal to the heat generated within the wire,

$$\dot{Q} = \dot{W}_e = VI = (8\text{ V})(10\text{ A}) = 80\text{ W}$$



The total thermal resistance is

$$R_{conv} = \frac{1}{h_o A_o} = \frac{1}{(24\text{ W/m}^2.°\text{C})[\pi(0.004\text{ m})(10\text{ m})]} = 0.3316\text{ °C/W}$$

$$R_{plastic} = \frac{\ln(r_2/r_1}{2\pi kL} = \frac{\ln(2/1)}{2\pi(0.15\text{ W/m.°C})(10\text{ m})} = 0.0735\text{ °C/W}$$

$$R_{total} = R_{conv} + R_{plastic} = 0.3316 + 0.0735 = 0.4051\text{ °C/W}$$

Then the interface temperature becomes

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{total}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{total} = 30°\text{C} + (80\text{ W})(0.4051\text{ °C/W}) = \mathbf{62.4°C}$$

The critical radius of plastic insulation is

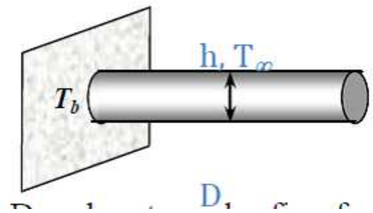
$$r_{cr} = \frac{k}{h} = \frac{0.15\text{ W/m.°C}}{24\text{ W/m}^2.°\text{C}} = 0.00625\text{ m} = 6.25\text{ mm}$$

Doubling the thickness of the plastic cover will increase the outer radius of the wire to 3 mm, which is less than the critical radius of insulation. Therefore, doubling the thickness of plastic cover will increase the rate of heat loss and decrease the interface temperature.

9. Obtain a relation for the fin efficiency for a fin of constant cross-sectional area *Ac*, perimeter *p*, length *L*, and thermal conductivity *k* exposed to convection to a medium at *T* with a heat transfer coefficient *h.* Assume the fins are sufficiently long so that the temperature of the fin at the tip is nearly *T*. Take the temperature of the fin at the base to be *Tb* and neglect heat transfer from the fin tips. Simplify the relation for (*a*) a circular fin of diameter *D* and (*b*) rectangular fins of thickness *t*..

Assumptions **1** The fins are sufficiently long so that the temperature of the fin at the tip is nearly . **2** Heat transfer from the fin tips is negligible

Analysis Taking the temperature of the fin at the base to be and using the heat transfer relation for a long fin, fin efficiency for long fins can be expressed as Tb



$$\eta_{\text{fin}} = \frac{\text{Actual heat transfer rate from the fin}}{\begin{array}{c}\text{Ideal heat transfer rate from the fin}\\ \text{if the entire fin were at base temperature}\end{array}}$$

$$= \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_{fin}(T_b - T_\infty)} = \frac{\sqrt{hpkA_c}}{hpL} = \frac{1}{L}\sqrt{\frac{kA_c}{ph}}$$

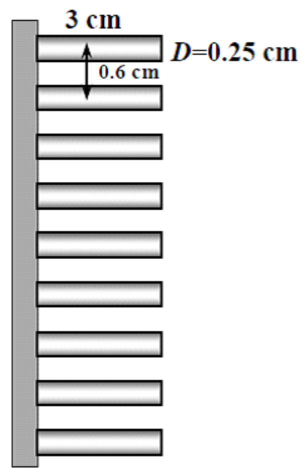
This relation can be simplified for a circular fin of diameter D and rectangular fin of thickness t and width w to be

$$\eta_{\text{fin,circular}} = \frac{1}{L}\sqrt{\frac{kA_c}{ph}} = \frac{1}{L}\sqrt{\frac{k(\pi D^2/4)}{(\pi D)h}} = \frac{1}{2L}\sqrt{\frac{kD}{h}}$$

$$\eta_{\text{fin,rectangular}} = \frac{1}{L}\sqrt{\frac{kA_c}{ph}} = \frac{1}{L}\sqrt{\frac{k(wt)}{2(w+t)h}} \cong \frac{1}{L}\sqrt{\frac{k(wt)}{2wh}} = \frac{1}{L}\sqrt{\frac{kt}{2h}}$$

10. A hot plate is to be cooled by attaching aluminum pin fins on one side. The rate of heat transfer from the 1 m by 1 m section of the plate and the effectiveness of the fins are to be determined.

Assumptions 1 Steady operating conditions exist. 2 The temperature along the fins varies in one direction only (normal to the plate). 3 Heat transfer from the fin tips is negligible. 4 The heat transfer coefficient is constant and uniform over the entire fin surface. 5 The thermal properties of the fins are constant. 6 The heat transfer coefficient accounts for the effect of radiation from the fins.



Properties The thermal conductivity of the aluminium plate and fins is given to be $k = 237$ W/m.°C.

Analysis Noting that the cross-sectional areas of the fins are constant, the efficiency of the circular fins can be determined to be

$$a = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi D^2/4}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(35\ \text{W/m}^2.°\text{C})}{(237\ \text{W/m.°C})(0.0025\ \text{m})}} = 15.37\ \text{m}^{-1}$$

$$\eta_{\text{fin}} = \frac{\tanh aL}{aL} = \frac{\tanh(15.37\ \text{m}^{-1} \times 0.03\ \text{m})}{15.37\ \text{m}^{-1} \times 0.03\ \text{m}} = 0.935$$

The number of fins, finned and unfinned surface areas, and heat transfer rates from those areas are

$$n = \frac{1\text{ m}^2}{(0.006\text{ m})(0.006\text{ m})} = 27{,}777$$

$$A_{\text{fin}} = 27777\left[\pi DL + \frac{\pi D^2}{4}\right] = 27777\left[\pi(0.0025)(0.03) + \frac{\pi(0.0025)^2}{4}\right] = 6.68\text{ m}^2$$

$$A_{\text{unfinned}} = 1 - 27777\left(\frac{\pi D^2}{4}\right) = 1 - 27777\left[\frac{\pi(0.0025)^2}{4}\right] = 0.86\text{ m}^2$$

$$\begin{aligned}\dot{Q}_{\text{finned}} &= \eta_{\text{fin}}\dot{Q}_{\text{fin,max}} = \eta_{\text{fin}} h A_{\text{fin}}(T_b - T_\infty) \\ &= 0.935(35\text{ W/m}^2.°\text{C})(6.68\text{ m}^2)(100-30)°\text{C} \\ &= 15{,}300\text{ W}\end{aligned}$$

$$\begin{aligned}\dot{Q}_{\text{unfinned}} &= hA_{\text{unfinned}}(T_b - T_\infty) = (35\text{ W/m}^2.°\text{C})(0.86\text{ m}^2)(100-30)°\text{C} \\ &= 2107\text{ W}\end{aligned}$$

Then the total heat transfer from the finned plate becomes

$$\dot{Q}_{\text{total,fin}} = \dot{Q}_{\text{finned}} + \dot{Q}_{\text{unfinned}} = 15{,}300 + 2107 = 1.74\times10^4\text{ W} = \mathbf{17.4\,kW}$$

The rate of heat transfer if there were no fin attached to the plate would be

$$A_{\text{no fin}} = (1\text{ m})(1\text{ m}) = 1\text{ m}^2$$

$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty) = (35\text{ W/m}^2.°\text{C})(1\text{ m}^2)(100-30)°\text{C} = 2450\text{ W}$$

Then the fin effectiveness becomes

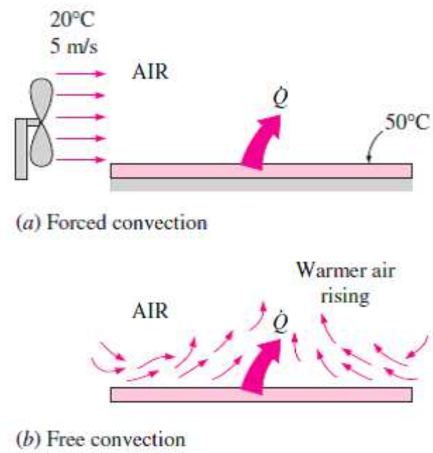
$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{17{,}400}{2450} = \mathbf{7.10}$$

UNIT II –
CONVECTION

INTRODUCTION

The preceding unit have considered the mechanism and calculation of conduction heat transfer. Convection was considered only insofar as it related to the boundary conditions imposed on a conduction problem. We now wish to examine the methods of calculating convection heat transfer and, in particular, the ways of predicting the value of the convection heat-transfer coefficient h. The subject of convection heat transfer requires an energy balance long with an analysis of the fluid dynamics of the problems concerned. Our discussion in this chapter will first consider some of the simple relations of fluid dynamics and boundary layer analysis that are important for a basic understanding of convection heat transfer. Next, we shall impose an energy balance on the flow system and determine the influence of the flow on the temperature gradients in the fluid. Finally, having obtained a knowledge of the temperature distribution, the heat-transfer rate from a heated surface to a fluid that is forced over it may be determined.

The process of heat transfer between a surface and a fluid flowing in contact with it is called convection. If the flow is caused by an **external device** like a pump or blower, it is termed as **forced convection**. If the flow is caused by the **buoyant forces** generated by heating or cooling of the fluid the process is called as natural or **free convection**.



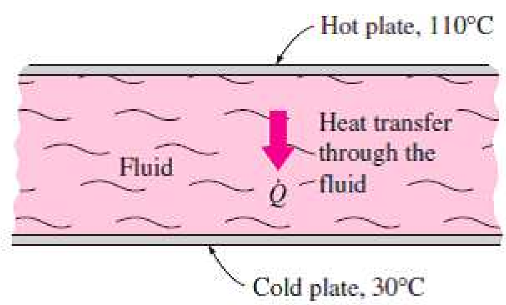


Fig. 1. Heat transfer through a fluid sandwiched between two parallel plates.

MECHANISM OF CONVECTION

In conduction, energy is transferred as heat either due to free electron flux or lattice vibration. There is no movement of mass in the direction of energy flow. In convection, energy flow occurs at the surface purely by conduction. But in the next layers both conduction and diffusion-mass movement in the molecular level or macroscopic level occurs. Due to the mass movement the rate the rate of energy transfer is higher. Higher the rate of mass movement, higher will be the heat flow rate.

Consider the cooling of a hot iron block with a fan blowing air over its top surface, as shown in Figure 2. We know that heat will be transferred from the hot block to the surrounding cooler air, and the block will eventually cool. We also know that the block will cool faster if the fan is switched to a higher speed. Replacing air by water will enhance the convection heat transfer even more.

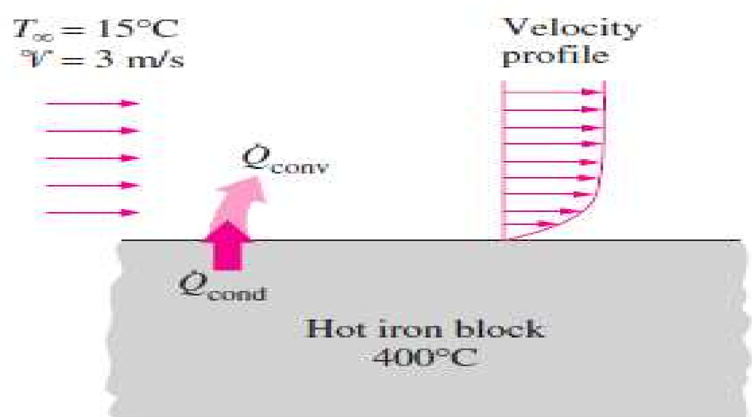


Fig. 2. The cooling of a hot block by forced convection.

Experience shows that convection heat transfer strongly depends on the fluid properties *dynamic viscosity*, *thermal conductivity k, density*, and *specific heat Cp,* as well as the *fluid velocity*. It also depends on the *geometry* and the *roughness* of the solid surface, in addition to the *type of fluid flow* (such as being streamlined or turbulent). Thus, we expect the convection heat transfer relations to be rather complex because of the dependence of convection on so many variables. This is not surprising, since convection is the most complex mechanism of heat transfer.

Despite the complexity of convection, the rate of convection heat transfer is observed to be proportional to the temperature difference and is conveniently expressed by **Newton's law of cooling** as

$$\dot{q}_{conv} = h(T_s - T_\infty) \qquad (W/m^2)$$

or

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \qquad (W)$$

where

h = convection heat transfer coefficient, $W/m^2 \cdot {}^\circ C$
A_s = heat transfer surface area, m^2
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C

Judging from its units, the **convection heat transfer coefficient** *h* can be defined as *the rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference.*

CLASSIFICATION OF FLUID FLOWS

Convection heat transfer is closely tied with fluid mechanics, which is the science that deals with the behavior of fluids at rest or in motion, and the interaction of fluids with solids or other fluids at the boundaries. There are a wide variety of fluid flow problems encountered in practice, and it is usually convenient to classify them on the basis of some common characteristics to make it feasible to study them in groups. There are many ways to classify the fluid flow problems, and below we present some general categories.

Viscous versus Inviscid Flow

When two fluid layers move relative to each other, a friction force develops between them and the slower layer tries to slow down the faster layer. This internal resistance to flow is called the **viscosity,** which is a measure of internal stickiness of the fluid. Viscosity is caused by cohesive forces between the molecules in liquids, and by the molecular collisions in gases. There is no fluid with zero viscosity, and thus all fluid flows involve viscous effects to some degree. Flows in which the effects of viscosity are significant are called **viscous flows.** The effects of viscosity are very small in some flows, and neglecting

those effects greatly simplifies the analysis without much loss in accuracy. Such idealized flows of zero-viscosity fluids are called frictionless or **inviscid flows.**

Internal versus External Flow

A fluid flow is classified as being internal and external, depending on whether the fluid is forced to flow in a confined channel or over a surface. The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is **external flow.** The flow in a pipe or duct is **internal flow** if the fluid is completely bounded by solid surfaces. Water flow in a pipe, for example, is internal flow, and air flow over an exposed pipe during a windy day is external flow (Fig. 3). The flow of liquids in a pipe is called *open-channel flow* if the pipe is partially filled with the liquid and there is a free surface. The flow of water in rivers and irrigation ditches are examples of such flows.

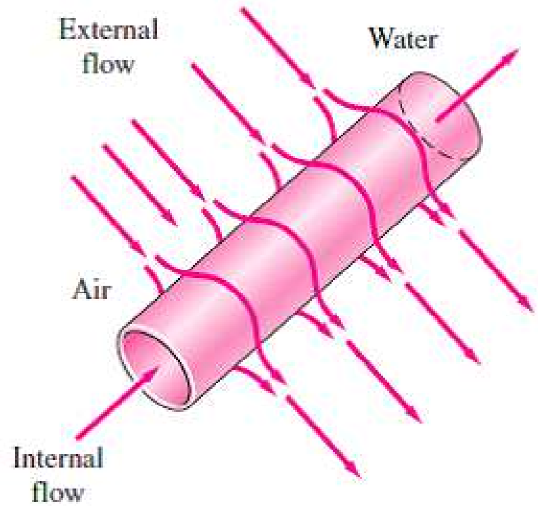


Fig. 3. Internal flow of water in a pipe and the external flow of air over the same pipe.

Compressible versus Incompressible Flow

A fluid flow is classified as being *compressible* or *incompressible,* depending on the density variation of the fluid during flow. The densities of liquids are essentially constant, and thus the flow of liquids is typically incompressible. Therefore, liquids are usually classified as *incompressible substances.* A pressure of 210 atm, for example, will cause the density of liquid water at 1 atm to change by just 1 percent. Gases, on the other hand, are highly compressible. A pressure change of just 0.01 atm, for example, will cause a change of 1 percent in the density of atmospheric air. However, gas flows can be treated as incompressible if the density changes are under about 5 percent, which is usually the case when the flow velocity is less than 30 percent of the velocity of sound in that gas (i.e., the Mach number of flow is less than 0.3). The velocity of sound in air at room temperature is 346 m/s. Therefore, the compressibility effects of air can be neglected at speeds under 100 m/s. Note that the flow of a gas is not necessarily a compressible flow.

Laminar versus Turbulent Flow

Some flows are smooth and orderly while others are rather chaotic. The highly ordered fluid motion characterized by smooth streamlines is called **laminar.** The flow of high-viscosity fluids such as oils at low velocities is typically laminar. The highly disordered fluid motion that typically occurs at high velocities characterized by velocity fluctuations is called **turbulent.** The flow of low-viscosity fluids such as air at high velocities is typically turbulent. The flow regime greatly influences the heat transfer rates and the required power for pumping.

Natural (or Unforced) versus Forced Flow

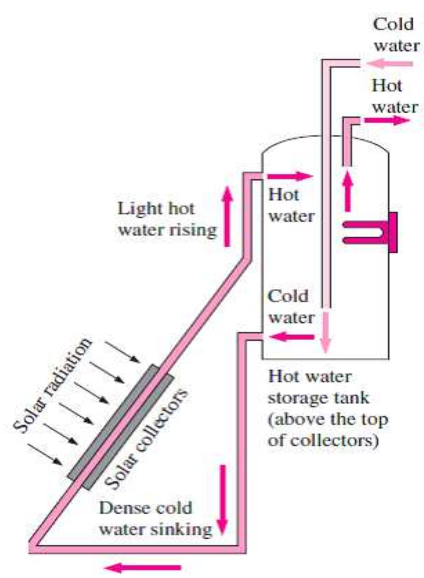


Fig. 4. Natural circulation of water in a solar water heater by thermosiphoning.

A fluid flow is said to be natural or forced, depending on how the fluid motion is initiated. In **forced flow,** a fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In **natural flows,** any fluid motion is due to a natural means such as the buoyancy effect, which manifests itself as the rise of the warmer (and thus lighter) fluid and the fall of cooler (and thus denser) fluid. This thermosiphoning effect is commonly used to replace pumps in solar water heating systems by placing the water tank sufficiently above the solar collectors (Fig. 4).

The Convection Boundary Layer

The concept of boundary layers is central to the understanding of convection heat transfer between a surface and a fluid flowing past it. In this section, velocity and thermal boundary layers are described, and their relationships to the friction coefficient and convection heat transfer coefficient are introduced.

6.1.1 The Velocity Boundary Layer

To introduce the concept of a boundary layer, consider flow over the flat plate of Figure 5.

When fluid particles make contact with the surface, their velocity is reduced significantly relative to the fluid velocity upstream of the plate, and for most situations it is valid to assume that the particle velocity is zero at the wall.1 These particles then act to retard the motion of particles in the adjoining fluid layer, which act to retard the motion of particles in the next layer, and so on until, at a distance *y* _ _ from the surface, the effect becomes negligible. This retardation of fluid motion is associated with *shear stresses* _ acting in planes that are parallel to the fluid velocity (Figure 5). With increasing distance *y* from the surface, the *x* velocity component of the fluid, *u*, must then increase until it approaches the free stream value *u*_. The subscript _ is used to designate conditions in the *free stream* outside the boundary layer.

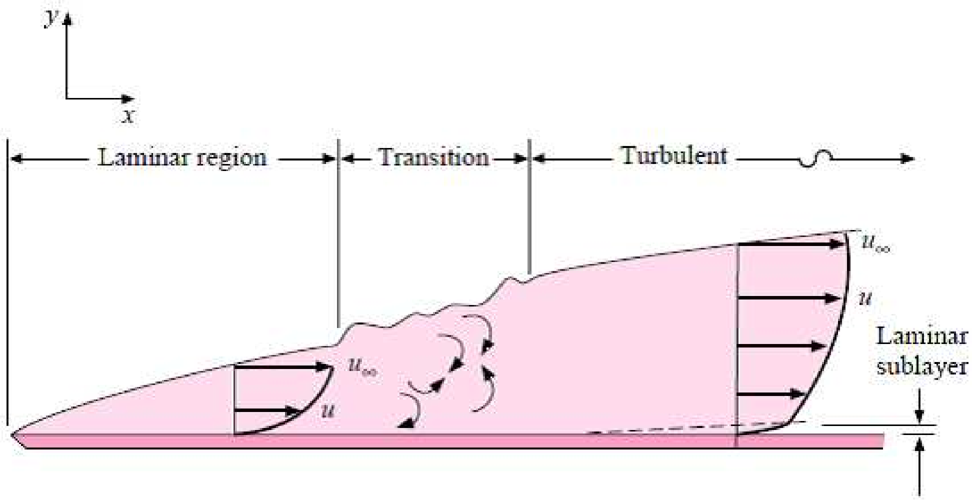


Fig. 5. Laminar velocity profile on a flat plate.

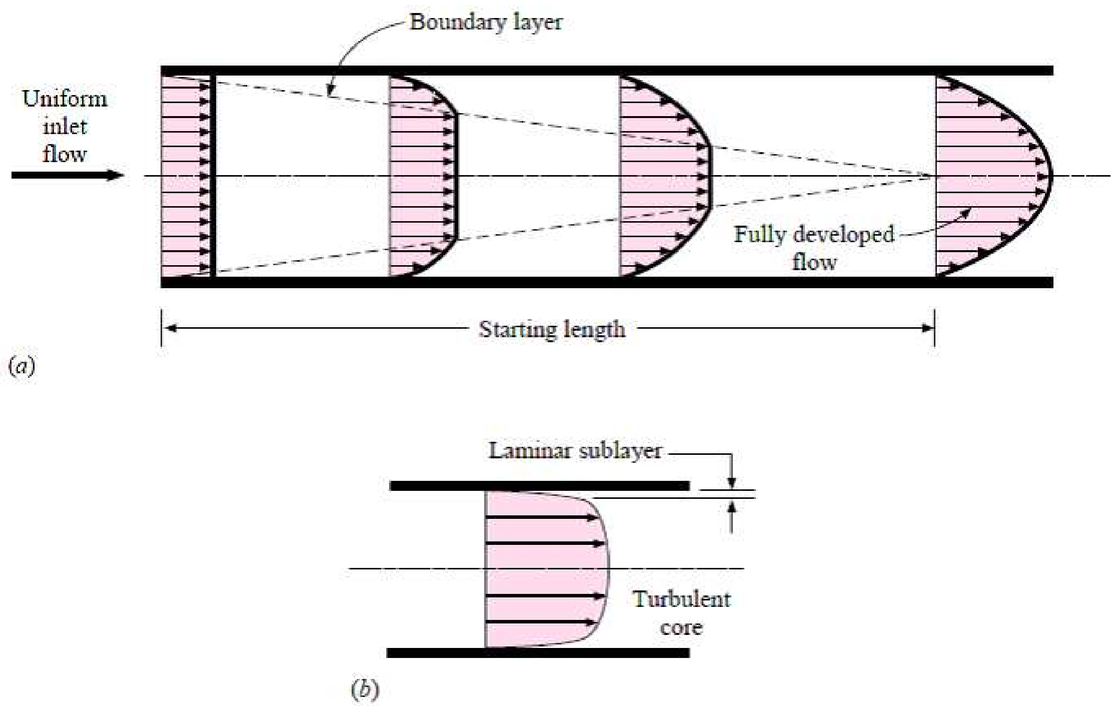
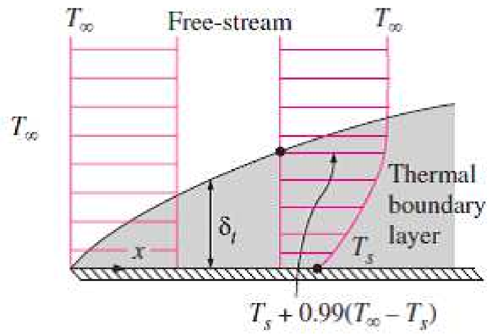


Fig. 6. Velocity profile for (*a*) laminar flow in a tube and (*b*) turbulent tube flow.

