UNIT

Unit One: Exploring What It Means To 'Do' Mathematics

From the module:
Teaching and Learning Mathematics in Diverse Classrooms

South African Institute for Distance Education (SAIDE)

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- RADMASTE Centre, University of the Witwatersrand (2006). *Number Algebra and Pattern* (EDUC 264).
- Stoker, J. (2001). *Patterns and Functions*. ACE Lecture Notes. RUMEP, Rhodes University, Grahamstown.

Contents

How the unit fits into the module

Overview of content of module

The module *Teaching and Learning Mathematics in Diverse Classrooms* is intended as a guide to teaching mathematics for in-service teachers in primary schools. It is informed by the inclusive education policy (Education White Paper 6 Special Needs Education, 2001) and supports teachers in dealing with the diversity of learners in South African classrooms.

In order to teach mathematics in South Africa today, teachers need an awareness of where we (the teachers and the learners) have come from as well as where we are going. Key questions are:

Where will the journey of mathematics education take our learners? How can we help them?

To help learners, we need to be able to answer a few key questions:

- What is mathematics? What is mathematics learning and teaching in South Africa about today?
- How does mathematical learning take place?
- How can we teach mathematics effectively, particularly in diverse classrooms?
- What is 'basic' in mathematics? What is the fundamental mathematical knowledge that all learners need, irrespective of the level of mathematics learning they will ultimately achieve?
- How do we assess mathematics learning most effectively?

These questions are important for all learning and teaching, but particularly for learning and teaching mathematics in diverse classrooms. In terms of the policy on inclusive education, all learners – whatever their barriers to learning or their particular circumstances in life – must learn mathematics.

The units in this module were adapted from a module entitled *Learning and Teaching of Intermediate and Senior Mathematics*, produced in 2006 as one of the study guide for UNISA's Advanced Certificate in Education programme.

The module is divided into six units, each of which addresses the above questions, from a different perspective. Although the units can be studied separately, they should be read together to provide comprehensive guidance in answering the above questions.

Unit 1: Exploring what it means to 'do' mathematics

This unit gives a historical background to mathematics education in South Africa, to outcomes-based education and to the national curriculum statement for mathematics. The traditional approach to teaching mathematics is then contrasted with an approach to teaching mathematics that focuses on 'doing' mathematics, and mathematics as a science of pattern and order, in which learners actively explore mathematical ideas in a conducive classroom environment.

Unit 2: Developing understanding in mathematics

In this unit, the theoretical basis for teaching mathematics – constructivism – is explored. Varieties of teaching strategies based on constructivist understandings of how learning best takes place are described.

Unit 3: Teaching through problem solving

In this unit, the shift from the rule-based, teaching-by-telling approach to a problem-solving approach to mathematics teaching is explained and illustrated with numerous mathematics examples.

Unit 4: Planning in the problem-based classroom

In addition to outlining a step-by-step approach for a problem-based lesson, this unit looks at the role of group work and co-operative learning in the mathematics class, as well as the role of practice in problem-based mathematics classes.

Unit 5: Building assessment into teaching and learning

This unit explores outcomes-based assessment of mathematics in terms of five main questions – Why assess? (the purposes of assessment); What to assess? (achievement of outcomes, but also understanding, reasoning and problem-solving ability); How to assess? (methods, tools and techniques); How to interpret the results of assessment? (the importance of criteria and rubrics for outcomes-based assessment) ; and How to report on assessment? (developing meaningful report cards).

Unit 6: Teaching all children mathematics

This unit explores the implications of the fundamental assumption in this module – that ALL children can learn mathematics, whatever their background or language or sex, and regardless of learning disabilities they may have. It gives practical guidance on how teachers can adapt their lessons according to the specific needs of their learners.

During the course of this module we engage with the ideas of three teachers - Bobo Diphoko, Jackson Segoe and Millicent Sekesi. Bobo, Jackson and Millicent are all teachers and close neighbours.

Bobo teaches Senior Phase and Grade 10-12 Mathematics in the former Model C High School in town;

Jackson is actually an Economics teacher but has been co-opted to teach Intermediate Phase Mathematics and Grade 10-12 Mathematical Literacy at the public Combined High School in the township;

Millicent is the principal of a small farm-based primary school just outside town. Together with two other teachers, she provides Foundation Phase learning to an average 200 learners a year.

Each unit in the module begins with a conversation between these three teachers that will help you to begin to reflect upon the issues that will be explored further in that unit. This should help you to build the framework on which to peg your new understandings about teaching and learning Mathematics in diverse classrooms.

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How this unit is structured

The unit consists of the following:

- Welcome to the unit – from the three teachers who discuss their challenges and discoveries about mathematics teaching.
- Unit outcomes.
- Content of the unit, divided into sections.
- A unit summary.
- Self assessment.
- References (sources used in the unit).

Unit One: Exploring What It Means To 'Do' Mathematics

Welcome

"Eish!" remarked Jackson, "This was a tough week. I started by trying to get my IP learners to identify number patterns and many of them seemed to battle to identify all but the most obvious patterns. Then I found myself having to repeat almost the same struggle with my Maths Lit learners. I ended up teaching almost the same content to both groups; it's like my Grade 10 learners missed out all the earlier work."

"You'll get used to it," responded Bobo. "Every time I want to teach a new concept, I find myself having to go back and revise things I thought they'd already mastered. I think you're right that whole chunks of Maths get left out in earlier grades. I think the problem starts with a poor grasp of the basics. I don't know what happens at the beginning."

"Well," replied Millicent, "we do our best. We do a lot of practical work with objects like bottle tops and stones to try to get them to really understand numbers and then we build on that to introduce them to basic operations. We seem to get on best when we don't tell them that they're doing Maths! It's like as soon as they hear the word "maths" they freeze up. Just this week, for example, I had my Grade 3s working in groups of six and pretending to be in a taxi. They had to take turns at being in the front seat and sorting out the change. When they thought they were playing, they seemed to get on OK but as soon as I gave them some sums to practice, they started to make mistakes."

Think about the following:

1 Have you ever started to teach a maths concept then found yourself having to go back to repeat something that you thought learners had already mastered? Does this happen in some areas more than in others? How do you respond?

2 Millicent makes an interesting point about learners’ attitudes. Have you ever had a similar experience? Why do you think this sometimes happens?

3 For Jackson, “doing maths” involves looking for number patterns among other things; for Millicent it involves working with concrete objects, playing roles and doing sums. What does “doing maths” mean to you? Do you think it means the same to your learners? How could you find out?

Comments

It would be useful to compare your responses to these questions with those of some of your colleagues. We suspect that there will be a lot of similarities but also a few differences. Come back to your responses to these questions once you have worked through Unit 1 to see if your thinking has changed in any way.

Unit outcomes

Upon completion of Unit One you will be able to:

Outcomes

- Discuss critically the thinking that the traditional approach to teaching mathematics rewards the learning of rules, but offers little opportunity to do mathematics.
- Explain the phrase ‘mathematics is a science of pattern and order’.
- Evaluate a collection of action verbs that is used to reflect the kind of activities engaged by the learners when doing mathematics.
- Construct a list of features of a classroom environment considered as important for learners engaged in doing mathematics.
- Think about appropriate and interesting activities to help learners explore the process of problem-solving through number patterns and logical reasoning.

An introduction to mathematics education

The word 'mathematics' comes from the Greek word máthema which means 'science, knowledge, or learning'; the word mathematikós means 'fond of learning'. Today, the term refers to a specific body of knowledge and involves the study of quantity, structure, space and change.

Mathematics education is the study of the practices and methods of teaching and learning mathematics. Not only does the term mathematics education refer to the practices in classrooms, but it also refers to an academic discipline.

Before we get into what it means to 'do' mathematics, let us have a brief look at the historical background to mathematics education. There have been so many changes in the curriculum and assessment in recent years that it is important to understand the context in which there has been the need for these changes.

The history of mathematics education

Mathematics is not a new discipline - it has been around for centuries. Elementary mathematics was part of the education system in most ancient civilisations, including Ancient Greece, the Roman Empire, Vedic society and ancient Egypt. At this time a formal education was usually only available to male children of wealthy families with status in the community.

In the times of Ancient Greece and medieval Europe the mathematical fields of arithmetic and geometry were considered to be 'liberal arts' subjects. During these times apprentices to trades such as masons, merchants and money-lenders could expect to learn practical mathematics relevant to their professions.

During the Renaissance in Europe mathematics was not considered to be a serious academic discipline because it was strongly associated with people involved in trade and commerce. Although mathematics continued to be taught in European universities, philosophy was considered to be a more important area of study than mathematics. This perception changed in the seventeenth century when mathematics departments were established at many universities in England and Scotland.

In the eighteenth and nineteenth centuries the industrial revolution led to an enormous increase in urban populations, and so basic numeracy skills, such as the ability to tell the time, count money and carry out simple arithmetic, became essential in this new urban lifestyle. This meant that the study of mathematics became a standard part of the school curriculum from an early age.

