Chapter 1

Units, Physical Quantities, and Vectors

PowerPoint® Lectures for
University Physics, Thirteenth Edition
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Lectures by Wayne Anderson

Goals for Chapter 1

- To learn three fundamental quantities of physics and the units to measure them
- To understand vectors and scalars and how to add vectors graphically
- To determine vector components and how to use them in calculations
- To understand unit vectors and how to use them with components to describe vectors
- To learn two ways of multiplying vectors

The nature of physics

- Physics is an *experimental* science in which physicists seek patterns that relate the phenomena of nature.
- The patterns are called *physical theories*.
- A very well established or widely used theory is called a *physical law* or *principle*.

Physics

Divided into five major areas

- Classical Mechanics
- Relativity
- Thermodynamics
- Electromagnetism
- Optics
- Quantum Mechanics

Standards and units

- Length, time, and mass are three *fundamental* quantities of physics.
- The *International System* (SI for *Système International*) is the most widely used system of units.
- In SI units, length is measured in *meters*, time in *seconds*, and mass in *kilograms*.

Fundamental Quantities and Their Units

Quantity	SI Unit
Length	meter
Mass	kilogram
Time	second
Temperature	Kelvin
Electric Current	Ampere
Luminous Intensity	Candela
Amount of Substance	mole

Unit prefixes

- Table 1.1 shows some larger and smaller units for the fundamental quantities.

Table 1.1 Some Units of Length, Mass, and Time

Length	Mass	Time
1 nanometer = 1 nm = 10^{-9} m (a few times the size of the largest atom)	1 microgram = 1 μg = 10^{-6} g = 10^{-9} kg (mass of a very small dust particle)	1 nanosecond = 1 ns = 10^{-9} s (time for light to travel 0.3 m)
1 micrometer = 1 μm = 10^{-6} m (size of some bacteria and living cells)	1 milligram = 1 mg = 10^{-3} g = 10^{-6} kg (mass of a grain of salt)	1 microsecond = 1 μs = 10^{-6} s (time for space station to move 8 mm)
1 millimeter = 1 mm = 10^{-3} m (diameter of the point of a ballpoint pen)	1 gram = 1 g = 10^{-3} kg (mass of a paper clip)	1 millisecond = 1 ms = 10^{-3} s (time for sound to travel 0.35 m)
1 centimeter = 1 cm = 10^{-2} m (diameter of your little finger)		
1 kilometer = 1 km = 10^{3} m (a 10-minute walk)		

Prefixes, cont.

TABLE 1.4

Prefixes for Powers of Ten

Power	Prefix	Abbreviation	Power	Prefix	Abbreviation
10^{-24}	yocto	y	10^{3}	kilo	k
10^{-21}	zepto	z	10^{6}	mega	M
10^{-18}	atto	a	10^{9}	giga	G
10^{-15}	femto	f	10^{12}	tera	T
10^{-12}	pico	p	10^{15}	peta	P
10^{-9}	nano	n	10^{18}	exa	E
10^{-6}	micro	μ	10^{21}	zetta	Z
10^{-3}	milli	m	10^{24}	yotta	Y
10^{-2}	centi	c			
10^{-1}	deci	d			

Unit consistency and conversions

- An equation must be *dimensionally consistent*. Terms to be added or equated must *always* have the same units. (Be sure you're adding "apples to apples.")
- Always carry units through calculations.
- Convert to standard units as necessary.

Conversion – Ex.1

- Express 200 cubic feet in cubic meters:

1ft = 0.305 m

$$200\ ft^3 = 200 \times \frac{(0.305\ m)^3}{(1 ft)^3} = 5.67\ m^3$$

- Express 75 mi/h in m/s

1 mi = 1609 m   1h = 3600 s

$$75.0 \frac{mi}{h}\ x\ \frac{1609\ m}{1\ mi} \times \frac{1\ h}{3600\ s} = 33.5\ m/s$$

Uncertainty and significant figures—Figure 1.7

- The uncertainty of a measured quantity is indicated by its number of *significant figures*.
- For multiplication and division, the answer can have no more significant figures than the *smallest* number of significant figures in the factors.
- For addition and subtraction, the number of significant figures is determined by the term having the fewest digits to the right of the decimal point.
- As this train mishap illustrates, even a small percent error can have spectacular results!

Estimates and orders of magnitude

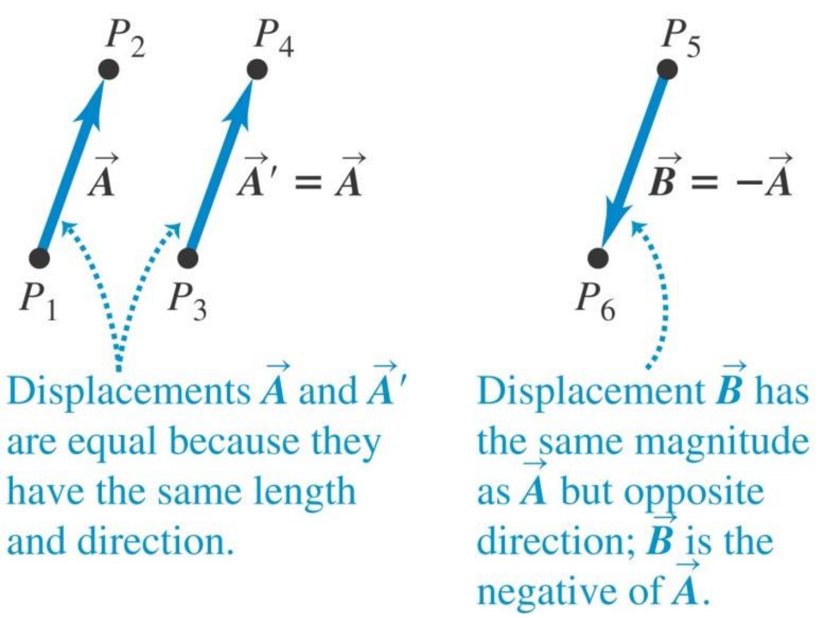
- An *order-of-magnitude estimate* of a quantity gives a rough idea of its magnitude.

Vectors and scalars

- A *scalar quantity* can be described by a *single number*.
- A *vector quantity* has both a *magnitude* and a *direction* in space.
- In this book, a vector quantity is represented in boldface italic type with an arrow over it: $\vec{\boldsymbol{A}}$.
- The magnitude of $\vec{\boldsymbol{A}}$ is written as A or $|\vec{\boldsymbol{A}}|$.

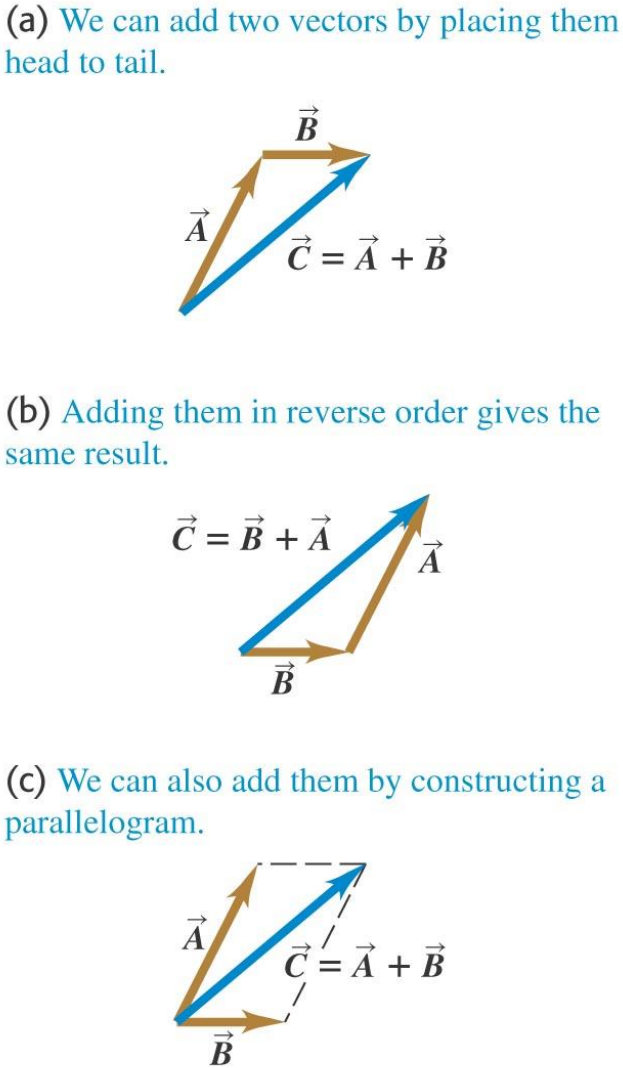
Drawing vectors—Figure 1.10

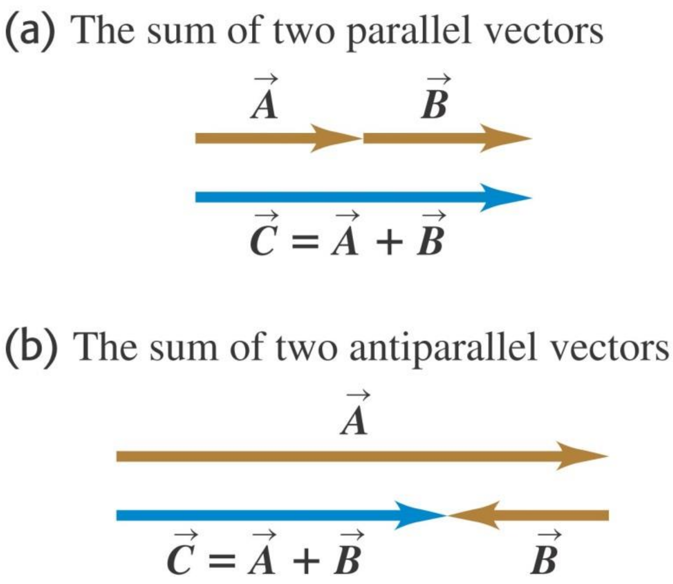
- Draw a vector as a line with an arrowhead at its tip.
- The *length* of the line shows the vector's *magnitude*.
- The *direction* of the line shows the vector's *direction*.