THERMAL BOUNDARY LAYER

Just as a velocity boundary layer develops when there is fluid flow over a surface, a *thermal boundary layer* must develop if the fluid free stream and surface temperatures differ. Consider flow over an isothermal flat plate. At the leading edge the *temperature profile* is uniform, with $T(y) = T$.



However, fluid particles that come into contact with the plate achieve thermal equilibrium at the plate's surface temperature.2 In turn, these particles exchange energy with those in the adjoining fluid layer, and temperature gradients develop in the fluid. The region of the fluid in which these temperature gradients exist is the thermal boundary layer, and its thickness _*t* is typically defined as the value of *y* for which the ratio [(*Ts* _ *T*)/(*Ts* _ *T*_)] _ 0.99. With increasing distance from the leading edge, the effects of heat transfer penetrate farther into the free stream and the thermal boundary layer grows. The relation between conditions in this boundary layer and the convection heat transfer coefficient may readily be demonstrated. At any distance *x* from the leading edge, the *local* surface heat flux may be obtained by applying Fourier's law to the *fluid* at *y* = 0.

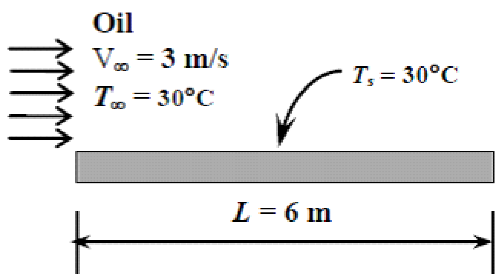
1. Hot engine oil flows over a flat plate. The temperature and velocity of the oil are 30 degree C&3 m/s respectively. The temperature of the plate is 30 degree C. compute the total drag force and the rate of heat transfer per unit width of the plate.

Assumptions 1 Steady operating condition exists. 2 The critical Reynolds number is Recr = 5*105. 3 Radiation effects are negligible.

Properties The properties of engine oil at the film temperature of (*Ts* + *T*□)/2 = (80+ 30)/2 =55□C = 328 K are

$\rho = 867 \text{ kg/m}^3 \qquad \upsilon = 123 \times 10^{-6} \text{ m}^2/\text{s}$

$k = 0.141 \text{ W/m.°C} \qquad \text{Pr} = 1505$



Analysis Noting that L = 6 m, the Reynolds number at the end of the plate is

$$\text{Re}_L = \frac{\mathbf{V}_\infty L}{\upsilon} = \frac{(3 \text{ m/s})(6 \text{ m})}{123 \times 10^{-6} \text{ m}^2/\text{s}} = 1.46 \times 10^5$$

which is less than the critical Reynolds number. Thus we have laminar flow over the entire plate. The average friction coefficient and the drag force per unit width are determined from

$$C_f = 1.328\,\mathrm{Re}_L^{-0.5} = 1.328(1.46\times10^5)^{-0.5} = 0.00347$$

$$F_D = C_f A_s \frac{\rho V_\infty^2}{2} = (0.00347)(6\times1\,\mathrm{m}^2)\frac{(867\ \mathrm{kg/m^3})(3\ \mathrm{m/s})^2}{2} = 81.3\,\mathrm{N}$$

Similarly, the average Nusselt number and the heat transfer coefficient are determined using the laminar flow relations for a flat plate,

$$Nu = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} = 0.664(1.46\times10^5)^{0.5}(1505)^{1/3} = 2908$$

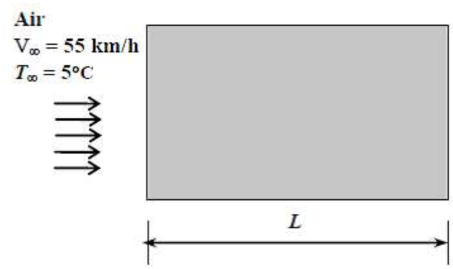
$$h = \frac{k}{L}Nu = \frac{0.141\ \mathrm{W/m.^\circ C}}{6\ \mathrm{m}}(2908) = 68.3\ \mathrm{W/m^2.^\circ C}$$

The rate of heat transfer is determined from Newton's law of cooling

2. Wind is blowing parallel to the wall of a house. The temperature and velocity of the air is 5oC& 55 km/hr. Calculate the rate of heat loss from the wall.

Assumptions 1 Steady operating condition exists. 2 The critical Reynolds number is $Re_{cr} = 5\times10^5$. 3 Radiation effects are negligible. 4 Air is an ideal gas with constant properties.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (12+5)/2 = 8.5°C$



$$k = 0.02428 \text{ W/m.°C}$$
$$\upsilon = 1.413\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7340$$

Analysis *Air flows parallel to the 10 m side*:

The Reynolds number in this case is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[(55\times1000/3600)\text{m/s}](10\text{ m})}{1.413\times10^{-5}\text{ m}^2\text{/s}} = 1.081\times10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar turbulent flow. Using the proper relation for Nusselt number, heat transfer coefficient then heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = (0.037\text{Re}_L^{0.8} - 871)\Pr^{1/3} = [0.037(1.081\times10^7)^{0.8} - 871](0.7340)^{1/3} = 1.336\times10^4$$

$$h = \frac{k}{L}Nu = \frac{0.02428 \text{ W/m.°C}}{10\text{ m}}(1.336\times10^4) = 32.43 \text{ W/m}^2\text{.°C}$$

$$A_s = wL = (4\text{ m})(10\text{ m}) = 40\text{ m}^2$$

$$\dot{Q} = hA_s(T_\infty - T_s) = (32.43\text{ W/m}^2.\text{°C})(40\text{ m}^2)(12 - 5)\text{°C} = 9081\text{ W} = \mathbf{9.08\,kW}$$

If the wind velocity is doubled:

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[(110\times1000/3600)\text{m/s}](10\text{ m})}{1.413\times10^{-5}\text{ m}^2\text{/s}} = 2.163\times10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, the average heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = (0.037\,\text{Re}_L{}^{0.8} - 871)\,\text{Pr}^{1/3} = [0.037(2.163\times10^7)^{0.8} - 871](0.7340)^{1/3} = 2.384\times10^4$$

$$h = \frac{k}{L}Nu = \frac{0.02428\text{ W/m°C}}{10\text{ m}}(2.384\times10^4) = 57.88\text{ W/m}^2.\text{°C}$$

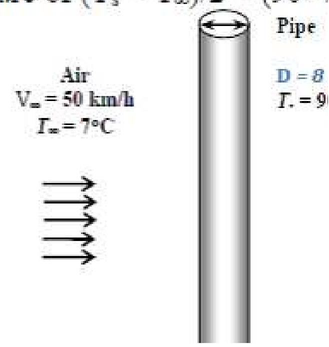
$$A_s = wL = (10\text{ m})(4\text{ m}) = 40\text{ m}^2$$

$$\dot{Q} = hA_s(T_\infty - T_s) = (57.88\text{ W/m}^2.\text{°C})(40\text{ m}^2)(12 - 5)\text{°C} = 16{,}206\text{ W} = \mathbf{16.21\,kW}$$

3. Air is flowing over the steam pipe having steam temperature of 90 oC .The velocity and temperature of the air are 7 degree C and 50 km/ hr respectively. Calculate rate of heat loss by the air on the steam pipe

Assumptions 1 Steady operating conditions exist. 2 Radiation effects are negligible. 3 Air is an ideal gas with constant properties.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (90+7$
$= 48.5°C$ are (Table A-15)



$$k = 0.02724 \text{ W/m.°C}$$
$$\upsilon = 1.784 \times 10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7232$$

Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(50 \text{ km/h})(1000 \text{ m/km})/(3600 \text{ s/h})](0.08 \text{ m})}{1.784 \times 10^{-5} \text{ m}^2\text{/s}} = 6.228 \times 10^4$$

The Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62 \text{Re}^{0.5} \text{Pr}^{1/3}}{\left[1 + (0.4/\text{Pr})^{2/3}\right]^{1/4}} \left[1 + \left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(6.228 \times 10^4)^{0.5}(0.7232)^{1/3}}{\left[1 + (0.4/0.7232)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{6.228 \times 10^4}{282{,}000}\right)^{5/8}\right]^{4/5} = 159.1$$

The heat transfer coefficient and the heat transfer rate become

$$h = \frac{k}{D} Nu = \frac{0.02724 \text{ W/m°C}}{0.08 \text{ m}}(159.1) = 54.17 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.08 \text{ m})(1 \text{ m}) = 0.2513 \text{m}^2$$

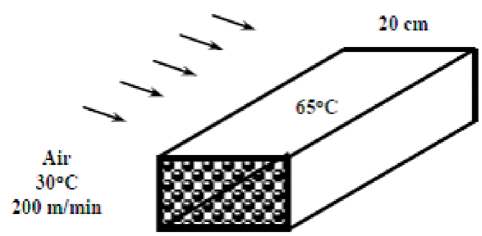
$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = (54.17 \text{ W/m}^2\text{.°C})(0.2513 \text{ m}^2)(90 - 7)°\text{C} = \mathbf{1130 \text{ W}} \text{ (per m length)}$$

4. The components of an electronic system located in a horizontal square duct (20cm×20 cm) is cooled by air flowing over the duct. The velocity and temperature of the air are 200 m/min & 30degree C. Determine the total power rating of the electronic device.

Assumptions 1 Steady operating condition exists. 2 Radiation effects are negligible. 3 Air is an ideal gas with constant properties. 4 The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (65+30)/2 = 47.5°C$ are

$$k = 0.02717 \text{ W/m.°C}$$
$$\upsilon = 1.774 \times 10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7235$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(200/60)\text{ m/s}](0.2\text{ m})}{1.774\times10^{-5}\text{ m}^2\text{/s}} = 3.758\times10^4$$

Using the relation for a square duct from Table 7-1, the Nusselt number is determined to be

$$Nu = \frac{hD}{k} = 0.102\,\text{Re}^{0.675}\,\text{Pr}^{1/3} = 0.102(3.758\times10^4)^{0.675}(0.7235)^{1/3} = 112.2$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02717\text{ W/m.°C}}{0.2\text{ m}}(112.2) = 15.24\text{ W/m}^2\text{.°C}$$

5. Water at 15ºC is to be heated to 65ºC by passing it over a bundle of 4-m-long 1-cm-diameter resistance heater rods maintained at 90ºC. Water approaches the heater rod bundle in normal direction at a mean velocity of 0.8 m/s. The rods arc arranged in-line with longitudinal and transverse pitches of *SL* = 4 cm and *ST* = 3 cm. Determine the number of tube rows *NL* in the flow direction needed to achieve the indicated temperature

rise.

Assumptions **1** steady operating condition exists. **2** The surface temperature of the rods is constant.

Properties The properties of water at the mean temperature of (15ºC +65ºC)/2=40ºC

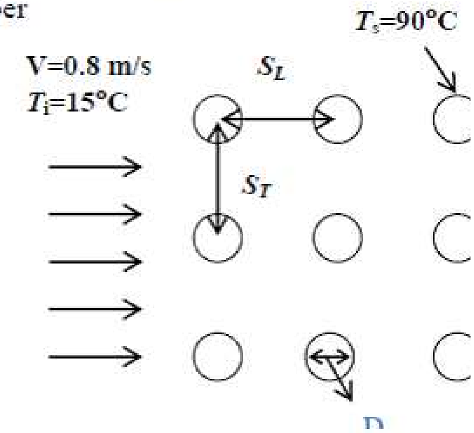
Analysis It is given that $D = 0.01$ m, $S_L = 0.04$ m and $S_T = 0.03$ m, and V = 0.8 m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become

$$V_{max} = \frac{S_T}{S_T - D} V = \frac{0.03}{0.03 - 0.01}(0.8\ \text{m/s}) = 1.20\ \text{m/s}$$

$$Re_D = \frac{\rho V_{max} D}{\mu} = \frac{(992.1\ \text{kg/m}^3)(1.20\ \text{m/s})(0.01\ \text{m})}{0.653 \times 10^{-3}\ \text{kg/m} \cdot \text{s}} = 18{,}232$$

The average Nusselt number is determined using the proper relation from Table 7-2 to be

$$Nu_D = 0.27 Re_D^{0.63} Pr^{0.36} (Pr/Pr_s)^{0.25}$$
$$= 0.27(18{,}232)^{0.63}(4.32)^{0.36}(4.32/1.96)^{0.25} = 269.3$$



Assuming that $N_L > 16$, the average Nusselt number and heat transfer coefficient for all tubes in the tube bank become

$$\text{Nu}_{D,N_L} = \text{Nu}_D = 269.3$$

$$h = \frac{Nu_{D,N_L} k}{D} = \frac{269.3(0.631 \text{ W/m}\cdot°\text{C})}{0.01\text{m}} = 16{,}994 \text{ W/m}^2\cdot°\text{C}$$

Consider one-row of tubes in the transpose direction (normal to flow), and thus take N_T Then the heat transfer surface area becomes

$$A_s = N_{tube}\pi DL = (1\times N_L)\pi(0.01\text{ m})(4\text{ m}) = 0.1257N_L$$

Then the log mean temperature difference, and the expression for the rate of heat tran become

$$\Delta T_{\ln} = \frac{(T_s - T_i) - (T_s - T_e)}{\ln[(T_s - T_i)/(T_s - T_e)]} = \frac{(90-15)-(90-65)}{\ln[(90-15)/(90-65)]} = 45.51°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (16{,}994 \text{ W/m}^2\cdot°\text{C})(0.1257N_L)(45.51°\text{C}) = 97{,}220N_L$$

The mass flow rate of water through a cross-section corresponding to $N_T = 1$ and the rate heat transfer are

$$\dot{m} = \rho A_c V = (999.1\text{ kg/m}^3)(4\times 0.03\text{m}^2)(0.8\text{ m/s}) = 95.91\text{ kg/s}$$

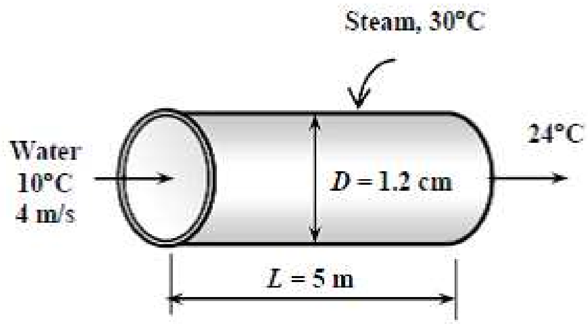
$$\dot{Q} = \dot{m}C_p(T_e - T_i) = (95.91\text{ kg/s})(4179\text{ J/kg.C})(65-15)°\text{C} = 2.004\times10^7\text{ W}$$

Substituting this result into the heat transfer expression above we find th e number of tul rows

$$\dot{Q} = hA_s\Delta T_{\ln} \rightarrow 2.004\times10^7\text{ W} = 97{,}220N_L \rightarrow N_L = \mathbf{206}$$

6. Cooling water available at 10°C is used to condense steam at 30°C in the condenser of a power plant at a rate of 0.15 kg/s by circulating the cooling water through a bank of 5-m-long 1.2-cm-internal-diameter thin copper tubes. Water enters the tubes at a mean velocity of 4 m/s, and leaves at a temperature of 24°C. The tubes are nearly isothermal at

30°C.Determine the average heat transfer coefficient between the water and the tubes, and the number of tubes needed to achieve the indicated heat transfer rate in the condenser.



Assumptions 1 Steady operating conditions exist. 2 The surface temperature of the pipe is constant. 3 The thermal resistance of the pipe is negligible.

Properties The properties of water at the average temperature of (10+24)/2=17□C are

$$\rho = 998.7 \text{ kg/m}^3$$
$$C_p = 4184.5 \text{ J/kg.°C}$$

Also, the heat of vaporization of water at 30°C is $h_{fg} = 2431 \text{ kJ/kg}$.

Analysis The mass flow rate of water and the surface area are

$$\dot{m} = \rho A_c V_m = \rho\left(\frac{\pi D^2}{4}\right)V_m$$

$$= (998.7 \text{ kg/m}^3)\frac{\pi(0.012 \text{ m})^2}{4}(4 \text{ m/s}) = 0.4518 \text{ kg/s}$$

The rate of heat transfer for one tube is

$$\dot{Q} = \dot{m}C_p(T_e - T_i) = (0.4518\ \text{kg/s})(4184.5\ \text{J/kg.°C})(24 - 10°\text{C}) = 26{,}468\ \text{W}$$

The logarithmic mean temperature difference and the surface area are

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{24 - 10}{\ln\left(\frac{30 - 24}{30 - 10}\right)} = 11.63°\text{C}$$

$$A_s = \pi DL = \pi(0.012\ \text{m})(5\ \text{m}) = 0.1885\ \text{m}^2$$

The average heat transfer coefficient is determined from

$$\dot{Q} = hA_s\Delta T_{\ln} \longrightarrow h = \frac{\dot{Q}}{A_s\Delta T_{\ln}} = \frac{26{,}468\ \text{W}}{(0.1885\ \text{m}^2)(11.63°\text{C})}\left(\frac{1\ \text{kW}}{1000\ \text{W}}\right) = \mathbf{12.1\ kW/m^2.°C}$$

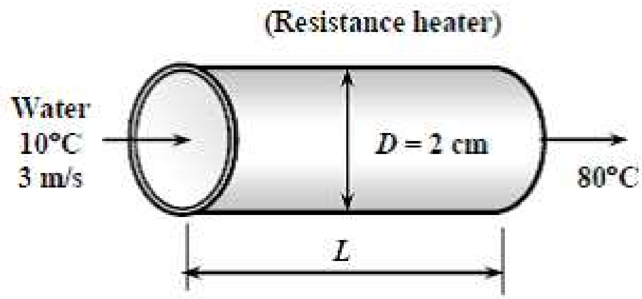
The total rate of heat transfer is determined from

$$\dot{Q}_{total} = \dot{m}_{cond}h_{fg} = (0.15\ \text{kg/s})(2431\ \text{kJ/kg}) = 364.65\ \text{kW}$$

Then the number of tubes becomes

$$N_{tube} = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{364{,}650\ \text{W}}{26{,}468\ \text{W}} = \mathbf{13.8}$$

7. Water is to be heated from 10°C to 80°C as it flows through a 2-cm-internal-diameter, 7- m-long tube. The tube is equipped with an electric resistance heater, which provides uniform heating throughout the surface of the tube. The outer surface of the heater is well insulated, so that in steady operation all the heat generated in the heater is transferred to the water in the tube. If the system is to provide hot water at a rate of 8 L/min, determine the power rating of the resistance heater. Also, estimate the inner surface temperature of the pipe at the exit.



Assumptions 1 Steady flow conditions exist. 2 The surface heat flux is uniform. 3 The inner surfaces of the tube are smooth.

Properties The properties of water at the average temperature of (80+10) / 2 = 45 deg C are

$$\rho = 990.1\ \text{kg/m}^3$$
$$k = 0.637\ \text{W/m.°C}$$
$$\upsilon = \mu / \rho = 0.602 \times 10^{-6}\ \text{m}^2\text{/s}$$
$$C_p = 4180\ \text{J/kg.°C}$$
$$\Pr = 3.91$$

Analysis The power rating of the resistance heater is

$$\dot{m} = \rho \dot{V} = (990.1\ \text{kg/m}^3)(0.008\ \text{m}^3\text{/min}) = 7.921\ \text{kg/min} = 0.132\ \text{kg/s}$$

$$\dot{Q} = \dot{m} C_p (T_e - T_i) = (0.132\ \text{kg/s})(4180\ \text{J/kg.°C})(80-10)\text{°C} = \mathbf{38{,}627\ W}$$

The velocity of water and the Reynolds number are

$$V_m = \frac{\dot{V}}{A_c} = \frac{(8\times10^{-3}/60)\ m^3/s}{\pi(0.02\ m)^2/4} = 0.4244\ m/s$$

$$Re = \frac{V_m D_h}{\upsilon} = \frac{(0.4244\ m/s)(0.02\ m)}{0.602\times10^{-6}\ m^2/s} = 14{,}101$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.02\ m) = 0.20\ m$$

which is much shorter than the total length of the duct. Therefore, we can assume full developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,Re^{0.8}\,Pr^{0.4} = 0.023(14{,}101)^{0.8}(3.91)^{0.4} = 82.79$$

Heat transfer coefficient is

$$h = \frac{k}{D_h}Nu = \frac{0.637\ W/m.°C}{0.02\ m}(82.79) = 2637\ W/m^2.°C$$

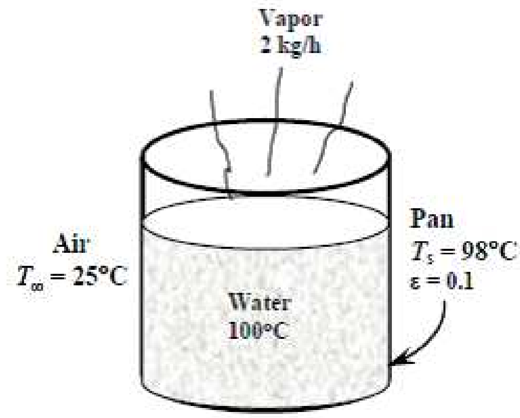
Then the inner surface temperature of the pipe at the exit becomes

$$\dot{Q} = hA_s(T_{s,e} - T_e)$$

$$38{,}627\ W = (2637\ W/m^2.°C)[\pi(0.02\ m)(7\ m)](T_s - 80)°C$$

$$T_{s,e} = \mathbf{113.3°C}$$

8. Water is boiling in a 12-cm-deep pan with an outer diameter of 25 cm that is placed on top of a stove. The ambient air and the surrounding surfaces are at a temperature of 25°C, and the emissivity of the outer surface of the pan is 0.95. Assuming the entire pan to be at an average temperature of 98°C, determine the rate of heat loss from the cylindrical side surface of the pan to the surroundings by (a) natural convection and (b) radiation. (c) If water is boiling at a rate of 2 kg/h at 100°C, determine the ratio of the heat lost from the side surfaces of the pan to that by evaporation of water. The heat of vaporization of water at 100°C is 2257 kJ/kg.



