# Using Classroom Talk to Support the Standards for Mathematical Practice

SAS Campus  Cary, NC
August 9, 2012

Triangle High Five Math Summit 2012

# Presenters

## Gemma Mojica

Gemma Mojica is an Assistant Professor of Mathematics Education at the University of North Carolina at Chapel Hill. She is a former Wake County middle school mathematics teacher and is Nationally Board Certified in early adolescence mathematics. Mojica received her master's and doctorate degrees in mathematics education from North Carolina State University. Her teaching and research focus on teacher education in elementary and middle school mathematics education.

gmojica@email.unc.edu

## Pat Sickles

Pat Sickles retired from Durham Public Schools, where she taught and was the Director of Secondary Mathematics. She currently works as an Educational Consultant in Mathematics and is an Adjunct Instructor in the Duke MAT program, the UNC Middle Grades Program and a consultant with the TAP Math MSP Grant. She received her bachelor's degree from the University of North Carolina at Greensboro and her master's in education from the University of North Carolina at Chapel Hill. Additionally, Sickles serves as the North Carolina Council of Teachers of Mathematics Central Region President.

pat@sickles.org

# Agenda

- Process for Statistical Investigation PCAI Model (30 min)
  - Typical Name Length Activity
- Promoting Discourse and the SMP (50 min)
- Short Break (10 min)
- Exploring Mean Absolute Deviation (15 min)
- Choosing an MVP Activity (15 min)
- Shooting the One-and-One Activity (25 min)
- Questions/comments (5 min)

# A word about the SMP chart …

The chart illustrating the eight Standards of Mathematical Practice will be distributed to all Triangle High Five math teachers during the coming school year. A preview copy will be available in each of the meeting rooms and on several of the large screens in the Networking Hall. This resource will be distributed by each district and will be available for your use very soon. Feel free to use it in our session.

# Conference presentations & handouts:

http://mathsummit2012pd.wikispaces.com/

gmojica@email.unc.edu

# Statistics

# Typical Name Length Activity

- What is the typical number of letters in the first names of students in your class?
- What is the typical number of letters in the last names of students in your class?

Adapted from the Connected Mathematics Project's (2002) *Data About Us*

# Typical Name Length Activity

- On your own …
  - Write the number of letters in your first name on a sticky note. Use your preferred name.
  - Write the number of letters in your last name on a ***different*** color sticky note.
- With a partner …
  - Discuss ways to organize the data so you can determine the typical name length of students' first and last names from your class.

# Typical Name Length Activity


- As a whole group …
  - Represent the class data using a line plot so you can determine the typical length of students' first names.
  - Represent the class data using a line plot so you can determine the typical length of students' last names.
- On your own …
  - Write some statements about your class data. Note any patterns you see.
- In a small group …
  - Share your observations with your partner and another pair. Answer the questions, and be prepared to share your arguments with the whole group.

# Questions to Guide Small Group Discussions

- What patterns did you notice about the data?
- What is the overall shape of the distribution?
- Where is the center of the distribution?
- How would you describe the spread of the distribution?
- Compare the typical name lengths for first and names. How are they similar? How are they different?
- If a new student joined our class today, what would you predict about the length of the student's first and last name?

# Process of Statistical Investigation

PCAI Model

- **P**ose a question
- **C**ollect data
- **A**nalyze data
- **I**nterpret results

(Graham, 1987)

The four components of the PCAI model may emerge linearly, or may include revisiting and making connections among the components.

# Posing a Question

Select a question that

- is motivated by describing summarizing, comparing, and generalizing data within a ***context***
- is measurable
- anticipates variability

# Posing a Question

Statistical questions

- focus on a census of the classroom in elementary school (GAISE Report, 2007)
- often require cycles of iteration with data collection to get the question "right"
  - *"How tall am I?"*
  - *"How tall are the people in my class?"*