Adding two vectors graphically

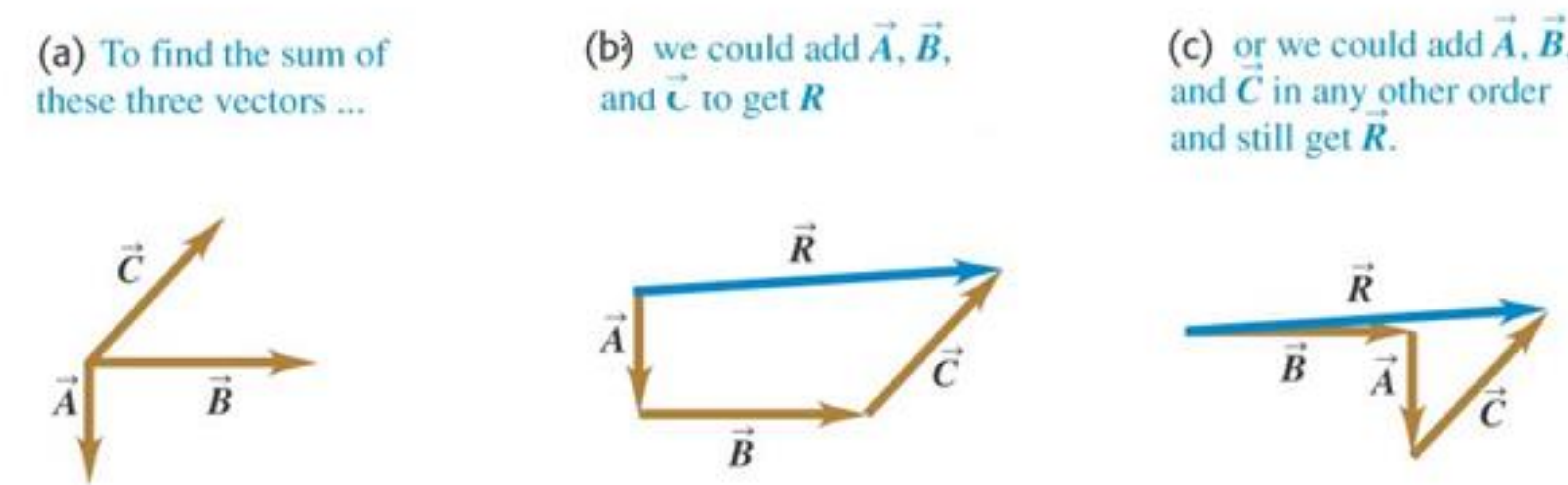
- Two vectors may be added graphically using either the *parallelogram* method or the *head-to-tail* method.





Adding more than two vectors graphically—

- To add several vectors, use the head-to-tail method.
- The vectors can be added in any order.



Negative of a Vector

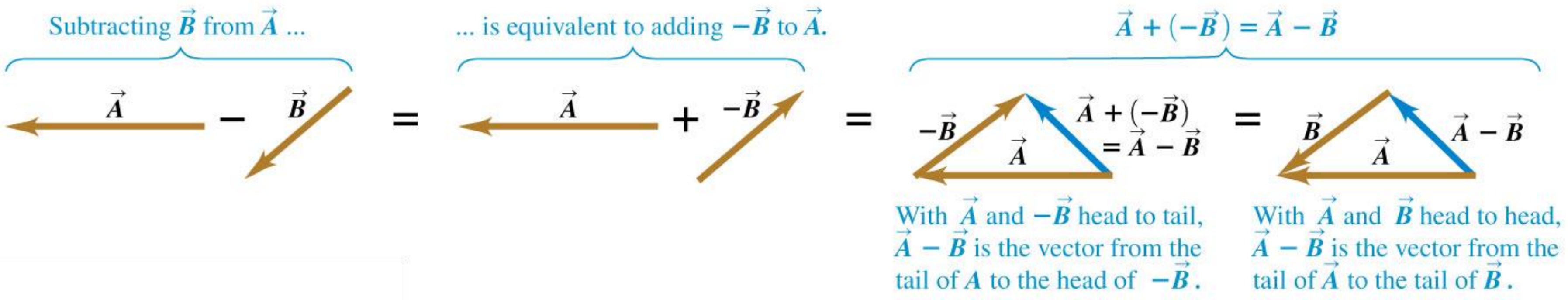
The negative of a vector is defined as the vector that, when added to the original vector, gives a resultant of zero

- Represented as $-\vec{\mathbf{A}}$
- $\vec{\mathbf{A}} + \left(-\vec{\mathbf{A}}\right) = 0$

The negative of the vector will have the same magnitude, but point in the opposite direction

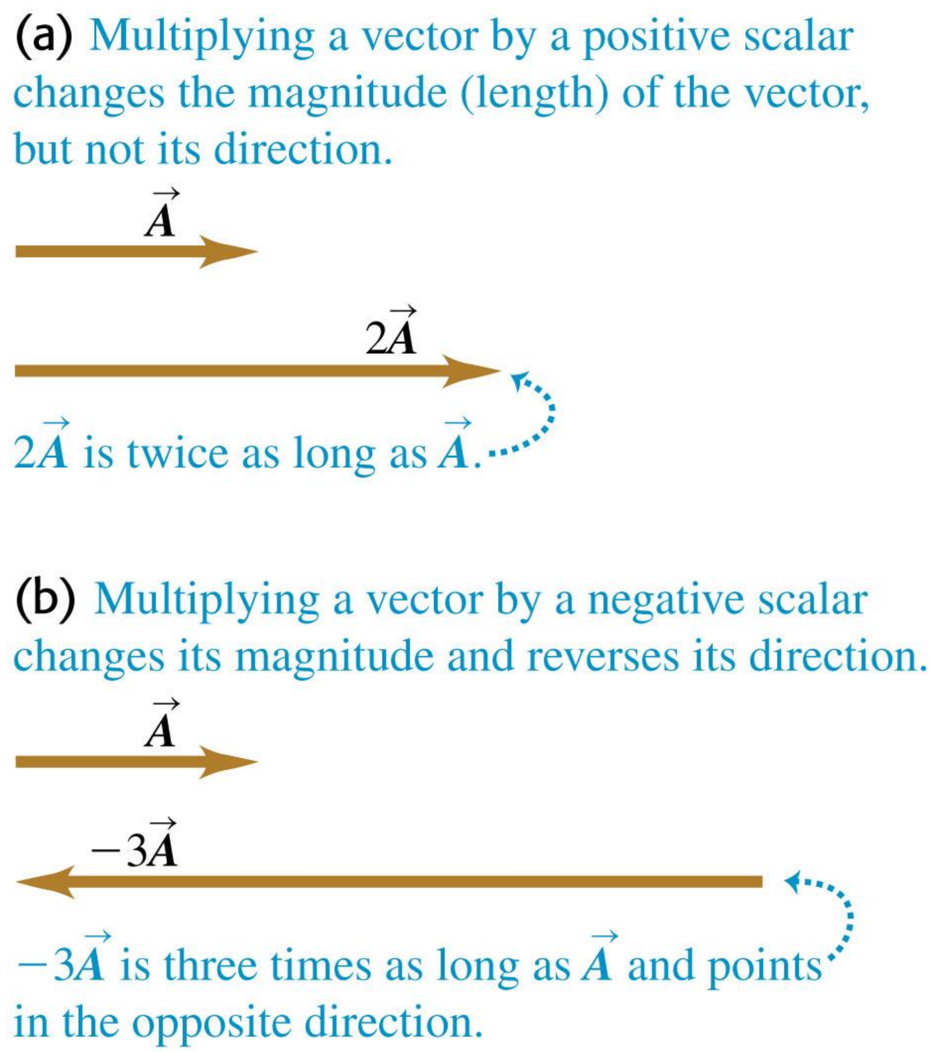
Subtracting vectors

- Figure shows how to subtract vectors.