Assumptions 1 Steady operating conditions exist. 2 Air is an ideal gas with constant properties. 3 The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of (Ts+T□)/2 = (98+ 25)/2 = 61.5 deg C are

$$k = 0.02819 \text{ W/m.°C}$$
$$\upsilon = 1.910\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7198$$
$$\beta = \frac{1}{T_f} = \frac{1}{(61.5+273)\text{K}} = 0.00299 \text{ K}^{-1}$$

Analysis (*a*) The characteristic length in this case is the height of the pan, $L_c = L = 0.12$ ı Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.00299 \text{ K}^{-1})(98-25 \text{ K})(0.12 \text{ m})^3}{(1.910\times10^{-5} \text{ m}^2\text{/s})^2}(0.7198) = 7.299\times10^6$$

We can treat this vertical cylinder as a vertical plate since

$$\frac{35L}{Gr^{1/4}} = \frac{35(0.12)}{(7.299\times10^6/0.7198)^{1/4}} = 0.07443 < 0.25 \quad \text{and thus } D \geq \frac{35L}{Gr^{1/4}}$$

Therefore,

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(7.299\times10^6)^{1/6}}{\left[1+\left(\frac{0.492}{0.7198}\right)^{9/16}\right]^{8/27}}\right\}^2 = 28.60$$

$$h = \frac{k}{L} Nu = \frac{0.02819 \text{ W/m.°C}}{0.12 \text{ m}}(28.60) = 6.720 \text{ W/m}^2.\text{°C}$$

$$A_s = \pi DL = \pi(0.25 \text{ m})(0.12 \text{ m}) = 0.09425 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (6.720 \text{ W/m}^2.\text{°C})(0.09425 \text{ m}^2)(98-25)\text{°C} = \mathbf{46.2 \text{ W}}$$

(b) The radiation heat loss from the pan is

$$\dot{Q}_{rad} = \varepsilon A_s \sigma(T_s{}^4 - T_{surr}{}^4)$$
$$= (0.10)(0.09425 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)\left[(98+273 \text{ K})^4 - (25+273 \text{ K})^4\right] = \mathbf{5.9 \text{ W}}$$

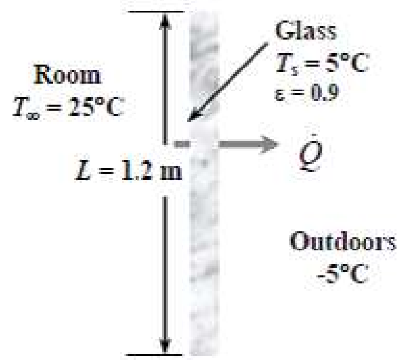
(c) The heat loss by the evaporation of water is

$$\dot{Q} = \dot{m}h_{fg} = (2/3600 \text{ kg/s})(2257 \text{ kJ/kg}) = 1.254 \text{ kW} = 1254 \text{ W}$$

Then the ratio of the heat lost from the side surfaces of the pan to that by the evaporation of water then becomes

$$f = \frac{46.2 + 5.9}{1254} = 0.042 = \mathbf{4.2\%}$$

9. Consider a 1.2-m-high and 2-m-wide glass window with a thickness of 6 mm, thermal conductivity k =0.78 W/m °C, and emissivity 0.9. The room and the walls that face the window are maintained at 25°C, and the average temperature of the inner surface of the window is measured to be 5°C. If the temperature of the outdoors is 5°C, determine (*a*) the convection heat transfer coefficient on the inner surface of the window, (*b*) the rate of total heat transfer through the window, and (*c*) the combined natural convection and radiation heat transfer coefficient on the outer surface of the window. Is it reasonable to neglect the thermal resistance of the glass in this case?



Assumptions 1 Steady operating conditions exist. 2 Air is an ideal gas with constant properties. 3 The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of (Ts+T□)/2 = (5+ 25)/2 = 15deg C are

$$k = 0.02476 \text{ W/m.°C}$$
$$\upsilon = 1.471\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7323$$
$$\beta = \frac{1}{T_f} = \frac{1}{(15+273)\text{K}} = 0.003472 \text{ K}^{-1}$$

Analysis (*a*) The characteristic length in this case is the height of the window. $L_c = L = 1.2 \text{ m}$. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003472 \text{ K}^{-1})(25-5 \text{ K})(1.2 \text{ m})^3}{(1.471\times10^{-5} \text{ m}^2\text{/s})^2}(0.7323) = 3.986\times10^9$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(3.986\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{0.7323}\right)^{9/16}\right]^{8/27}}\right\}^2 = 189.7$$

$$h = \frac{k}{L}Nu = \frac{0.02476 \text{ W/m.°C}}{1.2 \text{ m}}(189.7) = \mathbf{3.915 \text{ W/m}^2\text{.°C}}$$
$$A_s = (1.2 \text{ m})(2 \text{ m}) = 2.4 \text{ m}^2$$

(*b*) The sum of the natural convection and radiation heat transfer from the room to the window is

$$\dot{Q}_{\text{convection}} = hA_s(T_\infty - T_s) = (3.915 \text{ W/m}^2\text{.°C})(2.4 \text{ m}^2)(25-5)\text{°C} = 187.9 \text{ W}$$
$$\dot{Q}_{\text{radiation}} = \varepsilon A_s\sigma({T_{surr}}^4 - {T_s}^4)$$
$$= (0.9)(2.4 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(25+273 \text{ K})^4 - (5+273 \text{ K})^4] = 234.3 \text{ W}$$
$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{convection}} + \dot{Q}_{\text{radiation}} = 187.9 + 234.3 = \mathbf{422.2 \text{ W}}$$

(c) The outer surface temperature of the window can be determined from

$$\dot{Q}_{\text{total}} = \frac{kA_s}{t}(T_{s,i} - T_{s,o}) \longrightarrow T_{s,o} = T_{s,i} - \frac{\dot{Q}_{\text{total}} t}{kA_s} = 5°\text{C} - \frac{(346\text{ W})(0.006\text{ m})}{(0.78\text{ W/m.°C})(2.4\text{ m}^2)} = 3.65°\text{C}$$

Then the combined natural convection and radiation heat transfer coefficient on the outer window surface becomes

$$\dot{Q}_{\text{total}} = h_{\text{combined}} A_s (T_{s,o} - T_{\infty,o})$$

or $$h_{\text{combined}} = \frac{\dot{Q}_{\text{total}}}{A_s(T_{s,o} - T_{\infty,o})} = \frac{346\text{ W}}{(2.4\text{ m}^2)[3.65 - (-5)]°\text{C}} = \mathbf{20.35\ W/m^2.°C}$$

Note that $\Delta T = \dot{Q}R$ and thus the thermal resistance R of a layer is proportional to the temperature drop across that layer. Therefore, the fraction of thermal resistance of the glass is equal to the ratio of the temperature drop across the glass to the overall temperature difference,

$$\frac{R_{\text{glass}}}{R_{\text{total}}} = \frac{\Delta T_{\text{glass}}}{\Delta TR_{\text{total}}} = \frac{5 - 3.65}{25 - (-5)} = 0.045 \quad (\text{or } 4.5\%)$$

which is low. Thus it is reasonable to neglect the thermal resistance of the glass.

10. Nitrogen at a pressure of 0.1 atm flows over a flat plate with a free stream velocity of 8 m/s. The temperature of the gas is – 20°C. The plate temperature is 20°C. Determine the length for the flow to turn turbulent. Assume 5×10^5 as critical Reynolds number. Also determine the thickness of thermal and velocity boundary layers and the average convection coefficient for a plate length of 0.3 m.

Properties are to be found at film temperature.

Solution: Film temperature = (– 20 + 20)/2 = 0°C

As density and kinematic viscosities will vary with pressure, dynamic viscosity is ı from tables.

$$\mu = 16.67 \times 10^{-6} \text{ Ns/m}^2,\ k = 24.31 \times 10^{-3} \text{ W/mK}$$

$$Pr = 0.705,\ \rho = 1.250 \times 0.1 = 0.125 \text{ kg/m}^3$$

$$Re = \frac{u_\infty \rho x}{\mu} = 5 \times 10^5$$

$$\therefore \quad x = \frac{5 \times 10^5 \times 16.67 \times 10^{-6}}{8 \times 0.125} = 8.335 \text{ m}$$

Check for dimensions :

$$u_\infty = \frac{m}{s},\ \rho = \frac{\text{kg}}{m^3},\ x = m,\ \mu = \text{Ns/m}^2$$

$$\therefore \qquad \frac{m}{s}\frac{kgm}{m^3}\frac{m^2}{Ns} = \frac{kg\,.\,m}{s^2}\frac{1}{N} = \text{dimensionless}$$

As the density is low, the kinematic viscosity is higher and hence turbulence is supp for a longer distance.

At $x = 0.3$ m, the Reynolds number is less than 5×10^5 and so the flow is laminar

$$\boldsymbol{\delta_h} = 5x/Re^{0.5} = 5 \times 0.3/(8 \times 0.125 \times 0.3/16.67 \times 10^{-6})^{0.5}$$

$$= 5 \times 0.3/\sqrt{17996.4} = 0.01118 \text{ m or } \mathbf{11.18\ mm}$$

Thermal boundary

layer thickness $= \delta_h/Pr^{0.33} = 11.18/0.705^{0.33} = \mathbf{12.56\ mm}$

Average convection coefficient:

$$\overline{h} = \frac{0.644 \times 24.31 \times 10^{-3}}{0.3}\ (8 \times 0.125 \times 0.3/16.67 \times 10^{-6})^{0.5}\ (0.705)^{0.3}$$

$$= \mathbf{6.23\ W/m^2K,}$$

If the **pressure was atmospheric**, then boundary layers thickness is

$$\boldsymbol{\delta_h} = 5 \times 0.3/(8 \times 1.25 \times 0.3/16.67 \times 10^{-6})^{0.5} = \mathbf{3.54\ mm}$$

$$\boldsymbol{\delta_t} = 3.54/(0.705)^{0.3} = \mathbf{3.98\ mm}$$

$$\overline{h} = \frac{0.664 \times 24.31 \times 10^{-3}}{0.3}\ [(8 \times 1.25 \times 0.3/16.67 \times 10^{-6})^{0.5}\ (0.705)^{0.33}]$$

$$= \mathbf{20.32\ W/m^2K.}$$

UNIT III - PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGERS

BOILING AND CONDENSATION

We know from thermodynamics that when the temperature of a liquid at a specified pressure is raised to the saturation temperature Tsat at that pressure, *boiling* occurs.

Likewise, when the temperature of a vapor is lowered to T_{sat}, *condensation* occurs. In this chapter we study the rates of heat transfer during such liquid-to-vapor and vapor-to-liquid phase transformations.

Although boiling and condensation exhibit some unique features, they are considered to be forms of *convection* heat transfer since they involve fluid motion (such as the rise of the bubbles to the top and the flow of condensate to the bottom). Boiling and condensation differ from other forms of convection in that they depend on the *latent heat of vaporization* h_{fg} of the fluid and the *surface tension* _ at the liquid–vapor interface, in addition to the properties of the fluid in each phase. Noting that under equilibrium conditions the temperature remains constant during a phase-change process at a fixed pressure, large amounts of heat (due to the large latent heat of vaporization released or absorbed) can be transferred during boiling and condensation essentially at constant temperature. In practice, however, it is necessary to maintain some difference between the surface temperature T_s and T_{sat} for effective heat transfer. Heat transfer coefficients h associated with boiling and condensation are typically much higher than those encountered in other forms of convection processes that involve a single phase.

BOILING HEAT TRANSFER

Many familiar engineering applications involve condensation and boiling heat transfer. In a household refrigerator, for example, the refrigerant absorbs heat from the refrigerated space by boiling in the *evaporator* section and rejects heat to the kitchen air by condensing in the *condenser* section (the long coils behind the refrigerator). Also, in steam power plants, heat is transferred to the steam in the *boiler* where water is vaporized, and the waste heat is rejected from the steam in the *condenser* where the steam is condensed. Some electronic components are cooled by boiling by immersing them in a fluid with an

appropriate boiling temperature. Boiling is a liquid-to-vapor phase change process just like evaporation, but there are significant differences between the two. **Evaporation** occurs at the *liquid–vapor interface* when the vapor pressure is less than the saturation pressure of the liquid at a given temperature. Water in a lake at 20°C, for example, will evaporate to air at 20°C and 60 percent relative humidity since the saturation pressure of water at 20°C is 2.3 kPa and the vapor pressure of air at 20°C and 60 percent relative humidity is 1.4 kPa. Other examples of evaporation are the drying of clothes, fruits, and vegetables; the evaporation of sweat to cool the human body; and the rejection of waste heat in wet cooling towers. Note that evaporation involves no bubble formation or bubble motion.

FIGURE 10–1
A liquid-to-vapor phase change process is called *evaporation* if it occurs at a liquid–vapor interface and *boiling* if it occurs at a solid–liquid interface.

Boiling, on the other hand, occurs at the *solid–liquid interface* when a liquid is brought into contact with a surface maintained at a temperature *Ts* sufficiently above the saturation temperature Tsat of the liquid (Fig. 10–2). At 1 atm, for example, liquid water in contact with a solid surface at 110°C will boil since the saturation temperature of water at 1 atm is 100°C. The boiling process is characterized by the rapid formation of *vapor bubbles* at the solid–liquid interface that detach from the surface when they reach a certain size and attempt to rise to the free surface of the liquid. When cooking, we do not say water is boiling until we see the bubbles rising to the top. Boiling is a complicated phenomenon because of the large number of variables involved in the process and the complex fluid motion patterns caused by the bubble formation and growth.

As a form of convection heat transfer, the *boiling heat flux* from a solid surface to the fluid is expressed from Newton's law of cooling as

$$\dot{q}_{\text{boiling}} = h(T_s - T_{\text{sat}}) = h\Delta T_{\text{excess}} \qquad (\text{W/m}^2)$$

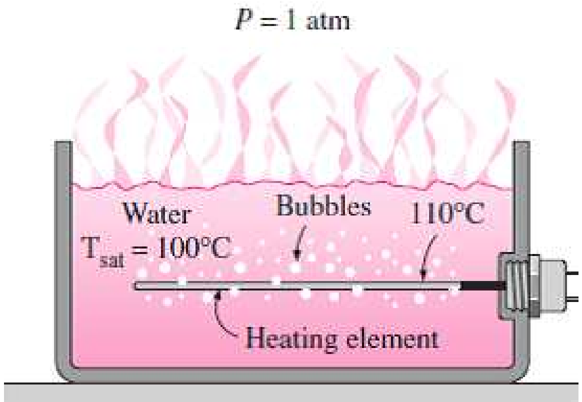


FIGURE 10–2
Boiling occurs when a liquid is brought into contact with a surface at a temperature above the saturation temperature of the liquid.

Boiling is classified as *pool boiling* or *flow boiling,* depending on the presence of bulk fluid motion (Fig. 10–3). Boiling is called **pool boiling** in the absence of bulk fluid flow and **flow boiling** (or *forced convection boiling*) in the presence of it. In pool boiling, the fluid is stationary, and any motion of the fluid is due to natural convection currents and the motion of the bubbles under the influence of buoyancy. The boiling of water in a pan on top of a stove is an example of pool boiling. Pool boiling of a fluid can also be achieved by placing a heating coil in the fluid. In flow boiling, the fluid is forced to move in a heated pipe or over a surface by external means such as a pump. Therefore, flow boiling is always accompanied by other convection effects.

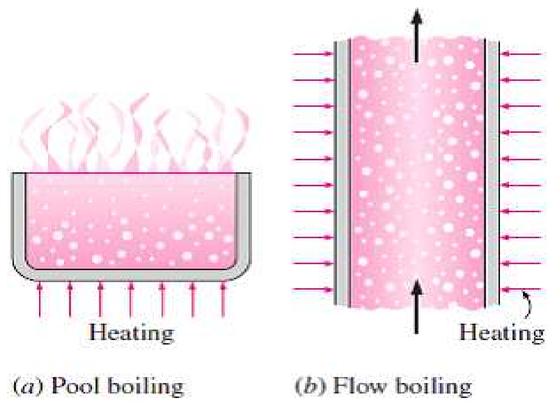


FIGURE 10–3
Classification of boiling on the basis of the presence of bulk fluid motion.

Pool and flow boiling are further classified as *subcooled boiling* or *saturated boiling,* depending on the bulk liquid temperature (Fig. 10–4). Boiling is said to be **subcooled** (or *local*) when the temperature of the main body of the liquid is below the saturation temperature Tsat (i.e., the bulk of the liquid is subcooled) and **saturated** (or *bulk*) when the temperature of the liquid is equal to Tsat (i.e., the bulk of the liquid is saturated). At the early stages of boiling, the bubbles are confined to a narrow region near the hot surface. This is because the liquid adjacent to the hot surface vaporizes as a result of being heated above its saturation temperature. But these bubbles disappear soon after they move away from the hot surface as a result of heat transfer from the bubbles to the cooler liquid surrounding them. This happens when the bulk of the liquid is at a lower temperature than the saturation temperature. The bubbles serve as "energy movers" from the hot surface into the liquid body by absorbing heat from the hot surface and releasing it into the liquid as they condense and collapse. Boiling in this case is confined to a region in the locality of the hot surface and is appropriately called *local* or *subcooled* boiling.

When the entire liquid body reaches the saturation temperature, the bubbles start rising to the top. We can see bubbles throughout the bulk of the liquid,

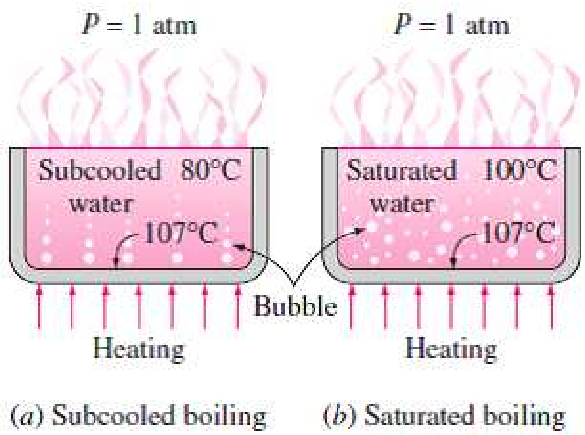


FIGURE 10–4
Classification of boiling on the basis of the presence of bulk liquid temperature.

POOL BOILING CURVE

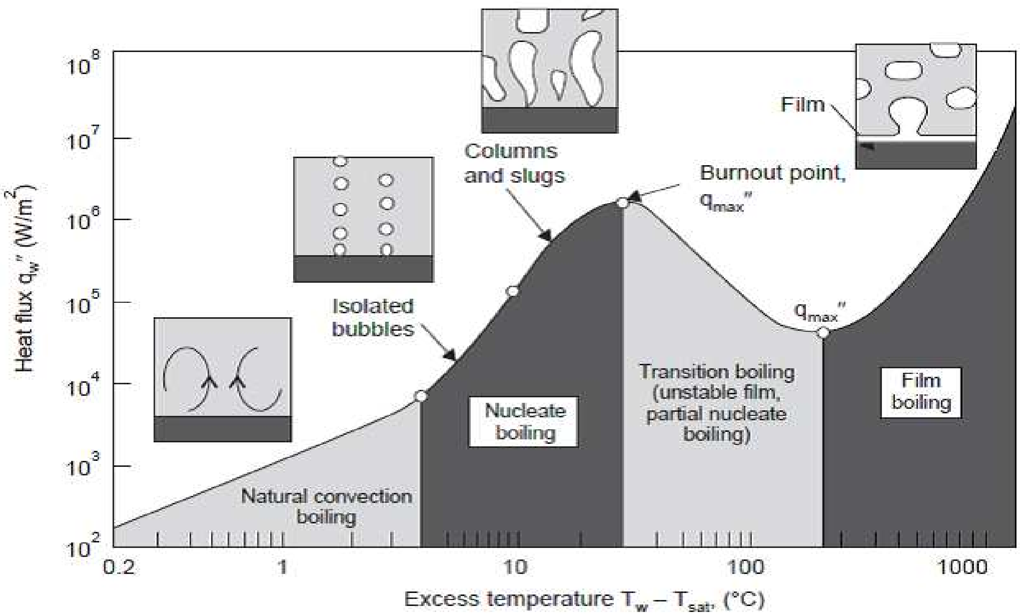
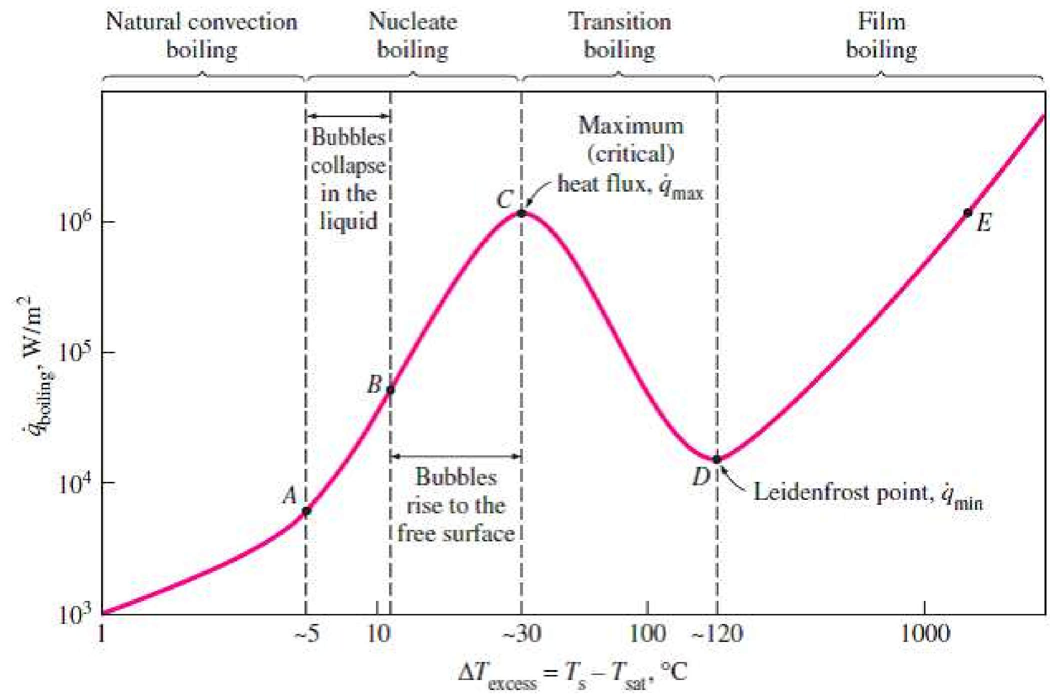


Fig. 11.1. The four regimes of pool boiling in water at atmospheric pressure.

1. Purely convective region	$\Delta T < 5°C$
2. Nucleate Boiling	$5 < \Delta T < 50°C$
3. Unstable (nucleate ⇔ film) boiling	$50°C < \Delta T < 200°C$
4. Stable film boiling	$\Delta T > 200°C$.



THE CORRELATIONS OBTAINED FOR NUCLEATE POOL BOILING IS GIVEN BY ROHSENOW (1952).

$$\frac{Q}{A} = q = \left[\frac{c_1\,\Delta T}{h_{fg}\,Pr^n\,C_{sf}}\right]^3 \mu_1\,h_{fg}\left[\frac{g(\rho_1-\rho_v)}{g_0\,\sigma}\right]^{0.5}$$

suffix *l* denotes liquid properties and *v* denotes vapour properties.

Where $c1$ — Specific heat of liquid J/kgK

ΔT — excess temperature °C or K, (difference)

hfg — specific enthalpy of evaporation J/kg

Pr — Prandtl number of liquid

n — constant equal to 1 for water and 1.7 for other fluids

Csf — surface factor shown in tabulation 11.1 and taken as 0.013 for other cases

μ1 — dynamic viscosity of the liquid kg/ms or Ns m2

ρ1 — density of the liquid kg/m3

ρv — density of vapour kg/m3

σ — surface tension-liquid-vapour interface N/m

g — gravitational acceleration m/s2

$g0$ — force conversion factor kgm/Ns2 = 1 in SI units.