# Collecting Data

Determine

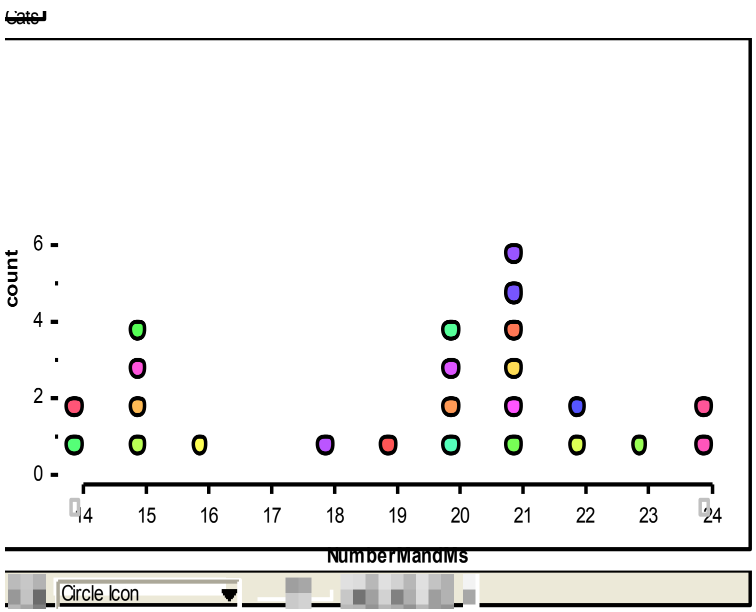
- the ***population***
  - full set of people or things that the study is designed to investigate
- methods of collecting data
  - ***sample***, a subset of the entire population
  - ***census***, the entire population
- if a sample will be collected, decide the size and how many
- if class data will be pooled
- consider representativeness and bias
  - ***random samples*** have characteristics that are representative of the population

# Collecting Data

*How do students see data?*

- Data as a ***pointer***
- Data as a ***case***
- Data as a ***classifier***
- Data as an ***aggregate***

(Konold & Higgins, 2003)



# Analyzing Data

Describe and summarize data

- using relevant summary statistics, such as the mean, median, mode and
- using tables, diagrams, graphs, or other representations

Describe variation

- *measurement variability*
- *natural variability*
- *induced variability*
- *sampling variability*

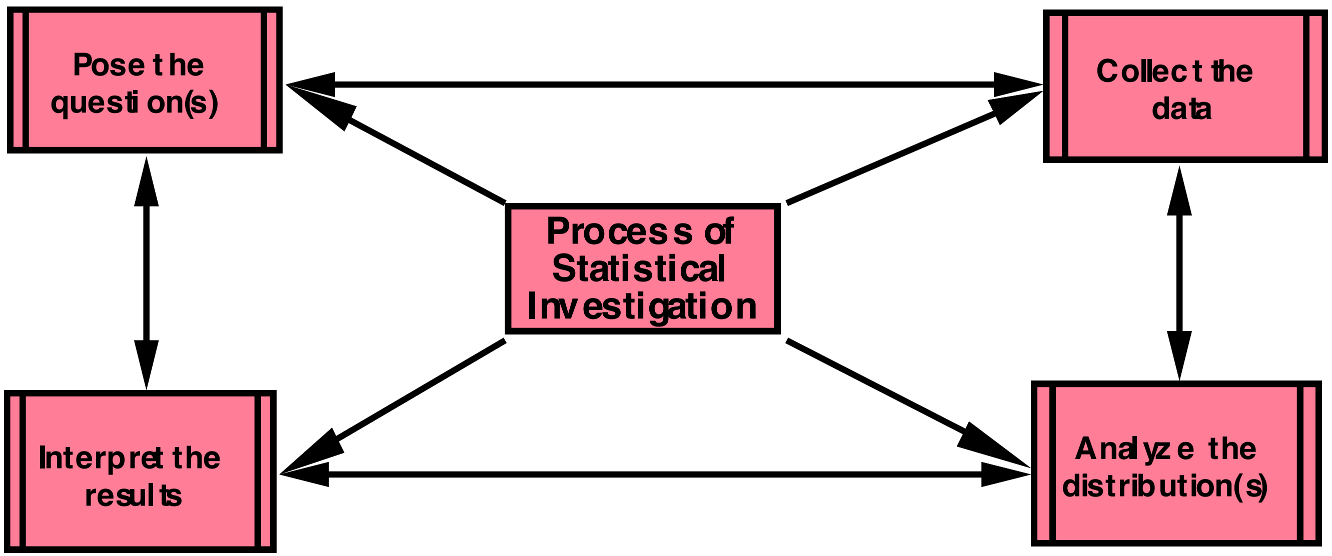
# Interpreting Data

- Relate analysis to original question and context
- Make decisions about the question posed within the context of the problem based on data collection and analysis

# Typical Name Length Activity

- What question did we investigate?
- How did we collect data?
- How did we organize and analyze the data?
- What interpretations did we make about the data?