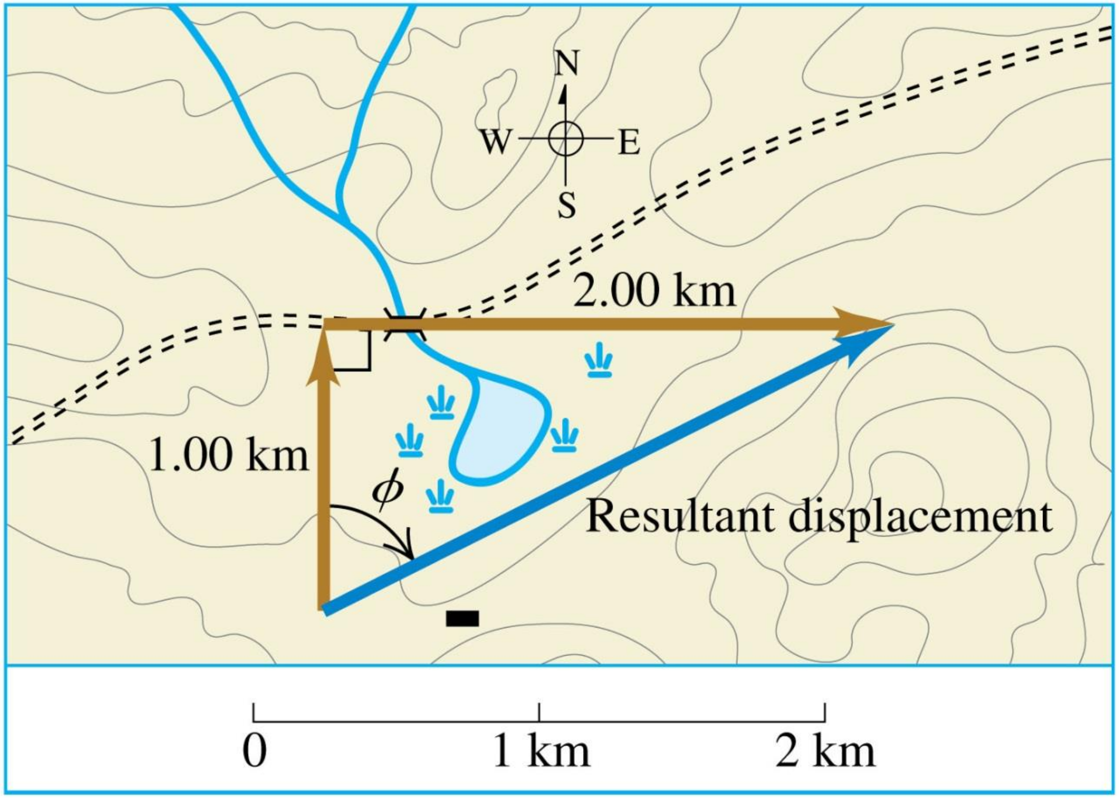
Multiplying a vector by a scalar

- If c is a scalar, the product $c\vec{\boldsymbol{A}}$ has magnitude $|c|A$.
- Multiplication of a vector by a positive scalar and a negative scalar.



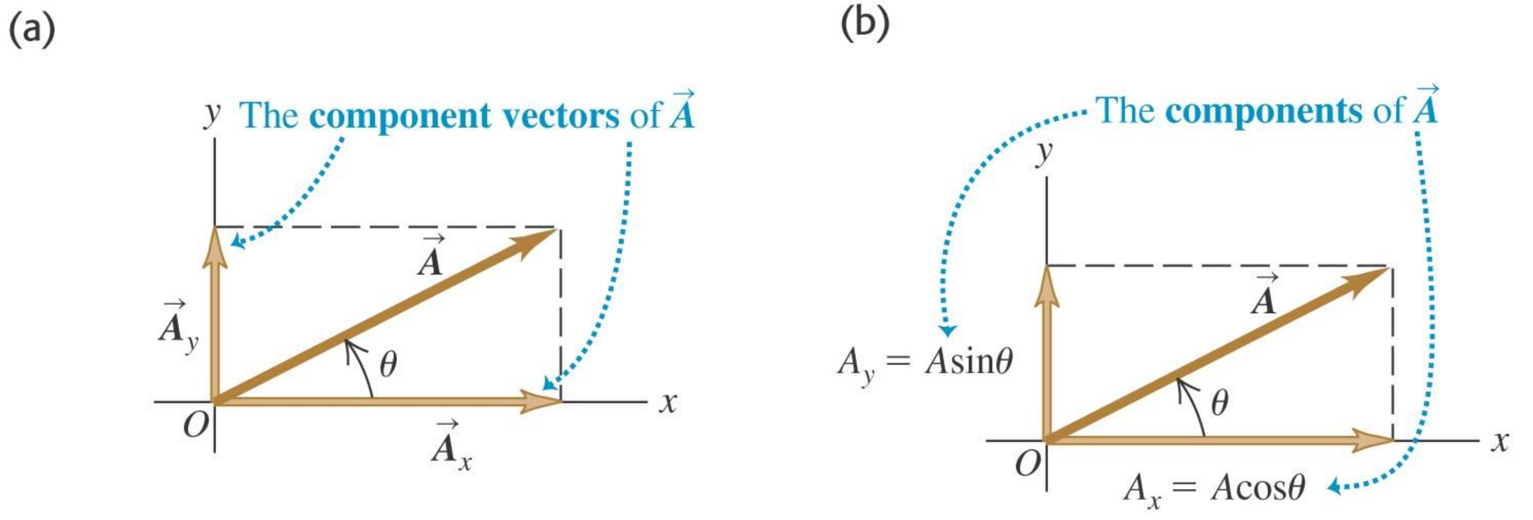
Addition of two vectors at right angles

- First add the vectors graphically.
- Then use trigonometry to find the magnitude and direction of the sum: $R = \sqrt{1\ km^2 + 2\ km^2} = 2.2$ km



Components of a vector—

- Adding vectors graphically provides limited accuracy. Vector components provide a general method for adding vectors.
- Any vector can be represented by an *x*-component A_x and a *y*-component A_y.
- Use trigonometry to find the components of a vector: $A_x = A\cos\theta$ and $A_y = A\sin\theta$, where θ is measured from the +*x*-axis toward the +*y*-axis.



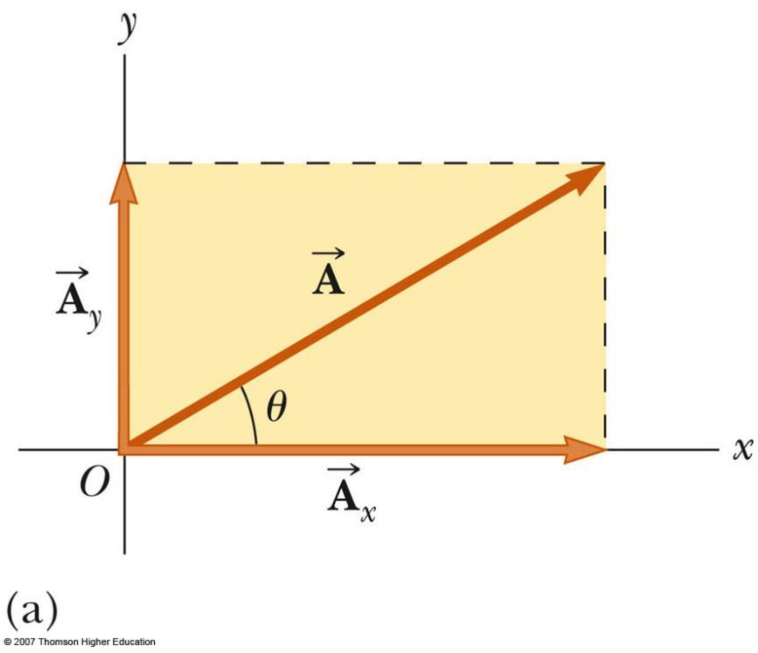
Components of a Vector

The x-component of a vector is the projection along the x-axis

$$A_x = A\cos\theta$$

The y-component of a vector is the projection along the y-axis

$$A_y = A\sin\theta$$



This assumes the angle θ is measured with respect to the positive direction of x-axis

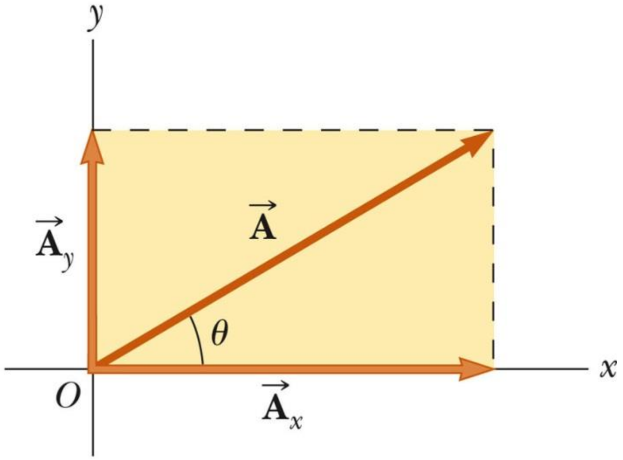
- If not, do not use these equations, use the sides of the triangle directly