This correlation is the result of a log plot of experimental results with parameters

$$\frac{q}{\mu_1 h_{fg}}\left[\frac{g_0}{g(\rho_1-\rho_v)}\right]^{0.5} \text{ and } \frac{C_1}{h_{fg}}\Delta T\frac{1}{Pr^{1.7}}.$$

1. **Water at** atmospheric pressure (saturation temperature = 100°C) is boiling on a brass surface heated from below. If the surface is at 108°C, determine the heat flux and compare the same with critical heat flux.

Solution: The property values are taken at the liquid temperature

$\rho_c = 961$ kg/m³, $h_{fg} = 2257$ kJ/kg, $\rho_v = 0.598$ kg/m³, $c = 4216$ J/kg K

$\mu_1 = 2.816 \times 10^{-4}$ kg/ms, $\sigma = 58.8 \times 10^{-3}$ N/m, $Pr = 1.74$

From table 11.1 $C_{sf} = 0.0060$. For water $n = 1$

using equation (11.2), (h_{fg} in J/kg)

$$\mathbf{q} = \left[\frac{c_1\,\Delta T}{c_{sf}\,h_{fg}\,Pr_1^n}\right]^3 \mu_1\,h_{fg}\left[\frac{g(\rho_1-\rho_v)}{g_0\,\sigma}\right]^{0.5}$$

$$= \left[\frac{4216\times 8}{0.0060\times 2257\times 10^3\times 1.74}\right]^3 .\ 2.816\times 10^{-4}\times 2257\times 10^3\left[\frac{9.81\,(961-0.598)}{1\times 58.8\times 10^{-3}}\right]^{0.5}$$

$= \mathbf{0.746 \times 10^6\ W/m^2}$

Critical heat flux is given by equation (11.3)

$$\mathbf{q_{cr}} = 0.149\,h_{fg}\,[\sigma\,g\,g_0\,(\rho_1-\rho_v)\,\rho_v^2]^{1/4}$$

$$= 0.149\times 2257\times 10^3\,[58.8\times 10^{-3}\times 9.81\times 1\,(961-0.598)\times 0.598^2]^{0.25}$$

$= \mathbf{1.262 \times 10^6\ W/m^2}$

The actual flux is less than the critical flux at $\Delta T = 8$°C and hence pool boiling exis The critical flux is found to occur at $\Delta T = 10.5$°C when substituted in equation (11.2). **Usi simplified expression,** equation (11.4)

$\mathbf{h} = 5.56\,(8)^3\,(1)^{0.4} = \mathbf{2846.72\ W/m^2\,K}$

$q = h\,\Delta T = 0.0228 \times 10^6$ W/m². A lower prediction.

2. Water is to be boiled at atmospheric pressure in a mechanically polished stainless steel pan placed on top of a heating unit. The inner surface of the bottom of the pan is maintained at 108°C. If the diameter of the bottom of the pan is 30 cm, determine (*a*) the rate of heat transfer to the water and (*b*) the rate of evaporation of water.

SOLUTION Water is boiled at 1 atm pressure on a stainless steel surface. The rate of heat transfer to the water and the rate of evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the heater and the pan are negligible

Properties The properties of water at the saturation temperature of 100°C are surface tension- 0.0589 N/m

$$\rho_l = 957.9 \text{ kg/m}^3 \qquad h_{fg} = 2257.0 \times 10^3 \text{ J/kg}$$
$$\rho_v = 0.6 \text{ kg/m}^3 \qquad \mu_l = 0.282 \times 10^{-3} \text{ kg} \cdot \text{m/s}$$
$$\text{Pr}_l = 1.75 \qquad C_{pl} = 4217 \text{ J/kg} \cdot {}^\circ\text{C}$$

$$\begin{aligned} q_{\text{nucleate}} &= \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2} \left[\frac{C_{pl}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \text{Pr}_l^n}\right]^3 \\ &= (0.282 \times 10^{-3})(2257 \times 10^3)\left[\frac{9.81 \times (957.9 - 0.6)}{0.0589}\right]^{1/2} \\ &\quad \times \left(\frac{4217(108 - 100)}{0.0130(2257 \times 10^3)1.75}\right)^3 \\ &= 7.20 \times 10^4 \text{ W/m}^2 \end{aligned}$$

The surface area of the bottom of the pan is

$$A = \pi D^2/4 = \pi(0.3 \text{ m})^2/4 = 0.07069 \text{ m}^2$$

Then the rate of heat transfer during nucleate boiling becomes

$$\dot{Q}_{\text{boiling}} = Aq_{\text{nucleate}} = (0.07069 \text{ m}^2)(7.20 \times 10^4 \text{ W/m}^2) = \mathbf{5093 \text{ W}}$$

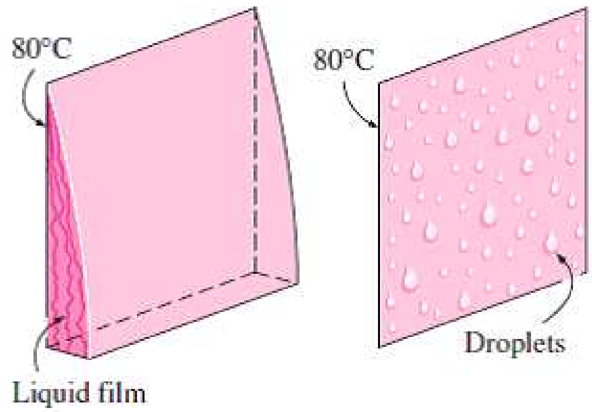
(*b*) The rate of evaporation of water is determined from

$$\dot{m}_{\text{evaporation}} = \frac{\dot{Q}_{\text{boiling}}}{h_{fg}} = \frac{5093 \text{ J/s}}{2257 \times 10^3 \text{ J/kg}} = \mathbf{2.26 \times 10^{-3} \text{ kg/s}}$$

That is, water in the pan will boil at a rate of more than 2 grams per second.

CONDENSATION HEAT TRANSFER

When saturated vapour comes in contact with a cooler surface, the vapour condenses into liquid. The surface temperature should be lower in this case as compared to the temperature of the vapour. The condensate generally moves down by gravity. **If the liquid wets the surface a thin layer of liquids forms over the surface and the film thickness increases along the downward direction. This type of condensation is known as filmwise condensation and this is the type encountered in most practical situations.** The film introduces a resistance to heat flow between the surface and the vapour. The heat transfer rate is reduced because of this resistance. **If the surface is nonwetting, then droplets form on the surface and these roll down individually.** The vapour is in direct contact with the surface over most of the area and heat transfer rates are much higher as there is very little resistance for heat flow between the vapour and the surface. **This type is known as dropwise condensation.** In practice no surface is found to continue as nonwetting over any length of time. So using the value of heat transfer coefficients assuming dropwise condensation for design purposes is not advisable.



(*a*) Film condensation

(*b*) Dropwise condensation

Saturated steam at a temperature of 65°C condenses on a vertical surface at 55°C. Determine the thickness of the condensate film at locations 0.2, 0.4, 0.6, 0.8, 1 m from the top. Also determine the condensate flow, the film Reynolds number, the local and average values of convective heat transfer coefficients at these locations. Also calculate the condensation numbers.

Solution: The property values for liquid should be taken at the film temperature = (55 + 65)/2 = 60°C. The liquid property values at 60°C are

$\rho_1 = 985\ \text{kg/m}^3, \quad k_1 = 0.6513\ \text{W/mK}, \quad c = 4183\ \text{J/kgK}$

$\mu_1 = 4.7083 \times 10^{-4}\ \text{kg/ms},$

h_{fg} at 65°C = 2346.2 kJ/kg, ρ_v 1/6.197 kg/m^3

Considering unit width: using eqn. (11.21).

$$\delta = \left[\frac{4\mu k x (T_g - T_w)}{g h_{fg}\, \rho_1 (\rho_1 - \rho_v)}\right]^{0.25} = \left[\frac{4 \times 4.7083 \times 10^{-4} \times 0.6513 \times 10x}{9.81 \times 2346.2 \times 10^3 \times 985(985 - 1/6.197}\right]^{0.25}$$

$$= \mathbf{1.53 \times 10^{-4}\ .\ x^{0.25}}$$

$$\mathbf{G} = m = \frac{\rho_1(\rho_1 - \rho_v) g\ .\ \delta^3}{3\mu} = \left[\frac{985\ (985 - 1/6.197)9.81}{3 \times 4.7083 \times 10^{-4}}\right] \delta^3$$

$$= 6.73 \times 10^9\ \delta^3\ \text{kg/m width} = \mathbf{0.024176 \times x^{0.75}}$$

$$\mathbf{Re_\delta} = \frac{4G}{\mu_1} = \frac{4}{4.7083 \times 10^{-4}}\ G = 5.718 \times 10^{13}\ .\ \delta^3 = \mathbf{205.39 \times x^{0.75}}$$

$$h_x = \left[\frac{\rho_1 (\rho_1 - \rho_v) g\ h_{fg}\ k^3}{4\mu_1 x (T_g - T_w)}\right]^{1/4} = 4254.33 \times \left(\frac{1}{x}\right)^{0.25}.\ \bar{h} = \frac{4}{3} h_L$$

$$\mathbf{CO} = 1.47\ Re_\delta^{-1/3} = \mathbf{3.816 \times 10^{-5}/\delta}$$

These values at various locations are tabulated below:

Distance m	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*
δ, mm	0.10238	0.12175	0.13474	0.14479	0.1531
m, kg/s/m	0.00723	0.01216	0.0165	0.02045	0.02418
Re_δ	61.42	103.31	140.18	173.74	205.4
h_x	6361.7	5349.5	4833.9	4498.4	4254.3
$\overline{\mathbf{h}}$	**8482.3**	**7132.7**	**6445.2**	**5997.9**	**5672.4**
CO	0.3730	0.3133	0.2830	0.2634	0.2491

The value of h_{fg} can be corrected to take care of under cooling.

HEAT EXCHANGER

The process of heat exchange between two fluids that are at different temperatures and separated by a solid wall occurs in many engineering applications. The device used to implement this exchange is termed a heat exchanger, and specific applications may be found in space heating and air-conditioning, power production, waste heat recovery, and chemical processing. In this chapter our objectives are to introduce performance parameters for assessing the efficacy of a heat exchanger and to develop methodologies for designing a heat exchanger or for predicting the performance of an existing exchanger operating under prescribed conditions.

Heat Exchanger Types

Heat exchangers are typically classified according to *flow arrangement* and *type of construction.* The simplest heat exchanger is one for which the hot and cold fluids move in the same or opposite directions in a *concentric tube* (or *double-pipe*) construction. In the *parallel-flow* arrangement of Figure 11.1*a*, the hot and cold fluids enter at the same end, flow in the same direction, and leave at the same end. In the *counterflow* arrangement of Figure 11.1*b*, the fluids enter at opposite ends, flow in opposite directions, and leave at opposite ends. Alternatively, the fluids may move in *cross flow* (perpendicular to each other), as shown by the *finned* and *unfinned* tubular heat exchangers of Figure 11.2. The two configurations are typically differentiated by an idealization that treats fluid motion over the tubes as *unmixed* or *mixed*. In Figure 11.2*a*, the cross-flowing fluid is said to be unmixed because the fins inhibit motion in a direction (y) that is transverse to the main-flow direction (x). In this case the cross-flowing fluid temperature varies with x and y. In contrast, for the unfinned tube bundle of Figure 11.2*b*, fluid motion, hence mixing, in the transverse direction is possible, and temperature variations are primarily in the main-flow direction. Since the tube flow is unmixed in either heat exchanger, both fluids are unmixed in the finned exchanger, while the cross-flowing fluid is mixed and the tube fluid is unmixed in the unfinned exchanger. The nature of the mixing condition influences heat exchanger performance. Another common configuration is the *shell-and-tube* heat exchanger [1]. Specific forms differ according to the number of shell-and-tube passes, and the simplest form, which involves single tube and shell *passes*, is shown in Figure 11.3. Baffles are usually installed to increase the convection coefficient of the shell-side fluid by inducing turbulence and a cross-flow velocity component relative to the tubes. In addition, the baffles physically support the tubes, reducing flow-induced tube vibration. Baffled heat exchangers with one shell pass and two tube passes and with two shell passes and four tube passes are shown in Figures 11.4*a* and 11.4*b*, respectively.

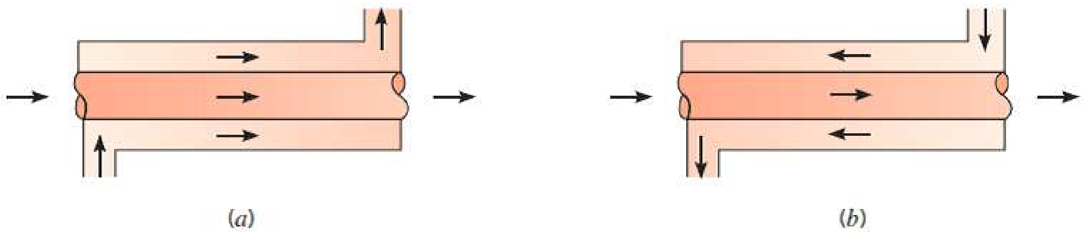


FIGURE 11.1 Concentric tube heat exchangers. (*a*) Parallel flow. (*b*) Counterflow.

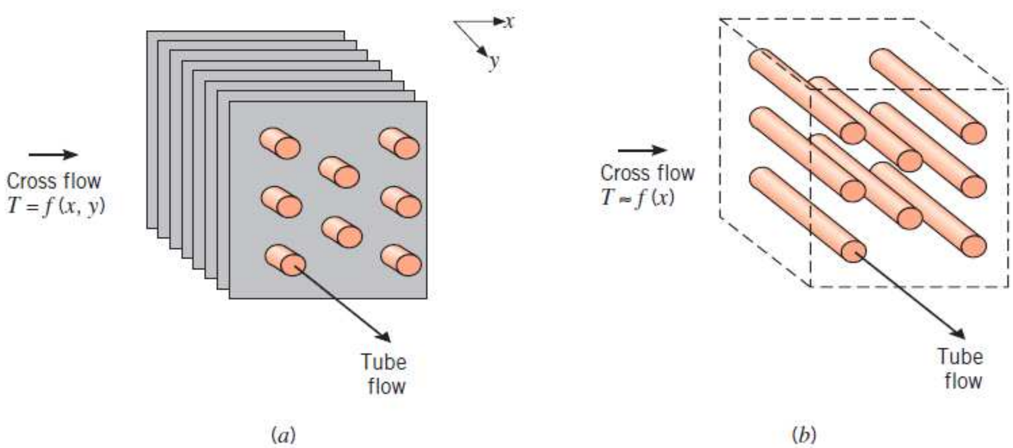


FIGURE 11.2 Cross-flow heat exchangers. (*a*) Finned with both fluids unmixed. (*b*) Unfinned with one fluid mixed and the other unmixed.

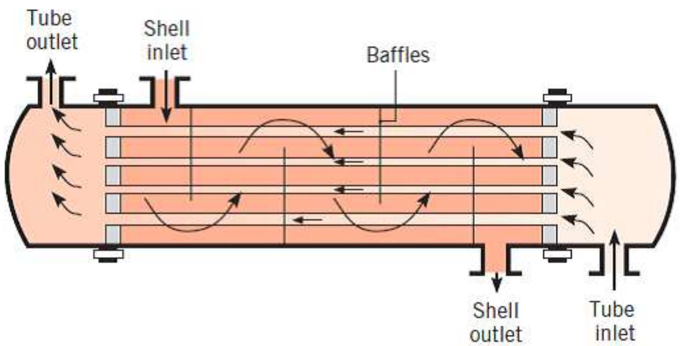


FIGURE 11.3 Shell-and-tube heat exchanger with one shell pass and one tube pass (cross-counterflow mode of operation).

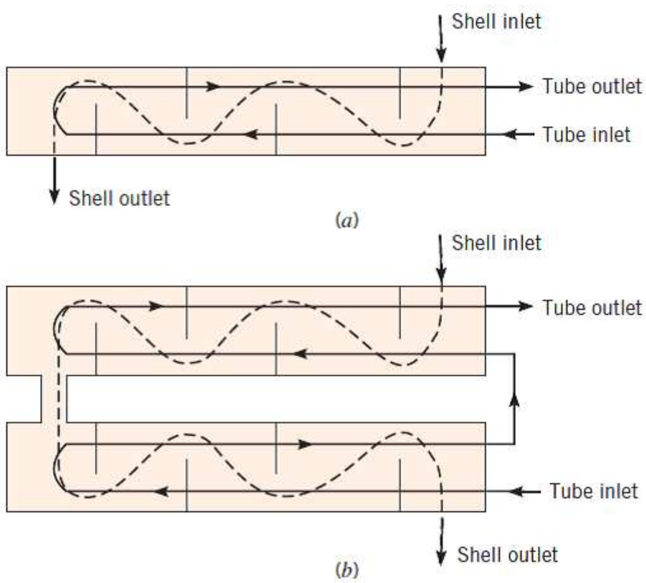


FIGURE 11.4 Shell-and-tube heat exchangers. (*a*) One shell pass and two tube passes. (*b*) Two shell passes and four tube passes.

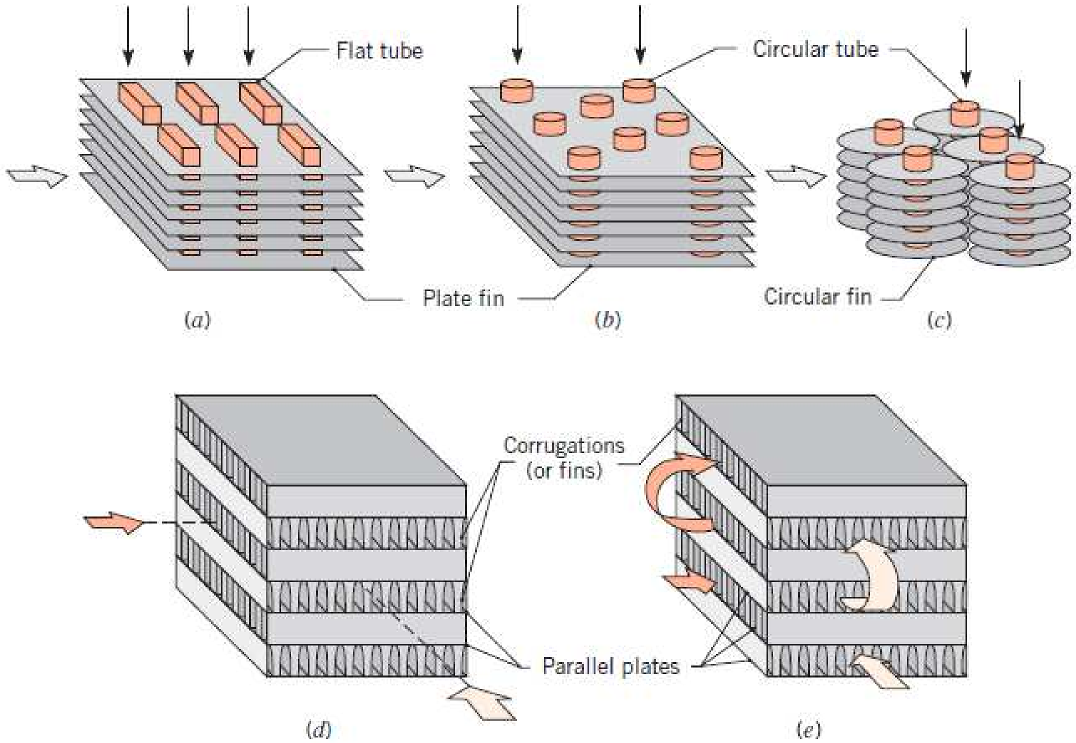


FIGURE 11.5 Compact heat exchanger cores. (*a*) Fin–tube (flat tubes, continuous plate fins). (*b*) Fin–tube (circular tubes, continuous plate fins). (*c*) Fin–tube (circular tubes, circular fins). (*d*) Plate–fin (single pass). (*e*) Plate–fin (multipass).

Determine the area required *in parallel flow heat exchanger to cool oil from 60°C to 30° C using water available at 20°C. The outlet temperature of the water is 26°C. The rate of* *flow of oil is 10 kg/s. The specific heat of the oil is 2200 J/kg K. The overall heat transfer coefficient U = 300 W/m2 K. Compare the area required for a counter flow exchager.*

Solution: The temperature variation for parallel flow is shown in Fig. 12.4 (*a*).

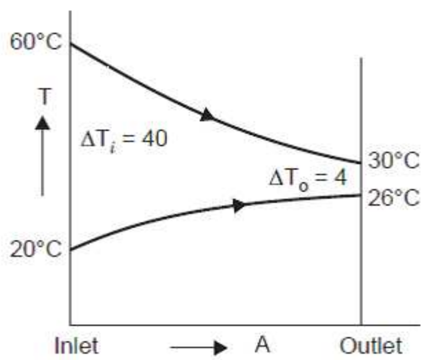


Fig. 12.4 (a) Parallel flow

$$\mathbf{Q} = m_h\, c_h\, (T_{hi} - T_{ho}) = 10 \times 2200\ (60 - 30)\ \text{J/s} = \mathbf{6{,}60{,}000\ W}$$

$$Q = U\,A\ (\text{LMTD})$$

$$\textbf{LMTD} = \frac{40 - 4}{\ln \frac{40}{4}} = \mathbf{15.635°C}$$

$$6{,}60{,}000 = 300 \times A \times 15.635 \quad \therefore \quad \mathbf{A = 140.71\ m^2}$$

As can be seen a single tube arrangement is impractical.

Counter flow:

The temperature variation is shown in Fig 12.4 (*b*)

$$\textbf{LMTD} = \frac{34 - 10}{\ln \frac{34}{10}} = \textbf{19.611°C}$$

$$\therefore \qquad \textbf{A = 112.18 m}^2$$

about 20% less.

The flow rate of water can also be determined as it will be a necessary data.

$$Q = m_c\, C_c\, (T_{co} - T_{ci})$$

$$6{,}60{,}000 = m_c \times 4180\,(26 - 20)$$

$$\mathbf{m_c = 26.32\ kg/s}$$

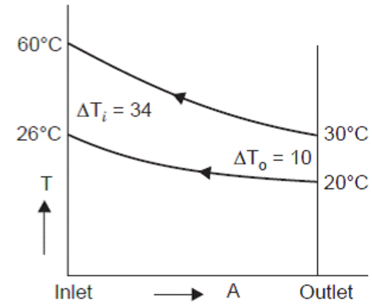


Fig. 12.4. (b) Counter flow

The counter flow arrangement provides more uniform temperature difference along the flow and hence a better rate of heat flow. The counter flow type can also be used to cool or heat over a wider range of temperatures. In the above case by increasing the area or by reducing flow the hot oil can in the limit be cooled to 20°C. Manipulation in the opposite direction can get the water heated to 60°C. This is not possible in the parallel flow where the exit temperature has to lie somewhere in between the two inlet temperatures. As far as possible counter flow is always used in heat exchanger designs.

An economiser in a boiler has flow of water inside the pipes and hot gases on the outside flowing across the pipes. The flow rate of gases is 2,000 tons/hr and the gases are cooled from 390°C to 200°C. The specific heat of the gas is 1005 J/kg K. Water is heated (under pressure) from 100°C to 220°C. Assuming an overall heat transfer coefficient of 35 W/m2 K, determine the area required. Assume that the air flow is mixed.