By the twentieth century mathematics was part of the core curriculum in all developed countries. However, diverse and changing ideas about the purpose of mathematical education led to little overall consistency in the content or methods that were adopted. At different times and in different cultures and countries, mathematical education has attempted to achieve a variety of different objectives. At one time or other these objectives have included the teaching of:

- basic numeracy skills to all school pupils
- practical mathematics to most pupils, to equip them to follow a trade or craft
- abstract mathematical concepts (such as set theory and functions)
- selected areas of mathematics (such as Euclidean geometry or calculus)
- advanced mathematics to learners wanting to follow a career in mathematics or science.

Mathematics education in South Africa today

At school level, mathematics is often viewed as empowering, and as a means of access to further education, and is offered at all grade levels. However, the level of success in mathematics education in South African schools is very low. In the 1998/99 repeat of the Third International Mathematics and Science Study (TIMSS – R), that was written by Grade 8 learners, South Africa was ranked last of the 38 nations who participated in the study for mathematics. This study included other developing countries. The South African learners scored the lowest across all five topics in mathematics. In the 2003 TIMSS study, South Africa was ranked last of 46 participating nations. This poor performance shows that the majority of South African learners in Grade 8 have not acquired basic knowledge about mathematics and lack the understanding of mathematical concepts expected at that level. This situation is compounded by a huge drop-out rate amongst learners and the fact that many mathematics teachers are not adequately qualified to teach the subject.

Outcomes Based Education (OBE) was introduced into South African schools in 1994. The curriculum changes brought about by OBE are currently being implemented in the FET band in schools nationally. This means that schools have moved out of the old system where mathematics in secondary schools was offered at two levels, namely Higher Grade and Standard Grade. During this time many schools only offered Standard Grade mathematics while some schools did not offer mathematics at all. The new curriculum was implemented in grade 10 in 2006, and the first grade 12's to write new curriculum exams will write in 2008. Within the structure of the OBE curriculum all schools now have to offer all learners mathematics up to Grade 12. The choice for learners will be between mathematics and mathematical literacy. Mathematics will suit those learners who wish to further their education in fields which require certain essential mathematical knowledge. Mathematical literacy provides an alternative which will equip learners with a more contextualised knowledge of mathematics related functions performed in everyday life.

The performance data for the Senior Certificate examination in 2003 showed that:

- Less than 60% of all candidates chose to do mathematics as an exam subject at either the Standard Grade or the Higher Grade level.
- Approximately 35% of all candidates passed mathematics with only a small fraction of these passing on HG.

There are a number of reasons for South Africa's poor performance in mathematics.

South Africa is one of the most complex and heterogeneous countries in the world. Van der Horst and McDonald (1997) point out educational problems which all contribute to the current crisis in education in South Africa. Some of these problems include:

- the challenge of providing equal access to schools
- the challenge of providing equal educational opportunities
- irrelevant curricula
- inadequate finance and facilities
- shortages of educational materials
- the enrolment explosion
- inadequately qualified teaching staff.

These problems imply that change is needed in the South African educational system.

Why is educational change needed in South Africa?

According to Van der Horst and McDonald (1997), as a result of the divisions which existed during the apartheid era, learners were not always taught to appreciate the different aspirations and perspectives of people who were different, and many did not receive adequate educational and training opportunities during this era. This disadvantaged them greatly. This means that educational change must provide equity in terms of educational provision and promote a more balanced view, by developing learners' critical thinking powers and problem-solving abilities.

There is a need for a people-centred, success-oriented curriculum that will grant people the opportunity to develop their potential to the full. The new curriculum in South Africa attempts to adequately cater for these needs. The philosophy that underpins the new curriculum is that of Outcomes Based Education (OBE).

What is outcomes-based education (OBE)?

OBE is a learner-centred, results-oriented approach to learning which is based on the following beliefs:

- All individual learners must be allowed to learn to their full potential.
- Success breeds further success. Positive and constructive ongoing assessment is essential in this regard.
- The learning environment is responsible for creating and controlling the conditions under which learners can succeed. The atmosphere must be positive and learning is active.
- All the different stakeholders in education such as the community, teachers, learners and parents share in the responsibility for learning.

This approach proposes a shift away from a content-based, exam-driven approach to schooling. Instead learners are required to achieve specific learning outcomes for different phases within each subject. Learner centred activities form an integral part of the new curriculum and the emphasis is on encouraging learners to be instruments of their own learning, whether they work individually or in groups.

Outcomes-based education can be described as an approach which requires teachers and learners to focus their attention on two things (Van der Horst, McDonald, 1997):

> *1. The desired end results of each learning process. These desired end results are called the outcomes of learning and learners need to demonstrate that they have attained them. They will therefore be continuously assessed to ascertain whether they are making any progress.*
>
> *2. The instructive and learning processes that will guide the learners to these end results.*

Activity 1	In the light of what you have read so far, reflect on your mathematics teaching and write answers to the following questions: 1 Write about your experiences as a teacher of mathematics. You should write about a page, describing at least one good experience and one bad experience. 2 Write a short paragraph saying why you have chosen to do an ACE in mathematics education. 3 Have you been aware of the importance of mathematics as a subject that can empower the learners in your classes? If yes, say how you have tried to help them be empowered. If no, say how you might encourage them in the future. 4 Write a short paragraph summarising the key things you know about the National Curriculum Statement (NCS) for the GET or FET curriculum, depending on the level at which you are teaching. ▪ What other course/s have you attended that relate to the new mathematics curricula? ▪ Do you have a copy of any of these curricula? If yes, have you read it? ▪ Do the other teachers at your school who teach mathematics know about the new curricula?

What is mathematics? People's views

Most people acknowledge that mathematics is an important subject at school. However very few really understand what mathematics is about and what it means to 'do' mathematics. People often define mathematics as being about a collection of 'rules': arithmetic computations, mysterious algebraic equations or geometric proofs that need to be learnt in order to pass an examination. In general, people tend to feel that they are 'no good at mathematics and that it is difficult'.

Such people often believe that:

- Mathematics requires a good memory
- Mathematics is based on memorisation of facts, rules, formulas and procedures
- You have to have a special brain to do mathematics
- Mathematics is not creative
- There is a best way to do a mathematics problem

- Every mathematics problem has only one correct answer and the goal is to find THE answer
- Mathematics problems are meant to be solved as quickly as possible
- Mathematics is all symbols and no words
- Boys are better at mathematics than girls
- School mathematics is useless
- Mathematics is exact and there is no room for innovation, estimation or intuition.

Much of this restricted (even negative) view of mathematics stems from very authoritarian (which some people have called 'traditional') approaches to the teaching of mathematics. In such 'traditional' teaching, the teacher 'tells' or explains a mathematical concept or idea to learners. The teacher 'tells' the learners how to 'use' a mathematical idea in a certain way in order to get a correct answer. The learners then practise the method and rely upon the teacher to tell them the correct answers. This way of teaching produces a follow-the-rules, computation-driven, answer-oriented view of mathematics. Learners exposed to this way of teaching accept that every problem has only one solution and that they cannot solve a problem without being told a 'solution method' before hand. The 'rules' often do not make sense to the learners and there is little excitement in lessons, particularly if you cannot remember the rule!

Some students doing a Post Graduate Certificate in Education (PGCE) at a South African university were asked to define mathematics. This is what they wrote:

Jay

Mathematics is the solving of numerical problems using specific rules and laws. Certain methods are applied to solve equations and obtain an answer which is either definitely right or wrong.

Richard

Mathematics is a discipline which uses one's cognitive abilities to solve numerical arguments. Mathematics deals with numerical properties of abstract ideas and physical things. Mathematics uses symbols to represent quantities and operations and functions. Operations manipulate numerical ideas and functions show the relationships between numerical ideas. Maths is an accepted method for interpreting the world. Maths is a universal language, maths is human numerical consciousness.

Jane

Mathematics is the means by which we get a hold on the real world and try to make sense of it. The emphasis in the phrase "doing mathematics" is on the verb "doing". This is not an exercise in semantics but, on the contrary, a confirmation of the ability of mathematics to provide the ropes of measurement for a myriad of real life problems. In short, mathematics is constructed by society as a tool to handle the abstract as well as the concrete.

Mpho

Mathematics is part of everyday life, whether apparent or inconspicuous.

Kelebogile

Mathematics is a group of concepts relating to numbers, patterns, shapes and their relationships or behaviour.

Thabo

Mathematics is the study of figures/numbers and how they are used in everyday life and their application to solve complex problems.

Refilwe

Mathematics is an idealised, abstract system of representation used to represent quantity. It is useful as it can be used to precisely model aspects of reality and make precise and accurate predictions.

An alternative view of mathematics is that it involves 'making sense' of mathematical ideas, patterns and information. This is reflected in some of the student definitions that you have just read. That it involves 'figuring out' how to approach problems; about finding and exploring regularity in patterns and making sense of relationships; about finding patterns and order all around us, for example in art, in buildings and in music. This is the view taken by the new mathematics curriculum in South Africa.

How does the new mathematics curriculum define mathematics?

The introduction to the National Curriculum Statement (NCS) for GET and FET mathematics defines mathematics based on certain characteristics of the discipline:

- Mathematics enables **creative and logical reasoning about problems** in the physical and social world and in the context of mathematics itself.
- It is a distinctly **human activity practised by all cultures**.
- Knowledge in the mathematical sciences is constructed through the **establishment of descriptive, numerical and symbolic relationships**.
- Mathematics is based on **observing patterns** which, with rigorous logical thinking, leads to theories of abstract relations.