Components of a Vector, 4

The components are the legs of the right triangle whose hypotenuse is the length of *A*

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$

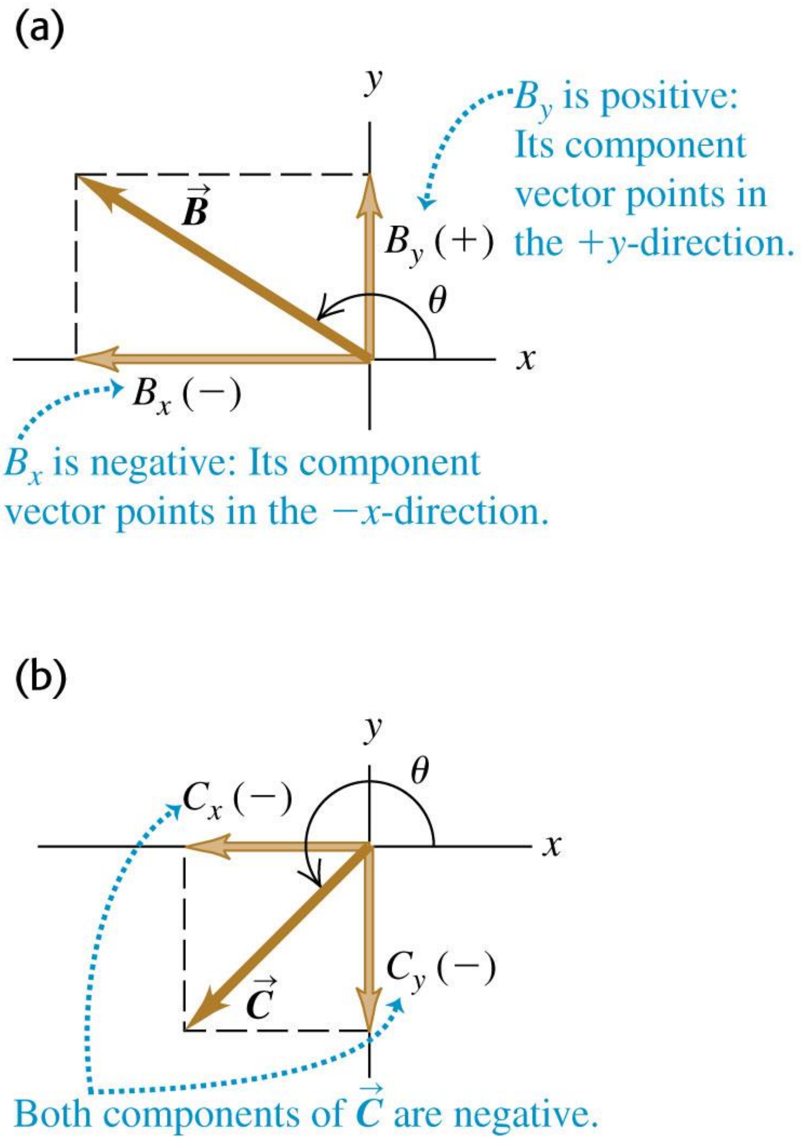
- May still have to find θ with respect to the positive x-axis



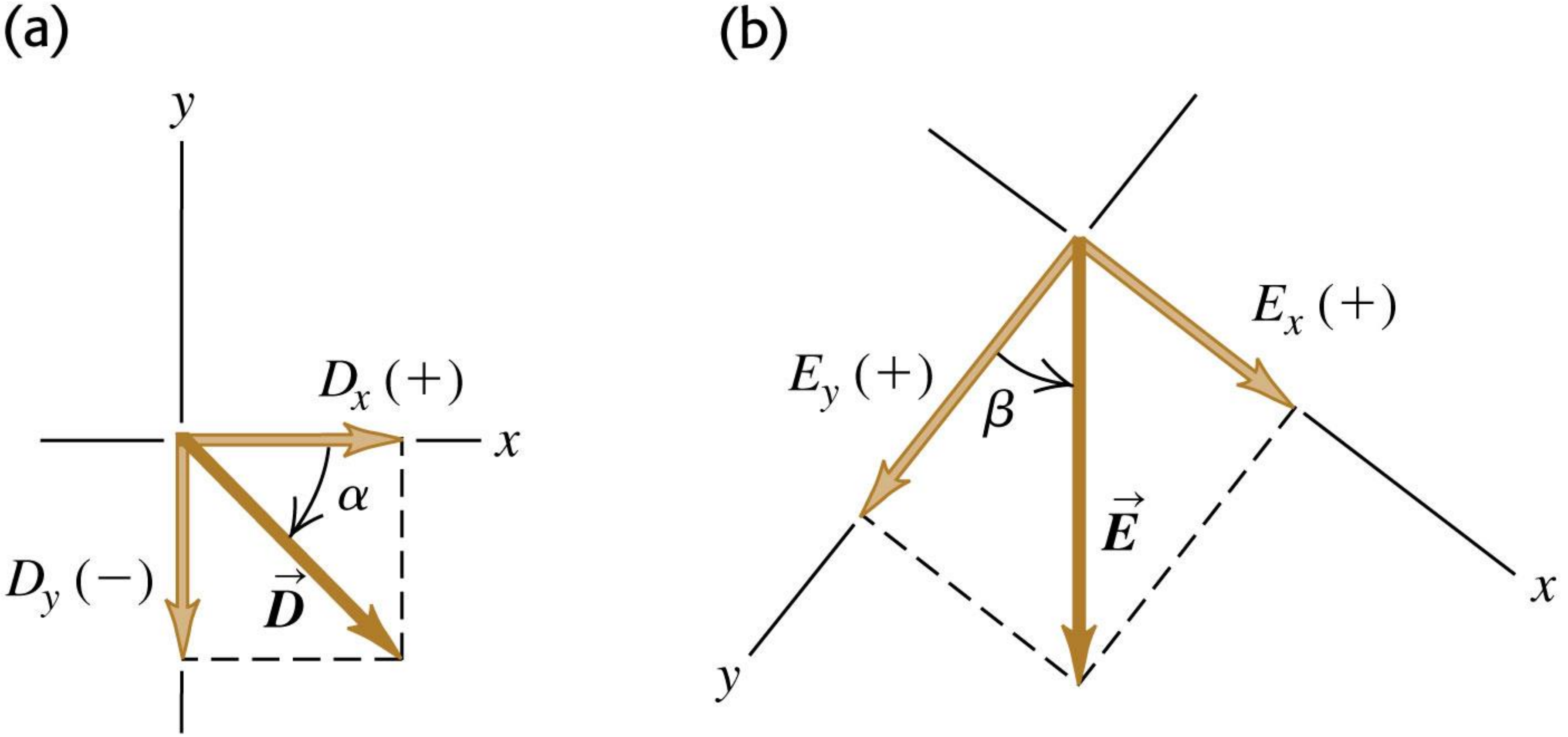
(a)

Positive and negative components—Figure 1.18

- The components of a vector can be positive or negative numbers, as shown in the figure.



