**Homework-8(1)**
**P8.3-1, 3, 8, 10, 17, 21, 24, 28,29**
**P8.4-1, 2, 5**

### Section 8.3: The Response of a First Order Circuit to a Constant Input

**P 8.3-1** The circuit shown in Figure P 8.3-1 is at steady state before the switch closes at time $t = 0$. The input to the circuit is the voltage of the voltage source, 12 V. The output of this circuit is the voltage across the capacitor, $v(t)$. Determine $v(t)$ for $t > 0$.

***Answer:*** $v(t) = 6 - 2e^{-1.33t}$ V for $t > 0$

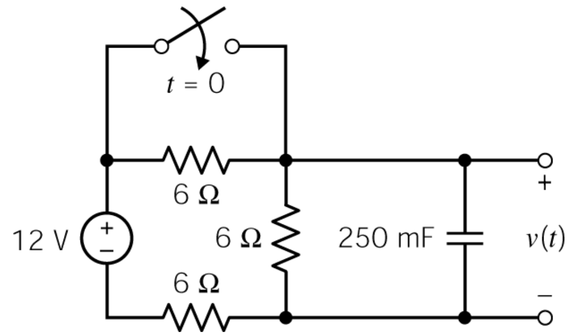
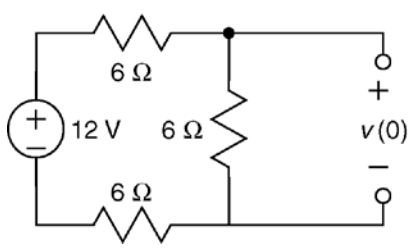


**Figure P 8.3-1**

**Solution:**

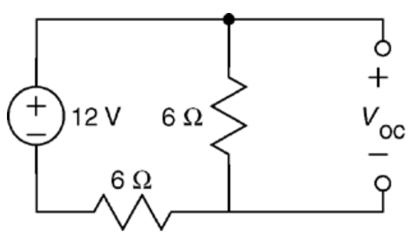



Here is the circuit before $t = 0$, when the switch is open and the circuit is at steady state. The open switch is modeled as an open circuit.

A capacitor in a steady-state dc circuit acts like an open circuit, so an open circuit replaces the capacitor. The voltage across that open circuit is the initial capacitor voltage, $v$ (0).

By voltage division

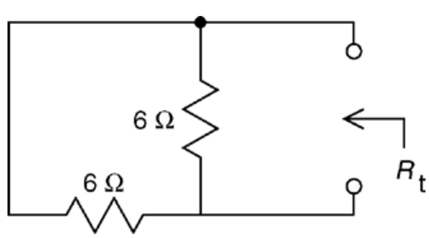
$$v(0) = \frac{6}{6+6+6}(12) = 4 \text{ V}$$




Next, consider the circuit after the switch closes. The closed switch is modeled as a short circuit.

We need to find the Thevenin equivalent of the part of the circuit connected to the capacitor. Here's the circuit used to calculate the open circuit voltage, $V_{oc}$.

$$V_{oc} = \frac{6}{6+6}(12) = 6 \text{ V}$$




Here is the circuit that is used to determine $R_t$. A short circuit has replaced the closed switch. Independent sources are set to zero when calculating $R_t$, so the voltage source has been replaced by a short circuit.

$$R_t = \frac{(6)(6)}{6+6} = 3 \ \Omega$$

Then $\tau = R_t\, C = 3(0.25) = 0.75\ \text{s}$

Finally, $v(t) = V_{oc} + \left(v(0) - V_{oc}\right) e^{-t/\tau} = 6 - 2\,e^{-1.33\,t}\ \text{V}\quad \text{for } t > 0$

**P 8.3-3** The circuit shown in Figure P 8.3-3 is at steady state before the switch closes at time $t = 0$. Determine the capacitor voltage, $v(t)$, for $t > 0$.

***Answer:*** $v(t) = -6 + 18e^{-6.67t}$ V for $t > 0$

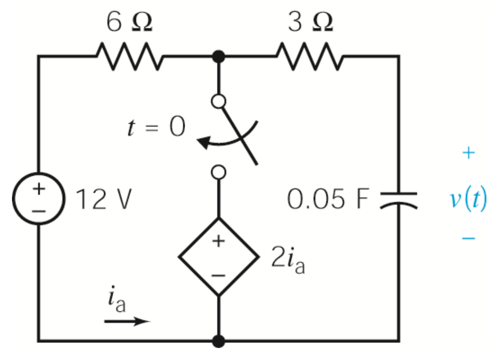
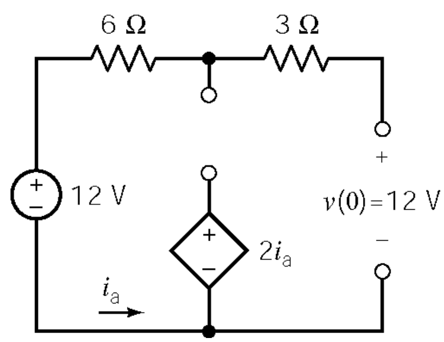
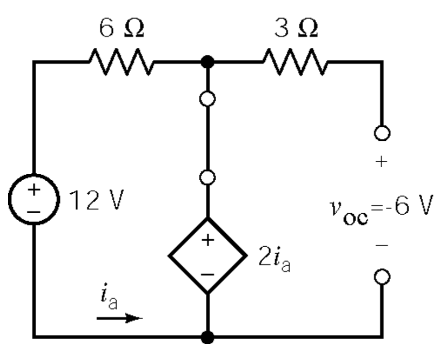


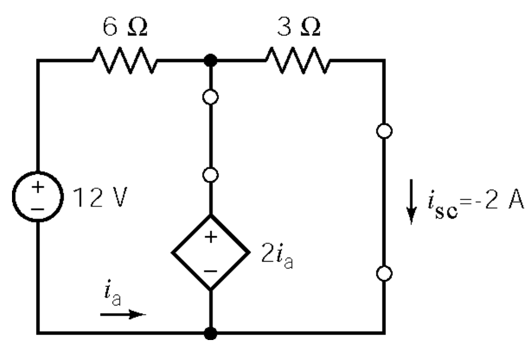

**Figure P 8.3-3**

**Solution:** Before the switch closes:




After the switch closes:




Therefore $R_t = \dfrac{-6}{-2} = 3\,\Omega$ so $\tau = 3(0.05) = 0.15\ \text{s}$.