- Mathematical problem solving enables us to understand the world and make use of that understanding in our daily lives.

- Mathematics is developed and contested over time through both language and symbols by social interaction and is thus open to change.

This means that mathematics:

- Is something that people create or invent. They can discuss and argue about their creation and in the process reach a shared understanding of what it is;
- Enables creative thinking;
- Enables logical thinking;
- Is based on patterns that lead to abstract ideas;
- Helps us solve problems in the real world;
- Helps us solve problems in the world of mathematics;
- Helps us better understand the world;
- Uses symbols and language to develop reasoning skills and to make meaning;
- Is always changing because the shared understanding reached may be overthrown or extended as the development of mathematics continues.

The NCS goes on to say how mathematics can help and empower learners and describes how the NCS can enable learners to achieve the critical and developmental outcomes for the discipline. It describes how the teacher and learners can ‘do’ mathematics in the classroom.

Activity 2

1 How do you experience NCS in your classroom practice? Reflect on your own practices and explain how you apply the above principles/guidelines:

- Are we developing learners who are confident and independent thinkers?
- Are we working at closing the gap between the classroom and real life in all its complexity?
- How would you know whether or not the outcomes have been achieved in a lesson?

2 As a teacher of mathematics, there must have been occasions when you have wondered 'What is mathematics?'

a Write a paragraph saying what you think mathematics is.

b Write a paragraph saying what you think mathematicians do.

c Ask three people in your community (a learner, a teacher who doesn't teach mathematics and another mathematics teacher) what they would answer to a) and b) above. Write their answers down.

d Write a paragraph describing the similarities and differences between the responses you received in c).

What does it mean to 'do' mathematics?

How would you describe what you are doing when you are doing mathematics? In the rest of this chapter we are going to explore what it means to 'do' mathematics. It is fine to come to this point with whatever beliefs were developed from your previous mathematics experiences. What we hope is that after you have worked through this unit you will have realised that it is not fine to accept outdated ideas about mathematics and expect to be a quality teacher. Combining the best of the old ideas with fresh ideas about teaching and learning will enable you to become a better quality mathematics teacher.

Fasheh's (1982) description of the teaching of mathematics is very similar to that given by many learners:

- The classroom is highly organised.
- The syllabus is rigid.
- The textbooks are smartly fixed.

- Mathematics is considered as a science that does not make mistakes. There is one correct answer to every question and one meaning to every word and that measuring is fixed for all people and for all times.

- Direct instruction remains the dominating mode of teaching in mathematics.

In this approach to teaching the learners are not 'doing' mathematics. No wonder many learners find mathematics a dull and unstimulating subject.

The students who gave definitions of mathematics above were also asked to say what "doing mathematics" involved. This is what they wrote:

Jay

When you do mathematics you are looking at an equation and identifying which method or rule you will need to apply in order to solve the equation and then applying these rules in order to obtain the correct answer.

Dikaledi

Doing mathematics is the writing of those descriptions. It is the using of mathematical language, to write "poetry" underpinned with the rules of logic and reason which not only describes but also solves and reasons a problem, and is verifiable.

Richard

When you do mathematics you analyse problems and use numerical methods to solve them. You apply definitions which are always true to reach reliable answers.

Akane

Doing mathematics amounts to mathematization of a real world problem, the provision of a solution to the problem, the explanation of the solution and the translation of mathematics into everyday language.

Mpho

Among other things one may do in mathematics, is data handling, spatial perception, data manipulation, formulae, interpretation of shapes and figures, estimation of distance, volume, area mass, to find a gradient, addition, subtraction, division and multiplication. A good mathematical mind is one that probes, questions and does not take all the content of text books without a proper dimensional analysis.

Thabo

When you do mathematics you do calculations, measurements, and estimations to actually solve a particular problem. The calculation done may solve problems like optimum amounts required to run the production process efficiently. When doing mathematics you can predict what the situation is going to be for a particular process. For example, looking at rate one may calculate projected figures for a product sale.

Refilwe

The process of doing mathematics involves representing the aspects of reality that relate to a certain problem in precise numerical terms. Mathematics can then be used to produce precise solutions that will be as accurate as the assumptions upon which your mathematical formulation was based.

These views show that the students who wrote them have been exposed to a range of different types of mathematics teaching styles, from the more formalised authoritarian to a more flexible constructivist approach. Spend a bit of time thinking about what each of them has said, and what this shows about the ways in which they have been taught mathematics.

Activity 3

Your classroom experience

1. Write down at least five ideas/viewpoints of your own about what it means to do mathematics. You should use only one minute to do this activity. Keep your list somewhere safe so that you can refer to it at a later date.
2. Reflect on your classroom practice in the light of the following questions. Is there a tendency in your mathematics teaching:
   - to reward only formal knowledge?
   - to memorize rules and procedures?
   - to spoon-feed (facts, rules, procedure)?
   - to devalue the learners' own way of making sense out of their own experiences, intuition and insight?
   - for learners to remain passive learners?

As you work through the rest of this study unit, you will be challenged to rethink and reconstruct your own understanding of what it means to know and do mathematics – so that learners with whom you work will have an exciting and more positive vision of mathematics. Doing mathematics (mathematization) will be eventful, compelling and creative.

Teachers need to have ideas about how to structure classrooms so that they can help learners develop. Since experience is a powerful teacher, it makes sense for learners to experience mathematical ways of thinking, reasoning, analyzing, abstracting, generalizing- all modelled on good instruction and doing mathematics (Evan & Lappan: 1994). Therein lies the challenge for South African mathematics teachers.

Contrasting perceptions of teaching school mathematics

The teaching of school mathematics is subject to the same pressures for changing practices as the teaching of other subjects. In classrooms across the world we see a continuum of practice ranging from a more traditional content- and rule-based approach in which teachers spoon-feed learners in the use of rules to get the “right answer” towards a more open-ended approach in which the process of learning is equally emphasised and in which children work in groups to find and talk about and sometimes find innovative solutions to real-life problems. There are probably times when the more traditional approach is appropriate and other times when the more open-ended approach is desirable. Outcomes-based education allows us to work across the continuum but many teachers will need to develop new teaching strategies to work within the more open-ended, problem-based end of the continuum. We must challenge ourselves to make mathematics an interesting way for learners to engage with and make meaning of their lived experiences now rather than pursuing mathematics as a compulsory subject for its own sake which might have some kind of benefit to them in the future.

Activity 4

The traditional approach: three simple examples

Three simple ‘problems’ (A, B, C) are given to you below (at the Intermediate Phase Level).

A. Work through these problems

Problem A:

In servicing a car the attendant used 45ℓ of petrol at R4,25/ℓ and 2 tins of oil at R8,50 each.

What was the total cost for petrol and oil?

Problem B:

The world’s record for the high jump in a recent year was 1,87 metres.

On Mars, this jump would be 2½ times as high. How much higher in metres will it be?

Problem C:

John covers ½ of a journey by car, 1/3 of the journey by bicycle, and walks the rest of the way.

- a) What part of the journey does he cover by car and bicycle?
- b) What part of the journey does he walk?

B. Indicate by means of a tick (✓) in the blocks given below which of the mathematical skills listed come to the fore for learners attempting each of these 'problems' using traditional approaches to unpacking and 'solving' the problem.

Skills acquired by learners	Problem A	Problem B	Problem C
Computational skills			
Using correct order of operations			
Formulating expressions as a mathematical model			
Estimation to assess reasonableness of answer			
Problem-solving thinking skills			
Investigatory skills			
Exploration of rules and logical thinking			
Self-discovery			

Now do the following.

1 Indicate whether the three problems (A, B, C) are typical of the stereotypical traditional way of questioning learners in South Africa.

2 Refer to the skills mentioned above and explain your response briefly.

3 What skills should we teach our learners now in order to prepare them to cope with the 21st century?

4 Make recommendations to your colleagues on the skills we should teach in mathematics.

Mathematics as a science of pattern and order

Van de Walle (2004: 13) uses a wonderfully simple description of mathematics found in the publication 'Everybody counts' (MSEB :1989):

> *Mathematics is the science of pattern and order.*

This immediately challenges the popular social view that mathematics is dominated by computation and rules that learners need not understand but must apply rigorously. As a science, mathematics is a process of figuring things out (formulating number patterns, investigating, exploring, conjecturing, generalising, inducing, deducing, etc.) or in general, making sense of things.

Van de Walle (2004: 13) continues :

> *Mathematics is, therefore, a science of things that* ***have a pattern of regularity and logical order****. Finding and exploring this regularity or order and then making sense of it is what doing mathematics is all about.*

The stereotypical traditional view emphasizes procedures and the solving of routine problems, with teachers showing and telling while learners listen and repeat.

The more progressive constructivist and OBE view is that of the learning of mathematics as a process (irrespective of the content material), emphasising meaningful development of concepts and generalisations, increasing the prospects of real problem-solving, open enquiry and investigation – characterised mainly by teachers challenging, questioning and guiding, with students doing, discovering and applying.

These views could be seen as extremes on a continuum. Teachers would locate themselves at a whole range of positions along this continuum, possibly tending more to one extreme than the other, but incorporating ideas from both sides in accordance with their belief and understanding of how one should teach mathematics in a meaningful way.