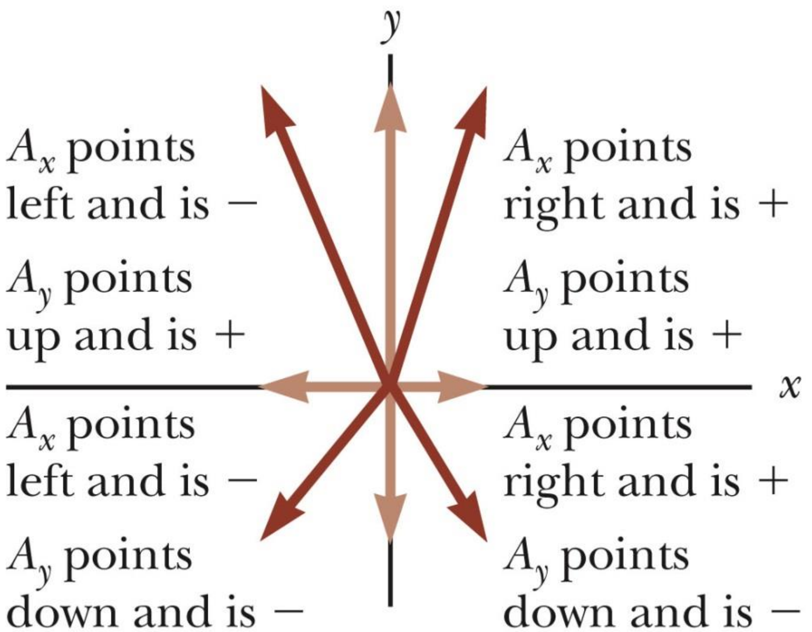
Finding components—Figure 1.19

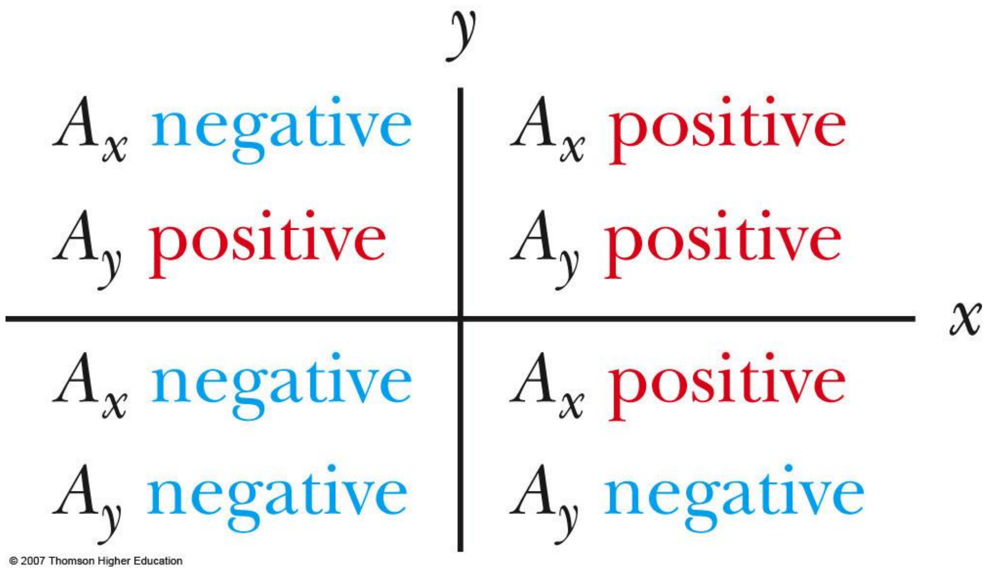


Components of a Vector, final

The components can be positive or negative

The signs of the components will depend on the angle





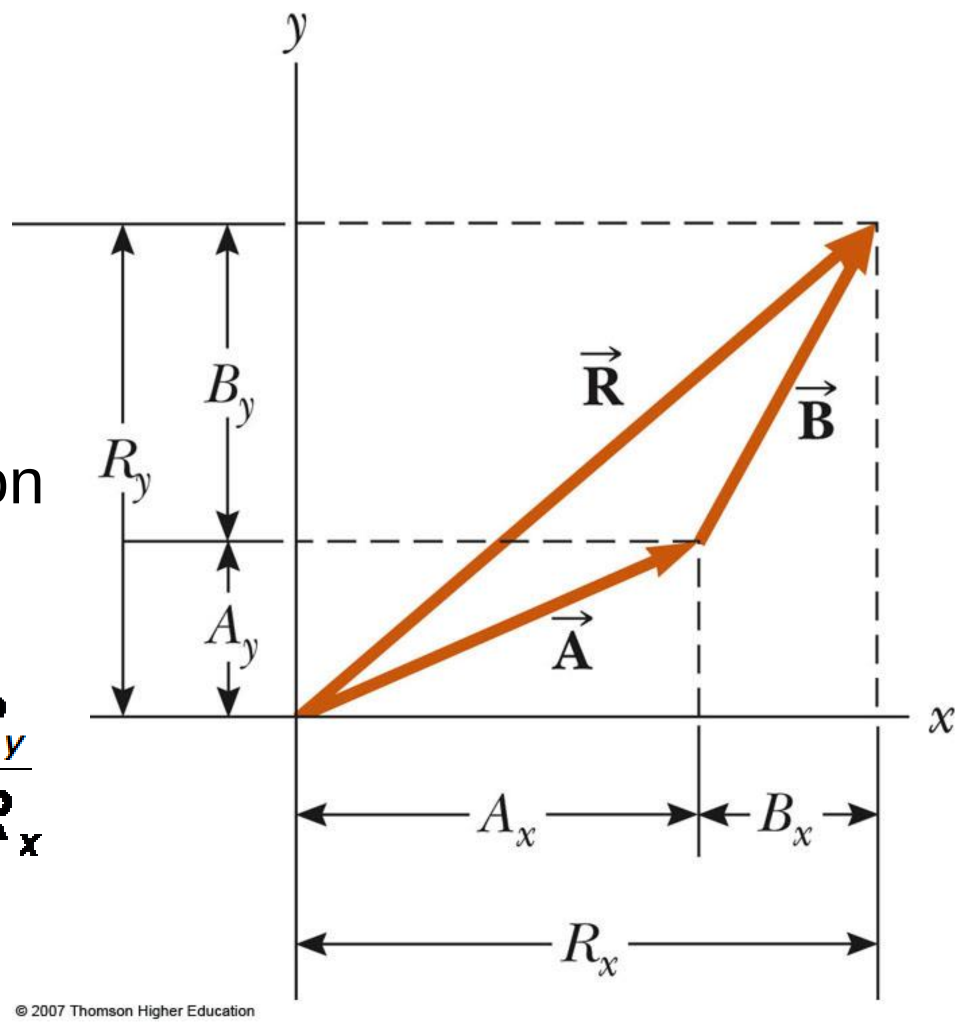
Adding Two Vectors Using Their Components

$$R_x = A_x + B_x$$

$$R_y = A_y + B_y$$

The magnitude and direction of resultant vectors are:

