Using Arrays for Multiplication in the Intermediate Phase

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INTRODUCTION

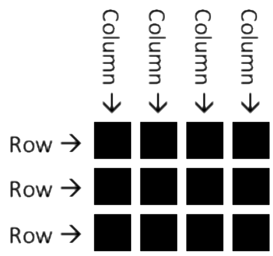
In their book *Maths for Mums and Dads*, Rob Eastaway and Mike Askew (2010) list some common problems that children have with multiplication:

- "Making mistakes using techniques they have learnt mechanically, without understanding what they were doing
- Thinking that multiplication means multiple adding (when often it is about ratios)
- Assuming that multiplying always makes things bigger (so they are stumped when they discover that multiplying by ½ makes things smaller)" (p. 141)

Learning multiplication with arrays can be a useful way to understand multiplication and division across both the Foundation and Intermediate Phases and to address some of the common problems described above. In the previous issue of LTM I explored some early contexts for introducing arrays in the Foundation Phase and shared an Array Scavenger Hunt activity. I also introduced the idea that arrays can be useful for conceptual understanding of (i) factors, (ii) distributive and commutative properties, and (iii) multi-digit multiplication. In this article I build on these ideas and share some further activities.

WHAT IS AN ARRAY?

Recall that an array is defined as a set of numbers or shapes laid out in a rectangle comprising rows and columns. For young learners, an array is a useful visual aid and typically consists of shapes rather than numbers. When describing an array we will use the convention of giving the number of rows first, followed by the number of columns. The array of squares shown alongside is a 3×4 array – i.e. it has 3 rows and 4 columns – and contains a total of 12 squares.



EXPLORING FACTORS AND THE COMMUTATIVE PROPERTY WITH ARRAYS

This activity uses arrays of playing cards to explore factors and the commutative property. If you wish, you could perhaps introduce correct mathematical terminology by using the terms *factor* and *commutative property*, although this is not recommended in the Foundation Phase.

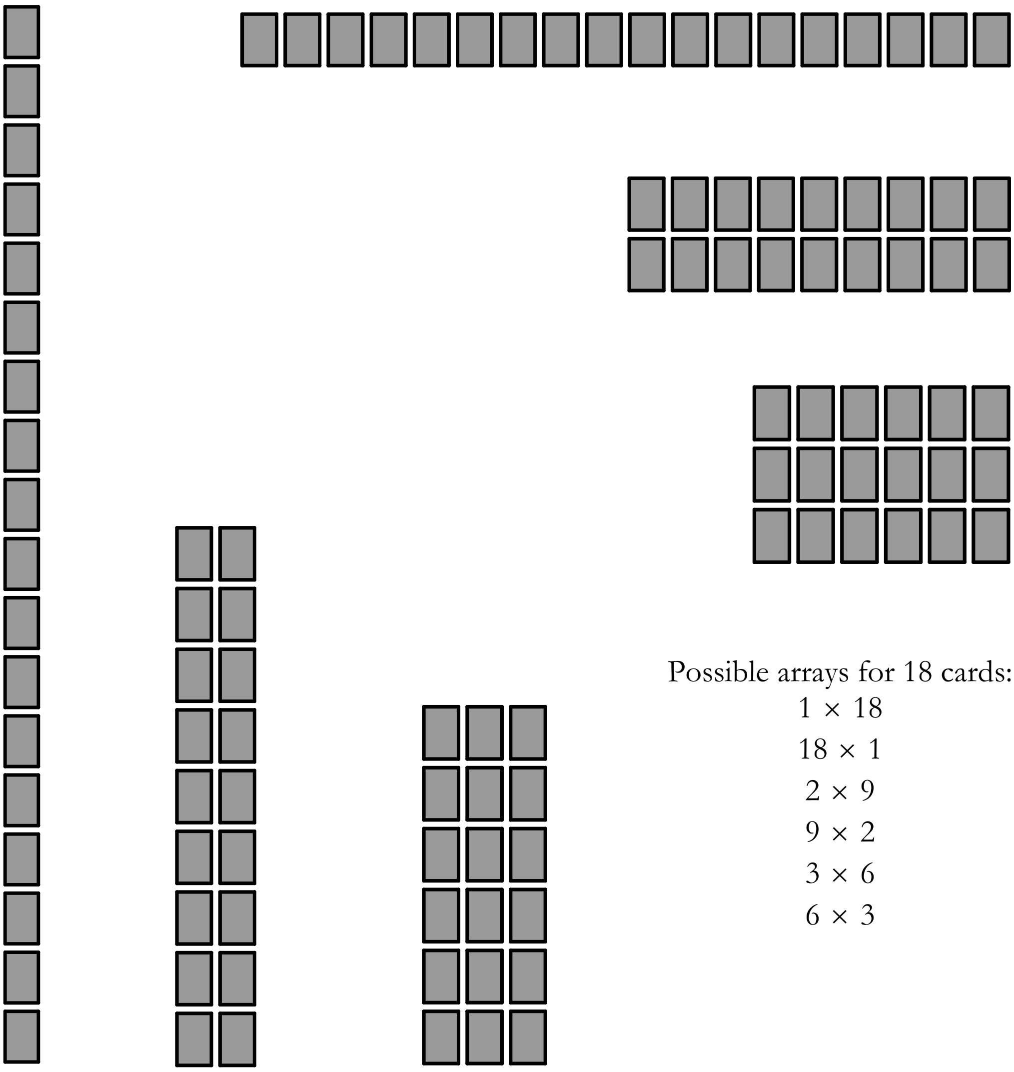
Divide packs of cards into piles of 18 and give a pile to each pair of learners. If you have sufficient counters or bottle tops you could use those instead as they will take up less space.