Finally, $v(t) = v_{oc} + (v(0) - v_{oc})\, e^{-t/\tau} = -6 + 18\, e^{-6.67\,t}\ \text{V}\quad \text{for } t > 0$

**P 8.3-8** The circuit shown in Figure P 8.3-8 is at steady state before the switch opens at time $t = 0$. The input to the circuit is the voltage of the voltage source, $V_s$. This voltage source is a dc voltage source; that is, $V_s$ is a constant. The output of this circuit is the voltage across the capacitor, $v_o(t)$. The output voltage is given by

$$v_o(t) = 2 + 8e^{-0.5t}\text{ V} \quad \text{for} \quad t > 0$$

Determine the values of the input voltage, $V_s$, the capacitance, $C$, and the resistance, $R$.

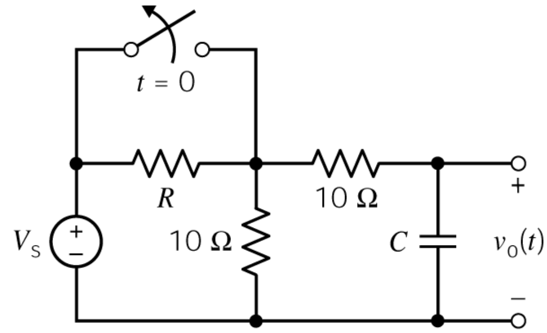
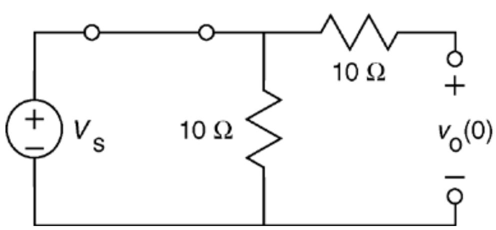



**Figure P 8.3-8**

**Solution:** Before the switch opens, the circuit will be at steady state. Because the only input to this circuit is the constant voltage of the voltage source, all of the element currents and voltages, including the capacitor voltage, will have constant values. Opening the switch disturbs the circuit. Eventually the disturbance dies out and the circuit is again at steady state. All the element currents and voltages will again have constant values, but probably different constant values than they had before the switch opened.




Here is the circuit before $t = 0$, when the switch is closed and the circuit is at steady state. The closed switch is modeled as a short circuit. The combination of resistor and a short circuit connected is equivalent to a short circuit. Consequently, a short circuit replaces the switch and the resistor $R$. A capacitor in a steady-state dc circuit acts like an open circuit, so an open circuit replaces the capacitor. The voltage across that open circuit is the capacitor voltage, $v_o(t)$.

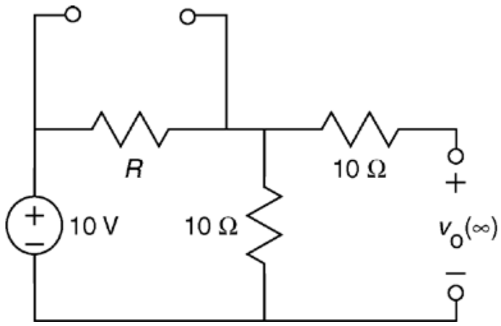
Because the circuit is at steady state, the value of the capacitor voltage will be constant. This constant is the value of the capacitor voltage just before the switch opens. In the absence of unbounded currents, the voltage of a capacitor must be continuous. The value of the capacitor voltage immediately after the switch opens is equal to the value immediately before the switch opens. This value is called the initial condition of the capacitor and has been labeled as $v_o(0)$. There is no current in the horizontal resistor due to the open circuit. Consequently, $v_o(0)$ is equal to the voltage across the vertical resistor, which is equal to the voltage source voltage. Therefore

$$v_o(0) = V_s$$

The value of $v_o(0)$ can also be obtained by setting $t = 0$ in the equation for $v_o(t)$. Doing so gives

$$v_o(0) = 2 + 8\,e^0 = 10\text{ V}$$

Consequently, $$V_s = 10\text{ V}$$




Next, consider the circuit after the switch opens. Eventually (certainly as $t \to \infty$) the circuit will again be at steady state. Here is the circuit at $t = \infty$, when the switch is open and the circuit is at steady state. The open switch is modeled as an open circuit. A capacitor in a steady-state dc circuit acts like an open circuit, so an open circuit replaces the capacitor. The voltage across that open circuit is the steady-state capacitor voltage, $v_o(\infty)$. There is no current in the horizontal resistor and $v_o(\infty)$ is equal to the voltage across the vertical resistor. Using voltage division,

$$v_o(\infty) = \frac{10}{R+10}(10)$$

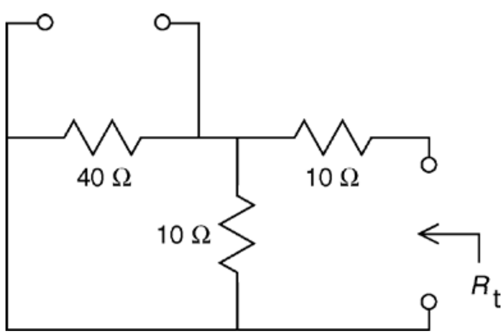
The value of $v_o(\infty)$ can also be obtained by setting $t = \infty$ in the equation for $v_o(t)$. Doing so gives

$$v_o(\infty) = 2 + 8\,e^{-\infty} = 2 \text{ V}$$

Consequently,

$$2 = \frac{10}{R+10}(10) \quad\Rightarrow\quad 2\,R + 20 = 100 \quad\Rightarrow\quad R = 40\,\Omega$$