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

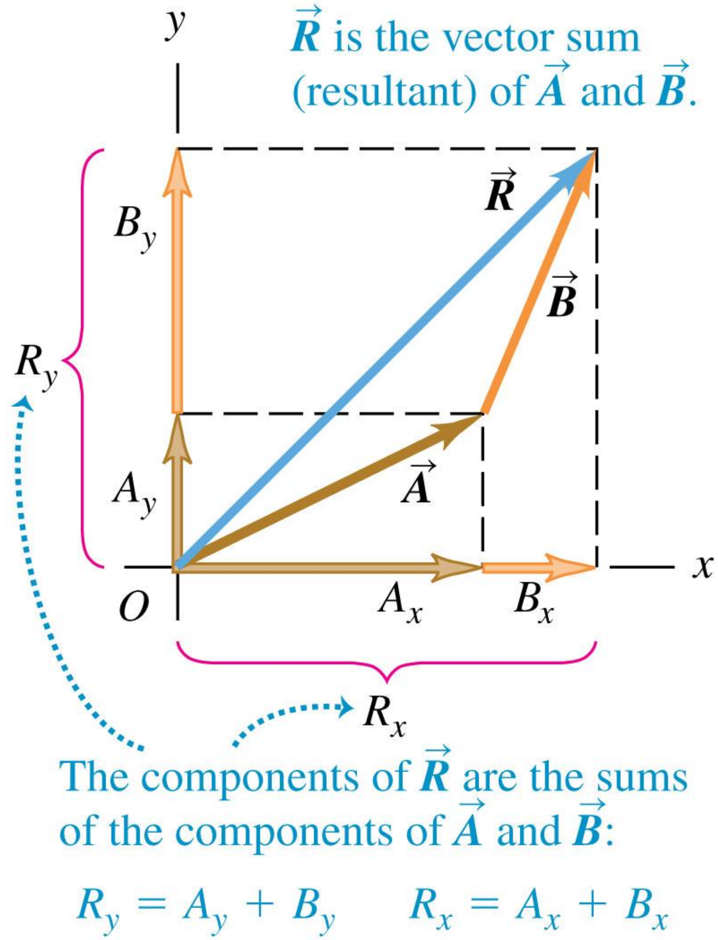


Adding vectors using their components

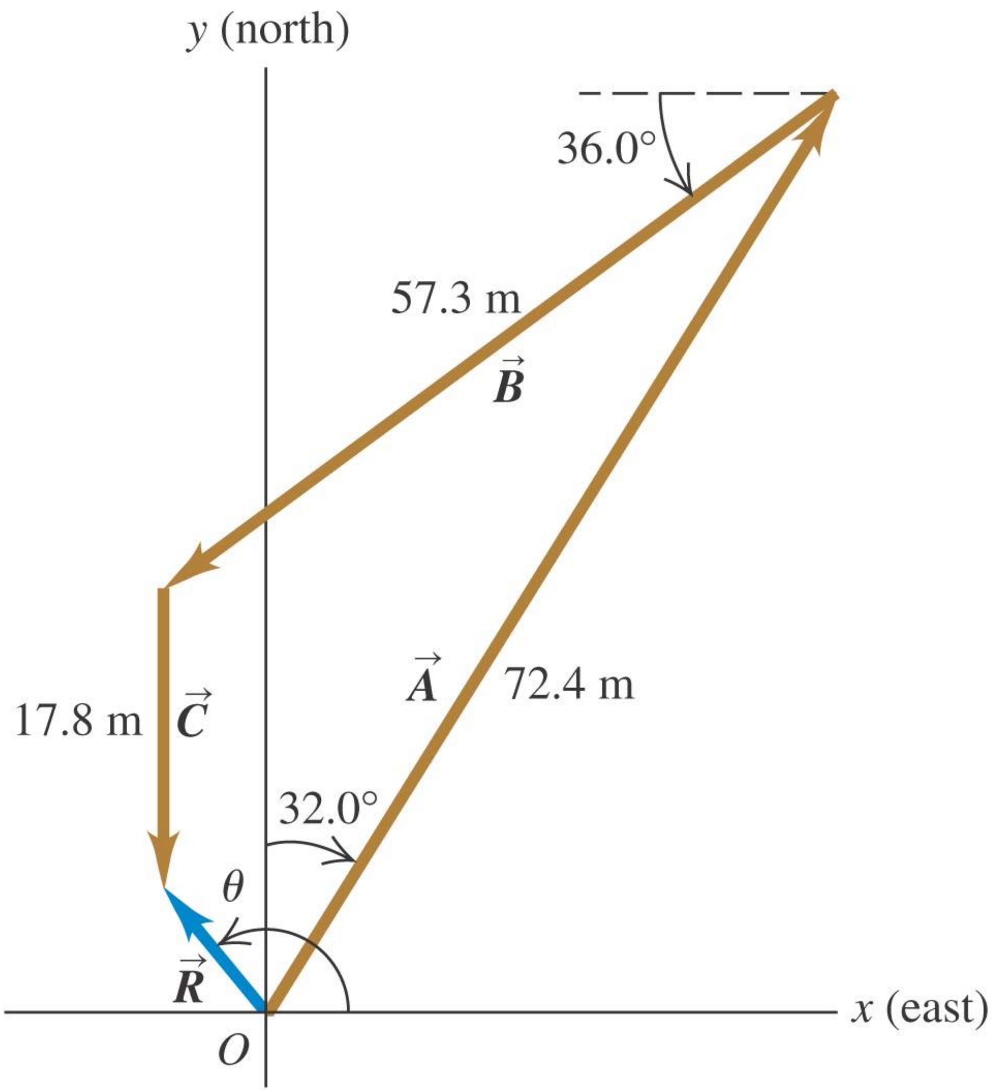
- For more than two vectors we can use the components of a set of vectors to find the components of their sum:

$$R_x = A_x + B_x + C_x + \cdots, \quad R_y = A_y + B_y + C_y + \cdots$$

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$



Adding vectors using their components—Ex. 2



Example 2

$A_x = 72.4\cos 58^0 = 38.37$

$A_y = 72.4\sin 58^0 = 61.40$

$B_x = 57.3\cos 216^0 = -46.36$

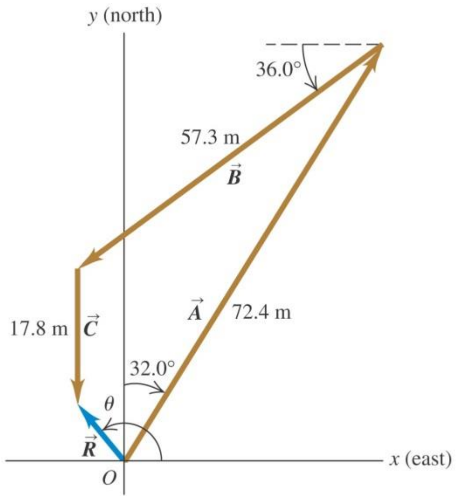
$B_y = 57.3\sin 216^0 = -33.68$

$C_x = 17.8\cos 270^0 = 0$

$A_x = 17.8\sin 270^0 = -17.8$

$R_x = 38.37 - 46.36 + 0 = -8.00$ second quadrant

$R_y = 61.4 - 33.68 - 17.8 = 9.92$

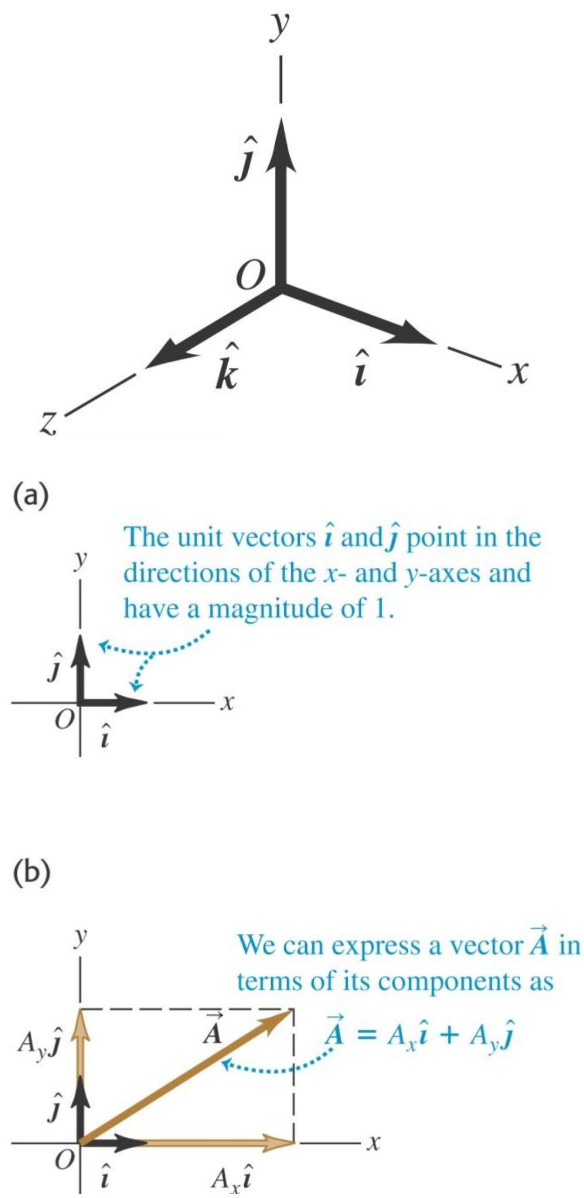


$$R = \sqrt{(-8)^2 + 9.92^2} = 12.7$$

$$\theta = 180^0 - \tan^{-1}\frac{9.92}{8.00} = 129^0$$

Unit vectors

- A *unit vector* has a magnitude of 1 with no units.
- The unit vector $\hat{\boldsymbol{\imath}}$ points in the +*x*-direction, $\hat{\boldsymbol{\jmath}}$ points in the +*y*-direction, and $\hat{\boldsymbol{k}}$ points in the +*z*-direction.
- Any vector can be expressed in terms of its components as $\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{\imath}} + A_y\,\hat{\boldsymbol{\jmath}} + A_z\,\hat{\boldsymbol{k}}$



Adding vectors using unit-vector notation

In three dimensions if $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$

then

$$\vec{\mathbf{R}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$

$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}} + \left(A_z + B_z\right)\hat{\mathbf{k}}$$

$$\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}} + R_z\hat{\mathbf{k}}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \qquad \theta = \cos^{-1}\frac{R_x}{R}, \mathit{etc}.$$

and so $R_x = A_x + B_x$, $R_y = A_y + B_y$, and $R_z = A_z + B_z$

Unit vector notation , adding vectors

In two dimensions, if $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$

then

$$\vec{\mathbf{R}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}} \qquad \vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$$

and so $R_x = A_x + B_x$, $R_y = A_y + B_y$,

The magnitude and direction are

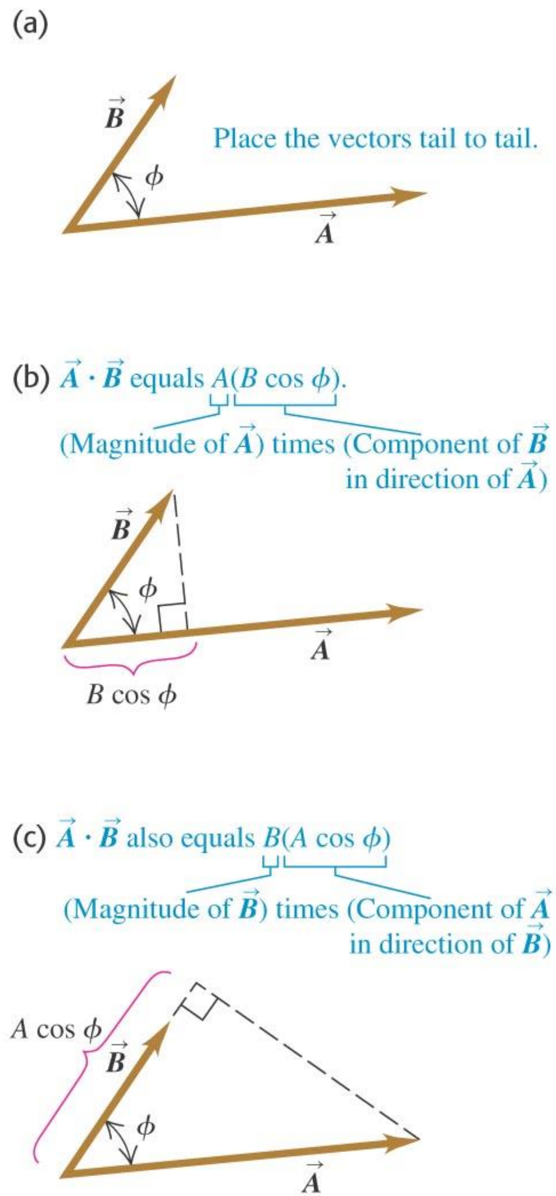
$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