Remind learners about the structure of an array: always a rectangle or square, equal items in each row, and equal items in each column.

Learners must then build as many different arrays as possible with ALL of their 18 cards. It is preferable to place the cards face down so that learners don't get distracted by the numbers.

After building each array, learners must write down the number of rows and columns the array has, as well as the total number of cards, which should always be 18.

Encourage the learners to be methodical in their exploration, making sure that they have all the possible combinations.



Possible arrays for 18 cards:
1 × 18
18 × 1
2 × 9
9 × 2
3 × 6
6 × 3

- After the pair work, ask learners to contribute different arrays. Write their contributions on the board. When you have all of them, re-write them in a logical order to show patterns and to further illustrate the idea of commutativity. Talk about the patterns.
- After all the arrays of 18 have been explored, split the cards into piles of 24 or 36 and repeat.

Arrays with 18 cards	**Arrays with 24 cards**	**Arrays with 36 cards**
1 × 18 ; 18 × 1	1 × 24 ; 24 × 1	1 × 36 ; 36 × 1
2 × 9 ; 9 × 2	2 × 12 ; 12 × 2	2 × 18 ; 18 × 2
3 × 6 ; 6 × 3	3 × 8 ; 8 × 3	3 × 12 ; 12 × 3
	4 × 6 ; 6 × 4	4 × 9 ; 9 × 4
		6 × 6

DISTRIBUTIVE PROPERTY ACTIVITIES

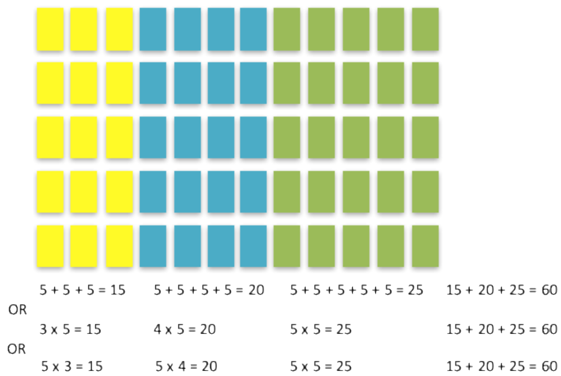
This is an introductory problem solving activity[1] that can be used to explore the idea of the distributive property with Grade 3 and 4 learners.

How many cards are needed?

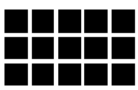
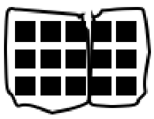
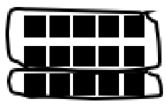
A soccer club wants to put a coloured "Thank You" card on each seat in a certain section of the stadium. They want to use a section of seats that has 5 rows. In each row they plan to put:

- 3 yellow cards
- then 4 blue cards
- then 5 green cards

How many cards will be needed altogether so that each seat in this section has a card on it? Draw the seats with the cards. Write number sentences to show how you found your answer.

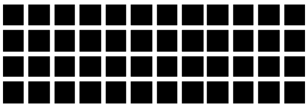
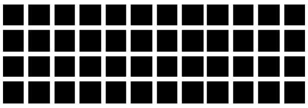
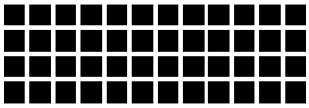
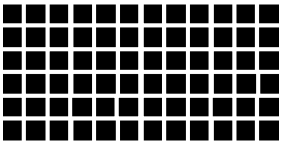
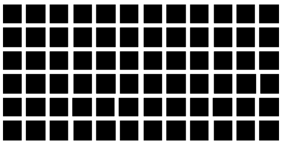
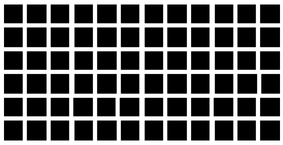
Possible solutions may look like this  	• Encourage learners to use repeated addition or multiplication strategies rather than counting the cards in ones. • Use learners' contributions to work towards using the bracket notation: $(5 \times 3) + (5 \times 4) + (5 \times 5)$ $15 + 20 + 25 = 60$ OR $(3 \times 5) + (4 \times 5) + (5 \times 5)$ $15 + 20 + 25 = 60$

Learners could then work on a worksheet such as the sample shown below to practice the distributive property further. These worksheets are available on the SANC project website[2]. These worksheet examples could be re-visited by asking the learners to split the arrays into 3 or 4 smaller arrays where possible.

