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University of Houston High School Mathematics Contest
Pre-Calculus Exam – Spring 2016

1. $\arctan(1)+\arctan(2)+\arctan(3)=$
   A) π
   B) $\frac{\pi}{2}$
   C) $\frac{\pi}{4}$
   D) $\frac{\pi}{3}$
   E) 2π
   F) none of these

2. $\log_2(\log_4(\log_8 64))=$
   A) 1
   B) -1
   C) $\frac{1}{2}$
   D) $\frac{1}{4}$
   E) $-\frac{1}{2}$
   F) none of these

3. Simplify $\cot(\arcsin(\tan(\arccos(x))))$.
   A) x
   B) $\frac{x}{\sqrt{x^2-1}}$
   C) $\frac{\sqrt{2x^2-1}}{\sqrt{x^2-1}}$
   D) $\frac{\sqrt{2x^2-1}}{\sqrt{1-x^2}}$
   E) $\frac{x}{\sqrt{1-x^2}}$
   F) none of these

4. Solve $3^{\sin(2\theta)} = 81^{\cos(\theta)}$ on the interval $-2\pi < \theta < 2\pi$.

   A) $\theta = \{0, \pi\}$

   B) $\theta = \left\{-\frac{3\pi}{2}, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$

   C) $\theta = \left\{-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$

   D) $\theta = \left\{-\frac{3\pi}{2}, -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$

   E) $\theta = \left\{-\frac{3\pi}{2}, -\frac{5\pi}{6}, -\frac{\pi}{2}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$

   F) none of these

5. List all x – intercepts for $f(x) = -3\cos\left(2x + \frac{\pi}{3}\right)$ on the interval $0 \le x < \frac{17\pi}{6}$.

   A) $\frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}$

   B) $\frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}, \frac{25\pi}{12}, \frac{31\pi}{12}$

   C) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}, \frac{13\pi}{6}$

   D) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$

   E) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\ \frac{7\pi}{3}$

   F) none of these

6. Suppose $f(x) = \log(3-2x) - \sqrt{1+x}$ and $g(x) = \frac{2x+1}{x-3}$, give the domain of $\left(\frac{f}{g}\right)(x)$.

   A) $\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{3}{2}, 3\right) \cup (3, \infty)$

   B) $\left[-1, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \frac{3}{2}\right)$

   C) $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \frac{3}{2}\right)$

   D) $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \frac{3}{2}\right) \cup \left(\frac{3}{2}, 3\right) \cup (3, \infty)$

   E) all real numbers

   F) none of these

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7. $\lim_{x\to 2^+} \frac{2-x}{|2-x|} =$

A) -1
B) 1
C) 0
D) does not exist
E) 2
F) none of these

8. Find the area of triangle XYZ if $\angle Y = 60°$, XY = 5 and YZ = 4.

A) 12
B) 10
C) $10\sqrt{3}$
D) $5\sqrt{3}$
E) 5
F) none of these

9. CDE is a triangle with $\angle C = 60^o$, $\angle D = 45^o$, and DE = 8 cm. Find DC.

A) $\frac{8\sqrt{6}}{3}$
B) $\frac{8\sqrt{2}}{3}$
C) $\frac{4\sqrt{6}}{3} + 4\sqrt{2}$
D) $\frac{8\sqrt{6}}{3} + 8\sqrt{2}$
E) $\frac{4\sqrt{6}}{3}$
F) none of these

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10. $\lim_{x\to\infty}\left(\sqrt{4x^2+3x+1}-2x\right)=$

A) $\frac{1}{4}$

B) $\frac{1}{3}$

C) $\frac{3}{4}$

D) $\frac{3}{2}$

E) 0

F) none of these

11. In which of the following situations would $f(g(x))$ always be an even function?

I. $f(x)$ and $g(x)$ are both odd

II. $f(x)$ even and $g(x)$ odd

III. $f(x)$ and $g(x)$ are both even

IV. $f(x)$ odd and $g(x)$ even

A) I only

B) II and III only

C) II, III and IV only

D) I, II, III and IV

E) I and IV only

F) none of these

12. Give the range of $g(x)=\sqrt{2-\sqrt{1-x^2}}$.

A) $\left[0,\sqrt{2}\right]$

B) $[0,1]$

C) $\left[0,\sqrt{2}\right)$

D) $\left[1,\sqrt{2}\right]$

E) $\left[1,\sqrt{2}\right)$

F) none of these

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13. Simplify $\left(\cos\left(\frac{\theta}{2}\right)+\sin\left(\frac{\theta}{2}\right)\right)^2$.

A) $1+\sin(\theta)$

B) $\cos(\theta)+\sin(\theta)$

C) $\cos(\theta)+1$

D) 1

E) $\frac{1}{2}$

F) none of these

14. The wheel on a car has a diameter of 52 cm and rotates at a rate of 600 rpm (rotations per minute). What is the approximate linear speed (in cm per second) of a point on the wheel?

A) 1633 cm/sec

B) 63 cm/sec

C) 816 cm/sec

D) 2543 cm/sec

E) 1540 cm/sec

F) none of these

15. QRST is a quadrilateral with QR = 12 cm, RS = $2\sqrt{43}$ cm, and ST = 6 cm. When diagonal $\overline{QS}$ is drawn, $\angle RQS = 60^o$ and $\angle QST = 45^o$. Find QT.