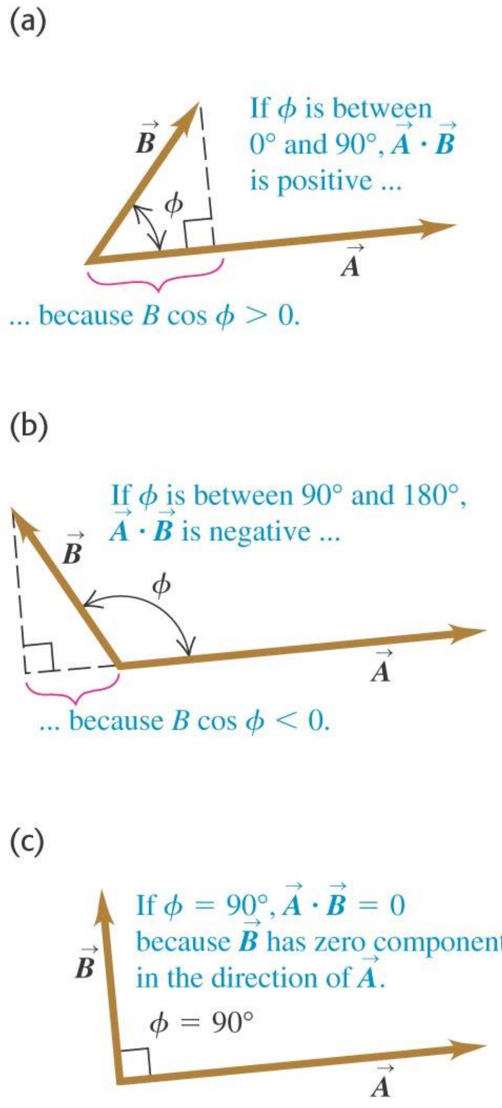
Example 3

If **A** = 24**i**-32**j** and B=24**i**+10**j**, what is the magnitude and direction of the vector **C** = **A**-**B**?

The scalar product

- The *scalar product* (also called the "dot product") of two vectors is
  $$\vec{A}\bullet\vec{B} = AB\cos\phi.$$
- Figures illustrate the scalar product.





Dot Products of Unit Vectors

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = 0$$

Using component form with vectors:

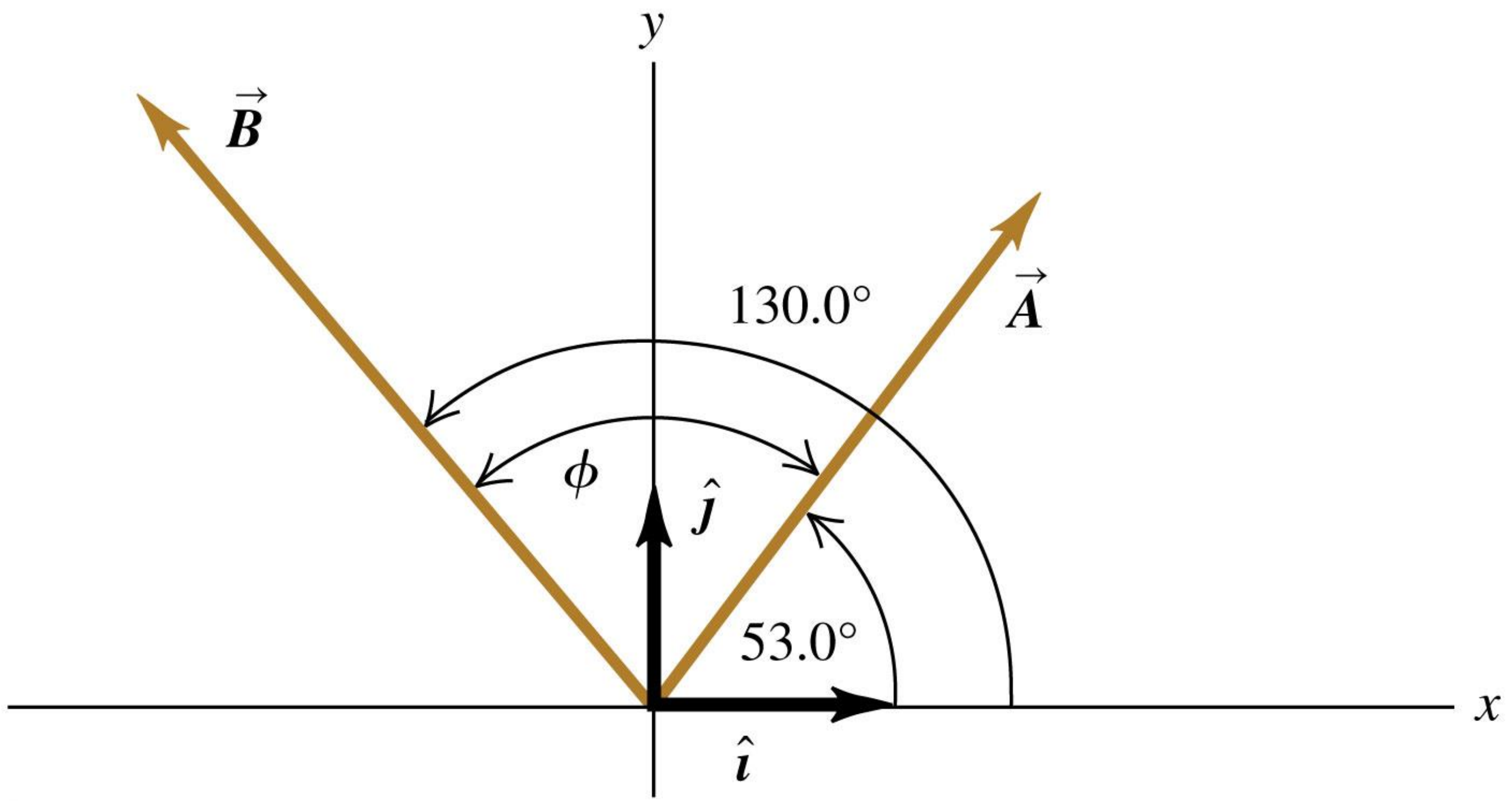
$$\vec{\mathbf{A}} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}}$$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z$$

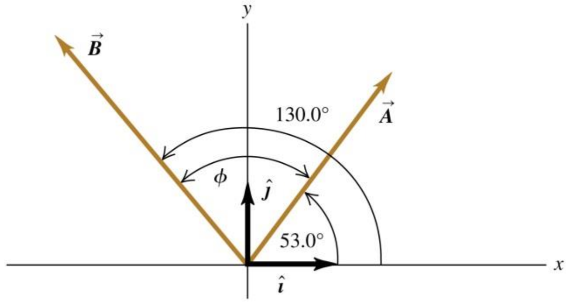
Calculating a scalar product

- $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\phi.$ $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z.$
- Find the scalar product $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$ of two vectors shown in the figure. The magnitudes of the vectors are: A = 4.00, and B = 5.00



Calculating a scalar product – Example 4

- $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\phi.$ $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z.$
- Find the scalar product $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$ of two vectors shown in the figure. The magnitudes of the vectors are: A = 4.00, and B = 5.00