# Concept Map – PCAI



# Discussion

- How does the process of statistical investigation, using the PCAI model, have the potential to support students' in engaging in the SMP?
- Which grade level standards have the potential to be addressed by the Typical Name Length activity?
- How might you modify the activity for your specific students?

# PROMOTING DISCOURSE IN THE MATHEMATICS CLASSROOM

# Making the Case for Meaningful Discourse: Standards for Mathematical Practice

- Standard 1: Explain the meaning and structure of a problem and restate it in their words
- Standard 3: Habitually ask "why"
  - Question and problem-pose
  - Develop questioning strategies ...
  - Justify their conclusions, communicate them to others and respond to the arguments of others
- Standard 6: Communicate their understanding of mathematics to others
  - Use clear definitions and state the meaning of the symbols they choose
- Standard 7: ...describe a pattern orally...
  - Apply and discuss properties

# Standards for Mathematical Practice

#3 **Construct viable arguments and critique the reasoning of others**

- Justify solutions and approaches
- Listen to the reasoning of others
- Compare arguments
- Decide if the arguments of others make sense
- Ask clarifying and probing questions

# Mathematical Discourse

"Teachers need to develop a range of ways of interacting with and engaging students as they work on tasks and share their thinking with other students.  This includes having a repertoire of specific kinds of questions that can push students' thinking toward core mathematical ideas as well as methods for holding students accountable to rigorous, discipline-based norms for communicating their thinking and reasoning."

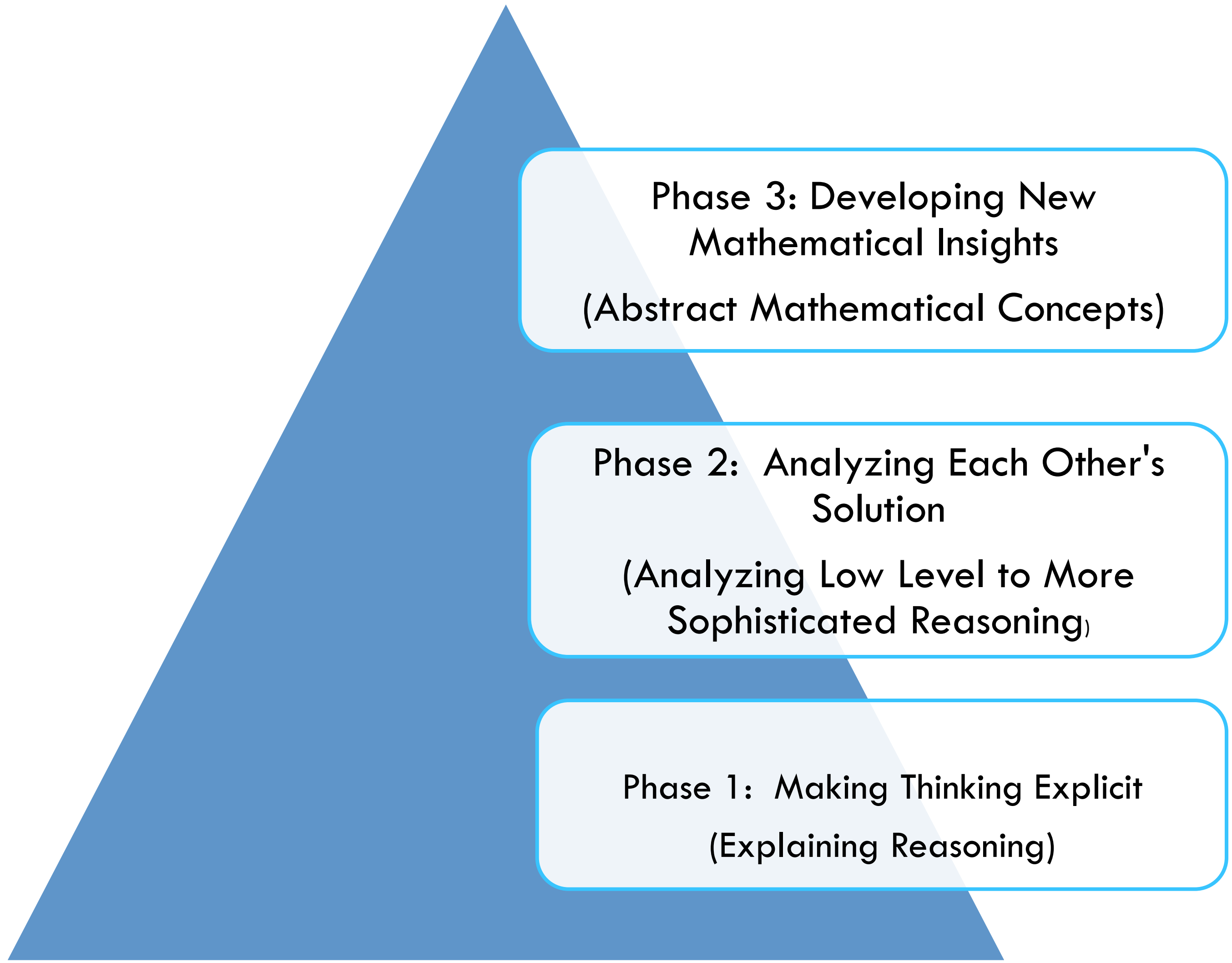
(Smith and Stein, 2011)