If learners do not understand how things work or have not achieved a proper conceptual understanding – that is they cannot see the pattern and order – they could make computational errors. These errors may be the result of different things, including simple carelessness, errors or incorrect understanding and misconceptions. Learners may think they are doing the right thing, based on a concept they have which they think is correct but it is actually incomplete or incorrect. Some examples of this are given in the next activity.

Activity 5

Computational errors and misconceptions

Learners make what appear to be computational errors as a result of a lack of understanding of how things work. Here are a few examples:

Computational/notational/conceptual errors

i $\frac{18}{85} = \frac{1}{5}$ $\qquad$ $\frac{1\cancel{8}}{\cancel{8}5} = \frac{1}{5}$ [Learner cancels the 8's]

ii $\begin{array}{r} 625 \\ +\ \underline{25} \\ 875 \end{array}$ [The learner does not understand the concept of place value].

iii 0,234 is bigger than 0,85 [Since 234 is bigger than 85]

iv 3 (4×5) = (3×4) (3×5) [Since 3 (4+5) = 3×4 + 3×5]

v 8 ÷ ½ = 4 [Since 8 ÷ 2 = 4 or 8 × ½ = 4]

vi $a \times a = 2a$ [Since $a + a = 2a$]

vii Half of 8 = 3 [Since half of the figure 8 is 3, if you cut an 8 in half vertically with a pair of scissors]

and so on.

1 Write down from your own experience a few more examples in which learners make computational errors as a result of a lack of understanding (misconceptions) of how procedures or rules actually work. Consult with other teachers of mathematics.

2 Discuss this with your colleagues and explore solutions to these problems.

3 Suggest the teaching strategies an innovative teacher could use to avoid such misconceptions from developing in learners.

Note: We encourage you to discuss this with your colleagues – we can learn a lot from each other.

'Doing' mathematics

Engaging in the science of pattern and order requires a good deal of effort and often takes time. This effort is well worth while, because of the quality of learning that it facilitates. It is more desirable that learners understand and are able to use the learning rather than merely seem to do so by answering artificially constructed 'test' questions. The next activity will illustrate this.

Activity 6

Problem-solving: number pattern activity

Try out the activity, if you can, with a senior phase class.

Reflect on the method and solution given in this activity to the following problem:

Ten cities in South Africa need to be directly connected to all other cities by a telephone line. How many direct connections are needed? (Paling & Warde: 1985).

One approach would be to follow the three steps given below – but you are at liberty to use any other problem-solving techniques. If you try this problem out with one of the classes that you are teaching, it will be interesting for you to observe the different strategies that your learners use. Remember not to guide them too closely, let them think the question through and think of how to go about drawing it up and finding the solution.

Step 1: UNDERSTAND THE PROBLEM: e.g. three or more cities are not situated in a straight line. What do we need to do about it? We need to make sure that every one of the ten cities is connected by a line, which we will use to represent a telephone connection.

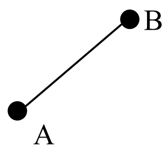
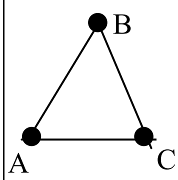
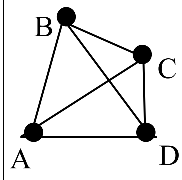
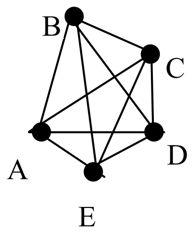
Reflection: To carry out this step the learners need to have the language skills to read and interpret the problem, they need to be able to visualise the problem, and then use their mathematical knowledge to move into the next step in which they represent the problem symbolically and numerically.

Step 2: DEVISE A PLAN: Reduce the problem to simpler terms –start with one city, and then two cities, three cities and so on.

Reflection: To carry out this step the learners need to use their mathematical knowledge to think about how to represent the problem symbolically and numerically. Here learners also need to use strategic reasoning. The idea to develop a pattern by building up the number of cities from one, to two, and then three, and so on, is essential to the solution to this problem. This is where we see the pattern element of the problem coming through.

Step 3: CARRY OUT THE PLAN: Use drawings and write down a sequence to establish the pattern, formulate conjectures, test conjectures and generalise.

Using drawings, as shown on the next page:

Number of cities	1	2	3	4	5
Drawing					
Number of connections	0	1	3	6	10
Establishing	0	0 + 1	0 + 1 + 2	0 + 1 + 2 + 3	0 + 1 + 2 + 3 + 4
a pattern (rule)	$\frac{1(1-1)}{2}=0$	$\frac{2(2-1)}{2}=1$	$\frac{3(3-1)}{2}=3$	$\frac{4(4-1)}{2}=6$	$\frac{5(5-1)}{2}=10$

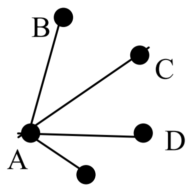
Reflection: To carry out this step the learners need to use their mathematical knowledge to represent the problem. Here they will use procedures and skills that they have been taught, but they will need to reason about the way in which they apply this knowledge. They will think about doing drawings of the first few cases, but as soon as they can see that there is a pattern emerging, they need to analyse the nature of the pattern. They can base their final solution on the basis of this pattern, by using the same reasoning to find the total number of connections of 6 cities, 7 cities..., and finally 10 cities. (Use drawings and test your conjecture/rules). When they do this, they are moving onto the next step.

Step 4: EVALUATE AND EXTEND THE PLAN FOR n CITIES: If there are n cities how many connections will there be?

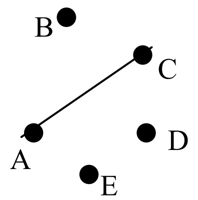
Let us use a simpler example again.

For 5 cities

Each city will be connected to 4 other cities
i.e. $(5 - 1) = 4$



There will be five such cases i.e. from A, B, C, D and E.



From this we have 5 (5 – 1) connections

The connection from A to C is the same as C to A.

This is the same for each case. So divide by 2.

We therefore get $\frac{5(5-1)}{2} = 10$

Now write down the number of connections for n cities.

This would give us $\frac{n(n-1)}{2}$ as a formula to work out the number of connections between n cities.

For 10 cities we therefore get $\frac{10(10-1)}{2} = 45$

Reflection: The solution of this problem illustrates the idea that mathematics can be seen as the "science of pattern and order". The pattern was established though drawings made of the connections between up to five cities. Using the drawings, a numeric pattern could be established, which could be used to work out how many connections there would be between ten cities. Learners will not all follow the same steps, or carry out the steps in the same order. As the teacher, you need to be flexible, and follow the learners' thinking. You need to probe and guide, without leading too explicitly, so that the learners are able to make connections and develop their mathematical understanding, in short, so that they can be involved in "doing mathematics".

Does the process of 'doing' mathematics (mathematising):

- Provide a real problem-solving situation?
- Encourage enquiry, exploration and investigation of numbers?
- Stimulate the learning of regularity and order of numbers?
- Require the teacher to guide and pose thought-provoking questions?
- Involve the learners in actively doing mathematics and discovering rules?

Perhaps you are wondering after working through that rather complex example, what mathematics teachers are supposed to do about basic skills? For example, you may be asking, don't learners need to count accurately, know the basic facts of addition, multiplication, subtraction and division of whole numbers, fractions, and decimals and so on?

The fact is, that when we teach an algorithm in mathematics (like long multiplication) and then give learners exercises to do in their books, our learners are not 'doing' mathematics. This doesn't mean that teachers should not give learners this kind of exercise, which is simply drill-work, but that **drill should never come before understanding**.

Repetitive drill of the bits and pieces is not 'doing' mathematics and will never result in understanding. Only when learners are capable of making sense of things by 'doing' mathematics in the classroom, are they being truly empowered.

The verbs of doing mathematics

Farrell and Farmer in *Systematic Instructions in Mathematics* (1980) state that:

> *Mathematics is a verb, as well as a noun.*

Two questions arise from this statement:

1 What do we do when we mathematise?

2 What do we obtain?

Mathematics is about processes (expressed in 'doing verbs') and it is also about products (expressed by nouns). In the table below a few examples are given.

Processes of Mathematics	Products of Mathematics
Generalising	Formula
Computing	Theorem
Assuming	Definition
Solving	Axiom
Proving	Corollary
Testing	Concepts (number etc.)

What verbs would you use to describe an activity in a classroom where learners are doing mathematics? Here is a list of verbs that can be associated with doing mathematics:

explore	represent	use
predict	solve	construct
justify	verify	explain
investigate	discover	justify
conjecture	develop	formulate

Study these verbs carefully – they describe what action or behaviour is expected from the learners when doing the classroom activity. If you look closely at the words, you will see that they are all action words that indicate that learners engaging in such activities would be actively involved in making sense and figuring things out. Learners cannot be passive observers and listeners when they are doing mathematics.

Activity 7

The verbs of doing mathematics

1 Reflect on the collection of 'action verbs' above. Do these verbs clearly indicate the type of action required of the learner during the process of mathematising?

2 Study the NCS for a grade to which you are currently teaching mathematics.

3 Are the Assessment Standards expressed in terms of the action verbs?

4 Make a list of the verbs that you find in the Assessment Standards.

5 Do these verbs clearly indicate the type of action required of the learner during the process of mathematising?

6 Give an example of a mathematical activity that demonstrates the action involved in each of these verbs.

Now that you have more understanding of what it means to do mathematics and the processes involved, you might like to take another look at Activity 4.