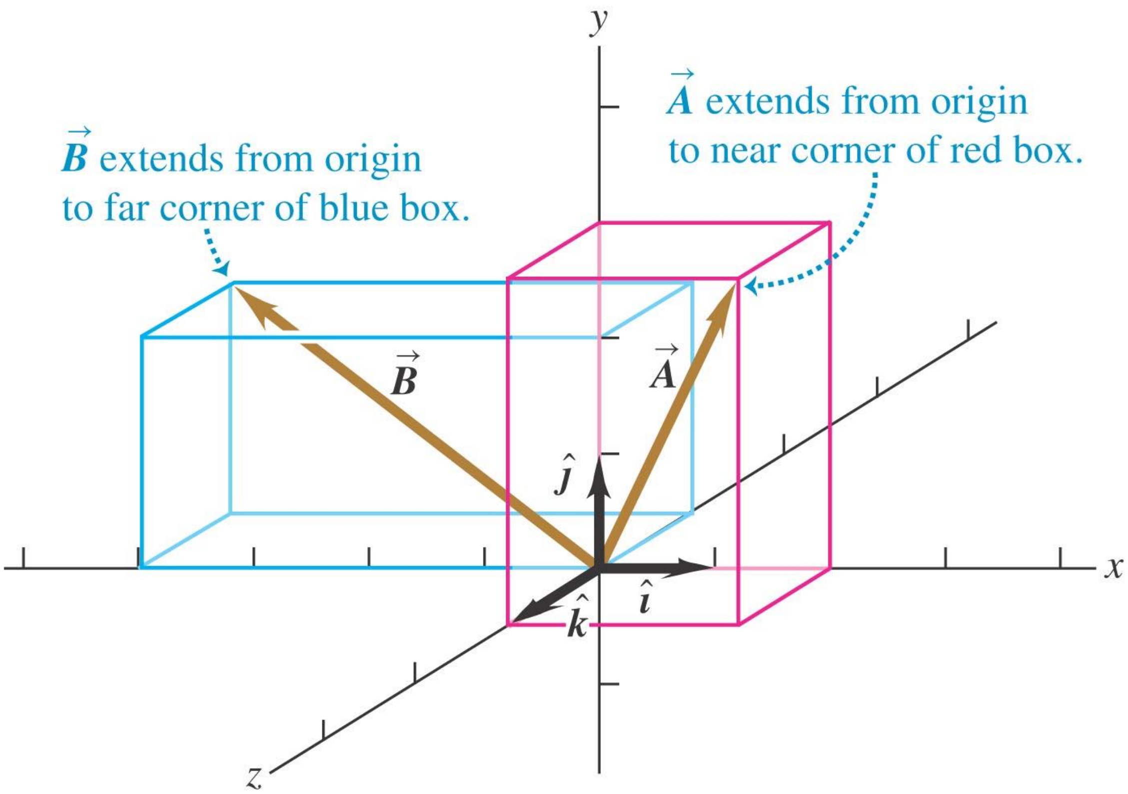


Finding an angle using the scalar product – Ex. 5

- Find the angle between the vectors.

$$\vec{A} = 2.00\hat{\imath} + 3.00\hat{\jmath} + 1.00\hat{k}$$
$$\vec{B} = -4.00\hat{\imath} + 2.00\hat{\jmath} - 1.00\hat{k}$$

- Use equation: $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\phi.$

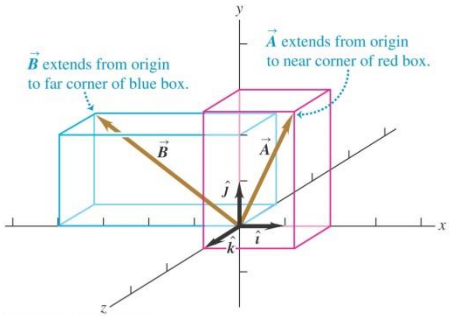


Finding an angle using the scalar product – Ex. 5

- Find the angle between the vectors.

$$\vec{A} = 2.00\hat{\imath} + 3.00\hat{\jmath} + 1.00\hat{k}$$
$$\vec{B} = -4.00\hat{\imath} + 2.00\hat{\jmath} - 1.00\hat{k}$$

- Use equation: $\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = AB\cos\phi.$



The Vector Product Defined

Given two vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$

The vector (cross) product of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is defined as a *third vector*, $\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}$

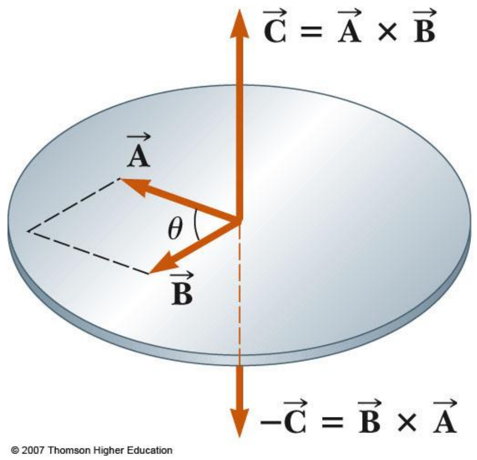
The magnitude of vector C is *AB* sin θ

- θ is the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$

More About the Vector Product

The direction of $\vec{C}$ is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$

The best way to determine this direction is to use the right-hand rule



Right-hand rule

Using Determinants

The components of cross product can be calculated as

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \hat{\mathbf{i}} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \hat{\mathbf{j}} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \hat{\mathbf{k}}$$

Expanding the determinants gives

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \left(A_y B_z - A_z B_y\right)\hat{\mathbf{i}} - \left(A_x B_z - A_z B_x\right)\hat{\mathbf{j}} + \left(A_x B_y - A_y B_x\right)\hat{\mathbf{k}}$$

If $A_z = 0$ and $B_z = 0$ then

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \left(A_x B_y - A_y B_x\right)\hat{\mathbf{k}}$$

$=$

Vector Product Example 6

Given $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}; \ \vec{\mathbf{B}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

Find $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \left(A_x B_y - A_y B_x\right)\hat{\mathbf{k}}$

Result $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) =$ [2*2-3*(-1)] $\hat{\mathbf{k}} = 7\hat{\mathbf{k}}$

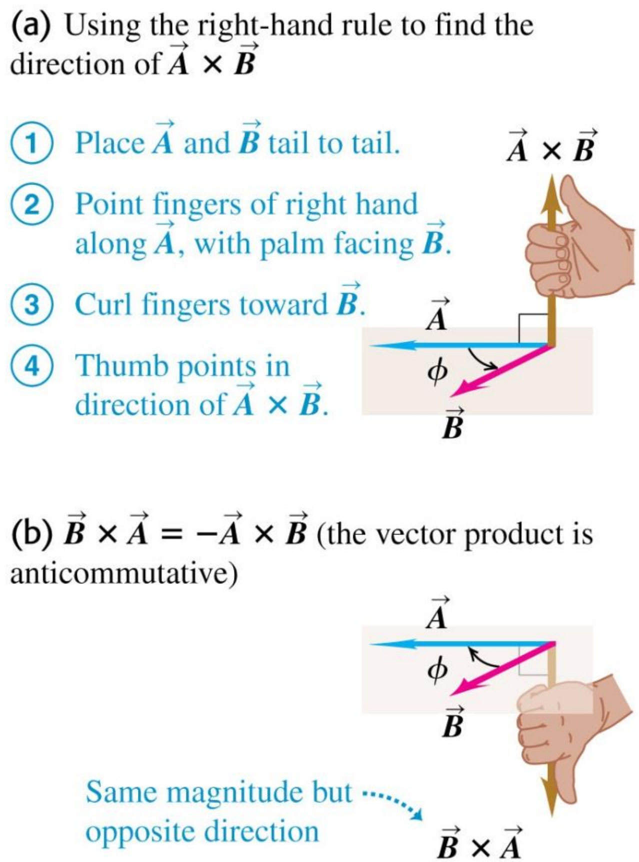
$=$

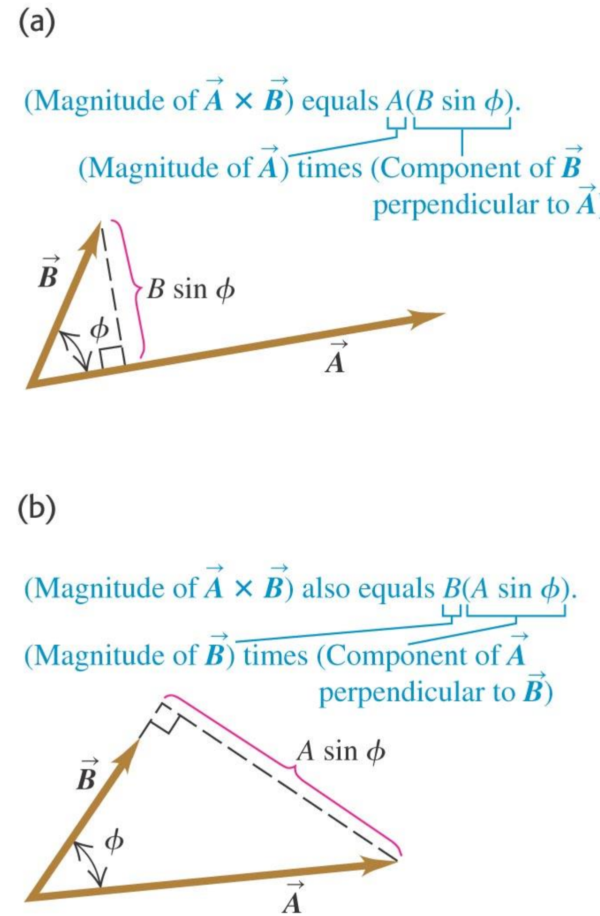
The vector product—Summary

- The vector product ("cross product") of two vectors has magnitude

$$|\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}}| = AB\sin\phi$$

and the *right-hand rule* gives its direction.





Calculating the vector product— ex. 6

- Vector $\vec{\mathbf{A}}$ has magnitude 6 units and is in the direction of the +*x* axis. Vector $\vec{\mathbf{B}}$ has magnitude 4 units and lies in the *xy* – plane making an angle of 30^0 with the x axis. Find the cross product $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$

Use $AB\sin\phi$ to find the magnitude and the right-hand rule to find the direction.