# Teacher Questioning

Teacher questioning has been identified as a critical part of teachers' work. The act of asking a good question is cognitively demanding, it requires considerable pedagogical content knowledge and it necessitates that teachers know their learners well.

(Boaler & Brodie, 2004, p.773)

## Levels of Analysis and Sense making




From *Whole Class Mathematics Discussions,* Lamberg, Pearson 2012

# Phase I:  Make Thinking Explicit

- Turn and talk:  What does this mean in the classroom?

# Phase 1: Make Thinking Explicit

Active Listening is an important part of understanding someone else's solution.

- Student should ask questions, if they are unclear about an idea presented.
- Teacher should monitor understanding of group by asking questions of explanation presented.

From *Whole Class Mathematics Discussions,* Lamberg, Pearson 2012

# Questions to Clarify Understanding of Explanation

- Does everyone understand _____’s solution?
- Who can explain what ______ is thinking?
- Who would you like to help you explain?
- Can someone explain what _____ is thinking?
- Anyone confused about what he/she is saying?

From *Whole Class Mathematics Discussions,* Lamberg, Pearson 2012

## Phase 2: Analyzing each other's solutions to make mathematical connections

When students are expected to analyze each other's solutions, they have to pay attention to what the students are saying.  In addition, they need to think if the student's explanation makes sense.

This requires students to make mathematical connections between ideas presented.

From *Whole Class Mathematics Discussions,* Lamberg, Pearson 2012

# PHASE 2: ANALYZING EACH OTHER'S SOLUTION

Teacher questions to promote analysis and reflection of each other's solutions:

- What do you see that is the same about these solutions?
- What do you see that is different about these solutions?
- How does this relate to ____?
- Ask students to think about how these strategies relate to the mathematical concept being discussed

From *Whole Class Mathematics Discussions,* Lamberg, Pearson 2012

# PHASE 3: DEVELOPING NEW MATHEMATICAL INSIGHTS (ABSTRACT MATHEMATICAL CONCEPTS)

*Teacher Questions to Promote Mathematical Insights*

- Ask students to summarize key idea.
- Ask questions:  Will the rule will work all the time? (Making generalizations)
- Introduce vocabulary or mathematical ideas within the context of conversation
- Ask students to solve a related problem that extends the insights they had gained from the discussion.
- Ask "What if" questions.

From *Whole Class Mathematics Discussions,* Lamberg, Pearson 2012

# In the Classroom

- Video:  Statistical Analysis to Rank Baseball Players
- www.teachingchannel.org

- What evidence of understanding is seen in the student discussions?

# Short Break

# 5 Practices for Orchestrating Productive Mathematics discussions

- Determine the goal—what you want the students to learn
- Choose a rich task to help students attain the goal

The Five Practices:

1. Anticipating
2. Monitoring
3. Selecting
4. Sequencing
5. Connecting

Adapted from *5 Practices for Orchestrating Productive Mathematics Discussions, Smith and Stein, NCTM, 2011*

# Anticipating

- Anticipating Students' Responses
  - What strategies are students likely to use to approach or solve a challenging, high-level mathematical task
  - How to respond to the work that students are likely to produce
  - Which strategies from student work will be most useful in addressing the mathematical goals

# Monitoring

- Monitoring is the process of paying attention to what and how students are thinking during the lesson
  - Students working in pairs or groups
  - Listening to and making note of what students are discussing and the strategies they are using
  - Asking questions of the students that will help them stay on track or help them think more deeply about the task

# Recording notes while monitoring

| Strategy | Who and What | Order |
|---|---|---|
| Number line | | |
| Comparing Decimal Equivalents | | |
| Comparing Fractional Equivalents | | |
| | | |

# Selecting and sequencing

- Selecting
  - This is the process of deciding the *what* and the *who* to focus on during the discussion
- Sequencing
  - What order will the solutions be shared with the class?

