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Usage of Combinatorics for the Construction of Meaning and Mathematical Thought Development in First Semester Engineering Students

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Abstract

This article is based on a research project that has as its main objective to construct individual and group didactic situations called ladders. These situations are based on the construction and solution of challenging, motivating, structured and graded problem sets that allow students to reach a higher competence in mathematics. Its function is to serve as a framework in the construction of mathematical concepts meanings that require analysis and autonomous thinking; the student is able to establish links between mathematics in her or his environment and situations that can be modeled scientifically. Furthermore, students mathematical thought strengthening is sought through the application of a learning methodology based on problem solving. This should lead to a better performance in engineering students

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studies and in their professional future; mathematics, thus, will serve as a key to enter the scientific, sociocultural and work world.

Keywords: Ladder, Thought development and problem solving

1 Introduction

In this article we will present the results of a research project based on the creation, application and evaluation of a didactic strategy based on the design of combinatorial problems called ladders, applied to first semester engineering students.
Four ladders were designed and applied, each one is an autonomous set of mathematical contents ordered with slow difficulty level increases that can be used by professors or students in their work inside and outside their classroom to facilitate not well established previous concepts acquisition on the selected themes. Students will be able to enrich, deepen and test their knowledge in combinatorics. The four themes have been classified according to the degree of usage and depth that first semester students might have in the future in the fields of computer science, engineering, physics, biology, statistics, social sciences and in any area in which they might be applied. These themes are:

1. Counting combinatorics. Basic counting principles.

2. Combinations and permutations without repetition.

3. Combinations and permutations with repetition.

4. Combinations and permutations with repetition.

Problems are designed so that students can develop a detailed argumentation of the solution including the justification and/or demonstration of each one of the mathematical processes or concepts students have used to solve the problems.

2 Methodology

The methodology is divided into three stages:

Ladders' design.
Ladders' application.
Results' analysis.

2.1 Ladders' design

2.1.1 Preliminary steps

A modeling methodology with a focus on argumentation was used. This methodology would allow the construction, application and evaluation of a didactic strategy to build basic concepts meanings in mathematics previous to calculus and the development of mathematical thought. The subjects of this study were first semester engineering students. The theme was combinatorics.

2.1.2 Ladders' design

The first part of the ladder presents a problem situation or an explanatory context in which the main theme and the specific items to be developed in or outside the classroom are introduced. Then, a solved problem is presented to be sure the introduced concepts are clear. The solved problems are presented so that students can propose good ideas to be applied in the problems they will have to tackle later on. Then, other two or three problems are presented to advance on some applications of the concepts presented earlier. Generally, each ladder has three subthemes. Finally, some challenge and application problems are posited so that students strengthen the acquired knowledge.

2.2 Ladders' design

Each of the proposed ladders, with the above mentioned themes, was developed in two stages with differential calculus students that had not seen these themes.
Two engineering differential calculus students groups were chosen. The first group, with 17 students, worked in teams of two or three students following the given instructions. The professor was given the ladders in advance so that she or he could study them to be able to contribute with suggestions when students started solving the problems. The same procedure was used with the second group; it had 26 students. Then, a survey to evaluate the activity was conducted both with students and professors.
The results of the problem sets and the applied survey determined the degree of acceptance and efficacy of the didactic proposal. The final ladders were enriched with the commentaries and suggestions of students and professors.

2.3 Results

2.3.1 Results obtained from the application of the methodology

The application of the four combinatorics ladders was conducted during the first semester of 2013. Ladder one corresponds to the basic principles of combinatorics. It is composed of 34 problems: 10 correspond to the addition principle, 14 to the multiplication principle and 5 to the inclusion - exclusion principle. There are also one application problem, 2 challenging problems and 2 concluding problems.

Nine groups were made and they worked for two hours in class and four hours outside the classroom. One group solved 30 of the 34 proposed problems. 2 groups solved more than half the problems. 2 groups solved more than 10 problems and the remaining 5 solved 10 problems. The solved problem average was 41%.

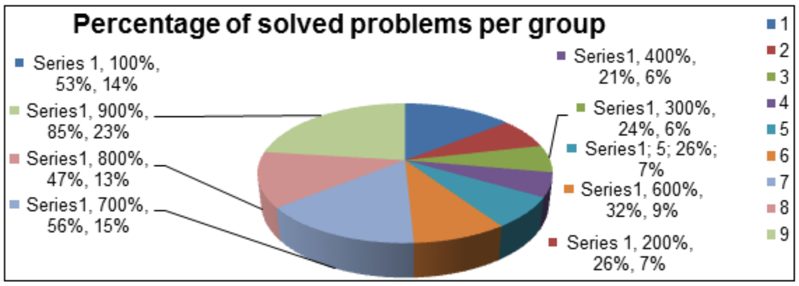

Figure 1:

All the groups used the definitions and properties seen in the ladder to solve the proposed problems. Five groups presented some kind of demonstration, conjecture or explanation in their answers. The other four gave numeric answers. We can see in figure 2 that 60% of the solved problems correspond to the sum principle, 32% to the multiplication principle, 5% to the inclusion exclusion principle, 1% to application problems and 1% to challenging problems. One possible explanation for the distribution of these percentages is the insufficient class time to solve the problems.