Finally, the exponential part of $v_o(t)$ is known to be of the form $e^{-t/\tau}$ where $\tau = R_t\,C$ and $R_t$ is the Thevenin resistance of the part of the circuit connected to the capacitor.




Here is the circuit that is used to determine $R_t$. An open circuit has replaced the open switch. Independent sources are set to zero when calculating $R_t$, so the voltage source has been replaced by a short circuit.

$$R_t = 10 + \frac{(40)(10)}{40+10} = 18\ \Omega$$

so

$$\tau = R_t\,C = 18\,C$$

From the equation for $v_o(t)$

$$-0.5\,t = -\frac{t}{\tau} \quad\Rightarrow\quad \tau = 2\text{ s}$$

Consequently,

$$2 = 18\,C \quad\Rightarrow\quad C = 0.111 = 111\text{ mF}$$

**P 8.3-10** A security alarm for an office building door is modeled by the circuit of Figure P 8.3-10. The switch represents the door interlock, and $v$ is the alarm indicator voltage. Find $v(t)$ for $t > 0$ for the circuit of Figure P 8.3-10. The switch has been closed for a long time at $t = 0^-$.

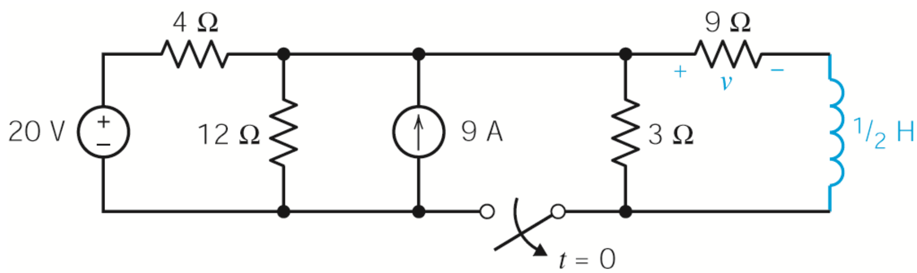
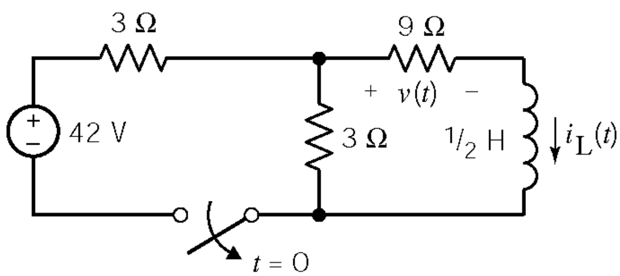
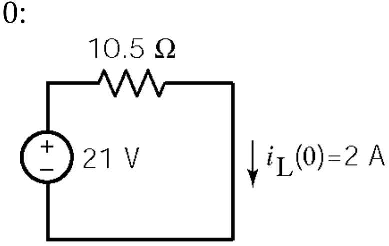
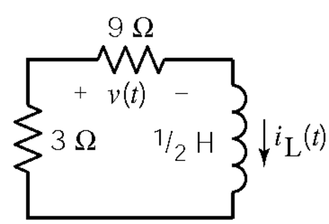



**Figure P 8.3-10**

**Solution:** First, use source transformations to obtain the equivalent circuit




for $t < 0$: 


for $t > 0$: 


So $i_L(0) = 2$ A, $I_{sc} = 0$, $R_t = 3+9 = 12\ \Omega$, $\tau = \dfrac{L}{R_t} = \dfrac{\frac{1}{2}}{12} = \dfrac{1}{24}$ s

and $i_L(t) = 2e^{-24t} \quad t > 0$

Finally $v(t) = 9\, i_L(t) = 18\, e^{-24t} \quad t>0$

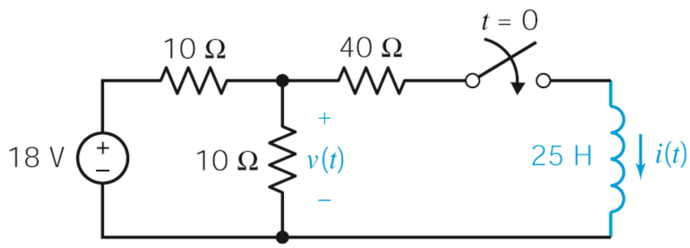
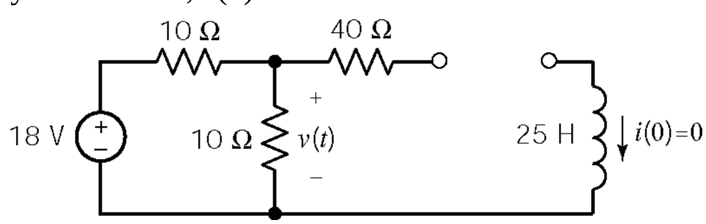