# Connecting

- Perhaps the most challenging part
- Teacher must ask the questions that will make the mathematics explicit and understandable
- Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations
  - Not just clarifying and probing

# Teacher Questioning

Teachers can effectively use questions during the whole class discussion to help students to make deeper mathematical connections.

# Moves to Guide Discussion and Ensure Accountability

- Revoicing
- Asking students to restate someone else's reasoning
- Asking students to apply their own reasoning to someone else's reasoning
- Prompting students for further participation
- Using wait time

Adapted from *5 Practices for Orchestrating Productive Mathematics Discussions,* Smith and Stein, NCTM, 2011

# Purposeful Discourse

- Through mathematical discourse in the classroom, teachers "empower their students to engage in , understand and own the mathematics they study."

(Eisenman, *Promoting Purposeful Discourse,* 2009)

# Exploring Mean Absolute Deviation

Nine people were asked, "How many people are in your family?" One result from the poll is that the average family size for the nine people was five.

Source: Kader, G. D. (1999). Means and MADs. *Mathematics Teaching in the Middle School, 4*(6), 398–403.

# Exploring Possible Distributions with a Mean Value of 5

- As a whole group …
  - Examine Distribution 1. Do you agree that this distribution has a mean of 5. Interpret the mean.
- On your own …
  - Explore other possible distributions for nine different family sizes so that the mean family size is 5. Create a line plot to show nine people's family size with a mean of 5 if the smallest family size is 2 and the largest family size is 11. Write a description of your strategy.
- In a small group …
  - Share your distributions and strategies.

# Ordering Distributions through Visual Examination

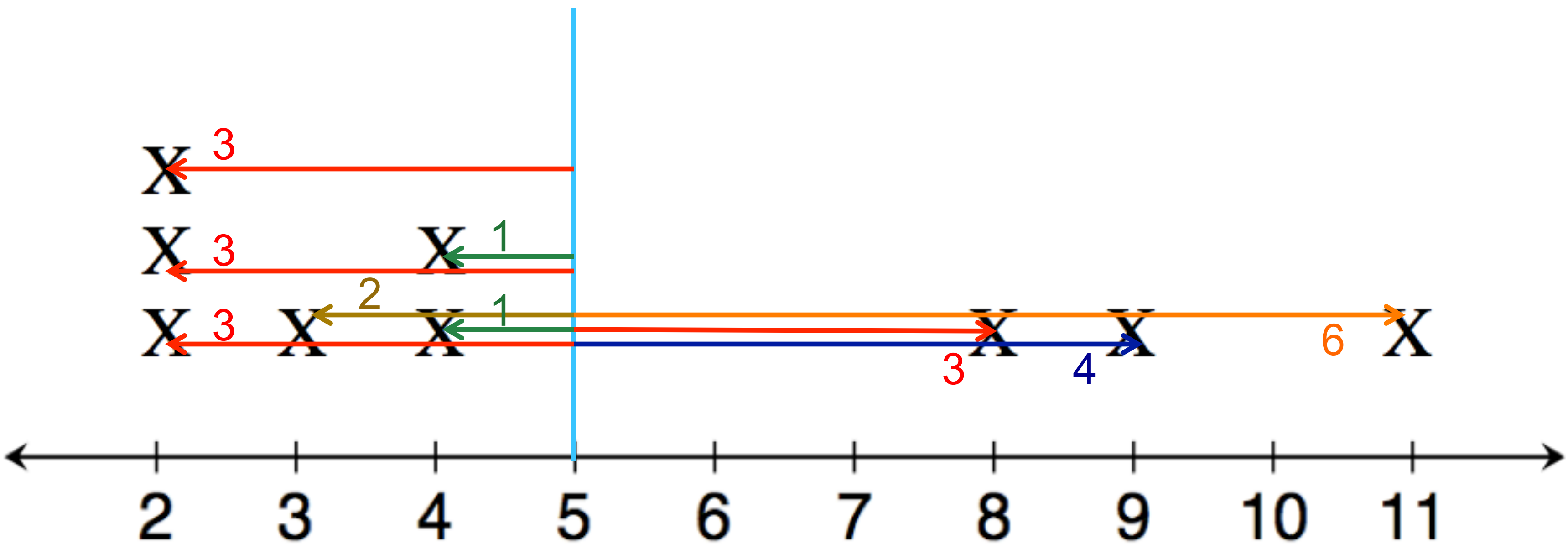
The eight distributions all have a mean of 5.