Solution: This is a cross flow heat exchanger

The flow rate of gas is $\frac{2000 \times 1000}{3600}$ kg/s = 555.6 kg/s

$$\mathbf{C_h = 5.583 \times 10^5\ W}$$

The flow rate of water $= \frac{555.6 \times 1005 \times (390 - 200)}{4180 \times (220 - 100)} = 211.49$ kg/s

$$\mathbf{C_c = 211.49 \times 4180 = 8.84 \times 10^5\ W}$$

$\therefore$ C_h is C_{min} and $\mathbf{C_{min}/C_{max} = 0.632}$

Effectiveness with C_h as $C_{min} = \frac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}} = \frac{390 - 200}{390 - 100} = \mathbf{0.6551}$

From the chart ($C_h = C_{min}$ mixed), NTU is read as 1.75

$$\therefore \quad \mathbf{A} = \frac{\text{NTU} \times C_{min}}{U} = \frac{1.75 \times 5.583 \times 10^5}{35} = \mathbf{27{,}915\ m^2}$$

Check using LMTD:

$$\textbf{LMTD} \text{ counter flow} = \frac{(390 - 220) - (200 - 100)}{\ln\left(\frac{390 - 220}{200 - 100}\right)} = \mathbf{131.92°C}$$

To determine correction factor,

$$\mathbf{P} = \frac{220 - 100}{390 - 100} = \frac{120}{290} = \mathbf{0.414},\ \mathbf{R} = \frac{390 - 200}{220 - 100} = \frac{190}{120} = \mathbf{1.583}$$

$$Q = F\,U\,A\,(\text{LMTD})$$

$\therefore$ $$A = Q/FU\,(\text{LMTD}) = \frac{2000 \times 1000}{3600} \times \frac{1005\,(390 - 200)}{35 \times 131.92 \times 0.8}$$

$\therefore$ **A = 28720 m^2** checks within reasonable limits.

The LMTD method is simpler in this case.

UNIT – IV - Thermal Radiation

Introduction

This mode of heat transfer defers from the other two modes in that radiation does not require the presence of material medium.

Ex: Heat transfer through a vacuum, from Sun to earth, a hot body inside a vacuum.

Thermal radiation is that electro magnetic radiation emitted by a body as a result of its temperature which is propagated at the speed of light (3×10^8 m/s). It is called as the "Stefan-Boltzmann Law". The total energy emitted by an ideal radiator or black body per unit time per unit area, E_b can be written as

E_b Black body radiation or emissive power (W/m^2)

σ Stefan Boltzman Constant (5.67×10^{-8} W/m^2K^4)

T Temperature in K

$$E_b = \sigma T^4$$

Black body

A body at a temperature above absolute zero emits radiation in all directions over a wide range of wave lengths. The amount of radiation emitted from a surface at a given wave length depends on the material surface condition and temperature. Therefore different bodies may emit different amount of radiation (Per unit surface area) even when they are at the same temperature. A black body is an ideal surface having the following properties

- It absorbs all the incident radiation regardless of wave length and direction.
- No surface can emit more energy than a black body for a given temperature and wave length
- Radiation emitted by a black body is a function of both wave length and temperature but independent of direction

Properties of radiation

When radiant energy strikes a material surface, part of the radiation is reflected, part is absorbed and a part is transmitted. If ρ is the fraction reflected, α is the fraction absorbed and τ is the fraction transmitted

$$\alpha + \rho + \tau = 1$$

But for most solid bodies $\tau = 0$

Emissivity

The ratio of the emissing power of a body to the emissive power of the black body at the same temperature is equal to absorptivity of the body.

$$\alpha = \frac{E}{E_b}$$

The ratio is defined as the emissivity of the body

$$\varepsilon = \frac{E}{E_b}$$

$$\therefore \alpha = \varepsilon$$

α is a function of body temperature and wave length but a grey body is defined as for which absorptivity and emmisivity are independent of wave length.

Radiation exchange between surfaces

A_2

A_1

Shape factor is defined as the fraction of energy leaving one surface, which reaches the other surface. The figure shows two black surfaces, with areas A_1 and A_2.

If F_{12} is the fraction of energy leaving surface 1, which reaches surface 2, and if F_{21} is the fraction of energy leaving surface 2 which reaches surface 1

The energy leaving surface 1 and reaching surface 2 = $E_{b1}A_1F_{12}$

The energy leaving surface 2 and reaching surface 2 = $E_{b2}A_2F_{21}$

The net energy exchange $Q_{1\text{-}2} = E_{b1}A_1F_{12} - E_{b2}A_2F_{21}$

For same temperature

$Q_{1\text{-}2} = 0$ therefore

$E_{b1}A_1F_{12} = E_{b2}A_2F_{21}$

$A_1F_{12} = A_2F_{21}$

In general $A_iF_{ij} = A_jF_{ji}$

f_{ij}` Geometric factor for surface i to surface j

Heat exchange between non-black bodies

The calculation of radiation heat transfer between black surfaces is relatively easy since all the radiant energy which strikes a surface is absorbed. The main problem is to find out the shape factors, but once this accomplished, the calculation of heat exchange is very simple.

When non-black bodies are involved, the situation is much more complex and for all the energy striking a surface will not be absorbed; part will be reflected back to another heat transfer surface, and part will be reflected back to another heat transfer surface and part may be reflected out of the system entirely.

To cope with the situation let us assume the surfaces considered in our analysis are diffusive and uniform in temperature and the reflective and emissive properties are constant over all the surfaces.

J_i radiosity, total radiation leaving a surface i per unit time and per unit area.

G_i irradiation, total energy incident upon surface i per unit time per unit area.

E_i emission, total radiation generated by surface i per unit time per unit area.

When no energy transmitted, the radiosity is,

$J = \varepsilon E_b + \rho G$

When trasmitivity is assumed zero,

$\rho = 1 - \alpha = 1 - \varepsilon$

So that

$$J = \varepsilon E_b + (1-\varepsilon)G \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The net energy leaving the surface

$$\frac{q}{A} = J - G = \varepsilon E_b + (1-\varepsilon)G - G = \varepsilon E_b - \varepsilon G \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Solving (1) and (2)

$$q = \frac{\varepsilon A}{1-\varepsilon}(E_b - J)$$

$$q = \frac{E_b - J}{1-\varepsilon \big/ \varepsilon A}$$

Element representing "Surface resistant" in radiation network method

Now consider he exchange of radiant energy by two surfaces A_1 and A_2, of that total radiation leaving A_1 and reaches A_2

$$J_1A_1F_{12}$$

And of that total radiation leaving A_2 and reaches A_1 is

$$J_2A_2F_{21}$$

The net interchange between the two surfaces is

$$q_{1\text{-}2} = J_1A_1F_{12} - J_2A_2F_{21}$$

But $\quad A_1F_{12} = A_2F_{21}$

Therefore

$$q_{1-2} = (J_1 - J_2)\, A_1F_{12}$$

$$q_{1-2} = \frac{(J_1 - J_2)}{1/A_1F_{12}}$$

We may construct the network elements

Element representing "Space resistant" in radiation network method

q_{1-2}

Ex: Two surfaces of area A_1 and A_2 of emissivities ε_1 and ε_2 which exchangeheat with each other and nothing else, would be represented by the following circuit diagram.

A radiation network for three surfaces which see each other and nothing else is

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1}$$

$$\frac{1}{A_1 F_{12}}$$

$$\frac{1-\varepsilon_2}{\varepsilon_2 A_2}$$

E_{b1}

J_2

J_1

$$\frac{1-\varepsilon_3}{\varepsilon_3 A_3}$$

$$\frac{1}{A_1 F_{13}}$$

$$\frac{1}{A_2 F_{23}}$$

J_3

E_{b3}

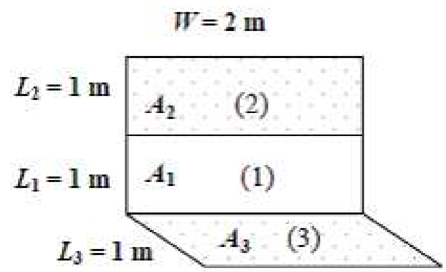
E_{b2}

1. Determine the view factors F_{13} and F_{23} between the rectangular surfaces

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis From Fig.

$$\left.\begin{array}{l}\frac{L_3}{W}=\frac{1}{2}=0.5\\ \frac{L1}{W}=\frac{1}{2}=0.5\end{array}\right\}F_{31}=0.24$$



and

$$\left.\begin{array}{l}\frac{L_3}{W}=\frac{1}{2}=0.5\\ \frac{L_1+L_2}{W}=\frac{2}{2}=1\end{array}\right\}F_{3\to(1+2)}=0.29$$

We note that $A_1 = A_3$. Then the reciprocity and superposition rules gives

$$A_1F_{13}=A_3F_{31}\longrightarrow F_{13}=F_{31}=\mathbf{0.24}$$

$$F_{3\to(1+2)}=F_{31}+F_{32}\longrightarrow\quad 0.29=0.24+F_{32}\longrightarrow F_{32}=0.05$$

Finally, $A_2=A_3\longrightarrow F_{23}=F_{32}=\mathbf{0.05}$

2. **2.** Consider a 4-m × 4-m × 4-m cubical furnace whose floor and ceiling are black and whose side surfaces are reradiating. The floor and the ceiling of the furnace are maintained at temperatures of 550 K and 1100 K, respectively. Determine the net rate of radiation heat transfer between the floor and the ceiling of the furnace.

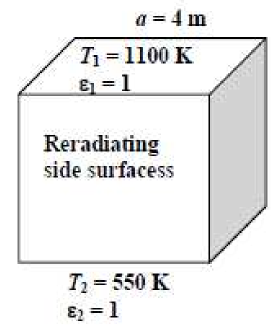
Assumptions 1 Steady operating condition exist 2 The surfaces are opaque, diffuse, and gray. 3 Convection heat transfer is not considered.

Properties The emissivities of all surfaces are = 1 since they are black or reradiating.

Analysis We consider the ceiling to be surface 1, the floor to be surface 2 and the side surfaces to be surface 3. The furnace can be considered to be three-surface enclosure with a radiation network shown in the figure. We assume that steady-state conditions exist. Since the side surfaces are reradiating, there is no heat transfer through them, and the entire heat lost by the ceiling must be gained by the floor. The view factor from the ceiling to the floor of the furnace is . Then the rate of heat loss from the ceiling can be determined from

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}}\right)^{-1}}$$

where



$$E_{b1} = \sigma T_1^4 = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1100\ \text{K})^4 = 83{,}015\ \text{W/m}^2$$
$$E_{b2} = \sigma T_2^4 = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(550\ \text{K})^4 = 5188\ \text{W/m}^2$$

and

$$A_1 = A_2 = (4\ \text{m})^2 = 16\ \text{m}^2$$
$$R_{12} = \frac{1}{A_1F_{12}} = \frac{1}{(16\ \text{m}^2)(0.2)} = 0.3125\ \text{m}^{-2}$$
$$R_{13} = R_{23} = \frac{1}{A_1F_{13}} = \frac{1}{(16\ \text{m}^2)(0.8)} = 0.078125\ \text{m}^{-2}$$

Substituting,

$$\dot{Q}_{12} = \frac{(83{,}015-5188)\ \text{W/m}^2}{\left(\dfrac{1}{0.3125\ \text{m}^{-2}}+\dfrac{1}{2(0.078125\ \text{m}^{-2})}\right)^{-1}} = 7.47\times10^5\ \text{W} = \mathbf{747\ kW}$$

3. A thin aluminum sheet with an emissivity of 0.15 on both sides is placed between two very large parallel plates, which are maintained at uniform temperatures $T1$ =900 K and $T2$ =650 K and have emissivities 0.5 and 2 ,0.8, respectively. Determine the net rate of radiation heat transfer between the two plates per unit surface area of the plates and Compare the result with that without the shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

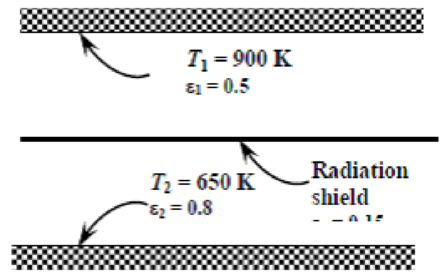
Properties The emissivities of surfaces are given to be emm1 = 0.5, emm2 = 0.8, and emm3 = 0.15.

Analysis The net rate of radiation heat transfer with a thin aluminum shield per unit area of the plates is the plates is

$$\dot{Q}_{12,\text{oneshield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{1}{\varepsilon_{3,1}}+\frac{1}{\varepsilon_{3,2}}-1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(900\ \text{K})^4-(650\ \text{K})^4]}{\left(\frac{1}{0.5}+\frac{1}{0.8}-1\right)+\left(\frac{1}{0.15}+\frac{1}{0.15}-1\right)}$$

$$= \mathbf{1857\ W/m^2}$$



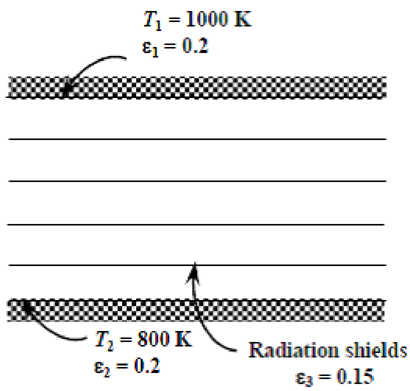
The net rate of radiation heat transfer between the plates in the case of no shield is

$$\dot{Q}_{12,no\,\text{shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(900\ \text{K})^4-(650\ \text{K})^4]}{\left(\frac{1}{0.5}+\frac{1}{0.8}-1\right)} = 12{,}035\ \text{W/m}^2$$

Then the ratio of radiation heat transfer for the two cases becomes

$$\frac{\dot{Q}_{12,\text{one shield}}}{\dot{Q}_{12,\text{no shield}}} = \frac{1857\ \text{W}}{12{,}035\ \text{W}} \cong \mathbf{\frac{1}{6}}$$

4. Two very large parallel plates are maintained at uniform temperatures of $T1$=1000 K and $T2$ = 800 K and have emissivities of 1 ,2 and 0.2, respectively. It is desired to reduce the net rate of radiation heat transfer between the two plates to one-fifth by placing thin aluminum sheets with anemissivity of 0.15 on both sides between the plates. Determine the number of sheets that need to be inserted.



Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.2$, and $\varepsilon_3 = 0.15$.

Analysis The net rate of radiation heat transfer between the plates in the case of no shield is

$$\dot{Q}_{12,no\,shield} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(1000\ \text{K})^4 - (800\ \text{K})^4]}{\left(\frac{1}{0.2} + \frac{1}{0.2} - 1\right)}$$

$$= 3720\ \text{W/m}^2$$

The number of sheets that need to be inserted in order to reduce the net rate of heat transfer between the two plates to onefifth can be determined from

$$\dot{Q}_{12,\text{shields}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + N_{\text{shield}}\left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$\frac{1}{5}(3720\ \text{W/m}^2) = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(1000\ \text{K})^4 - (800\ \text{K})^4]}{\left(\frac{1}{0.2} + \frac{1}{0.2} - 1\right) + N_{\text{shield}}\left(\frac{1}{0.15} + \frac{1}{0.15} - 1\right)} \longrightarrow N_{\text{shield}} = 2.92 \cong \mathbf{3}$$

5. ***T2 = 800 K***

emm2 = 0.2

T1 = 1000 K

emm1 = 0.2

Radiation shields

emm3 = 0.15

A2-m-internal-diameter double-walled spherical tank is used to store iced water at 0°C. Each wall is 0.5 cm thick, and the 1.5-cm-thick air space between the two walls of the tank is evacuated in order to minimize heat transfer. The surfaces surrounding the evacuated space are polished so that each surface has an emissivity of 0.15. The temperature of the outer wall of the tank is measured to be 20°C. Assuming the inner wall of the steel tank to be at 0°C, determine (*a*) the rate of heat transfer to the iced water in the tank and (*b*) the amount of ice at 0°C that melts during a 24-h period.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.

Properties The emissivities of both surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.15$.

Analysis (*a*) Assuming the conduction resistance s of the walls to be negligible, the rate of heat transfer to the iced water in the tank is determined to be

$$A_1 = \pi D_1^2 = \pi(2.01\text{ m})^2 = 12.69\text{ m}^2$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_2^4 - T_1^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)^2}$$

$$= \frac{(12.69\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(20+273\text{ K})^4 - (0+273\text{ K})^4]}{\dfrac{1}{0.15} + \dfrac{1-0.15}{0.15}\left(\dfrac{2.01}{2.04}\right)^2}$$

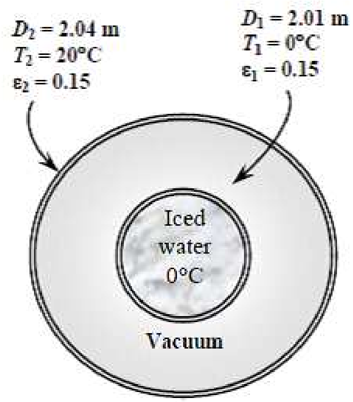
$$= \mathbf{107.4\text{ W}}$$

(*b*) The amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.1074\text{ kJ/s})(24\times3600\text{ s}) = 9275\text{ kJ}$$

The amount of ice that melts during this period then becomes

$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{9275\text{ kJ}}{333.7\text{ kJ/kg}} = \mathbf{27.8\text{ kg}}$$



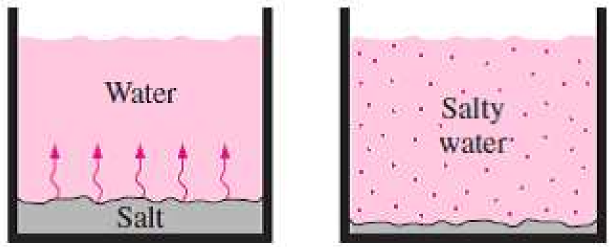
UNIT- V- MASS TRANSFER

Introduction

Mass transfer can result from several different phenomena. There is a mass transfer associated with convection in that mass is transported from one place to another in the flow system. This type of mass transfer occurs on a macroscopic level and is usually treated in the subject of fluid mechanics. When a mixture of gases or liquids is contained such that there exists a concentration gradient of one or more of the constituents across the system, there will be a mass transfer on a microscopic level as the result of diffusion from regions of high concentration to regions of low concentration. In this chapter we are primarily concerned with some of the simple relations that may be used to calculate mass diffusion and their relation to heat transfer. Nevertheless, one must remember that the general subject of mass transfer encompasses both mass diffusion on a molecular scale and the bulk mass transport that may result from a convection process.

Not only may mass diffusion occur on a molecular basis, but accelerated diffusion rates will also occur in turbulent-flow systems as a result of the rapid-eddy mixing processes, just as these mixing processes created increased heat transfer and viscous action in turbulent flow.

Although beyond the scope of our discussion, it is well to mention that mass diffusion may also result from a temperature gradient in a system; this is called *thermal diffusion.* Similarly, a concentration gradient can give rise to a temperature gradient and a consequent heat transfer. These two effects are termed *coupled phenomena* and may be treated by the methods of irreversible thermodynamics.



(*a*) Before (*b*) After

Whenever there is concentration difference of a physical quantity in a medium, nature tends to equalize things by forcing a flow from the high to the low concentration region.

ANALOGY BETWEEN HEAT AND MASS TRANSFER

We have spent a considerable amount of time studying heat transfer, and we could spend just as much time (perhaps more) studying mass transfer. However, the mechanisms of heat and mass transfer are analogous to each other, and thus we can develop an understanding of mass transfer in a short time with little effort by simply drawing *parallels* between heat and mass transfer. Establishing those "bridges" between the two seemingly unrelated areas will make it possible to use our heat transfer knowledge to solve mass transfer problems. Alternately, gaining a working knowledge of mass transfer will help us to better understand the heat transfer processes by thinking of heat as a massless substance as they did in the nineteenth century. The short-lived caloric theory of heat is the origin of most heat transfer terminology used today and served its purpose well until it was replaced by the kinetic theory. Mass is, in essence, energy since mass and energy can be converted to each other according to Einstein's formula $E = mc^2$, where c is the speed of light. Therefore, we can look at mass and heat as two different forms of energy and exploit this to advantage without going overboard.

PROPERTIES OF MIXTURE

In a mixture consisting of two or more materials the mass per unit volume of any component is called mass concentration of that component. If there are two components *A* and *B*, then the mass concentration of *A* is

$$m_a = \frac{\text{mass of } A \text{ in the mixture}}{\text{volume of the mixture}}$$

and concentration of B,

$$m_b = \frac{\text{mass of } B \text{ in the mixture}}{\text{volume of the mixture}}.$$

The total mass concentration is $ma + mb$, which is also the density of the mixture. Mass concentration can also be expressed in terms of individual and total densities of the mixture *i.e.*,

$$m_a = \frac{\rho_A}{\rho}$$

where ρa is the density of A in the mixture and ρ is the density of the mixture. It is more convenient to express the concentration in terms of the molecular weight of the component.

Mole fraction *Na* can be expressed as

$$N_a = \frac{\text{Number of moles of component } A}{\text{Total number of moles in the mixture}}.$$

Number of Mole = mass/molecular weight

For gases as $\rho_i = \dfrac{P}{R_i T}$

or $N_i = \dfrac{P_i}{\Re T}$

where $\Re$ is universal gas constant.

At the temperature T of the mixture then

$$N_i \propto P_i$$

where $C_a = \dfrac{N_a}{Nt} = \dfrac{P_a}{P_T}$

where P_a is the partial pressure of A in the mixture and P_T is the total pressure of the mixture. C_a is the mole concentration of A in the mixture.

Also $C_a + C_b = 1$ for a two component mixture.

DIFFUSION MASS TRANSFER

Diffusion mass transfer occurs without macroscopic mass motion or mixing. A lump of sugar dropped into a cup of tea will dissolve by diffusion even if left unstirred. But it will take a long time for the sugar to reach all of the volume in the cup. However it will diffuse into the volume by and by. Consider a chamber in which two different gases at the same pressure and temperature are kept separated by a thin barrier. When the barrier is removed, the gases will begin to diffuse into each others volume. After some time a steady condition of uniform mixture would be reached. This type of diffusion can occur in solids also. The rate in solids will be extremely slow. Diffusion in these situations occurs at the molecular level and the governing equations are similar to those in heat conduction where energy transfer occurs at the molecular level.

The basic law governing mass transfer at the molecular diffusion level is known as **Fick's law.** This is similar to the Fourier heat conduction law. In Mass transfer, molal quantities are more convenient to use as compared to mass units, because mass transfer is due to the movement of molecules as discrete quantities. Hence it is convenient to use number of moles, or molar concentration instead of density etc.

FICK'S LAW OF DIFFUSION

The Fick's law can be stated as

$$N_a = -D_{ab}\frac{dC_a}{dx} \quad \ldots\ldots.1$$

Where N_a—> number of moles of '*a*' diffusing perpendicular to area *A*, moles/m^2 sec

D_{ab}—> Diffusion coefficient or mass diffusivity, m^2/s, *a* into *b*

Ca —> mole concentration of '*a*' moles/m^3

x —> diffusion direction

The diffusion coefficient is similar to thermal diffusivity, α and momentum diffusivity *v*.