Activity 8

Action verbs used to identify the process skills in mathematics

1 Work through Activity 6 again.

2 Use appropriate action verbs to write down three mathematical process skills that learners could actively acquire through this problem-solving activity.
Begin with: The learners should be able to ……………

What is basic mathematics?

What is 'basic' in mathematics is always a matter of public discussion and debate. Is it

- mastering the basic operations?
- demonstrating achievement of the Learning Outcomes for mathematics (in the NCS)?
- becoming mathematically literate?
- recognising that mathematics is a part of human creative activity?

A simple challenge, no matter what position you take on this question, is that mathematics must make sense.

The implications of this statement are:

- Every day learners must experience that mathematics makes sense.
- Learners must come to believe that they are capable of making sense of mathematics.
- Teachers must stop teaching by telling and start letting learners make sense of the mathematics they are learning.
- To this end, teachers must believe in their learners – all of them!

This is a profound challenge – mathematics is for everyone without exception. Everyone has to learn that mathematics makes sense and to make sense of mathematics.

Activity 9

What is basic mathematics?

Van de Walle (2004:15) says

> *All learners are capable of learning all of the mathematics we want them to learn, and they can learn it in a meaningful manner that makes sense to them if they are given the opportunity to do so*

1 Do you agree with Van de Walle's position on what he considers basic in mathematics? Do you find it realistic or revolutionary? Discuss your opinion with your colleagues.

2 What do you think Van de Walle would say about OBE and its Expanded Opportunity Assessment Strategy (projects, investigations, group-work, oral assessment, peer-assessing and so on)?

3 What do your learners think is basic in mathematics? Ask them, and then compare their ideas with yours.

The following list is what the course developers think is 'basic in mathematics' - fundamental mathematical knowledge for all learners. Essential skills were itemised from each of the Learning Outcomes (LOs) in the NCS for GET Mathematics (Grade R-9).

1 Number concept and operations – learning outcome 1 (LO1)

- Place value

Operations (conceptual understanding giving rise to understanding of different strategies for all operations.)

2 Pattern (LO2)

- Core concepts – recognition of patterns, using words and drawings to describe and analyse patterns, generalise rules for patterns, generate other patterns
- Link working with patterns to everyday situations

3 Space and shape (LO 3)

- 2-D and 3-D perception. Up to Van Hiele first three levels: learners should be able to visualise, recognise and describe shapes
- Look at properties of shapes. No calculations of area and volume, simply recognition of shapes, drawing, describing and naming of polygons and polyhedra

4 Measurement (LO4)

- Concept formation using developmental activities and Piaget's conservation tests
- The process of quantifying - length, mass, capacity, volume, area, time
- The stages in the teaching of measurement

5 Data (LO5)

- Collecting, recording, ordering, presenting and analysing
- Awareness of statistics presented in everyday life

This content will be found integrated into the guide, throughout the various units of the guide. Depending on your own current knowledge of mathematics, you may find some of the mathematical content presented in this guide challenging. You need to make the necessary effort to understand the mathematical content, because in order to teach mathematics, you need to have a sound understanding of the fundamental mathematical content yourself.

An environment for doing mathematics

It is the job of the teacher to ensure that every child learns to do mathematics, but for this there has to be the right environment.

An environment for doing mathematics is one in which learners are allowed to engage in investigative processes where they have the time and space to explore particular cases (problems). Then they can move slowly towards establishing, through discovery and logical reasoning, the underlying regularity and order (in the form of rules, principles, number patterns and so on).

Learners can create a 'conjecturing atmosphere' in the classroom if the teacher provides appropriate tasks and promotes learner thinking and discussion around these tasks. This atmosphere is one in which the rightness or wrongness of answers is not the issue, but rather an environment which encourages learners to make conjectures (guesses) as to the regularity (sameness) they see and to discuss these conjectures with others without fear of being judged wrong or stupid, to listen to the ideas expressed by others and to modify their conjectures as a result.

The mathematical processes involved in doing mathematics are best expressed by the action verbs. They require reaching out, taking risks, testing ideas and expressing these ideas to others. (In the traditional classroom these verbs take the form of: listening, copying, memorising, drilling and repeating - passive activities with very little mental engagement, involving no risks and little initiative.)

The classroom must be an environment where every learner is respected regardless of his or her perceived 'cleverness', where learners can take risks without fear that they will be criticised if they make a mistake. It should be an environment in which learners work in groups, in pairs or individually, but are always sharing ideas and engaged in discussion.

The following activity illustrates how even the most apparently 'routine' problem can be tackled in a variety of ways.

Activity 10

Doing mathematics: informal methods

Reflect on the following informal strategies attempted by learners at an Intermediate Phase/Senior Phase and then answer the questions below:

MULTIPLICATION AS REPEATED ADDING

43×6:	or	Adding speeded by doubling		
$40 + 3$		$43 \text{ x } 6$:	43	
$40 + 3$			$\underline{43}$	
$40 + 3$			86	(2x43)
$40 + 3$			$\underline{86}$	
$40 + 3$			172	(4x43)
$40 + 3$			$\underline{86}$	
$240 + 18 = 258$			258	(6x43)

DIVISION AS REPEATED SUBTRACTION:

$564 \div 18$:	10×18	$= 180$
	10×18	$= \underline{180}$
		360
	10×18	$= 180$
		$\underline{540}$
	$\underline{1 \times 18}$	18
	31	556

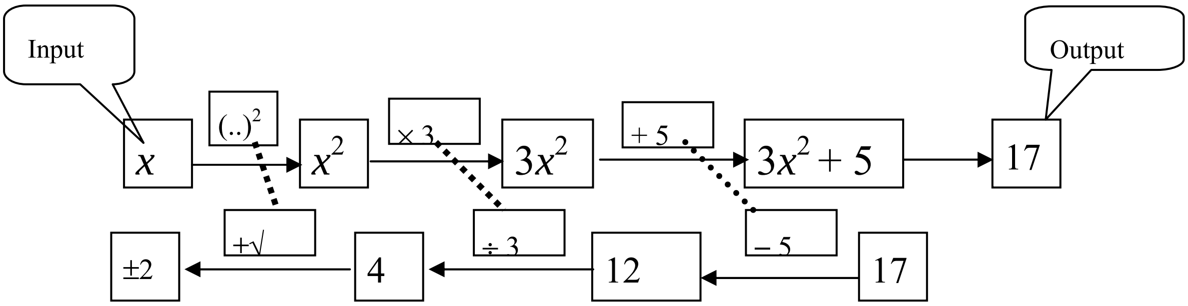
Remainder:

$$\begin{array}{r} 564 \\ -\ \underline{556} \\ 8 \end{array}$$

$564 \div 18 = 31$ remainder 8

USING A REVERSE FLOW DIAGRAM TO SOLVE AN EQUATION

Solve for x: $3x^2 + 5=17$



1 Reverse the flow diagram (see the dotted lines to indicate the inverse operations). Start with the output and apply inverse operations. What do you find?

2 Do you agree that the use of informal strategies where learners wrestle towards solutions is never a waste of time? Motivate your response.

3 Compare the above non-routine strategies with the recipe-type routine methods and explain which offer better opportunities for 'doing mathematics' discussions, developing reasons, testing reasons and offering explanations.

4 Have you come across some interesting non-routine methods used by learners in a particular situation? If you have, describe some of these examples. You could also discuss them with your fellow mathematics teachers.

What qualities does the teacher need in order to create an environment in which learners feel safe and stimulated to 'do' mathematics? The following activity helps to describe these.

Activity 11

An environment for doing mathematics

Read through the following motivational dialogue between Mr Bright and Mr Spark, two mathematics teachers. Describe the features of a classroom environment which you consider as important for learners to be engaged in doing mathematics.

Mr Bright:	A teacher of 'doing' mathematics needs to be enthusiastic, committed and a master of his or her subject.
Mr Spark:	He or she needs to have a personal and easier feel for doing mathematics to create the right environment in the classroom.
Mr Bright:	Teachers should provide activities designed to provide learners with opportunities to engage in the science of pattern and order.
Mr Spark:	Yes, a real opportunity to do some mathematics!
Mr Bright:	We need to develop this technique and discover as much as we can in the process.
Mr Spark:	Let us invite Mr Pattern and perhaps Mr Order and some of the other mathematics teachers.
Mr Bright:	Yes! We would all be actively and meaningfully engaged in doing mathematics and respect and listen to the ideas put forward by the others.
Mr Spark:	We shall challenge each other's ideas without belittling anyone.

Having thought about the conditions necessary for doing mathematics, we can now begin to think about the kinds of activities that will promote doing mathematics.

Examples of doing mathematics

In this part of the unit, we invite you to explore patterns in mathematics through a number of different mathematics problems. Mathematics gives us (and our learners) ways to think about the world. Selection and design of activities for use in a mathematics classroom is discussed in detail in this guide in Unit 4. Here we simply give examples of doing mathematics relating to each of the LOs in the NCS. This will give you the opportunity to do mathematics, reflect on what you have done, and think about appropriate and interesting activities to help learners explore the process of problem-solving through number patterns and logical reasoning.