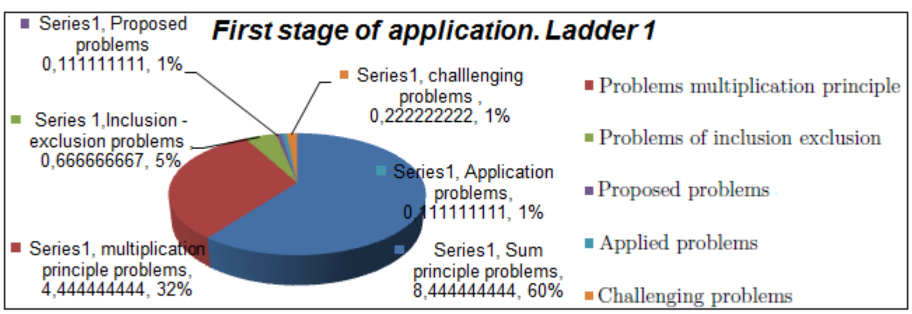

Figure 2:

Ladder 2 treats applications of binomial coefficients. It has 14 problems of enumerative combinatorics, 6 of the development of Newton' s binom and a challenge problem for a total of 21 problems proposed for this ladder. 7 groups of students worked this problem sets.

The results obtained by the work groups in ladder 2 show that 40% of the problems were answered correctly.

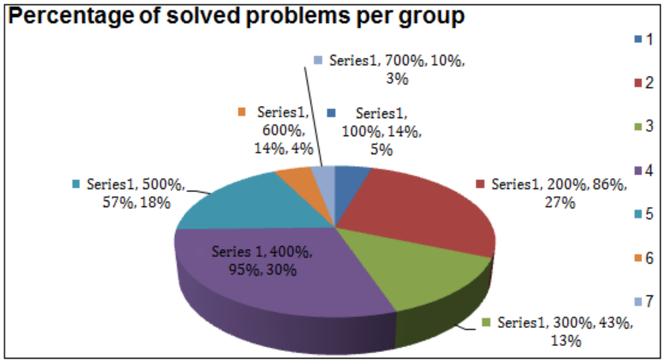

Figure 3:

From the solved problems, 66% posit arguments that use definitions and properties from the concepts already seen.

Ladder 3 proposes 26 permutation problems, 10 correspond to the ordering of elements in a set, 7 to fixing and unfixing, 5 to permutation cycles and 4 to challenging problems.

The information obtained in the application of ladder 3 shows that students only worked in 47% of the proposed problems.

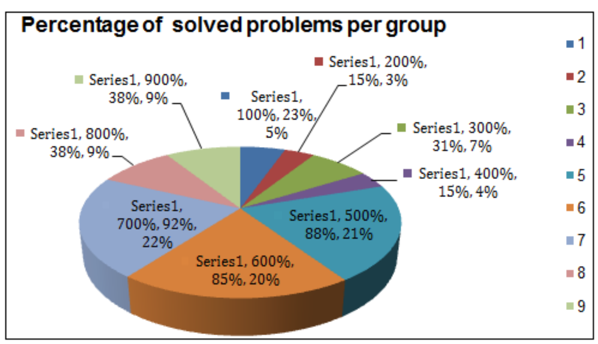

Figure 4:

We can also see that in three of the groups one of the problems was solved with the usage of the properties and definitions presented in the ladder.

In ladder 4 14 are double counting and box principle problems, from these, 11 are pigeonhole principle problems, 2 are challenging problems and 1 is a proposed problem. The obtained results show that students solved 90% of the proposed problems. 9 groups used the concepts and problems developed to solve the problems.

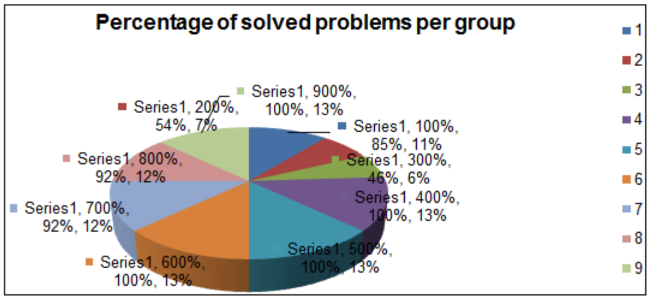

Figure 5:

3 Conclussions

The design, application and analysis of the four proposed ladders on combinatorics allowed us to observe how students mastered these themes. This could be seen in the great acceptance and enthusiasm they showed when solving the problems; they could construct meaning and develop their mathematical thought by using combinatorics.
With the data obtained from the problems solutions we could find that the objective of constructing a didactic strategy that allows students to better their competence in mathematics is based precisely in the low degree of argumentation in the given answers in which they just offer a procedural or numeric answer. We can conclude from that fact that the construction of the ladders by itself, with their learning methodology based on problem solving, is a useful tool to reach the goals and that around these ladders we can structure a specific didactic approach that can contribute to the development and construction of meaning.

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