Number of moles multiplied by the molecular mass (or more popularly known as molecular weight) will provide the value of mass transfer in kg/s. Equation 1 can also be written as

$$\frac{m_a}{A} = -D_{ab}\cdot\frac{d\rho_a}{dx} \quad \ldots\ldots2$$

but this form is not as popular as the more convenient equation (1). The conduction equation similar to this is

$$\frac{Q}{A} = -\left(\frac{k}{\rho c}\right) \cdot \frac{d(\rho c T)}{dx} \quad .....3$$

$k/\rho c$ is thermal diffusivity α and ρc is the heat capacity (energy density) for unit volume.

The derivation of the general mass diffusion equation is similar to that of the general heat conduction equation with C_a replacing T and D replacing $k/\rho c$.

The general mass diffusion equation for the species A under steady state condition is given by equation (4)

$$\frac{\partial^2 C_a}{dx^2} + \frac{\partial^2 C_a}{\partial y^2} + \frac{\partial^2 C_a}{\partial z^2} = \frac{1}{D}\frac{\partial C_a}{\partial \tau} \quad ..... 4$$

Generation of mass of the species '*A*' by chemical reaction is not considered in the equation. However an additive term N_a/D on the LHS will take care of this similar to heat generation term q/k.

The solutions for this equation are also similar to the solutions of the general conduction equation. However there exist some differences. These are

(*i*) While heat flow is in one direction, the mass of one species flows opposite to the flow of the other component of the mixture. (here two component mixture is considered).

(*ii*) Even while one component alone diffuses under certain circumstances, a bulk flow has to be generated as otherwise a density gradient will be created spontaneously, which is not possible. For example when water evaporates into an air body over water surface, an equal quantity of air cannot enter the water phase. The density gradient created is dispersed by some mixture moving away from the surface maintaining a balance. This is termed as bulk flow.

The value of *Dab* for certain combinations of components are available in literature. It can be proved that *Dab*= *Dba*. When one molecule of '*A*' moves in the *x* direction, one molecule of '*B*' has to move in the opposite direction. Otherwise a macroscopic density gradient will develop, which is not sustainable, (*A* is area)

$$\frac{N_a}{A} = -D_{ab}\frac{dC_a}{dx}$$

$$\frac{N_b}{A} = -D_{ba}\frac{dC_b}{dx} = -D_{ba}\frac{d(1-C_a)}{dx} = D_{ba}\frac{dC_a}{dx}$$

as $\frac{N_a}{A} = -\frac{N_b}{A}$ and so $D_{ab} = D_{ba}$

EQUIMOLAL COUNTER DIFFUSION

The total pressure is constant all through the mixture. Hence the difference in partial pressures will be equal. The Fick's equation when integrated for a larger plane volume of thickness *L* will give

$$\frac{N_a}{A} = D_{ab}\frac{(C_{a1}-C_{a2})}{L}$$

$$\frac{N_b}{A} = D_{ba}\frac{(C_{b2}-C_{b1})}{L}$$

$$\frac{N_b}{A} = -\frac{N_a}{A}, \text{ and } (C_{a1}-C_{a2}) = (C_{b2}-C_{b1}).$$

Dab equals *Dba*

Where *Ca*1 and *Cb*1 are the mole concentrations at face 1 and *Ca*2 and *Cb*2 are mole concentrations at face 2 which is at a distance L from the first face.

When applied to gases,

$$\frac{N_a}{A} = \frac{D}{\Re T} \cdot \frac{P_{a1} - P_{a2}}{(x_2 - x_1)}$$

Where $Pa1$ and $Pa2$ are partial pressures of component 'A' at $x1$ and $x2$ and $\Re$ is the universal gas constant in J/kg mol K. T is the temperature in absolute units. The distance should be expressed in metre.

The partial pressure variation and diffusion directions are shown in Fig 1.

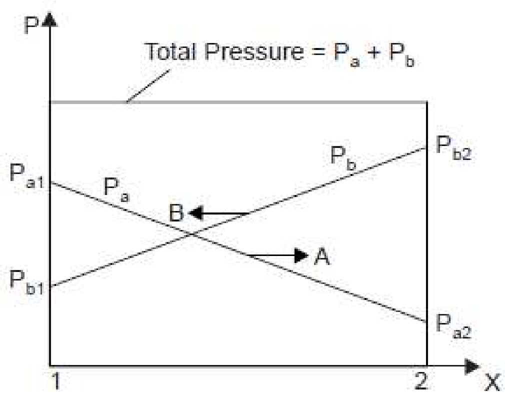


Fig. 1. Partial Pressure variation of components in equimolal counter diffusion.

1. *In order to avoid pressure build up ammonia gas at atmospheric pressure in a pipe is vented to atmosphere through a pipe of 3 mm dia and 20 m length. Determine the mass of ammonia diffusing out and mass of air diffusing in per hour.* Assume D = 0.28 × 10–4 m2/s, M = 17 kg/kg mole

Solution: *P_{NH3} in pipe = 1 atm*

P_{NH3} at the outlet = 0

$$m_{NH_3} = \frac{D.A.}{\Re T} \frac{P_{NH_3 1} - P_{NH_3 2}}{L} \times M$$

$$= 0.28 \times 10^{-4} \times \frac{\pi}{4}(0.003)^2 \times \frac{(1.013 \times 10^5 - 0)}{20} \times 3600 \times 17/8315$$

$$= \mathbf{7.38 \times 10^{-6}\ kg/hr.}$$

$$m_{air}.\ N_B = -N_A = -7.38 \times 10^{-6} \times 28.97/17$$

$$= \mathbf{-1.26 \times 10^{-5}\ kg/hr.}$$

$$M_{air} = 28.97\ \text{kg/kg mole.}$$

STATIONARY MEDIA WITH SPECIFIED SURFACE CONCENTRATION

In the diffusion of gas from containers, there is diffusion of gas from inside to the outside without the metal molecules diffusing into the gas. In these cases the concentration of gas at the surfaces should be known. The solubility of the gas in the surface determines the concentration at the surface.

These cases are similar to conduction through the medium. In these cases the temperature potential in conduction is replaced by concentration potential ($Ca1 - Ca2$) for component A. The flow rate can be obtained as in the case of conduction.

$Na = (Ca1 - Ca2)/R$.

Where R is the resistance of diffusion. The resistance in the case of plane wall is

$$R_p = \frac{L}{D_{ab} A}$$

For hollow cylindrical configuration.

$$R_{cyl} = \frac{\ln(r_2 / r_1)}{2\pi D_{ab} L}$$

For hollow sphere, $R_{sp} = \frac{1}{4\pi D_{ab}} \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$

These equations can be derived from the general equation in Cartesian, cylindrical and spherical coordinate systems.

2. Hydrogen stored in a vessel diffuses through the steel wall of 20 mm thickness. The molar concentration at the inner surface is 2 kg mol/m^3. At the other surface it is zero. Assuming plane wall condition and Dab = 0.26 × 10–12 m^2/s, determine the mass of hydrogen diffused per 1 m^2.

Solution: $$N_a = -D_{ab} \cdot \frac{C_{a2} - C_{a1}}{L} = D_{ab} \cdot \frac{C_{a1} - C_{a2}}{L} = D_{ab} \cdot \frac{C_{a1}}{L}$$

$$= 0.26 \times 10^{-12} \times \frac{2}{0.02} = 2.6 \times 10^{-11} \text{ kg mol/s m}^2.$$

For H_2, molecular weight is 2.

∴ mass diffused = 2 × 2.6 × 10^{-11} = **5.2 × 10^{-11}** kg/m^{2}s.

DIFFUSION OF ONE COMPONENT INTO A STATIONARYCOMPONENT OR

UNIDIRECTIONAL DIFFUSION

In this case one of the components diffuses while the other is stationary. For steady conditions the mass diffused should be absorbed continuously at the boundary. In certain cases this is not possible. The popular example is water evaporating into air. In this case, as mentioned earlier, a bulk motion replaces the air tending to accumulate at the interface without being absorbed, causing an increase in the diffusion rate. The diffusion equation for gases can be derived as (with '*a*' as the diffusing medium and P = total pressure)

$$\frac{N_a}{A} = \frac{P}{\Re T} \cdot \frac{D}{(x_2 - x_1)} \cdot \ln\left(\frac{P - P_{a2}}{P - P_{a1}}\right)$$

For liquids (considering 'a' as diffusing medium)

$$\frac{N_a}{A} = \frac{D \, . \, C}{(x_2 - x_1)} \, . \ln\left(\frac{C - C_{a2}}{C - C_{a1}}\right)$$

1. Oxygen is diffusing in a mixture of oxygen-nitrogen at 1 std atm, 25C. Concentration of oxygen at planes 2 mm apart are 10 and 20 volume % respectively. Nitrogen is non-diffusing.

- Derive the appropriate expression to calculate the flux oxygen. Define units of each term clearly.
- Calculate the flux of oxygen. Diffusivity of oxygen in nitrogen = 1.89 * 10 $^{-5}$ m^2/sec.

Solution:

Given:

$D_{AB} = 1.89 * 10^{-5}$ m^2/sec

$P_t = 1$ atm $= 1.01325 * 10^5$ N/m^2

T = 25C = 273 + 25 = 298 K

z = 2 mm = 0.002 m

$P_{A1} = 0.2 * 1 = 0.2$ atm (From Ideal gas law and additive pressure rule)

$P_{A2} = 0.1 * 1 = 0.1$ atm

Substituting these in equation (6)

$$N_A = \frac{\left(1.89 * 10^{-5}\right)\left(1.01325 * 10^5\right)}{(8314)(298)(0.002)} \ln\left(\frac{1-0.1}{1-0.2}\right)$$

$$= 4.55 * 10^{-5} \text{ kmol/m}^2\text{.sec}$$

2. A vertical glass tube 3 mm in diameter is filled with liquid toluene to a depth of 20mm from the top openend. After 275 hrs at 39.4 C and a total pressure of 760 mm Hg the level has dropped to 80 mm from the top. Calculate the value of diffusivity.

Given Data:

vapor pressure of toluene at 39.4C = 7.64 kN / m^2,

density of liquid toluene = 850 kg/m^3

Molecular weight of toluene = 92

($C_6H_6CH_3$)

$$C = \frac{P}{RT} = \frac{1.01325 * 10^5}{8314 * (273 + 39.4)}$$

$= 0.039 \text{ k mol /m}^3$

Therefore

$$D_{AB} = \frac{850 * 0.9618}{92 * 0.039 * (0.0754 - 0) * 275 * 3600} \left(\frac{0.08^2 - 0.02^2}{2} \right)$$

$= 1.5262 * 10^{-3} (0.08^2 - 0.02^2)$

$= 9.1572 * 10^{-6} \text{ m}^2\text{/sec.}$

3. Methane diffuses at steady state through a tube containing helium. At point 1 the partial pressure of methane is $p_{A1} = 55$ kPa and at point 2, 0.03 m apart $P_{A2} = 15$ KPa. The total pressure is 101.32 kPa, and the temperature is 298 K. At this pressure and temperature, the value of diffusivity is $6.75 * 10^{-5}$ m^2/sec.

- calculate the flux of CH_4 at steady state for equimolar counter diffusion.
- Calculate the partial pressure at a point 0.02 m apart from point 1.

Calculation:

For steady state equimolar counter diffusion, molar flux is given by

Therefore;

$$N_A = \frac{6.75 * 10^{-5}}{8.314 * 298 * 0.03} (55 - 15) \frac{kmol}{m^2 . \sec}$$

$$= 3.633 * 10^{-5} \frac{kmol}{m^2 \sec}$$

And from (1), partial pressure at 0.02 m from point 1 is:

$$3.633 * 10^{-5} = \frac{6.75 * 10^{-5}}{8.314 * 298 * 0.02}(55 - p_A)$$

$p_A = 28.33$ kPa.

4. In a gas mixture of hydrogen and oxygen, steady state equimolar counter diffusion is occurring at a total pressure of 100 kPa and temperature of 20C. If the partial pressures of oxygen at two planes 0.01 m apart, and perpendicular to the direction of diffusion are 15 kPa and 5 kPa, respectively and the mass diffusion flux of oxygen in the mixture is 1.6 * 10^{-5} kmol/m^2.sec, calculate the molecular diffusivity for the system.

Solution:

For equimolar counter current diffusion:

$$N_A = \frac{D_{AB}}{RTz}(p_{A1} - p_{A2}) \text{ ------------------------ (1)}$$

N_A = molar flux of A (1.6 * 10^{-5} kmol/m^2.sec):

D_{AB} = molecular diffusivity of A in B

R = Universal gas constant (8.314 kJ/kmol.k)

T = Temperature in absolute scale (273 + 20 = 293 K)

z = distance between two measurement planes 1 and 2 (0.01 m)

P_{A1} = partial pressure of A at plane 1 (15 kPa); and

P_{A2} = partial pressure of A at plane 2 (5 kPa)

Substituting these in equation (1)

$$1.6 * 10^{-5} = \frac{D_{AB}}{(8.314)(293)(0.01)}(15-5)$$

Therefore, $D_{AB} = 3.898 * 10^{-5}\ m^2/sec$

5. A tube 1 cm in inside diameter that is 20 cm long is filled with Co_2 and H_2 at a total pressure of 2 atm at 0C. The diffusion coefficient of the $Co_2 - H_2$ system under these conditions is 0.275 cm^2/sec. If the partial pressure of Co_2 is 1.5 atm at one end of the tube and 0.5 atm at the other end, find the rate of diffusion for:

- steady state equimolar counter diffusion ($N_A = -N_B$)
- steady state counter diffusion where $N_B = -0.75\ N_A$, and
- steady state diffusion of Co_2 through stagnant H_2 ($N_B = 0$)

Given: $D_{AB} = 0.275\ cm^2/sec = 0.275 * 10^{-4}\ m^2/sec$; T = 0C = 273 k

$$N_A = \frac{0.275 * 10^{-4}}{8314 * 273 * 0.2}\left(1.5 * 1.01325 * 10^5 - 0.5 * 1.01325 * 10^5\right)$$

$$= 6.138 * 10^{-6}\ \frac{k\ mol}{m^2\ sec}$$

Rate of diffusion = $N_A S$

Where S is surface area

Therefore rate of diffusion = $6.138 * 10^{-6} * \ r^2$

$= 6.138 * 10^{-6} * \ (0.5 * 10^{-2})^2$

$= 4.821 * 10^{-10}$ k mol/sec

$= 1.735 * 10^{-3}$ mol/hr.

ii) $$N_A = -C D_{AB} \frac{d y_A}{d z} + y_A \left(N_A + N_B\right)$$

given: $N_B = - 0.75 N_A$

Therefore $$N_A = -C D_{AB} \frac{d y_A}{d z} + y_A \left(N_A - 0.75 N_A\right)$$

$$= -C D_{AB} \frac{d y_A}{d z} + 0.25 y_A N_A$$

$$N_A - 0.25 \ y_A N_A = - C D_{AB} \frac{d y_A}{d z}$$

$$N_A d z = -C D_{AB} \frac{d y_A}{1 - 0.25 y_A}$$

for constant N_A and C

$$N_A \int_{z_1}^{z_2} dz = -CD_{AB} \int_{y_{A1}}^{y_{A2}} \frac{dy_A}{1-0.25\, y_A}$$

$$\left[\int \frac{dx}{a+bx} = \frac{1}{b} \ln(a+bx) \right]$$

$$N_A z = (-C D_{AB}) \left(\frac{-1}{0.25} \right) \left[\ln(1-0.25\, y_A) \right]_{y_{A1}}^{y_{A2}}$$

$$N_A = -\frac{4\, CD_{AB}}{z} \ln\left(\frac{1-0.25\, y_{A2}}{1-0.25\, y_{A1}} \right) \quad \text{---------------------------------- (2)}$$

Given:

$$C = \frac{p}{RT} = \frac{2 * 1.01325 * 10^5}{8314 * 273} = 0.0893 \ K\, mol/m^3$$

$$y_{A1} = \frac{p_{A1}}{P} = \frac{1.5}{2} = 0.75$$

$$y_{A2} = \frac{p_{A2}}{P} = \frac{0.5}{2} = 0.25$$

Substituting these in equation (2),

$$N_A = \frac{4 * 0.0893 * 0.275 * 10^{-4}}{0.2} \left[\ln \frac{1 - 0.25 * 0.25}{1 - 0.25 * 0.75} \right]$$

$$= 7.028 * 10^{-6} \frac{kmol}{m^2 \sec}$$

Rate of diffusion = N_A S = $7.028 * 10^{-6} * \; * (0.5 * 10^{-2})^2$

$= 5.52 * 10^{-10}$ kmol/sec

$= 1.987 * 10^{-3}$ mol/hr.

iii) $$N_A = -CD_{AB} \frac{d y_A}{d z} + y_A \left(N_A + N_B \right)$$

Given: $N_B = 0$

Therefore $$N_A = -CD_{AB} \frac{d y_A}{d z} + y_A N_A$$

$$N_A \int_{Z_1}^{Z_2} d z = - CD_{AB} \int_{y_{A1}}^{y_{A2}} \frac{d y_A}{1 - y_A}$$

$$= \frac{CD_{AB}}{Z} \ln \left(\frac{1 - y_{A2}}{1 - y_{A1}} \right)$$

$$= \frac{0.0893 * 0.275 * 10^{-4}}{0.2}\left[\ln\left(\frac{1-0.25}{1-0.75}\right)\right]$$

$$= 1.349 * 10^{-5} \frac{kmol}{m^2.sec}$$

Rate of diffusion = 1.349 8 10^{-5} * * (0.5 * 10^{-2})^2

= 1.059 Kmol / sec = 3.814 mol/hr

CONVECTIVE MASS TRANSFER

In the study of convective heat transfer, the heat flux is connected to heat transfer coefficient as

$$Q/A = q = h(t_s - t_m) \text{ ---------------------- (4.1)}$$

The analogous situation in mass transfer is handled by an equation of the form

$$N_A = k_c(C_{As} - C_A) \text{ ---------------------- (4.2)}$$

The molar flux N_A is measured relative to a set of axes fixed in space. The driving force is the difference between the concentration at the phase boundary, C_{AS} (a solid surface or a fluid interface) and the concentration at some arbitrarily defined point in the fluid medium, C_A. The convective mass transfer coefficient k_C is a function of geometry of the system and the velocity and properties of the fluid similar to the heat transfer coefficient, h.

Significant Parameters in Convective Mass Transfer

Dimensionless parameters are often used to correlate convective transfer data. In momentum transfer Reynolds number and friction factor play a major role. In the correlation of convective heat transfer data, Prandtl and Nusselt numbers are important. Some of the same parameters, along with some newly defined dimensionless numbers, will be useful in the correlation of convective mass-transfer data.

The molecular diffusivities of the three transport process (momentum, heat and mass) have been defined as:

$$\text{Momentum diffusivity} \quad \nu = \frac{\mu}{\rho} \quad \text{------------------------------ (4.3)}$$

$$\text{Thermal diffusivity} \quad \alpha = \frac{k}{\rho C_p} \quad \text{--------------------------- (4.4)}$$

and

$$\text{Mass diffusivity } D_{AB} \quad \text{--------------------------- (4.5)}$$

It can be shown that each of the diffusivities has the dimensions of L^2 / t, hence, a ratio of any of the two of these must be dimensionless.

The ratio of the molecular diffusivity of momentum to the molecular diffusivity of heat (thermal diffusivity) is designated as the Prandtl Number

$$\frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}} = \text{Pr} = \frac{\nu}{\alpha} = \frac{C_p \mu}{K} \quad \text{------------------------ (4.6)}$$

The analogous number in mass transfer is Schmidt number given as

$$\frac{\text{Momentum diffusivity}}{\text{Mass diffusivity}} = Sc = \frac{\nu}{D_{AB}} = \frac{\mu}{\rho D_{AB}} \quad \text{-------------- (4.7)}$$

The ratio of the molecular diffusivity of heat to the molecular diffusivity of mass is designated the Lewis Number, and is given by

$$\frac{\text{Thermal diffusivity}}{\text{Mass diffusivity}} = Le = \frac{\alpha}{D_{AB}} = \frac{k}{\rho C_p D_{AB}} \quad \text{------------- (4.8)}$$

Lewis number is encountered in processes involving simultaneous convective transfer of mass and energy.

Let us consider the mass transfer of solute A from a solid to a fluid flowing past the surface of the solid. For such a case, the mass transfer between the solid surface and the fluid may be written as

$$N_A = k_c \left(C_{As} - C_{A\infty}\right) \quad \text{---------------------- (4.1 a)}$$

Since the mass transfer at the surface is by molecular diffusion, the mass transfer may also described by

$$N_A = -D_{AB} \left.\frac{d C_A}{d y}\right|_{y=0} \quad \text{------------------------- (4.9)}$$

When the boundary concentration, C_{As} is constant, equation (4.9) may be written as

$$N_A = -D_{AB} \left.\frac{d \left(C_A - C_{As}\right)}{d y}\right|_{y=0} \quad \text{---------------------- (4.10)}$$

Equation (4.1a) and (4.10) may be equated, since they define the same flux of component A leaving the surface and entering the fluid

$$k_c \left(C_{As} - C_{A\infty}\right) = -D_{AB} \frac{d}{dy} \left(C_A - C_{As}\right) \Bigg|_{y=0} \quad \text{-------------}$$

(4.11)

This relation may be rearranged into the following form:

$$\frac{k_c}{D_{AB}} = - \frac{d\left(C_A - C_{As}\right)/dy}{\left(C_A - C_{A\infty}\right)} \Bigg|_{y=0} \quad \text{--------------------} \ (4.12)$$

Multiplying both sides of equation(4.12) by a characteristic length, L we obtain the following dimensionless expression:

$$\frac{k_c L}{D_{AB}} = - \frac{d\left(C_A - C_{As}\right)/dy \,\big|_{y=0}}{\left(C_{AS} - C_{A\infty}\right)/L} \quad \text{-----------------} \ (4.13)$$

The right hand side of equation (4.13) is the ratio of the concentration gradient at the surface to an overall or reference concentration gradient; accordingly, it may be considered as the ratio of molecular mass-transport resistance to the convective mass-transport resistance of the fluid. This ratio is generally known as the Sherwood number, Sh and analogous to the Nusselt number Nu, in heat transfer.

Application of Dimensionless Analysis to Mass Transfer

One of the method of obtaining equations for predicting mass-transfer coefficients is the use of dimensionless analysis. Dimensional analysis predicts the various dimensionless parameters which are helpful in correlating experimental data.

There are two important mass transfer processes, which we shall consider, the transfer of mass into a steam flowing under forced convection and the transfer of mass into a phase which is moving as the result of natural convection associated with density gradients.

6. A stream of air at 100 kPa pressure and 300 K is flowing on the top surface of a thin flat sheet of solid naphthalene of length 0.2 m with a velocity of 20 m/sec. The other data are:

Mass diffusivity of naphthalene vapor in air = $6 * 10^{-6}$ m^2/sec

Kinematic viscosity of air = $1.5 * 10^{-5}$ m^2.sc

Concentration of naphthalene at the air-solid naphthalene interface = $1 * 10^{-5}$ kmol/m^3

Calculate:

- the overage mass transfer coefficient over the flat plate
- the rate of loss of naphthalene from the surface per unit width

Note: For heat transfer over a flat plate, convective heat transfer coefficient for laminar flow can be calculated by the equation.

$$Nu = 0.664 \; Re_L^{1/2} \; Pr^{1/3}$$

you may use analogy between mass and heat transfer.