Read carefully through each problem so that you are sure you understand what is required. None of these problems require any sophisticated mathematics, not even algebra. Don't be passive! Express your ideas. Get involved in doing mathematics!

LO1: Doing an activity involving operations

This activity deals with content assessment standards that relate to mental operations from LO1. It also covers the assessment standards relating to pattern identification from LO2.

Activity 12

Exploration – product of numbers

Young children learning about their basic number facts could be confronted with the following observation: In a sum, when you make the first number one more and the second number one less, you still get the same answer. For example:

$7 + 7 = 14$ and $8 + 6 = 14$

$5 + 5 = 10$ and $6 + 4 = 10$.

What can you find out about this?

Your task here is to examine what happens when you change addition to multiplication in this exploration. Consider the following examples:

A	B
$7 \times 7 = 49$	$8 \times 6 = 48$
$8 \times 8 = 64$	$9 \times 7 = 63$
$9 \times 9 = 91$	$10 \times 8 = 80$
$10 \times 10 = 100$	$11 \times 9 = 99$
.	.
.	.
.	.

What happens to the product when you **increase** the first number by 1 and **decrease** the second number by 1 in column A? Compare the products in column B and identify the pattern. State the pattern in your own words.

How do these results differ when the two factors are 1 apart?

Example A	B
$6 \times 7 = 42$	$7 \times 6 = 42$
$7 \times 8 = 56$	$8 \times 7 = 56$
$8 \times 9 = 72$	$9 \times 8 = 72$
.	.

Compare the products in column A to the products in column B and identify the change (if any).

How do these results differ when the two factors are 2 apart or 3 apart?

Example (the numbers are two apart)

A	B
$6 \times 4 = 24$	$7 \times 3 = 21$
$9 \times 7 = 63$	$10 \times 6 = 60$
$10 \times 8 = 80$	$11 \times 7 = 77$
.	.
.	.
.	.

How do the results differ from the results above?

What if you adjust the factors up and down by 4?

Examples:

A	B
$7 \times 3 = 21$	$8 \times 2 = 16$
$9 \times 5 = 45$	$10 \times 4 = 40$
.	.
.	.
.	.

Find the difference in the products between column A and column B.

Does it make any difference to the results if you use big numbers instead of small ones?

LO2: Doing an activity involving patterns

This activity deals with the assessment standards relating to pattern identification from LO2. It also deals with the content assessment standards that relate to mental operations from LO1.

Activity 13

Start and jump numbers, searching for numbers (adapted from Van de Walle)

You will need to make a list of numbers that begin with a 'start number' and increase it by a fixed amount which we will call the 'jump number'. First try 3 as the start number and 5 as a jump number. Write the start number first and then 8, 13 and so on 'jumping' by 5 each time until your list extends to about 130.

Your task is to examine this list of numbers and find as many patterns as you possibly can. Share your ideas with the group, and write down every pattern you agree really is a pattern.

Here are some suggestions to guide you:

- Look for alternating patterns
- Look for repeating patterns
- Investigate odd and even numbers
- What is the pattern in the units place?
- What is the pattern in the tens place?
- What happens when you go over 100?
- What happens when you add the digits in the numbers?
- Extend the pattern where you see a gap in the following table.

Which of the following are number patterns (in the columns)?

Complete the table by filling in the empty spaces, and then analyse the numbers in each column:

Separate the consecutive terms		Add terms (in columns 1 and 2)	Subtract terms (in columns 1 and 2)	Multiply terms (in columns 1 and 2)
3	8	11	5 (or -5)	24
13	18	31	5	234
23	28	51	5	644
33	38	71	5	1254
43	48	91	5	2064
53	58	111	5	3074
63	68	131	5	
73	78	151	5	

LO3: Doing an activity involving space and shape

This activity deals with the assessment standards relating to geometric shapes and compass directions from LO3. It also deals with the content assessment standards that relate to measurement (length and area) from LO4. This activity shows how maths can be used to solve real life problems.

Activity 14

Gardening problem

Jamal had a garden in the shape of a square.

Due to the construction of a new road the garden will lose a 3 metre long strip on the south side. Jamal wants to know if he can make up for this difference by adding an extra 3 metres on the east side.

1. Work out the area of the original garden.
2. Cut out a template to represent the area of the garden. Use a scale of 1 cm = 1 m. Mark off the centimetres with a ruler.
3. Cut off a 3 metre strip from the south side of your template.
4. Work out the area of the strip that has been lost.
5. Attach the strip to the east side.
6. What has Jamal not considered if he wants to keep his garden rectangular?
7. To ensure that the garden remains rectangular, what will the area of the strip on the east side need to be for this to be possible?
8. What is the area of the new garden?
9. Does this area differ from the original area at all? If so in what way?
10. Explain the reason for your answer above.

LO4: Doing an activity involving measurement

This activity deals with the content assessment standards relating to measurement (time) from LO4. It also deals with the assessment standards relating problem solving with rates from LO1. This is another activity that shows how maths can be used to solve real life problems.

Activity 15

Two machines, one job (adapted from Van de Walle)

Ron's Recycle Shop was started when Ron bought a used paper-shredding machine. Business was good, so Ron bought a new shredding machine. The old machine could shred a truckload of paper in 4 hours. The new machine could shred the same truckload in only 2 hours. How long will it take to shred a truckload of paper if Ron runs both shredders at the same time?

Make a serious attempt to figure out a solution. (You could use drawings or counters, coins and so on). If you get stuck consider:

- Are you overlooking any assumptions made in the problem?
- Do the machines run at the same time?
- Do they run as fast when working together as when they work alone?
- Does it work to find the average here? Explain your answer.
- Does it work to use ratio and proportion here? Explain your answer.

LO5: Doing an activity involving data handling

This activity deals with the content assessment standards relating to representation of data from LO5. This is another activity that shows how maths can be used when thinking about real life problems.

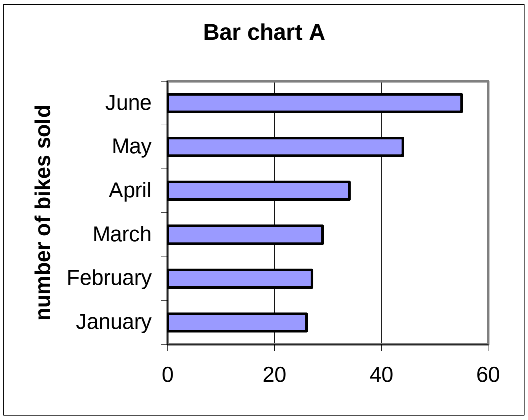
Activity 16

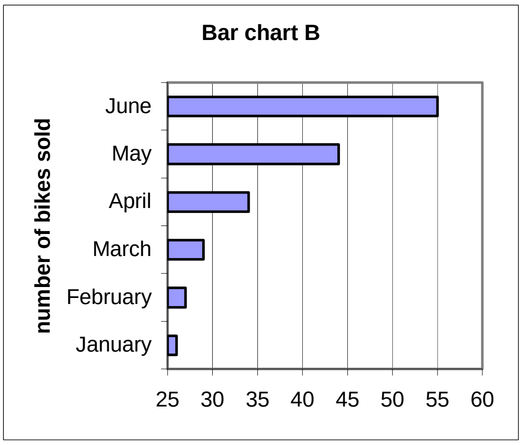
Interpreting bar graphs

Jairos sells bicycles. He is thinking about expanding his business and needs to borrow money from the bank. He wants to show the bank manager that his sales are growing fast. These are his sales for the last 6 months:

Month	Jan	Feb	Mar	Apr	May	June
Number of bikes sold	26	27	29	34	44	55

Jairos draws two graphs as shown below:





1 In what way are the graphs different?

2 Do you think the graphs tell the same story? Why / why not?

3 Which chart do you think Jairos should show to the bank manager? Why?

Reflecting on doing mathematics

Even if you worked hard on the above activities, you may not have found all of the patterns or solutions. The important thing, however, is to make an effort and take risks. Engaging with mathematics as the science of pattern and order is rewarding, but requires effort. As you will have noticed, the examples cover a wide range of mathematical content, all of which fall under into the bigger picture of doing mathematics. As you reflect on your experience, ask yourself the following questions:

- What difficulties did you encounter?
- How did you overcome them?
- If you did not overcome the difficulty, can you identify WHY NOT?
- Having identified why you were unable to solve the problem, are you able to get to the root of this problem? You may need to engage with colleagues or refer to books, and if so it is important that you do so, since as a teacher of mathematics, you will always have to master the content that you need to present.
- What methods did you use that were successful?
- Why were they successful?
- How would you like to present these activities (or other similar activities, appropriate to the grades which you teach) to your learners?
- How would you ensure that your learners are able to successfully solve the problems given to them?

This will help you to develop your ability to do mathematics and to teach mathematics in a developmental way. Instead of concentrating on explaining rules and procedures, a developmental approach to teaching mathematics is learner-centred and allows learners to grapple with ideas, discuss and explain solutions, challenge their own ideas and the ideas of others. Reflective thinking is the most important underlying tool required to construct ideas, develop new ideas and to connect a rich web of interrelated ideas.

In the rest of this unit we give further examples with a focus on pattern activities that you could do and reflect on – first by yourself, and then with your learners. In other units that follow you will be given the opportunity to try out problems relating to the other LOs from the mathematics curriculum.