	This array shows…	**Split the array into 2 smaller arrays and add them together. Use brackets to split up your sums.**	**Can you do it another way?**
EXAMPLE	 3 rows by 5 columns = 15	 $(3 \times 3) + (3 \times 2)$ $9 + 6 = 15$	 $(2 \times 5) + (1 \times 5)$ $10 + 5 = 15$

[1] Source: Benson, C. C., Wall, J. J., & Malm, C. (2013). The distributive property in Grade 3? *Teaching Children Mathematics*, *19*(8), 498–506.

[2] http://www.ru.ac.za/sanc/mathsclubs/clubresources/activities/#mult

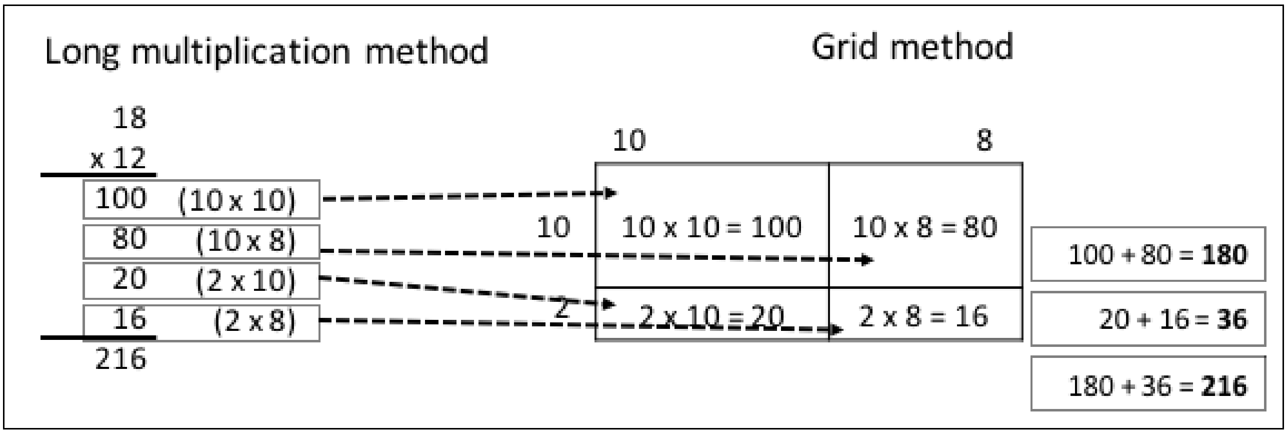
This array shows…	**Split the array into 2 smaller arrays and add them together. Use brackets to split up your sums.**	**Can you do it another way?**
 __ rows by __ columns = ___		
 __ rows by __ columns = ___		

LINKING ARRAYS TO THE LONG MULTIPLICATION ALGORITHM USING THE GRID METHOD

As I pointed out in my previous article, the area model provides a useful transition from concrete representations of arrays to a more abstract representation which discourages learners from counting the individual items in the array. The area model is also a useful interim step between arrays and more formal long multiplication algorithms and provides learners with a way to visualise a multiplication problem. Instead of using (and drawing) all the individual items in an array, learners can be encouraged to represent the items conceptually using boxes to represent 'invisible' items. This can then be extended to an *area model* by breaking each number into its *place value components.*

> *"For those children who struggle with multiplication, the grid-method approach ensures that they have a technique that they can understand… it's more of a case of building it up in stages so that understanding how it works is as important as being able to do it."* (Eastaway & Askew, 2010, p. 151)

The figure below shows the links between the area model approach and the long multiplication method for the example 18 × 12.



GRID METHOD LEARNER ACTIVITIES

Learners will need a great deal of practice with using the grid method to multiply. It may help them to do some initial work with array representations as blocks or dots as shown in some of the previous activities before moving on to this method. I suggest starting with 2-digit by 1-digit problems, working gradually up to 3-digit by 2-digit problems as shown in the examples below. Multiplication beyond 3-digits by 2-digits becomes cumbersome using this method, so try not to use those as examples. Master copies of these worksheets for practicing this method can be found on the SANC Project website[3]. Once learners understand the grid method and are fluent in using it they can then be introduced to the long multiplication method. By carefully showing the connections between the two methods, learners can be assisted to understand how and why the formal algorithm works.

Grid Method: 2 x 1 digit

Example
16 x 6 = 96

X	6
10	60
6	36
Answer →	96

Grid Method: 2 x 2 digits

Example
16 x 26 = 416

X	10	6	Add up ...
20	200	120	320
6	60	36	96
Answer →			416

Grid Method 3 x 1 digit

Example
163 x 6 = 978

X	100	60	3
6	600	360	18
Answer →			978

Grid Method: 3 x 2 digits

Example
163 x 16 = 2608

X	100	60	3	Add up ...
10	1000	600	30	1630
6	600	360	18	978
Answer →				2608

SOURCES

Eastaway, R., & Askew, M. (2010). *Maths for mums and dads: Take the pain out of maths homework*. London: Square Peg.

ACKNOWLEDGEMENT

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[3] http://www.ru.ac.za/sanc/mathsclubs/clubresources/activities/#mult