Solution:

Given: Correlation for heat transfer

$$Nu = 0.664\ \mathrm{Re}_L^{1/2}\ \mathrm{Pr}^{1/3}$$

The analogous relation for mass transfer is

$$Sh = 0.664\ \mathrm{Re}_L^{1/2}\ Sc^{1/3} \quad \text{--(1)}$$

where

Sh = Sherwood number = kL/D_{AB}

Re_L = Reynolds number = L/

Sc = Schmidt number = / (D_{AB})

k = overall mass transfer coefficient

L = length of sheet

D_{AB} = diffusivity of A in B

 = velocity of air

 = viscosity of air

 = density of air, and

/ = kinematic viscosity of air.

Substituting for the known quantities in equation (1)

$$\frac{k(0.2)}{6 * 10^{-6}} = 0.664 \left(\frac{(0.2)(20)}{1.5 * 10^{-5}}\right)^{1/2} \left(\frac{1.5 * 10^{-5}}{6 * 10^{-6}}\right)^{1/3}$$

k = 0.014 m/sec

Rate of loss of naphthalene = k ($C_{Ai} - C_A$)

$$= 0.014\ (1 * 10^{-5} - 0) = 1.4024 * 10^{-7}\ \text{kmol/m}^2\ \text{sec}$$

Rate of loss per meter width = $(1.4024 * 10^{-7})\ (0.2) = 2.8048 * 10^{-8}$ kmol/m.sec

$$= 0.101\ \text{gmol/m.hr.}$$

7. The mass flux from a 5 cm diameter naphthalene ball placed in stagnant air at 40C and atmospheric pressure, is $1.47 * 10^{-3}$ mol/m ^{2}. sec. Assume the vapor pressure of naphthalene to be 0.15 atm at 40C and negligible bulk concentration of naphthalene in air. If air starts blowing across the surface of naphthalene ball at 3 m/s by what factor will the mass transfer rate increase, all other conditions remaining the same?

For spheres :

$$Sh = 2.0 + 0.6\ (Re)^{0.5}\ (Sc)^{0.33}$$

Where Sh is the Sherwood number and Sc is the Schmids number. The viscosity and density of air are $1.8 * 10^{-5}$ kg/m.s and 1.123 kg/m ^{3}, respectively and the gas constant is 82.06 cm ^{3} . atm/mol.K.

Given:

$$N = 1.47 * 10^{-3}\ \frac{mol}{m^2.\sec} = \frac{K_c}{RT}\ \Delta \bar{p}_A$$

$$\frac{k_c}{RT}\left(\frac{0.15}{1} - 0\right) = 1.47 * 10^{-3} * 10^{-4}\ \frac{mol}{cm^2.\sec}$$

$$k_c = \frac{1.47 * 10^{-7}}{0.15} * 82.06 * (273 + 40)$$

= 0.0252 cm/sec

$k_c = 2.517 * 10^{-4}$ m/sec ------------------------------(3)

Estimation of D_{AB}:

From (2),

$$\frac{2.517 * 10^{-4} * 5 * 10^{-2}}{D_{AB}} = 2$$

(since v = 0)

Therefore $D_{AB} = 6.2925 * 10^{-6}$ m²/sec.

And

$$\frac{k_c * 5 * 10^{-2}}{6.2925 * 10^{-6}} = 2 + 0.6 \left(\frac{5 * 10^{-2} * 3 * 1.123}{1.8 * 10^{-5}}\right)^{0.5} \left(\frac{1.8 * 10^{-}}{1.123 * 6.2925}\right.$$

7946 k_c = 2 + 0.6 * (96.74) * (1.361)

k_c = 0.0102 m/sec. --- (4)

$$\frac{(4)}{(3)} \Rightarrow \frac{N_{A2}}{N_{A1}} = \frac{0.0102}{2.517 * 10^{-4}} = 40.5$$

Therefore, rate of mass transfer increases by 40.5 times the initial conditions.

8. A solid disc of benzoic acid 3 cm in diameter is spin at 20 rpm and 25C. Calculate the rate of dissolution in a large volume of water. Diffusivity of benzoic acid in water is 1.0 * 10^{-5} cm^2/sec, and solubility is 0.003 g/cc. The following mass transfer correlation is applicable:

Sh = 0.62 $Re^{1/2}$ $Sc^{1/3}$

Where $Re = \frac{D^2 \omega \rho}{\mu}$ and is the angular speed in radians/time.

Given:

Sh = 0.62 $Re^{1/2}$ $Sc^{1/3}$

(i.e.) $$\frac{k_c D}{D_{AB}} = 0.62 \left(\frac{D^2 \omega \rho}{\mu}\right)^{\frac{1}{2}} \left(\frac{\mu}{\rho D_{AB}}\right)^{\frac{1}{3}}$$ ------------------ (3)

1 rotation = 2 radian

Therefore 20 rotation per minute = 20 * 2 radian/min

$$= \frac{20}{60} * 2\pi \ radian/\sec$$

For water = 1 g/cm^3 = 1 centipoise = 0.01 g/cm.sec.

From (3),

$$k_c = 0.62\ D_{AB} \left(\frac{\omega\,\rho}{\mu}\right)^{\frac{1}{2}} \left(\frac{\mu}{\rho\, D_{AB}}\right)^{\frac{1}{3}}$$

$$= 0.62 * 1.0 * 10^{5} * \left(\frac{(40\,\pi/60) * 1}{0.01}\right)^{\frac{1}{2}} \left(\frac{0.01}{1 * 1.0 * 10^{-5}}\right)^{\frac{1}{3}}$$

$= 8.973 * 10^{-4}$ cm/sec.

From (2),

$N_A = 8.973 * 10^{-4} (0.003 - 0)$

$= 2.692 * 10^{-6}$ g/cm^2.sec

From (1),

$N_A S = N_A * (2r^2)$

$= 2.692 * 10^{-6} * (2 * 1.5^2)$

$= 3.805 * 10^{-5}$ g/sec

$= 0.137$ g/hr.

TWO MARKS QUESTIONS AND ANSWERS

UNIT – I

1. State Newton's law of convection.

Heat transfer from the moving fluid to solid surface is given by the equation

$$Q=h \times A \times (T_w-T)$$

This equation is referred to as Newton's law of cooling.

Where,

h -Local heat transfer coefficient in W/m^2K

A -Surface area in m^2

T_w -Surface (or) Wall temperature in K

T - Temperature of fluid in K

2. Write the three dimensional heat transfer poisson and Laplace equation in Cartesian Co-ordinates.

Poissons equation

+ + + = 0 (or)

Laplace equation

+ + = 0 (or)

3. Define thermal conductivity.

Thermal conductivity is defined as the *ability of a substance to conduct heat*. It also defined as the amount of heat conducted through a body of unit area and unit thickness in unit time temperature difference. Its unit is W/m K or W/m°C.

4. What are the modes of heat transfer?

- Conduction
- Convection
- Radiation

5. List out few types of fins?

- Uniform straight fin
- Tapered straight fin
- Splines
- Annular fin
- Pin fins

6. Define Fourier law of conduction?

Rate of heat conduction is proportional to the area measured normal to the direction of heat flow and to the temperature gradient in that direction.

$$Q = -k\,A$$

7. Define fin efficiency.

The efficiency of a fin is defined as the ration of actual heat transferred to the maximum possible heat transferred by the fin

8. What is lumped system analysis?

In a Newtonian heating or cooling process the temperature throughout the solid is considered to be uniform at a given time. Such an analysis is called lumped heat capacity analysis.

9. What is the main advantage of parabolic fins?

A fin of parabolic profile is very effective in the sense that it dissipates the maximum amount of heat at minimum material cost.

10. Differentiate between semi-infinite and infinite solids.

A solid which extends itself infinitely in all direction of space is termed as an infinite solid. For infinite solid, $0.1 < Bi < 100$.

If an infinite solid is split in the middle by a plane, each half is known as semi-infinite solid. For semi infinite solid, $Bi = \alpha$.

11. What is the main advantage of a parabolic fin?

A fin of parabolic profile is very effective in the sense that it dissipates the maximum amount of heat at minimum material cost.

12. What are the factors affecting thermal conductivity?

(i) Moisture (ii) Density of material (iii) Pressure (iv) Temperature (v) Structure of material

13. What are Heissler charts?

In Heissler chart, the solutions for temperature distributions and heat flows in plane walls, long cylinders and spheres with finite internal and surface resistance are presented. Heissler charts are nothing but a analytical solutions in the form of graphs.

14. Define overall heat transfer coefficient

The overall heat transfer U gives the heat transmitted per unit area per unit time per degree temperature difference between the bulk fluids on each side to the metal or solid.

15. What are the mechanisms in heat transfer through solids?

- Latice Vibration
- Transport of Free Electrons

16. What is thermal contact resistance?

Due to the reduced area and presence of voids, a large resistance to heat flow occurs at the interface. This resistance is known as thermal contact resistance.

UNIT – II

1. Define prandtl number and Grashoff number.

Prandtl number:

It is the ratio of the momentum diffusivity to the thermal diffusivity.

Grashof number

It is defined as the ratio of product of inertia force and buoyancy force to the square of viscous force.

2. Define the term 'Boundary Layer' (Thermal).

It is an imaginary layer in which temperature is less than 99%of free stream temperature. In thermal boundary layer heat transfer occurs.

3. What is convection heat transfer?

Convection is a process of heat transfer that will occur between a solid surface and a fluid medium when they are at different temperatures.

4. Differentiate b/w free and forced convection?

If the fluid motion is produced due to change in density resulting from temperature gradients, the mode of heat transfer is said to be free or natural convection.

If the fluid motion is artificially created by ends of an external force like a blower or fan, that type of heat transfer is known as forced convection.

5. State Buckingham-II theorem.

Buckingham II theorem states as follows: " IF there are n variables in a dimensionally homogeneous equation and if these contain m fundamental dimensions, then the variables are arranged into (n - m) dimensions terms. These dimensionless terms are called II terms ".

6. Explain the significance of Fourier number.

It is defined as the ratio of characteristic body dimension to temperature wave penetration depth in time.

=

It signifies the degree of penetration of heating or cooling effect of a solid.

7. What is dimensional analysis?

Dimensional analysis is a mathematical technique which makes use of the study of the dimensions for solving several engineering problems.

It is used in forced and free convection heat transfer.

8. Define bulk temperature.

The fluid properties involve in Nusselt number expressions are evaluated at film temperature which is called bulk temperature.

T_f – Film temperature in ºC

Ts- Surface temperature in ºC

- Fluid temperature in ºC

9. Define displacement thickness.

It is the thickness measured perpendicular to the plate to from plate to boundary layer. Due to this thickness, the main stream is displaced from plate surface to boundary layer.

(or)

It is the additional thickness which should be added to compensate for the reduction in flow rate on account of boundary layer formation.

10. Indicate the concept or significance of boundary layer.

In the boundary layer concept the flow field over a body is divided into two regions.

- A thin region near the body called the boundary layer, where the velocity and the temperature gradients are large.
- The region outside the boundary layer where the velocity and the temperature gradient are very nearly equal to their free stream values.

11. Define lower critical Reynolds number

It defines the limit below which all turbulence, no matter how severe, entering the flow from any source will eventually be damped out by viscous action.

12. What is boundary layer?

A layer adjacent to the boundary is called boundary layer.

13. What are the factors that change the boundary layer from laminar to turbulent?

Turbulence in ambient flow, surface roughness, pressure gradient, plate curvature and temperature difference between fluid and boundary.

17. Give few examples of free convection

- The cooling of transmission lines, electrical transformers and rectifiers The heating of rooms by use of radiators.
- The heat transfer from hot pipes and ovens surrounded by cooler air Cooling of reactor core.

UNIT – III

1. What is burnout point? Why is it called so?

A point at which the heat flow is maximum is known as burnout point. Once we cross this point, large temperature difference is required to get the same heat flux and most material may burn at this temperature.

2. Define NTU of a heat exchanger. Is it correct to say that, larger the NTU, larger the heat exchanger will be?

The heat exchanger effectiveness is defined as the ratio of actual heat transfer to the maximum possible heat transfer.

Effectiveness =

Number of Transfer Units (NTU) =

3. Differentiate between pool and flow boiling.

Pool boiling:

If heat is added to a liquid from a submerged solid surface, the boiling process is referred to as pool boiling. In this case the liquid above the hot surface is essentially stagnant and its motion near the surface is due to free convection and mixing induced by bubble growth and detachment.

Flow boiling:

Flow boiling or forced convection boiling may occur when a fluid is forced through a pipe or over a surface which is maintained at a temperature higher than the saturation temperature of the fluid.

4. What do you understand by fouling and heat exchanger effectiveness?

Fouling factor:

We know, the surfaces of heat exchangers do not remain clean after it has been in use for some time. The surfaces become fouled with scaling or deposits. The effect these deposits affecting the value of overall heat transfer co-effecient. This effect is taken care of by introducing an additional thermal resistance called the fouling resistance.

Effectiveness:

The heat exchanger effectiveness is defined as the ratio of actual heat transfer to the maximum possible heat transfer.

=

5. What is the difference between boiling and condensation?

Boiling

The change of phase from ***liquid to vapour*** state is known as boiling.

Condensation

The change of phase from ***vapour to liquid*** state is known as condensation.

6. What is meant by compact heat exchanger?

There are many special purpose heat exchangers called compact heat exchangers. They are generally employed when convective heat transfer Co-efficient associated with one of the fluids is much smaller than that associated with the other fluid.

7. List few applications of phase change heat transfer processes.

- Thermal and nuclear power plant.
- Refrigerating systems.
- Process of heating and cooling.
- Heating of metal in furnaces.
- Air conditioning systems.

8. Give the classification of heat exchangers in the basis of working principle?

There are several types of heat exchangers which may be classified on the basis of,

- Nature of heat exchange process
- Relative direction of field motion
- Design and constructional features
- Physical state of fluids

9. What is meant by film wise condensation?

The liquid condensate wets the solid surface, spreads out and forms a continuous film over the entire surface is known as film wise condensation.

10. Compare parallel flow and counter flow heat exchanger?

In this type of heat exchanger, *hot and cold fluids move in the same direction*.

In this type of heat exchanger, hot and cold fluids move in parallel but *opposite direction*.

11. What is fouling? What is its effect on the heat exchanger?

After a period of operation, the surfaces of heat exchanger may be coated with some deposits. Also the surfaces may be corroded as a result of the interaction between the fluids and material of the heat exchanger. This coating gives additional resistance to the heat flow. This resistance should be taken into consideration to design the heat exchanger. To calculate the overall heat transfer coefficient for a heat exchanger, the fouling factor or fouling resistance should be considered.

12. How the film wise condensation is different from drop wise condensation?

In film wise, condensation wets the surface forming a continuous film which covers the entire surface. In drop wise, the droplets of various sizes which fall down the surface in random fashion.

UNIT – IV

1. What is black body?

Black body is an ideal surface having the following properties.

- A black body absorbs all incident radiation, regardless of wave length and direction.
- For a prescribed temperature and wave length, no surface can emit more energy than black body.

2. What is irradiation and radiosity?

Irradiation (G):

It is defined as the total radiation incident upon a surface per unit time per unit area. It is expressed in W/m^2.

Radiosity (J):

It is used to indicate the total radiation leaving a surface per unit time per unit area. It is expressed in W/m^2.

3. Name the laws of variation used in heat transfer analysis.

- Plank's law
- Wien's displacement law
- Stefan – Bottzman law
- Kirchoff's law.

4. Define 'Thermal Radiation'.

The heat is transferred from one body to another without any transmitting medium is known as radiation.

5. What is meant by Radiosity (J).

It is used to indicate the total radiation leaving a surface per unit time per unit area. It is expressed in W/m^2.

6. State Wien's displacement law.

The Wien's law gives the relationship between temperature and wave length corresponding to the maximum spectral emissive power of the black body at that temperature

7. What are radiation shield?

Radiation shields constructed from low emissivity (high reflective) materials. It is used to reduce the net radiation transfer between two surfaces.

8. Define emissive power?

The emissive power is define as the total amount of radiation emitted by a body over unit time and unit area.

It is expressed in W/m^2.

9. What is the purpose of radiation shield?

Radiation shields constructed from low emissivity (high reflective) materials. It is used to reduce the net radiation transfer between two surfaces.

10. State the kirchoff's law.

- The emissivity of a block body is equal to its absorpitivity when the body remains in thermal equilibration with its surroundings.

- The law states that at any temperature, the ratio of total emissive power to the total absorpitivity is a constant for all substance which are in thermal equilibrium with their environment.

11. State Stephen Boltzmann law and planks law. How they are related?

Stephen Boltzmann Law: The emissive power of a black body is proportional to absolute temperature of the fourth power.

$$Eb \propto T4$$

$$Eb = \sigma T4$$

Where, σ = 5.67x10^{-8} W/m^{2} K^{-4}

Planck's law:

$$(E_\lambda)b = \frac{C_1 \lambda^{-5}}{e^{\left(\frac{C_2}{\lambda T}\right)} - 1}$$

Both are related by the following equation

$$E_b = \int_0^\infty (E_\lambda)_b \, d\lambda$$

12. Discuss the radiation characteristics of carbondioxide and water vapour.

The CO2 and H2O both absorb and emit radiation over certain wavelength regions called absorption bands. The radiation in these gases is a volume phenomenon. The emissivity of CO2 and the emissivity of H2O at a particular temperature increases with partial pressure and mean beam length.

13. What is meant by shape factor and mention its physical significance.

The shape factor is defined as “The fraction of the radiation energy that is diffused from one surface element and strikes the other surface directly with no intervening reflections”. It is represented by Fy. Other names for radiation shape factor are view factor, angle factor and configuration factor. The shape factor is used in the analysis of radiative heat exchange between two surfaces.

UNIT – V

1. State Fick's law of diffusion.

The diffusion rate is given by the Fick's law, which states that molar flux of an element per unit area is directly proportional to concentration gradient.

$$= -$$

Where,

- Molar flux -

D_{ab} - Diffusion co-efficient of species a and b, m^2/s

- Concentration gradient, kg/m^3

2. What do you understand by steady state molecular diffusion?

The transport of water on a microscopic level as a result of diffusion from a region of higher concentration to a region of lower concentration in a mixture of liquids or gases is known as molecular diffusion.

3. Define Scherwood Number.

It is defined as the ratio of concentration gradients at the boundary.

h_m - Mass transfer co – efficient, m/s

D_{ab} – Diffusion co –efficient, m^2/s

X – Length, m

4. What is convective mass transfer?

Convective mass transfer is a process of mass transfer that will occur between a surface and a fluid medium when they are at different concentrations.

7. What is free convective mass transfer?

If the fluid motion is produced due to change in density resulting from concentration gradients, the mode of mass transfer is said to be free or natural convective mass transfer. *Example : Evaporation of alcohol.*

8. Define force convective mass transfer.

If the motion is artificially created by means of an external force like a blower or fan, that type of mass transfer is known as convective mass transfer. *Example : The evaporation of water from an ocean when air flows over it.*

5. Define mole fraction and mass concentration.

$Y_A = \;=$

Mass concentration of species, P_A is defined as the mass of A per unit volume of the mixture.

16 MARKS QUESTIONS

UNIT – I

1. A Cold storage room has walls made of 23cm brick on the outside, 8cm of plastic foam and finally 1.5cm of wood on inside. The outside and inside air temperatures are 22°C and -2°C respectively. The inside and outside heat transfer co-efficient are 29 and 12 W/m^2 K. The thermal conductivities of brick, foam and wood are 0.98, 0.02 and 0.12 W/m K respectively. If the total wall area is 90 m^2, Determine the rate of heat removed by refrigeration and temperature of the inside surface of the brick. **(May 11)**

2. Derive the heat conduction equation in cylindrical co-ordinates using an elemental Volume for a stationary isotropic solid. **(May 07,May 10)**

3. A composite wall consists. of 2.5 cm thick layer of building brick, k = 355 W/mK and 3.2 mm thick plaster, k = 0.110W/mK. An insulating material of k = 0.08 W/mK is to be

added to reduce the heat transfer through the wall by 40%. Find its thickness. **(Dec 11)**

4. A reactor's wall 320 mm thick is made up of an inner layer of fire brick (k = 0.84 W/m° C) covered with a layer of insulation (k = 0.16 W/m° C). The reactor operates at a 25 ° C. Determine the thickness of the firebrick and insulation which gives minimum heat loss. Calculate the heat loss presuming that the insulating material has a maximum temperature of 1200° C. **(Dec 08)**

5. A composite wall is formed of a 2.5 cm copper plate (k = 355 W/m K), A 3.2 mm layer of asbestos (k = 0.110 W/m K) and a 5 cm layer of fiber plate (k = 0.049 W/m K). The wall is subjected to an overall temperature difference of 560°C (560°C on the Cu plate side and 0°C on the fiber plate side). Estimate the heat flux through this composite all and the interface temperature between asbestos and fiber plate. **(May 08)**

6. An exterior wall of house may be approximated by a 0.1 m layer of common brick (k = 0.7 W/m° C) followed by a 0.04 m layer of gypsum plaster (k = 0.48 W/m° C). What is the thickness of loosely packed rock wool insulation (k = 0.065 W/m° C) should be added to reduce the heat loss or gain through the wall by 80 %. (**May 04, Dec 06)**

7. A composite wall consist of 10 cm thick layer of building brick, k = 0.7 W/m K and 3 cm thick plaster, k = 0.5 W/m K. An insulating material of k = W/m K is to be added to reduce the heat transfer through the wall by 40%. Find its thickness. **(Dec 04, Dec 05)**

8. A furnace wall consists of three layers. The inner layer of 10 cm thickness is made of firebrick *(k* =1.04 W/m K). The intermediate layer of 25 cm thickness is made of masonry brick (k = 0.69 W/m K) followed by a 5 cm thick concrete wall (k = 1.37 W/m K). When the furnace is in continuous operation the inner surface of the furnace is at 800° C while the outer concrete surface is at 50°C. Calculate the rate of heat loss per unit area of the wall, the temperature at the interface of the firebrick and masonry brick and the temperature at the interface of the masonry brick and concrete. **(May 06)**

9. A 150 mm steam pipe has inside diameter of 120 mm and outside diameter of 160 mm.

It is insulated at the outside with asbestos. The steam temperature is 150º C and the air temperature is 20º C h (steam) = 100 W/m² C, h (air) = 100 W/m² C, K (asbestos) = 0.8 W/mº C and K (steel) = 42 W/mº C. How thick the asbestos should be provided in order to limit the heat losses to 2.1 Kw/m². **(Dec 12)**

10. An aluminium pipe carries steam at 110° C The pipe and K = 185 W/mº C. Has an inner diameter of 100 mm and outer diameter of 120 mm. The pipe is located in a room where the ambient air temperature is 30° C and the convective heat transfer co-efficient between the pipe and air is 15 W/m² C. Determine the heat transfer rate per unit length of pipe. To reduce the heat loss from the pipe, it is covered with a 50 mm thick layer of insulation (K = 0.20 W/m °C). Determine the heat transfer rate per unit length from the insulated pipe. Assume that the convective resistance of the steam is negligible. **(May 12)**

11. A steel tube with 5 cm ID, 7.6 cm OD and k = 15 W/mº C is covered with an insulative covering of thickness 2 cm and k = 0.2 W/mº C. A hot gas at 330º C with h = 400 W/m² º C flows inside the tube. The outer surface of the insulation is exposed to cooler air at 30º C with h = 60 W/m² º C. Calculate the heat loss from the tube to the air for 10 m of the tube and the temperature drops resulting from the thermal resistance of the hot gas flow, the steel tube, the insulation layer and outside air. **(May 05, May 09)**

12. An electrical wire of 10 m length and 1 mm diameter dissipates 200 W in air at 25°C. The convection heat transfer coefficient between the wire surface and air is 15 W/m2K. Calculate the critical radius of insulation and also determine the temperature of the wire if it is insulated to the critical thickness of insulation. **(May 06, Dec 06)**

13. A 3 cm OD steam pipe is to be covered with two layers of insulation each having a thickness of 2.5 cm. The average thermal conductivity of one insulation is 5 times that of the other. Determine the percentage decrease heat transfer if better insulating material is next to pipe than it is the outer layer. Assume that the outside and inside temperatures of composite insulation are fixed. **(May 07)**

14. A pipe consists of 100 mm internal diameter and 8 mm thickness carries steam at 170°C. The convective heat transfer coefficient on the inner surface of pipe is 75 W/m2C.