**Figure P 8.3-17**

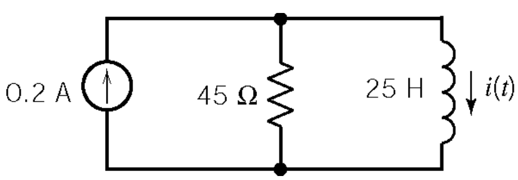
**P 8.3-17** The circuit shown in Figure P 8.3-17 is at steady state before the switch closes. The response of the circuit is the voltage $v(t)$. Find $v(t)$ for $t > 0$.
***Hint:*** After the switch closes, the inductor current is $i(t) = 0.2(1 - e^{-1.8t})$ A

***Answer:*** $v(t) = 8 + e^{-1.8t}$ V

**Solution:** Immediately before $t = 0,\ i(0) = 0$.



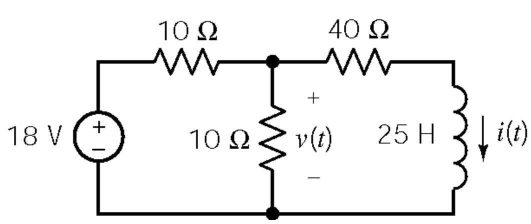

After $t = 0$, replace the circuit connected to the inductor by its Norton equivalent to calculate the inductor current:




$$I_{sc} = 0.2 \text{ A},\ R_t = 45\ \Omega,\ \tau = \frac{L}{R_{th}} = \frac{25}{45} = \frac{5}{9}$$

$$\text{So } i(t) = 0.2\,(1 - e^{-1.8t}) \text{ A}$$

Now that we have the inductor current, we can calculate $v(t)$:




$$\begin{aligned} v(t) &= 40\, i(t) + 25 \frac{d}{dt} i(t) \\ &= 8(1 - e^{-1.8t}) + 5(1.8)e^{-1.8t} \\ &= 8 + e^{-1.8t} \text{ V} \quad \text{for } t > 0 \end{aligned}$$

**P8.3-21** The circuit in Figure P8.3-21 at steady state before the switch closes. Determine an equation that represents the capacitor voltage after the switch closes.

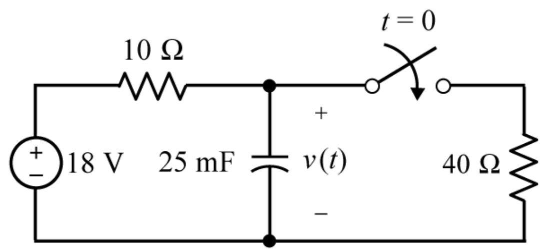



**Figure P8.3-21**

**Solution:**

For $t < 0$, the switch is open and the capacitor acts like an open circuit because the circuit is at steady state. Consequently, the current in the 10 Ω resistor is 0A and so the voltage across this resistor is 0 V. KVL gives $v(t) = 18$ V. Immediately before the switch opens we have $v(0-) = 18$ V. The capacitor voltage does not change instantaneously so $v(0+) = v(0-) = 18$ V.

For $t > 0$, the Thevenin equivalent of the part of the circuit connected to the capacitor is characterized by

$R_t = 10 \| 40 = 8\ \Omega$ and, using voltage division, $v_{oc} = \dfrac{40}{10+40}(18) = 14.4$ V.

$A = v_{oc} = 14.4$ V, $B = v(0+) - v_{oc} = 18 - 14.4 = 3.6$ V and $a = \dfrac{1}{\tau} = \dfrac{1}{R_t\, C} = \dfrac{1}{8(0.025)} = 5\ \dfrac{1}{\mathrm{s}}$

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**P8.3-24** Consider the circuit shown in Figure 8.3-24a and corresponding plot of the inductor current shown in Figure 8.3-24b. Determine the values of $L$, $R_1$ and $R_2$.

***Answer:*** $L = 4.8$ H, $R_1 = 200\ \Omega$ and $R_2 = 300\ \Omega$.

***Hint:*** Use the plot to determine values of $D$, $E$, $F$ and a such that the inductor current can be represented as

$$i(t) = \begin{cases} D & \text{for } t \le 0 \\ E + F\, e^{-at} & \text{for } t \ge 0 \end{cases}$$