A) $2\sqrt{58-21\sqrt{2}}$

B) $14\sqrt{43}$

C) $\sqrt{232-42\sqrt{2}}$

D) $84\sqrt{3}$

E) $14\sqrt{3}+84\sqrt{2}$

F) none of these

16. One foci on the hyperbola $\left(\frac{x}{a}\right)^2-\left(\frac{y}{2}\right)^2=1$ is the point $\left(2\sqrt{3},0\right)$. Find a.

A) 12

B) 4

C) $2\sqrt{2}$

D) $\sqrt{3}$

E) 0

F) none of these

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17. Find the tenth term of $8,4,2,...$

A) $\frac{1}{64}$

B) -12

C) $\frac{1}{1024}$

D) $\frac{1}{16}$

E) -16

F) none of these

18. Find the sum: $\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{3^n}$

A) $\frac{4}{3}$

B) $\frac{3}{4}$

C) $-\frac{4}{3}$

D) $-\frac{3}{4}$

E) does not exist

F) none of these

19. Suppose g is a function such that $g(g(y)) = y$. Find $\underbrace{g(g(g(g(\cdots(g(y)\cdots)}_{g \text{ appears } n \text{ times}}$.

A) $\begin{cases} y & n \, odd \\ g(y) & n \, even \end{cases}$

B) $\begin{cases} y & n \, even \\ g(y) & n \, odd \end{cases}$

C) y

D) $g(y)$

E) cannot be determined

F) none of these

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20. The expression $n^4 + 2n^3 + n^2$ is equivalent to

A) $\sum_{k=1}^{n} 4k^3$

B) $\sum_{k=1}^{n} 2k^3$

C) $\sum_{k=1}^{n} 6k^2$

D) $\sum_{k=1}^{n} 4k^2$

E) $\sum_{k=1}^{n} 2k^4$

F) none of these

21. Solve for x: $e^x - 2 = e^{-x} - 2e^{-2x}$

A) 0

B) $\{1, \ln(2)\}$

C) $\ln\left(\frac{1+\sqrt{2}}{2}\right)$

D) $\left\{0, \ln\left(\frac{1+\sqrt{2}}{2}\right)\right\}$

E) $\ln\left(1+\sqrt{2}\right)$

F) none of these

22. Find the limit of the sequence: $\left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, ...\right\}$.

A) 1

B) 2

C) $\sqrt{2}$

D) $\frac{3}{2}$

E) does not exist

F) none of these

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23. Give the Cartesian coordinate form of $r = \cos(2\theta)$.

A) $x^2 - y^2 = 2$

B) $\sqrt{x^2 + y^2} = x^2 - y^2$

C) $\sqrt{\left(x^2 + y^2\right)^3} = x^2 - y^2$

D) $\left(x^2 + y^2\right)^3 = x^2 - y^2$

E) $x^2 - y^2 = 2\sqrt{x^2 + y^2}$

F) none of these

24. Give the horizontal asymptote for $f(x) = \left|\arctan(x-3)\right| + 2$.

A) $y = 0$

B) $y = \dfrac{\pi}{2}$

C) $y = 2$

D) $y = \dfrac{3-\pi}{2}$

E) $y = \dfrac{4+\pi}{2}$

F) none of these

25. Solve for x: $4^x - 2^x = 3$

A) $\log_2\left(1+\sqrt{13}\right) - 1$

B) $\log_2\left(1+\sqrt{13}\right)$

C) $\log_2\left(2+\sqrt{13}\right)$

D) $\log_4\left(1+\sqrt{7}\right) - 1$

E) $\log_4\left(1+\sqrt{7}\right)$

F) none of these

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26. Which of the following polynomials has $\sqrt{3}-\sqrt{2}$ as one of its roots?

A) x^4-5x^2+6
B) x^4-10x^2+1
C) x^3-2x^2+6
D) $3x^3-12x^2-1$
E) x^4-2x^3+6
F) none of these

27. If $f(x)=x^2-x$, then $f(x+1)=$

A) $f(x)$
B) $-f(x)$
C) $f(x-1)$
D) $f(-x)$
E) $f(x)+1$
F) none of these

28. Simplify: $\sqrt{\sec^2\left(\tan^{-1}\left(\frac{\pi}{4}\right)\right)-1}$

A) $\frac{\pi^2}{16}$
B) 1
C) -1
D) $\frac{\pi}{4}$
E) 0
F) none of these

29. Give an equation in Cartesian coordinates for the curve parameterized by $x=3-2\sec(t),\ y=\tan(t)+1$.

A) $\frac{(x-3)^2}{4}-(y-1)^2=1$
B) $\frac{(x-3)^2}{2}+(y-1)^2=1$
C) $(3-2x)^2+(y-1)^2=1$
D) $(3-x)^2+(y-1)^2=4$
E) $(3-x)^2-(y-1)^2=4$
F) none of these

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30. A party of hikers walks 8 kilometers from camp on course 30°, then turns and walks 6 kilometers on a course 135°. Find the magnitude of the net displacement from camp. (Note that course is measured clockwise from due north.)

A) $\sqrt{100+24\sqrt{2}-24\sqrt{6}}$

B) 10

C) $\sqrt{100-24\sqrt{2}+24\sqrt{3}}$

D) $\sqrt{100-12\sqrt{2}+12\sqrt{3}}$

E) $\sqrt{112}$

F) $\sqrt{100-24\sqrt{2}-24\sqrt{6}}$

G) none of these

END OF EXAM ☺