Exploring pattern in Mathematics

The ability to recognise patterns, use words and drawings to describe and analyse patterns, generalise rules for patterns, and generate other patterns is fundamental to understanding mathematics. Learners need to be able to describe patterns observed using words or mathematical symbols. It is important for learners to recognise patterns by looking for common differences. Learners also need to be able to extend numeric and geometric patterns, which helps them recognise a variety of relationships in the patterns and make connections between mathematical topics. Identifying patterns like repeat patterns and growing patterns helps learners become aware of the structures of various pattern types. Teachers need to show their learners how to make generalisations for patterns and understand these generalisations.

Repeating patterns

Van de Walle (2004) says that

> *Identifying and extending patterns is an important process in algebraic thinking. Simple repetitive patterns can be explored as early as kindergarten. Young children love to work with patterns such as those made with coloured blocks, connecting cubes and buttons.*

Learning Outcome 2 in the NCS has several assessment standards that refer to pattern work as it progresses through the GET phase. Patterns may be geometric, numeric or algebraic. Working with patterns develops the logical reasoning skills of the learners, and leads naturally into thinking algebraically. The work on patterns that follows is taken from the RADMASTE materials for the Number Algebra and Pattern module of the WITS GET Mathematics ACE.

Learners can work independently or in small groups to extend (continue) the patterns given on strips. To do this activity with your class you should prepare enough pattern strips for the whole class.

Van de Walle points out,

> *The core of a repeating pattern is the* ***shortest string of elements that repeats.***

Each pattern must repeat completely and never be partially shown. In mathematics there is a convention that if a repetition of a pattern is seen three times, the observer can assume she/he has identified a repeating pattern. Here is an example of a visual pattern strip in which the pattern is repeated three times.

↗	↖	↗	↖	↗	↖

The core of the pattern shown above has two elements. Having each of the arrows in the first two frames in a different colour would highlight the repeating elements.

Here is another easy pattern to continue. The core has three elements, but the detail on the shapes might make it more difficult for young learners to identify.

⊕	∅	∅	⊕	∅	∅	⊕	∅	∅

This pattern below has four elements in its core.

®	®	)	Δ	®	®	)	Δ	®	®	)	Δ

The strip below with letters A and B translates the pattern above it from one medium to another: geometric to variable. Using variables, learners can identify similar types of patterns. The two patterns below are the same type of pattern. They both follow a sequence of a, b, b, a, b, b, a, b, b…

⊙	⬣	⬣	⊙	⬣	⬣	⊙	⬣	⬣
A	B	B	A	B	B	A	B	B

Activity 17

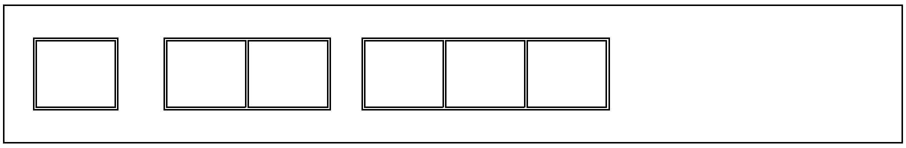
Try this with your class and write a report. Your report should include copies of all of the patterns you used in the lesson. Your report should cover at least the following points:

- Describe what you did.
- How did your learners respond?
- Were there any changes you needed to make?
- Why did you make the changes?
- Any other important observations or findings.

1. An overhead projector may be used to display a numbered set of different patterns. Teach the learners to use the A, B, C… method of reading a pattern. Half the class can close their eyes, while the other half can read aloud the pattern you point to. The learners who had their eyes closed must then open them and select the correct pattern/s.
2. Suggest an alternative way to present this lesson for a teacher who does not have an overhead projector in his/her class.

Growing patterns

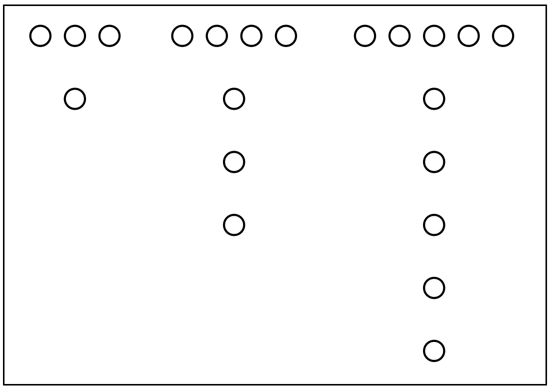
These patterns below are made up of geometric shapes, but they also have elements that can be counted. They pictorially illustrate sequences of numbers. You could supply the learners with pattern cards of the type below and have them copy and extend the pattern given in the first three frames. Let them explain why their extension is appropriate, by determining how each frame in the overall pattern differs from the preceding frame. For example, the simple pattern below begins with one brick and increases by one brick from frame to frame, representing the sequence 1; 2; 3; …



Learners should be given time to study the patterns. They can then extend the patterns, giving explanations of why their extension follows from the given sequence. The use of language to explain the extension is important as it develops the learners' mathematical reasoning.

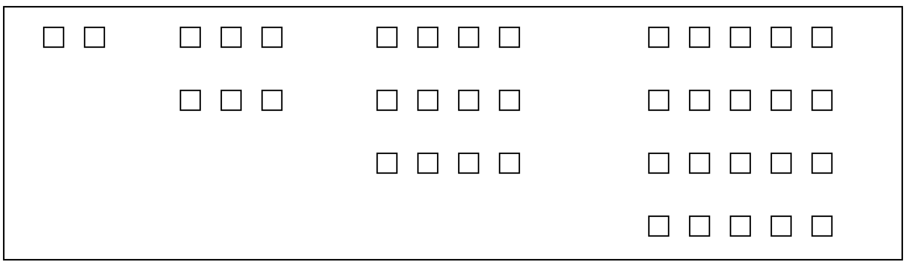
The next pattern is more complex. The pattern illustrated includes two sequences.

- The horizontal shapes increase from three shapes, by one shape from frame to frame, representing the sequence 3; 4; 5, …
- The vertical shapes increase from one shape, by two shapes from frame to frame, representing the sequence 1; 3; 5, …
- So the full pattern grows as a sum of the two: (1 + 3), (3 + 4), (5 + 5), …
- The next picture in the pattern will have 7 dots going down and 6 dots going across, and can be written numerically as (7 + 6).

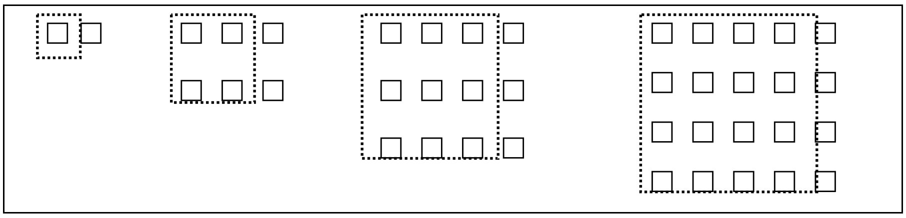


Patterns can be identified (and hence extended) in different ways. This illustrates how different people may see the same pattern in different ways. You need to listen carefully to your learners' explanations to assess whether or not they are valid. They might use different reasoning to you, but still be reasoning correctly. Look at the example of the pattern below. It can be used to illustrate two different relationships.

One explanation could be that from frame to frame we add one row and one column to the display.



Another explanation could be that in each frame we make a bigger square and add a column. The squares have been enclosed in dotted lines to illustrate this explanation.



We can also express this geometric pattern numerically. It is a good idea to use a table to write up the pattern. This helps the observer to identify the pattern, as it presents the information neatly and accessibly.

The frame number and the number of blocks in the frame are tabulated in order to calculate the number of blocks in successive frames. If a general formula can be found, then any term in the sequence can be found, using the general formula.

Let's say we want to find out the number of blocks in the 8th display of this pattern.

Frame number:	1	2	3	4	……	8
Number of blocks:	2	6	12	20	……	?

+ 4 **+ 6** **+ 8**

Using the table we can look at the number of blocks in each successive display and identify the pattern in which they are growing. Here we see that the displays are growing by adding successive even numbers, starting with 4.
We can use this to write out the numeric pattern.

Frame 1: 2

Frame 2: 2 + 4 = 6

Frame 3: 6 + 6 = 12

Frame 4: 12 + 8 = 20

Frame 5: 20 + 10 = 30

Frame 6: 30 + 12 = 42

Frame 7: 42 + 14 = 56

Frame 8: 56 + 16 = 72

So the 8th frame will have 72 blocks in the display.

Learners up to grade 7 could use the method above to determine the numbers of blocks in the display. Learners in Grade 8 or 9 might be ready to use the second explanation of the visual pattern to generate a simple rule for the successive terms in this pattern. The rule can easily be generalized, and expressed algebraically. This algebraic rule can then be used to find the number of blocks in any display.

Frame number	Number of blocks	Pattern
1	2	$(1 \times 1) + 1 = 1 + 1 = 2$
2	6	$(2 \times 2) + 2 = 4 + 2 = 6$
3	12	$(3 \times 3) + 3 = 9 + 3 = 12$
4	20	$(4 \times 4) + 4 = 16 + 4 = 20$
5	30	$(5 \times 5) + 5 = 25 + 5 = 30$
6	42	$(6 \times 6) + 6 = 36 + 6 = 42$
7	56	$(7 \times 7) + 7 = 49 + 7 = 56$
8	72	$(8 \times 8) + 8 = 64 + 8 = 72$
General (nth term)	B	$(n \times n) + n = B$

In conclusion to this brief introduction to exploring pattern in mathematics, try out the following activities which are taken from the RUMEP lecture notes on Patterns and Functions. First see if you can do them, and then see how your learners manage them. They may surprise you!