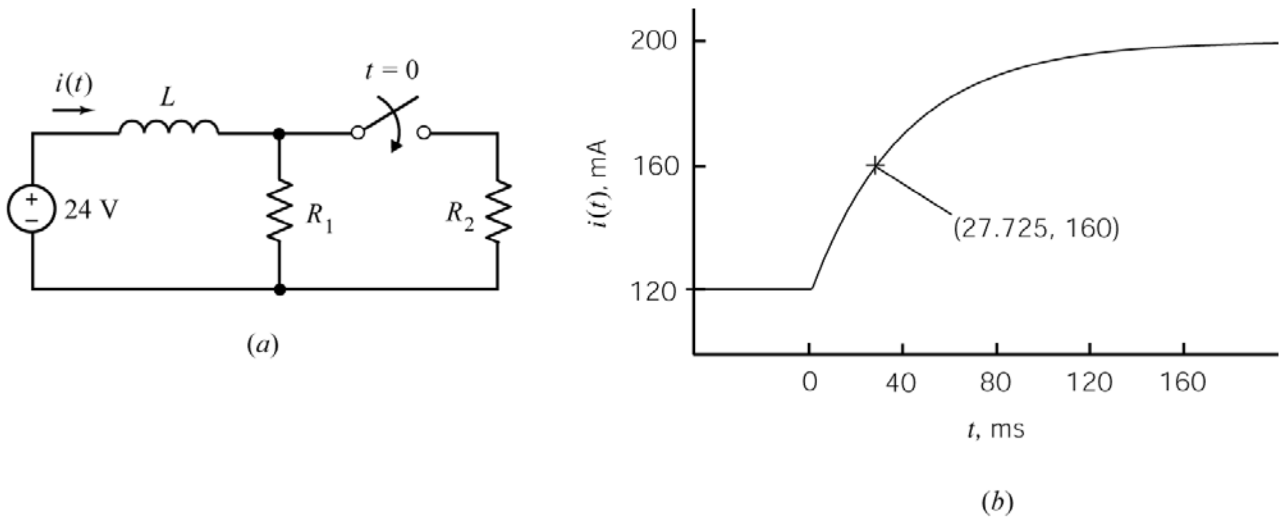
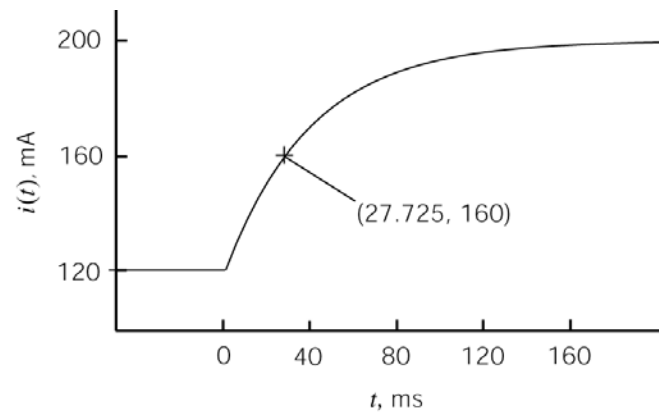



**Figure 8.3-24**

**Solution:** From the plot




$D =\ i(t)$ for $t < 0$ =120 mA = 0.12 A,

$E + F = i(0+)$ = 120 mA = 0.12 A

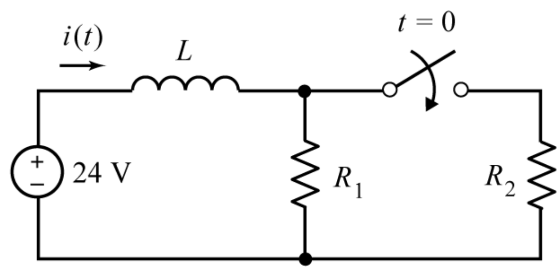
and

$$E = \lim_{t \to \infty} i(t) = 200 \text{ mA} = 0.2 \text{ A} .$$

The point labeled on the plot indicates that $i(t)$ = 160 mA when $t$ = 27.725 ms = 0.027725 s. Consequently

$$160 = 200 - 80\, e^{-a(0.027725)} \quad \Rightarrow \quad a = \frac{\ln\left(\dfrac{160-200}{80}\right)}{-0.027725} = 25\ \frac{1}{\text{s}}$$

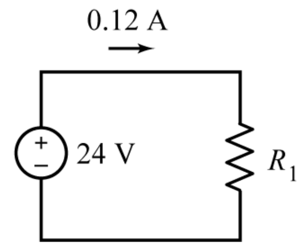
Then



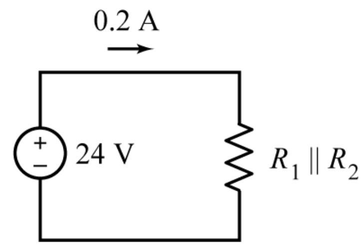

$$i(t) = \begin{cases} 120 \text{ mA} & \text{for } t \le 0 \\ 200 - 80\, e^{-25t} \text{ mA} & \text{for } t \ge 0 \end{cases}$$

When $t < 0$, the circuit is at steady state so the inductor acts like a short circuit.

$$R_1 = \frac{24}{0.12} = 200\ \Omega$$




As $t \to \infty$, the circuit is again at steady state so the inductor acts like a short circuit.




$$R_1 \parallel R_2 = \frac{24}{0.2} = 120\ \Omega$$

$$120 = 200 \parallel R_2 \quad \Rightarrow \quad R_2 = 300\ \Omega$$

Next, the inductance can be determined using the time constant:

$$25 = a = \frac{1}{\tau} = \frac{R_1 \parallel R_2}{L} = \frac{120}{L} \quad \Rightarrow \quad L = \frac{120}{25} = 4.8\ \text{H}$$

**P8.3-28** After time $t$ = 0, a given circuit is represented by the circuit diagram shown in FigureP8.3-28.

a.) Suppose that the inductor current is

$$i(t) = 21.6 + 28.4e^{-4t} \quad \text{mA} \qquad \text{for } t \ge 0$$

Determine the values of $R_1$ and $R_3$.

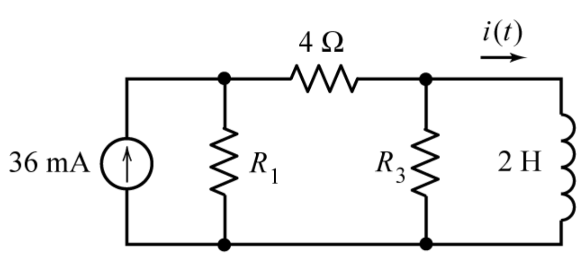



**Figure P8.3-28**

b.) Suppose instead that $R_1 = 16\ \Omega$, $R_3 = 20\ \Omega$ and the initial condition is $i(0) = 10$ mA. Determine the inductor current for $t \ge 0$.

**Solution:** The inductor current is given by $i(t) = i_{sc} + \left(i(0) - i_{sc}\right)e^{-at}$ for $t \ge 0$ where $a = \dfrac{1}{\tau} = \dfrac{R_t}{L}$.