- Of the distributions, which shows data values that differ the least from the mean?
- Of the distributions, which shows data values that differ the most from the mean?
- On the basis of how different the data values appear to be from the mean, how would you order (from least to most) the other six distributions? Explain your reasoning.
- Why might using visualization alone be challenging?

# Quantifying Deviations from the Mean

$$\text{Mean Abolute Deviation} = \frac{\text{total distance (of all values from the mean)}}{\text{number of values}}$$

$$\text{Mean Abolute Deviation} = \frac{\text{sum }(|\text{deviations from the mean}|)}{\text{number of values}}$$




$$\text{MAD} = \frac{3+3+3+2+1+1+3+4+6}{9} = \frac{26}{9} = 2.89$$

# Interpreting the MAD

- What does the MAD tell us about the spread of the data?
- What are the benefits and drawbacks of using the MAD?

| Distribution | MAD |
| --- | --- |
| 1 | 0.00 |
| 2 | 2.89 |
| 3 | 2.44 |
| 4 | 1.78 |
| 5 | 2.44 |
| 6 | 2.67 |
| 7 | 2.22 |
| 8 | 4.00 |

# Choosing an MVP

Our school's basketball coach plans to give an award to the MVP. She is having a difficult time deciding which player is most worthy of the award. Based on the data, determine which of the three players should be given the MVP award.

Adapted from Mathscape's (2002) *Looking Behind the Numbers*

# Introducing the MVP Activity

http://mmmproject.org/lbn/mainframeS.htm

# Choosing an MVP

Help the basketball coach determine which of the three players should be given the MVP award. Write a letter to the coach to convince her of your argument.

| Game | Player A | Player B | Player C |
|---|---|---|---|
| 1 | 12 | 18 | 24 |
| 2 | 13 | 21 | 14 |
| 3 | 12 | 15 | 14 |
| 4 | 14 | 13 | 22 |
| 5 | 11 | 16 | 25 |
| 6 | 20 | 18 | 16 |
| 7 | 15 | 18 | 11 |

Adapted from Mathscape's (2002) *Looking Behind the Numbers*

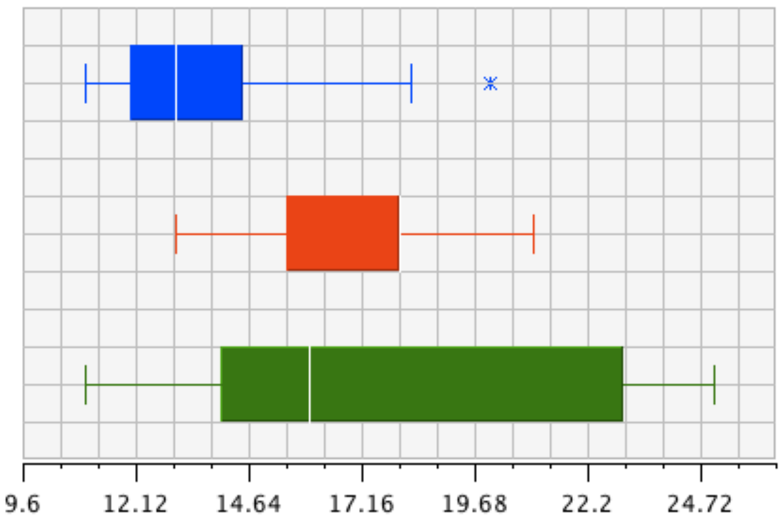
# Resources for Box Plots


NCTM's Illuminations (Advanced Data Grapher)
http://illuminations.nctm.org/ActivityDetail.aspx?ID=220

# Shodor (Box Plot)

[http://www.shodor.org/interactivate/activities/BoxPlot/](http://www.shodor.org/interactivate/activities/BoxPlot/)

Select a data set: My Data




Points Per Basketball Game

Do not use median for quartile range (TI-83 method)

Use median for quartile range only for data sets with an odd number of data points (Tukey method)