The pipe is insulated by two layers of insulation. The first layer of insulation is 46 mm in thickness having thermal conductivity of 0.14 W/m° C. The second layer of insulation is also 46 mm in thickness having thermal conductivity of 0.46 W/m° C. Ambient air temperature = 33°C. The convective heat transfer coefficient from the outer surface of pipe = 12 W/m^2C. Thermal conductivity of steam pipe = 46 W/m° C. Calculate the heat loss per unit length of pipe and determine the interface temperatures. Suggest the materials used for insulation. **(Dec 07)**

15. A steel ball of 5 cm diameter was initially at 450°C and is suddenly placed in environment at 100°C, heat transfer coefficient between the steel ball and the fluid is 10 W/m^2 K, For steel CP = 0.46 kj/kg K , ρ=7800 kg/m^3. K= 35W/m K. Calculate time required for the ball to reach a temperature of 150°C Also find the rate of cooling after 12 hr. **(May 10)**

16. Heat is conducted through a tapered circular rod of 200 mm length. The ends Aand B having diameters 50 mm and 25 mm are maintained at 27° C and 227 ° C respectively, K (rod material) = 40 W/m° C . Find (i) Heat conducted through the rod, and (ii) The temperature at midpoint of the end. Assume there is no temperature gradient at a particular cross section and there is no heat transfer through the peripheral surface. **(May 12)**

17. A 25 mm diameter rod of 360 mm length connects two heat sources maintained at 127° C and 227° C respectively. The curved surface of the rod is losing heat to the surrounding air at 27° C. The heat transfer co-efficient is 10 W/m^2 °C. Calculate the loss of heat from the rod if it is made of copper (k = 335 W/m° C) and steel (k = 40 W/m° C) . **(Dec 08)**

18. Circumferential aluminum fins of rectangular profile (1.5 cm wide and 1 mm thick) are fitted on a 90 mm engine cylinder with a pitch of 10 mm. The height of the cylinder is 120 mm. The cylinder base temperature before and after fitting the fins are 200° C and 150° C respectively. Take ambient at 30° C and h (average) = 100 W/m^2 K. Estimate the heat dissipated from the finned and un finned surface areas of cylinder body. **(Dec 04)**

19. A Cylinder 1 m long and 5 cm in diameter is placed in an atmosphere at 45° C. It is provided with 10 longitudinal straight fins of material having k = 120 W/m K. The height of 0.76 mm thick fins is 1.27 cm from the cylinder surface. The heat transfer co-efficient between cylinder and atmosphere air is 17 W/m^2K. Calculate the rate of heat transfer and temperature at the end of fins if surface temperature of cylinder is 150° C. **(May 05)**

20. A slab of Aluminum 10 cm thick initially at 500°C is suddenly immersed in a liquid at 100°C for which the convection heat transfer co-efficient is 1200 W/m^2K. Determine the temperature at a center line and the surface 1 minute after the immersion. Also calculate the energy removed per unit area from the plate during 1 minute of immersion. Take P = 2700 bar, Cp = 0.9 kJ/kg. OK, k=215W/m K, ά = 8.4X 10-5 m^2/s. **(Dec 12)**

21. An Aluminum plate (k = 160 W/m^2 C , ρ = 2790 kg/m^3, CP = 0.88 kJ/ kg° C) of thickness L = 3 cm and at a uniform temperature of 225° C is suddenly immersed at time t = 0 in a well stirred fluid maintained at a constant temperature $T\alpha$ = 25° C. Take h = 320 W/m^2 ° C. Determine the time required for the centre of the plate to reach 50° C. **(Dec 05)**

22. A slab of Aluminum 5 cm thick initially at 200°C is suddenly immersed in a liquid at 70°C for which the convection heat transfer co-efficient is 525 W/m^2K. Determine the temperature at a depth of 12.5 mm from one of the faces 1 minute after the immersion. Also calculate the energy removed per unit area from the plate during 1 minute of immersion. Take P = 2700 bar, Cp = 0.9 kJ/kg. OK, k=215W/m K, ά = 8.4X 10-5 m^2/s. **(May 07)**

23. A large iron plate of 10 cm thickness and originally at 800°C is suddenly exposed to an environment at 0°C where the convection coefficient is 50 W/m2K. Calculate the temperature at a depth of 4 cm from one of the faces 100 seconds after the plate is exposed to the environment. How much energy has been lost per unit area of the plate during this time? **(May 06)**

UNIT – II

1. Air at 200 kPa and 200°C is heated as it flows through a tube with a diameter of 25 mm at a velocity of 10 m./sec. The wall temperature is maintained constant and is 20°C above the air temperature all along the length of tube. Calculate:

i) The rate of heat transfer per unit length of the tube.

ii) Increase in the bulk temperature of air over a 3 m length of the tube. **(Dec 12)**

2. A 0.5 m high flat plate of glass at 93º C is removed from an annealing furnace and hung vertically in the air at 28º C, 1 atm. Calculate the initial rate of heat transfer to the air. The plate is 1 m wide. **(Dec 12)**

3. A fine wire having a diameter of 0.02 mm is maintained at a constant temperature of 54º C by an electric current. The wire is exposed to air at 1 atm. And 0º C. Calculate the electric power necessary to maintain the wire temperature if the length is 50 cm. **(Dec 12)**

4. Air stream at 27° C is moving at 0.3 m/s across a 100 W electric bulb at 127° C If the bulb is approximated by a 60mm diameter sphere, estimate the heat transfer rate and percentage of power lost due to convection. **(May 12)**

5. Atmospheric air fat 150° C flows with a velocity of 1.25 m/s over a 2m long flat plate whose temperature is 25° C. Determine the average heat transfer co-efficient and rate of heat transfer for a plate width 0.5 m. **(May 11)**

6. A 6 m long section of an 8 cm diameter horizontal hat water pipe passes through a large room in which the air and walls are at 20° C. The pipe surface as at 70° C and the emissivity of the pipe surface is 0.7. Find the rate of heat loss from the pipe by natural convection and radiation. **(May 11)**

7. Engine oil at 60°C flows with a velocity of 2 m/s over a 5 m long flat plate whose temperature is 20°C. Determine the drag force exerted by oil on the plate and the rate of heat transfer for 1m. **(Dec 11)**

8. A metallic cylinder of 12.7 mm diameter and 94 mm length is heated internally by an electric heater and its surface is cooled by air. The free stream air velocity and

temperatures are respectively 10 m/s and 26.2°C. Under steady operating conditions, heat dissipated by the cylinder is 39.1 W and it's avenge surface temperature is 128.4°C. Determine the convection heat transfer coefficient from the above experiment. Also find the convection heat transfer coefficient from an appropriate correlation and compare both. **(Dec 11)**

9. Air at 20° C at 3 m/s flows over a thin plate of 2m long and 1m wide. Estimate the boundary layer thickness at the trailing edge, total drag force, mass flow of air between X = 30cm and X = 80 cm. Take v = 15 X 106 and ρ = 1.17 kg/m3. **(May 10)**

10. Calculate the convective heat transfer from a radiator 0.5 m wide and 1 m high at 84° C in a room at 20° C. Treat the radiator as a vertical plate. **(May 10)**

11. Cylindrical cans of 150 mm length and 65 mm diameter are to be cooled from an initial temperature of 20°C by placing them in a cooler containing air at a temperature of 1°C and a pressure of 1 bar. Determine the cooling rates when the cans are kept in horizontal and vertical positions. **(May 09,May 08)**

12. Engine oil (k = 0.14 W/m K, v = 80 X 10-6 m2/s) flows with a mean velocity of 0.2 m/s inside a 1.25 cm diameter tube which is electrically heated at the wall at a uniform rate of 2.45 Kw/m2 . The heat transfer is taking place in the fully developed region . Calculate the temperature difference between the tube wall surface and the mean flow temperature. **(May 09)**

13. Air at 20° C and at a pressure of 1 bar is flowing over a flat plate at a velocity of 3m/s. If the plate is 280 mm wide and 56°C. Calculate the following x =280 mm.

i) Boundary layer thickness. ii) Local friction co-efficient.

iii) Average friction co-efficient.

iv) Thickness of thermal boundary layer.

v) Local convective heat transfer co-efficient.

vi) Average convective heat transfer co-efficient. vii) Rate of heat transfer by convection.

viii) Total drag force on the plate. **(Dec 08)**

14. A Cylindrical body of 300mm diameter and 1.6 m height is maintained at a constant temperature is 36.5° C. The surrounding temperature is 13.5° C. Find the amount of heat generated by the body per hour If CP = 0.96 kJ/kg° C; ρ = 1.025 kg/m3; k = 0.0892 W/m° c, v = 15.06 X 10-6 m²/s and β = 1/298 K-1. Assume Nu = 0.12 (Gr.Pr)1/3 **(Dec 08)**

15. Air at 400 K and 1 atm pressure flows at a speed of 1.5 m/s over a flat plate of 2 m long. The plate is maintained at a uniform temperature of 300 K. If the plate has a width of 0.5 m, estimate the heat transfer coefficient and the rate of heat transfer from the air stream to the plate. Also estimate the drag force acting on the plate.**(May 08)**

16. Explain for fluid flow along a flat plate:

(1) Velocity distribution in hydrodynamic boundary layer

(2) Temperature distribution in thermal boundary layer

(3) Variation of local heat transfer co-efficient along the flow. **(May 07)**

17. The water is heated in a tank by dipping a plate of 20 cm X 40 cm in size. The temperature of the plate surface is maintained at 100°C. Assuming the temperature of the surrounding water is at 30° C, Find the heat loss from the plate 20 cm side is in vertical plane.**(May 07)**

18. Air at 20° C is flowing along a heated plate at 134° C at a velocity of 3m/s. The plate is 2 m long and 1.5m wide. Calculate the thickness of hydrodynamic boundary layer and skin friction co-efficient at 40 cm from the leading edge of the plate. The kinematic viscosity of air at 20° c is 15.06 X 10-6 m²/s.**(Dec 05, Dec 06)**

19. Atmospheric air at 275 K and a free stream velocity of 20 m/s flows over a flat plate

1.5 m long that is maintained at a uniform temperature of 325 K. Calculate the average heat transfer coefficient over the region where the boundary layer is laminar, the average heat transfer coefficient over the entire length of the plate and the total heat transfer rate from the plate to the air over the length 1.5 m and width 1 m. Assume transition occurs at Rec = 2xl05.**(May 06)**

20. Write down the momentum equation for a steady, two dimensional flow of an incompressible, constant property Newtonian fluid in the rectangular coordinate system and mention the physical significance of each term. **(May 06)**

21. A large vertical plate 5 m high is maintained at 100°C and exposed to air at 30°C. Calculate the convection heat transfer coefficient. **(May 06)**

22. A steam pipe 10 cm outside diameter runs horizontally in a room at 23°C. Take the outside surface temperature of pipe as 165°C. Determine the heat loss per unit length of the pipe. **(Dec 05)**

UNIT – III

1. Water is boiled at a rate of 30 kg/h in copper pan, 300 in diameter, at atmospheric pressure. Estimate the temperature of the bottom surface of the pan assuming nucleate boiling conditions **(May 12)**

2. Hot oil with a capacity rate of 2500 W/K flows through a double pipe heat exchanger. It enters at 360° C and leaves at 300° C Cold fluid temperature enters at 30° C and leaves at 200° C. If the overall heat transfers co-efficient is 800W/m² K, Determine the heat exchanger area required for (i) parallel flow and (ii) counter flow. **(May 12)**

3. The bottom of copper pan, 300 mm in diameter is maintained at 120° C by an electric heater. Calculate the power required to boil water in this pan. What is the evaporation rate? Estimate the critical heat flux**.(Dec12)**

4. Water at the rate of 4 kg/s is heated from 40° C to 55° C in a shell and tube heat exchanger. On shell side one pass is used with water as heating fluid (m = 2 kg/s), entering the exchanger at 95° C The overall heat transfer co-efficient is 1500 W/m² ° C and average water velocity in the 2 cm diameter tubes is 0.5m/s. Because of space limitations the tube length must not exceed 3 m. Calculate the number of tube passes keeping in mind the design constraint. **(Dec 12)**

5. Dry steam at 2.45 bar condenses on a vertical tube of height of 1 m at 117° C. Estimate the thickness of the condensate film and local heat transfer co-efficient at a distance 0.2m from the upper end of the plate. **(May 10)**

6. A 10 by 10 array of horizontal tubes of 1.27 cm diameter is exposed to pure steam at atmospheric pressure. If the tube wall temperature is 98°C, estimate the mass of steam

condensed assuming a tube length of 1.5 m. **(Dec 11)**

7. In a cross flow heat exchanger, air is heated by water. Air enters the exchanger at 15°C and a mass flow rate of kg/s while water enters at 90°C and a mass flow rate of 0.25 kg/s. The overall heat transfer coefficient is 250 W/m .K. If the exchanger has a heat transfer area of 8.4 m2, find the exit temperatures of both the fluids and the total heat transfer rate. **(Dec 11)**

8. Define LMTD for a parallel flow heat exchanger stating the assumptions. **(May 10)**

9. Explain the various regimes of pool boiling. **(May 07,May 09)**

10. Water is to be boiled at atmospheric pressure in a mechanically polished stainless steel pan placed on top of a heating unit. The inner surface of the bottom of the pan is maintained at lO8°C. The diameter of the bottom of the pan is 30 cm. Assuming Csf = 0.0130. Calculate (i) the rate of heat transfer to the water and ii) the rate of evaporation of water. **(May 08)**

11. A vertical plate 0.5 m2 in area at temperature of 92°C is exposed to steam at atmospheric pressure. If the steam is dry and saturated estimate the heat transfer rate and condensate mass per hour. The vertical length of the plate is 0.5 m. Properties of water at film temperatures of 96°C can be obtained from tables.**(May 07)**

12. Hot exhaust gases which enters a finned tube cross flow heat exchanger at 300°C and leave at 100°c, are used to heat pressurized water at a flow rate of 1 kg/s from 35 to 125° C. The exhaust gas specific heat is approximately 1000 J/kg.K, and the overall heat transfer co-efficient based on the gas side surface area is Uh = 100W/m2K. Determine the requi red gas side surface area Ah using the NTU method. Take Cp,c at Tc = 80°C is 4197 J/kg.K and Cp,h = 1000 J/kg.K . **(May 07)**

UNIT – IV

1. A gray, diffuse opaque surface (α = 0.8) is at 100° C and receives an irradiation 100W/m². If the surface area is 0.1 m². Calculate

i) Radiosity of the surface

ii) Net radiative heat transfer rate from the surface

iii) Calculate above quantities, if surface is black. **(Dec 12)**

2. Emissivities of two large parallel plate maintained at 800° C and 300° C and 0.3 and 0.5 respectively Find the net heat exchange per square meter of these plates. **(Dec 12)**

3. Two rectangles 50X 50 cm are placed perpendicular with common edge. One surface has T1 = 1000 K; ε = 0.6; While the other surface is insulated and in radiant balance with a large surrounding room at 300 K . Determine the temperature of the insulated surface and heat lost by the surface at 1000 K. **(Dec 12)**

4. Two black square plates of size 1.0 by 1.0 m are placed parallel to each other at a

distance of 0.4 m. One plate is maintained at a temperature of 900° C and the other at 400° C. Find the net exchange of energy due to radiation between the two plates. **(May 12)**

5. The surfaces of a doubled walled spherical vessel used for storing liquid oxygen are covered with a layer of silver having, an emissivity of 0.03.The temperature of the outer surface of the inner wall is 153° C and the temperature of the inner surface of the outer wall is 27° C. The spheres are 210 mm and 300 mm in diameter, with the space between them evacuated. Calculate the radiation heat transfer through the walls into the vessel and the rate of the evaporation of liquid oxygen if its rate of vaporization is 220kJ/kg.**(May 12)**

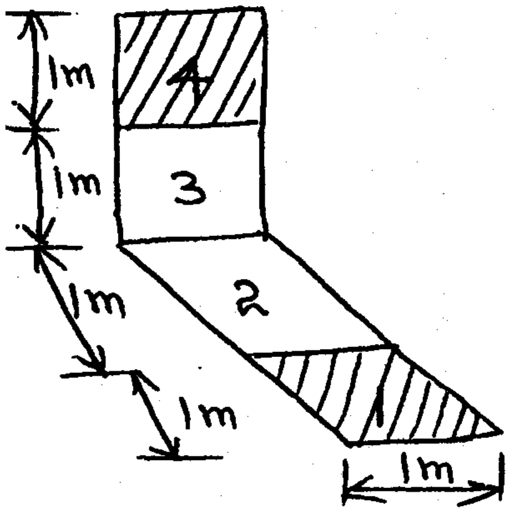
6. A furnace is approximated as an equilateral triangular duct of sufficient length so that end effects can be neglected. The hot wall of the furnace is maintained at 900 K and has an emissivity of 0.8. The cold wall is at 400 K and has the same emissivity. Find the net radiation heat flux leaving the wall. Third wall of the furnace may be assumed as a reradiating surface. **(Dec 11)**

7. Consider two concentric cylinders having diameters 10 cm and 20 cm and a length of 20 cm. designating the open ends of the cylinders as surfaces 3 and 4, estimate the shape factor, F3-4. **(Dec 11)**

8. Two very large parallel planes exchange heat by radiation. The emissivites of the planes are respectively 0.8 and 0.3. To minimize the radiation exchange between the planes; a polished aluminium radiation shield is placed between them. If the emissivity of the shield is 0.04 on both sides, find the percentage reduction in heat transfer rate. **(May 11)**

9. Two parallel plates of 1 x 1 m spaced 0.5 m a part in a very large room whose walls are at 27° C. The plates are at 900° C and 400° C with emissivities 0.2 and 0.5 respectively. Find the net heat transfer to each plate and to the room. **(May 10)**

10. Determine the view factor F 1-2 for the figure shown below. **(Dec 08)**



11. Determine the radiant heat exchange in Wm² between two large parallel steel plates of emissivities 0.8 and 0.5 held at temperatures of 1000K and 500 K respectively, If a thin copper plate of emissivity 0.1 is introduced as a radiation shield between the two plates. **(Dec 08)**

12. Show-from energy-balance consideration that the radiation heat transfer from a plane composite surface area A4 and made up of plane surface areas A2 and A3 to a plane surface area Al is given by: A4F41=A3F31+A2F21 & F14=F12+F13. **(May 08)**

13. A surface at l000 K with emissivity of 0.10 is protected from a radiation flux of 1250 W/m² by a shield with emissivity of 0.05. Determine the percentage cut off and the shield temperature. Assume shape factor as 1. **(May 07)**

14. Two large parallel planes at 800 K and 600 K have emissivities of 0.5 and 0.8

respectively. A radiation shield having an emissivity of 0.1 on one side and an emissivity of 0.05 on the other side is placed between the plates. Calculate the heat transfer rate by radiation per square meter with and without radiation shield. Common on the results. **(May 04)**

UNIT - V

1. Dry air at 27 degree Cel. and 1 atm flows over a wet flat plate 50 cm long at a velocity of 50 m/s. Calculate the mass transfer co-efficient of water vapour in air at the end of the plate. Take the diffusion coefficient of water vapour in air is Das = 0.26 X IO-a m2/s. **(Dec 07, May 10,May 12)**

2. Helium gas at 2500 and a pressure of 4 bar is stored in a spherical silica container of 150 mm inside diameter and 3 mm wall thickness. What is the intial rate leakage for the system? **(May 12)**

3. The tire tube of a vehicle has a surface area 0.62 m2 and wall thickness 12 mm. The tube has air filled in it at a pressure 2.4 x 105 N/m2• The air pressure drops to 2.3 x 105 N/m2 in 10 days. The volume of air in the tube is 0.034 m3• Calculate the diffusion coefficient of air in rubber at the temperature of 315K. Gas constant value = 287. Solubility of air in rubber tube = 0.075m3 of air/m3 of rubber tube at one atmosphere **(Dec 12)**

4. Dry air at 200C [ρ = 1.2 kg/m3, v = 15 x 10-6 m2/s. D = 4.2 x 10-5 m2/s] flows over a flat plate of length 50cm which is covered with a thin layer of water at a velocity of 1m/s. Estimate the local mass transfer co-efficient at a distance of 10cm from the leading edge and the average mass transfer co-efficient. **(May 06)**

5. Air at 1 atm and 250C containing small quantities of iodine flow with a velocity of 6.2 m/s inside a 35 mm diameter tube. Calculate mass transfer co-efficient for iodine. The thermo physical properties of air are v = 15.5 x 10-6 m2/s, D = 0.82 x 10-5m2/s. **(May 06)**

6. An open pan 20cm in diameter and 8cm deep contains water at 250C and is exposed to

dry atmospheric air. If the rate of diffusion of water vapour is 8.54 x 10-4 kg/h, estimate the diffusion co-efficient of water in air. **(May 05)**

7. Air at 20° C with D = 4.166 x10-5 m^2/s flows over a tray (length 320 mm, width 420mm) full of water with a velocity of 2.8 m/s. The total pressure of moving air 1 atm and the partial pressure of water present in the air is 0.0068 bar. If the temperature on the water surface is 15 ° C, Calculate the evaporation rate of water. **(May 08)**

8. A vessel contains binary mixture of O_2 and N_2 with partial pressure in the ratio 0.21 and 0.79 at 15° C. The total pressure of the mixture is 1.1 bar. Calculate the following

i) Molar concentrations

ii) Mass densities

iii) Mass fractions

iv) Molar fraction for each species **(Dec 08)**

FOR FURTHER READING

1. **Yunus A. Çengel.** Heat Transfer: A Practical Approach. Mcgraw-Hill (Tx).

2. **Theodore L. Bergman, Adrienne S. Lavine, *Frank P. Incropera*, David P. Dewitt.** Fundamentals of Heat and Mass Transfer. John Wiley & Sons.

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5. **M. N. Ozisik.** Heat Transfer. McGraw-Hill.