**a**. Comparing this to the given equation gives $21.6 = i_{sc} = \dfrac{R_1}{R_1 + 4}(36) \quad \Rightarrow \quad R_1 = 6\ \Omega$ and $4 = \dfrac{R_t}{2} \quad \Rightarrow \quad R_t = 8\ \Omega$. Next $8 = R_t = \left(R_1 + 4\right) \| R_3 = 10 \| R_3 \quad \Rightarrow \quad R_3 = 40\ \Omega$.

**b.** $R_t = (16+4) \| 20 = 10\ \Omega$ so $a = \dfrac{1}{\tau} = \dfrac{10}{2} = 5$ s. also $i_{sc} = \dfrac{16}{16+4}(36) = 28.8$ mA. Then $i(t) = i_{sc} + \left(i(0) - i_{sc}\right)e^{-at} = 28.8 + (10 - 28.8)e^{-5t} = 28.2 - 18.8e^{-5t}$.

**P8.3-29** Consider the circuit shown in Figure P8.3-29.

a.) Determine the time constant, τ, and the steady state capacitor voltage, $v(\infty)$, when the switch is **open**.

b.) Determine the time constant, τ, and the steady state capacitor voltage, $v(\infty)$, when the switch is **closed**.

Answers: a.) τ = 3 s and $v(\infty)$ = 24 V; b.) τ = 2.25 s and $v(\infty)$ = 2 V;

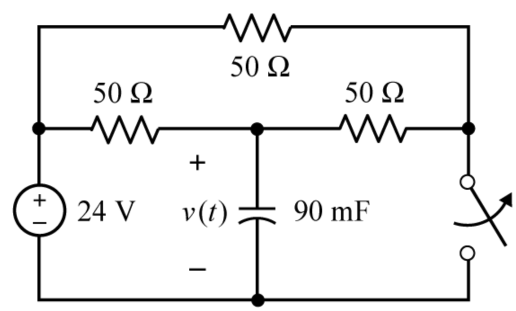
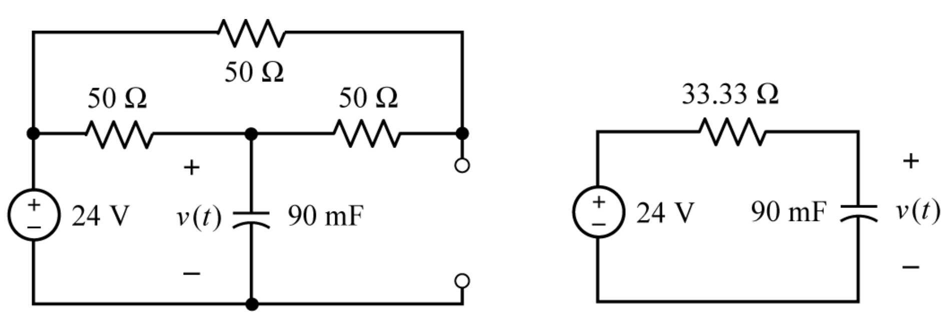



**Figure P8.3-29**

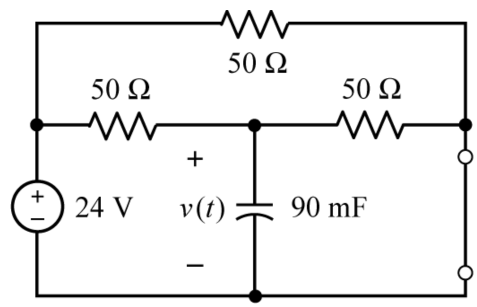
**Solution: a.)** When the switch is open we have



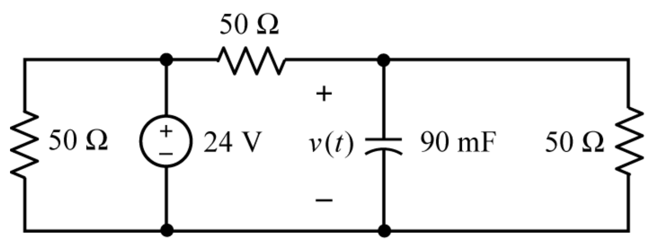
After replacing series and parallel resistors by equivalent resistors, the part of the circuit connected to the capacitor is a Thevenin equivalent circuit with $R_t = 33.33\ \Omega$. The time constant is $\tau = R_t C = 33.33(0.090) = 3$ s.

Since the input is constant, the capacitor acts like an open circuit when the circuit is at steady state. Consequently, there is zero current in the 33.33 Ω resistor and KVL gives $v(\infty) = 24$ V.

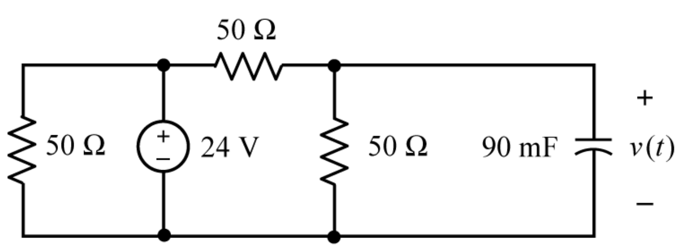
**b.)** When the switch is closed we have

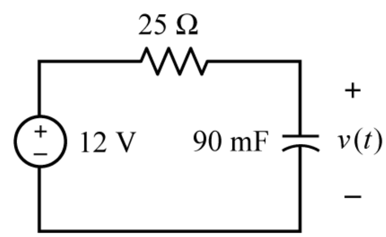



This circuit can be redrawn as




Now we find the Thevenin equivalent of the part of the circuit connected to the capacitor:




So $R_t = 25\ \Omega$ and

$$\tau = R_t C = 25(0.090) = 2.25 \text{ s}$$

Since the input is constant, the capacitor acts like an open circuit when the circuit is at steady state. Consequently, there is zero current in the 25 Ω resistor and KVL gives $v(\infty) = 12$ V.