Horizontal Scale

0.84

Set Horizontal Scale

Auto Scale

Graph All Data

Graph By Category

Describe Your Data: Points Per Basketball Game

Tabular Data

| Category | N | MIN | Q1 | MEDIAN | Q3 | MAX |
|---|---|---|---|---|---|---|
| Player A | 7 | 11.000 | 12.000 | 13.000 | 14.5 | 20.000 |
| Player B | 7 | 13.000 | 15.5 | 18.000 | 18.000 | 21.000 |
| Player C | 7 | 11.000 | 14.000 | 16.000 | 23.000 | 25.000 |

# SMP #3



- What types of arguments do you expect students to make based on the data?
- Watch the videos (#5, 9, 10).
  - How did students engage in the SMP?
  - Did anything surprise you?
  - Based on our discussions in the last component on discourse, how might teachers support students in engaging in productive mathematical discourse?

http://mmmproject.org/lbn/mainframeS.htm

# Student Argument

# Student Argument

# Probability

# Shooting the One-and-One

Emma plays basketball for her middle school team. During a recent game, Emma was fouled. She was in a one-and-one free-throw situation. This means Emma will have the opportunity to try one shot. If she makes the first shot, she gets to try a second shot. If she misses the first shot, she is done and does not get to try a second shot. Emma's free throw average is 60%.

(adapted from the Connected Mathematics Project's *What Do You Expect?*)

# Making Predictions

1. Which of the following do you think is most likely to happen?
   - Emma will score 0 points, missing the first shot.
   - Emma will score 1 point. That is, she will make the first shot and miss the second shot.
   - Emma will score 2 points by making two shots.
2. Record your prediction before you analyze the situation. Justify your response?
   - How can students create, or simulate, an experiment to find the likelihood of this probabilistic situation?

     What is the experimental probability that Emma will score 0 points? That she will score 1 point? That she will score 2 points?

# Simulating the Free Throws

| With technology | Without technology |
|---|---|
| | |

**With technology**

- Graphing calculator
- Excel spreadsheet
- *Probability Explorer*
- *Fathom*
- Applets on the internet
- Other ideas?

**Without technology**

- Spinners
- 10-sided die
- Other ideas?

# Calculating the Experimental Probabilities ($n$ = 20)

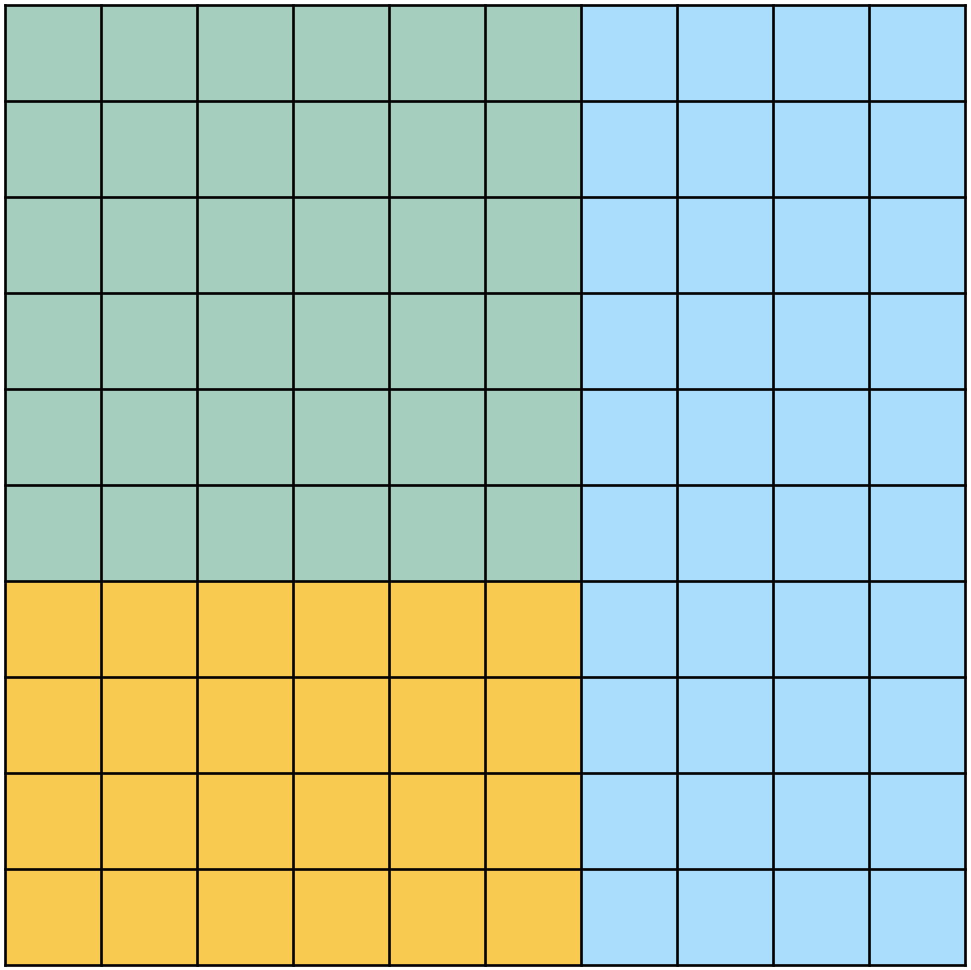
How do we interpret the data collected from the experiment?