The solutions to these activities are not given in this guide. You may need to do extra research and discuss these activities with colleagues to think more deeply about how to do these activities and how they would benefit your learners. Remember that such activity on your behalf will enrich your teaching of mathematics and would be very worthwhile.

The selection and design of activities is also not discussed here, since this theory is discussed in unit 4 of this guide. Here we simply provide you with some activities to further your own understanding of mathematical pattern work. As a teacher, you should think not only about the solutions to the given problems, but also about the wording and presentation of the problems. In unit 6 of this guide we discuss the complex issue of teaching mathematics to classes with diverse learner groups. As you work though the activities which follow, you could also start to think about which of them would be accessible to all learners and which would not, and why this is the case. You might even begin to think about how to adapt the activities to suit the needs of all learners, though further guidance on how to do this is given in unit 6.

Activity 18

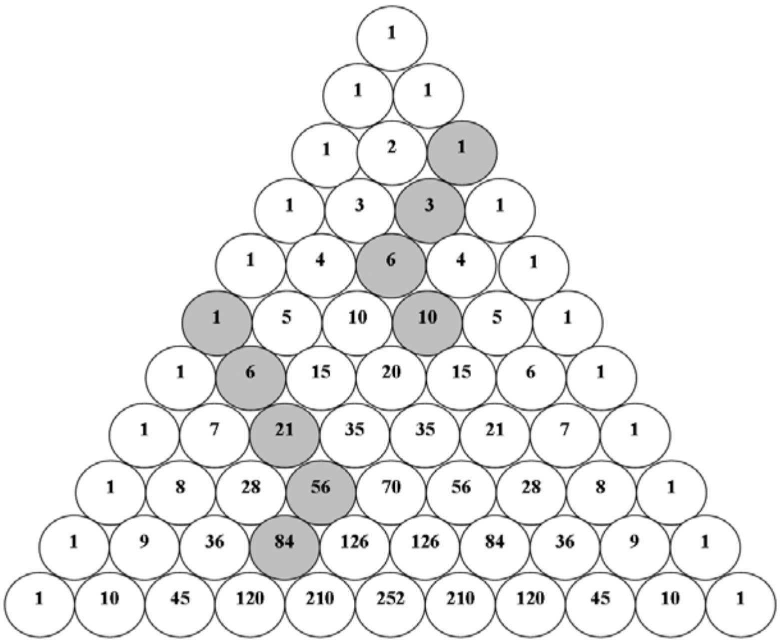
Patterns

1 Pascal's Triangle and the Leg-Foot Pattern

Pascal's Triangle is a fascinating display of numbers, in which many patterns are embedded. In Western writings the Pascal Triangle was named after Blaise Pascal, who was a famous French mathematician and philosopher. Chinese mathematicians knew about Pascal's Triangle long before Pascal was born, so it is also called a Chinese Triangle. It was documented in Chinese writings 300 years before Pascal was born.

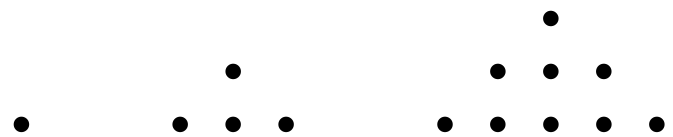
In the triangle below, some "leg-foot "patterns have been shaded. Can you shade more of the patterns that make Leg-foot in this Pascal's Triangle? First look at the two examples that have been done for you. Then shade some more leg-foot patterns in the triangle.

- Now look at the numbers in the patterns that you have shaded.
- Can you see a relationship between the numbers in the shaded in the leg and the foot of the leg-foot patterns that you have shaded? Describe this relationship in words.
- Write a numeric rule for the relationship you have identified.



2 Study the following patterns and then extend them by drawing in the next two stages.

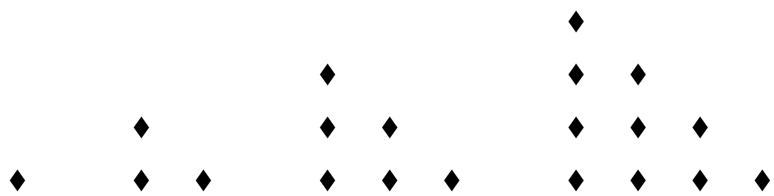
1 2 3



1 2 3

- Draw a table to display the information above:
- What type of numbers are these?
- Use the following to show different representations of square numbers
  - i Square grid
  - ii Isometric dotty paper
  - iii Square dotty paper

3 The following pattern will help Sipho to calculate the number of blocks he will need to build the stairs of his house.

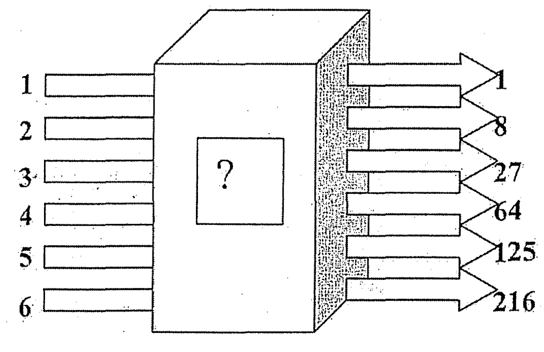


- Help him find the number of blocks he will use for 15 steps.
- Extend this pattern by drawing the next 2 stages of these steps.
- Enter your data in the following table:

Step	1	2	3	4	5	8	15	20	100
Number of blocks	1	3	6						

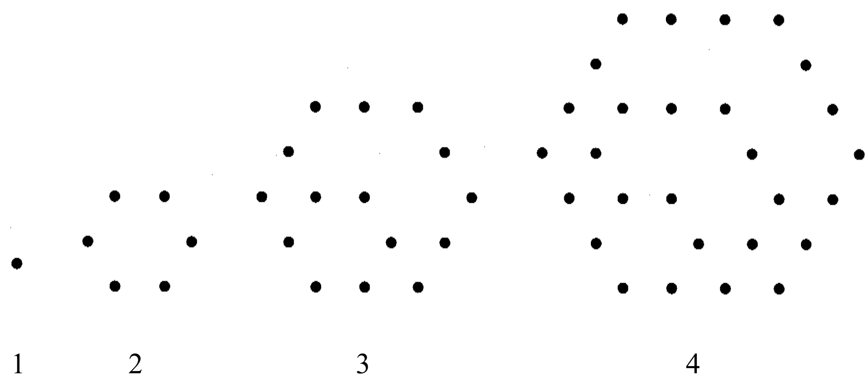
- Find and explain a rule that generates the above pattern.
- What type of numbers are these?
- Use the following to show different representations of triangular numbers:
  - i Square grid
  - ii Isometric dotty paper
  - iii Square dotty paper

4 The following function machine creates a number pattern.



- Investigate the rule that it uses. Write up your findings.
- What type of numbers are these?

5 Study the following number pattern and then complete the table that follows:



Stage	1	2	3	4	5	8	15	20	100
Number of dots	1	6	15	28					

- Investigate a general rule that generates the above pattern.
- What type of numbers are these?

On the following page is a blank sheet of grid paper for you to use if you would like to.

Unit summary

This study unit is designed to change the way in which learners perceive their role in the mathematics classroom – from being **passive** recipients of mathematics, involving facts, skills and knowledge to becoming **active participants** in ‘doing’ mathematics and doing the spadework for the creation of mathematical problems in the future. Accomplishing such a task has called for the development of stimulating activities and explanations in mathematics that would involve **all** learners, provide opportunities for more mathematical communication in the classroom, and link creative thinking with mathematical content.

Through the practice of explanation, investigation and ‘doing’ mathematics in general, learners will begin to experience the full impact of the process of solving problems and thereafter, generating problems. If we are truly committed to the notion that mathematics is for everyone, then we must begin to look for alternative methods to the stereotypical **traditional approach** for facilitating learning in the classroom for **all** learners. Perhaps the creative aspect of ‘doing’ mathematics might be the key that will open doors to mathematical learning for previously uninterested learners. Simultaneously, active participation in problem-solving and problem investigation might serve to cultivate the talents of learners who maintain an interest in **‘doing’** mathematics.

Self assessment

Assessment

Tick the boxes to assess whether you have achieved the outcomes for this unit. If you cannot tick the boxes, you should go back and work through the relevant part in the unit again.

I am able to:

#	Checklist	☑
1	Critically discuss the thinking that the traditional approach to teaching mathematics rewards the learning of rules, but offers little opportunity to "do" mathematics.	❑
2	Explain the term 'mathematics as a science of pattern and order'.	❑
3	Evaluate a collection of action verbs that is used to reflect the kind of activities engaged by the learners when doing mathematics.	❑
4	Construct a list of features of a classroom environment considered as important for learners engaged in doing mathematics.	❑
5	Think about appropriate and interesting activities to help learners explore the process of problem solving through number patterns and logical reasoning.	❑

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