## Section 8-4: Sequential Switching

**P 8.4-1** The circuit shown in Figure P 8.4-1 is at steady state before the switch closes at time $t$ = 0. The switch remains closed for 1.5 s and then opens. Determine the capacitor voltage, $v(t)$, for $t > 0$.

***Hint:*** Determine $v(t)$ when the switch is closed. Evaluate $v(t)$ at time $t$ = 1.5 s to get $v(1.5)$. Use $v(1.5)$ as the initial condition to determine $v(t)$ after the switch opens again.

***Answer:*** $v(t)=\begin{cases} 5+5e^{-0.5t}\ \text{V} & \text{for } 0<t<1.5\text{ s} \\ 10-2.64e^{-2.5(t-1.5)}\ \text{V} & \text{for } 1.5\text{ s}<t \end{cases}$

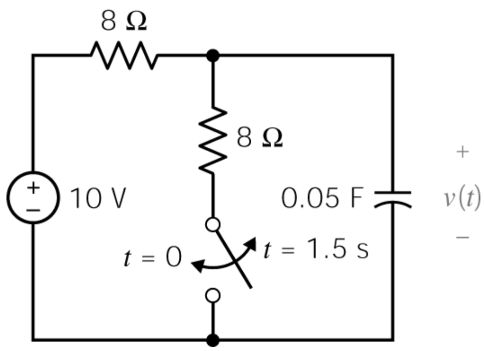
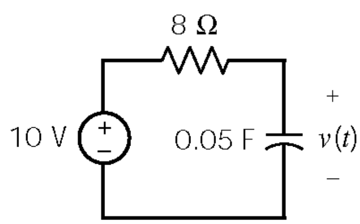


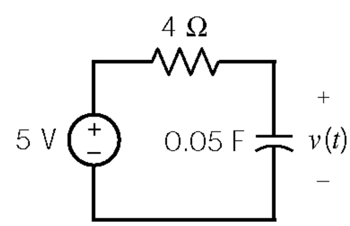

**Figure P 8.4-1**

**Solution:**

Replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit to get:




$t<0$ and $t>1.5$ s




$0<t<1.5$ s

Before the switch closes at t = 0 the circuit is at steady state so $v(0)$ = 10 V. For $0 < t < 1.5$s, $v_{oc}$ = 5 V and $R_t$ = 4 Ω so $\tau = 4\times 0.05 = 0.2$ s . Therefore

$$v(t)=v_{oc}+(v(0)-v_{oc})\,e^{-t/\tau}=5+5e^{-5t}\ \text{V} \quad \text{for } 0<t<1.5\text{ s}$$

At t =1.5 s, $v(1.5)=5+5e^{-0.05(1.5)}=5$ V . For 1.5 s < $t$, $v_{oc}$ = 10 V and $R_t$ = 8 Ω so $\tau = 8\times 0.05 = 0.4$ s . Therefore

$$v(t)=v_{oc}+(v(1.5)-v_{oc})\,e^{-(t-1.5)/\tau}=10-5\,e^{-2.5(t-1.5)}\ \text{V} \quad \text{for } 1.5\text{ s}<t$$

Finally

$$v(t)=\begin{cases} 5+5\,e^{-5t}\ \text{V} & \text{for } 0<t<1.5\text{ s} \\ 10-5\,e^{-2.5(t-1.5)}\ \text{V} & \text{for } 1.5\text{ s}<t \end{cases}$$

**P 8.4-2** The circuit shown in Figure P 8.4-2 is at steady state before the switch closes at time $t$ = 0. The switch remains closed for 1.5 s and then opens. Determine the inductor current, $i(t)$, for $t > 0$.

***Answer:*** $v(t) = \begin{cases} 2 + e^{-0.5t} \text{ A} & \text{for } 0 < 1 < 1.5 \text{ s} \\ 3 - 0.53 e^{-0..667(t-1.5)} \text{ A} & \text{for } 1.5 \text{ s} < t \end{cases}$

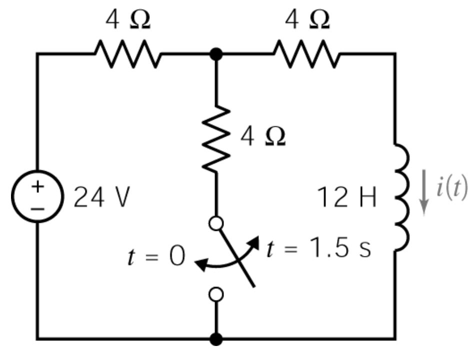
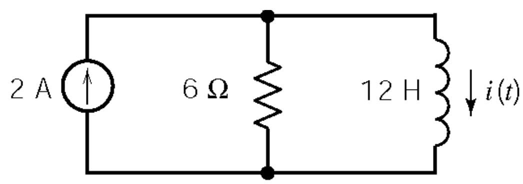


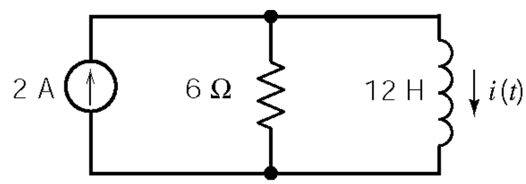

**Figure P 8.4-2**

**Solution:**

Replace the part of the circuit connected to the inductor by its Norton equivalent circuit to get:




$t < 0$ and $t > 1.5$ s




$0 < t < 1.5$ s

Before the switch closes at $t$ = 0 the circuit is at steady state so $i(0)$ = 3 A. For $0 < t < 1.5$s, $i_{sc}$ = 2 A and $R_t$ = 6 Ω so $\tau = \frac{12}{6} = 2$ s. Therefore

$$i(t) = i_{sc} + (i(0) - i_{sc})\, e^{-t/\tau} = 2 + e^{-0.5t} \text{ A} \quad \text{for } 0 < t < 1.5 \text{ s}$$

At $t$ =1.5 s, $i(1.5) = 2 + e^{-0.5(1.5)} = 2.47$ A. For 1.5 s < $t$, $i_{sc}$ = 3 A and $R_t$ = 8 Ω so $\tau = \frac{12}{8} = 1.5$ s.