- 2 points: 6/20 = 0.30
- 1 point: 8/20 = 0.40
- 0 points: 5/20 = 0.25

D19 fx 0

| | A | B | C | D |
|---|---|---|---|---|
| 1 | Free Throw 1 | Free Throw 2 | Interpretation | Points |
| 2 | 0 | 2 | Make, Make | 2 |
| 3 | 1 | 6 | Make, Miss | 1 |
| 4 | 0 | 8 | Make, Miss | 1 |
| 5 | 4 | 9 | Make, Miss | 1 |
| 6 | 3 | 4 | Make, Make | 2 |
| 7 | 1 | 5 | Make, Make | 2 |
| 8 | 9 | | Miss | 0 |
| 9 | 5 | 8 | Make, Miss | 1 |
| 10 | 5 | 3 | Make, Make | 2 |
| 11 | 0 | 8 | Make, Miss | 1 |
| 12 | 1 | 9 | Make, Miss | 1 |
| 13 | 3 | 3 | Make, Make | 2 |
| 14 | 8 | 0 | Miss | 0 |
| 15 | 1 | 8 | Make, Miss | 1 |
| 16 | 4 | 7 | Make, Miss | 1 |
| 17 | 9 | | Miss | 0 |
| 18 | 2 | 4 | Make, Make | 2 |
| 19 | 6 | | Miss | 0 |
| 20 | 6 | | Miss | 0 |

# Using an Area Model to Represent Theoretical Probabilities



**<u>2 Points</u>**

Make, Make

.6(.6) = 0.36

**<u>1 Point</u>**

Make, Miss

.6(.4) = 0.24

**<u>0 Points</u>**

Miss

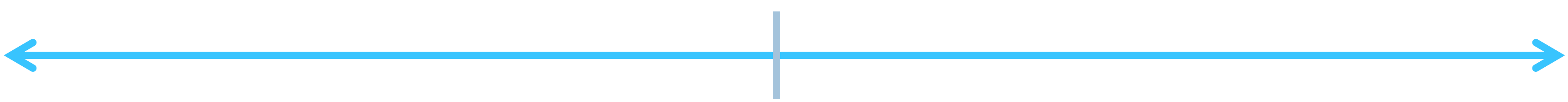
.4

# Comparing Experimental & Theoretical Probabilities

- How do the three theoretical probabilities compare with the three experimental probabilities?
- Under what conditions should the probabilities be similar? Different?
- What pedagogical decisions do teachers need to make in order to help students develop a robust understanding of the key concepts in the CCSSM?

# Probability

- Probability: likelihood of an event
- Probability of an event's occurrence is expressed as a value ranging from 0 to 1



| | | |
|---|---|---|
| 0 | ½ (or 0.5) | 1 |
| 0% | 50% | 100% |
| Impossible | Equally likely as unlikely | Certain |

- Probability of an event is $\dfrac{\text{\# of outcomes that result in event}}{\text{\# of all possible outcomes}}$

# Some Terminology

- Probability: likelihood of an event
  - Probabilities are between 0% and 100% (or 0 and 1)
  - Theoretical and experimental
    - Theoretical
    - Experimental

$$P(E) = \frac{\#\ \text{of specified outcomes}}{\text{Total}\ \#\ \text{of outcomes}}$$

- Independent outcomes: the outcome of one event has no influence on the outcome of another event
- Dependent outcomes: the outcome of one event influences the outcome of another event

# Questions/ Comments