Therefore

$$i(t) = i_{sc} + (i(1.5) - i_{sc})\, e^{-(t-1.5)/\tau} = 3 - 0.53\, e^{-0.667(t-1.5)} \text{ V} \quad \text{for } 1.5 \text{ s} < t$$

Finally

$$i(t) = \begin{cases} 2 + e^{-0.5\,t} \text{ A} & \text{for } 0 < t < 1.5 \text{ s} \\ 3 - 0.53\, e^{-0.667(t-1.5)} \text{ A} & \text{for } 1.5 \text{ s} < t \end{cases}$$

**P 8.4-5** The circuit shown in Figure P 8.4-5 is at steady state before the switch opens at $t = 0$. The switch remains open for 0.5 second and then closes. Determine $v(t)$ for $t \geq 0$.

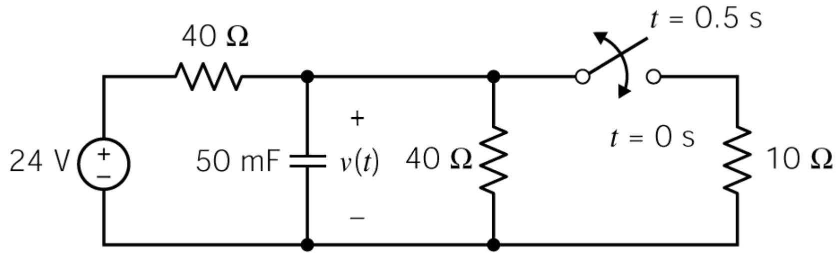
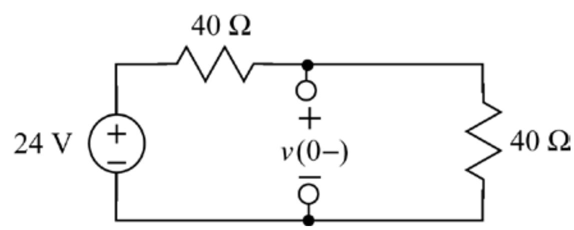



**Figure P 8.4-5**

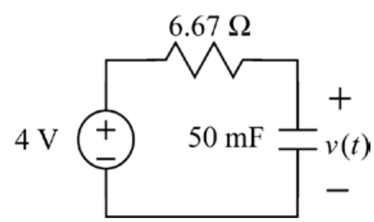
**Solution:**

The circuit is at steady state before the switch closes. The capacitor acts like an open circuit. The initial condition is

$$v(0+) = v(0-) = \left(\frac{40}{40+40}\right)24 = 12 \text{ V}$$




After the switch closes, replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit.




Recognize that $R_t = 6.67\ \Omega$ and $v_{oc} = 4$ V

The time constant is

$$\tau = R_t C = (6.67)(0.05) = 0.335 \text{ s} \quad \Rightarrow \quad \frac{1}{\tau} = 2.988 \approx 3\ \frac{1}{\text{s}}$$

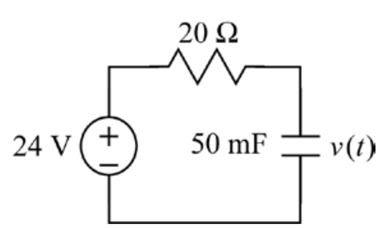
The capacitor voltage is

$$v(t) = \left(v(0+) - v_{oc}\right)e^{-t/\tau} + v_{oc} = (12-4)e^{-3t} + 4 = 4 + 8e^{-3t} \text{ V} \quad \text{for } 0 \geq t \geq 0.5 \text{ s}$$

When the switch opens again at time $t = 0.5$ the capacitor voltage is

$$v(0.5+) = v(0.5-) = 4 + 8e^{-3(0.5)} = 5.785 \text{ V}$$

After time $t = 0.5$ s, replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit.



Recognize that $R_t = 20\ \Omega$ and $v_{oc} = 12$ V

The time constant is

$$\tau = R_t C = 20(0.05) = 1 \quad \Rightarrow \quad \frac{1}{\tau} = 1\ \frac{1}{\text{s}}$$

The capacitor voltage is

$$v(t) = \left(v(0.5+) - v_{oc}\right) e^{-(t-0.5)/\tau} + v_{oc} = (5.785 - 12)e^{-10(t-0.5)} + 12$$
$$= 12 - 6.215e^{-10(t-0.5)} \text{ V} \quad \text{for } t \ge 0.5 \text{ s}$$

so

$$v(t) = \begin{cases} 12 \text{ V} & \text{for } t \ge 0 \\ 4 + 8e^{-3t} \text{ V} & \text{for } 0 \le t \le 0.5 \text{ s} \\ 12 - 6.215e^{-(t-0.5)} \text{ V} & \text{for } t \ge 0.5 \text{ s} \end{cases}$$