Valuation Risk and Asset Pricing*

Rui Albuquerque[†], Martin Eichenbaum[‡], Victor Luo,[§] and Sergio Rebelo[¶]

December 2015

Abstract

Standard representative-agent models fail to account for the weak correlation between stock returns and measurable fundamentals, such as consumption and output growth. This failing, which underlies virtually all modern asset-pricing puzzles, arises because these models load all uncertainty onto the supply side of the economy. We propose a simple theory of asset pricing in which demand shocks play a central role. These shocks give rise to valuation risk that allows the model to account for key asset pricing moments, such as the equity premium, the bond term premium, and the weak correlation between stock returns and fundamentals.

J.E.L. Classification: G12.
Keywords: Equity premium, bond yields, risk premium.

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*We benefited from the comments and suggestions of Fernando Alvarez, Ravi Bansal, Frederico Belo, Jaroslav Borovička, John Campbell, John Cochrane, Lars Hansen, Anisha Ghosh, Ravi Jaganathan, Tasos Karantounias, Howard Kung, Junghoon Lee, Dmitry Livdan, Jonathan Parker, Alberto Rossi, Costis Skiadas, Ivan Werning, and Amir Yaron. We thank Robert Barro, Emi Nakamura, Jón Steinsson, and José Ursua for sharing their data with us and Benjamin Johannsen for superb research assistance. Albuquerque gratefully acknowledges financial support from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement PCOFUND-GA-2009-246542. A previous version of this paper was presented under the title "Understanding the Equity Premium Puzzle and the Correlation Puzzle," http://tinyurl.com/akfmvxb.

[†]Boston College, CEPR, and ECGI.
[‡]Northwestern University, NBER, and Federal Reserve Bank of Chicago.
[§]Northwestern University.
[¶]Northwestern University, NBER, and CEPR.

1. Introduction

In standard representative-agent asset-pricing models, the expected return to an asset reflects the covariance between the asset's payoff and the agent's stochastic discount factor. An important challenge to these models is that the correlation and covariance between stock returns and measurable fundamentals, especially consumption growth, is weak at both short and long horizons. Cochrane and Hansen (1992), Campbell and Cochrane (1999), and Cochrane (2001) call this phenomenon the *correlation puzzle*. More recently, Lettau and Ludvigson (2011) document this puzzle using different methods. According to their estimates, the shock that accounts for the vast majority of asset-price fluctuations is uncorrelated with consumption at virtually all horizons.

The basic disconnect between measurable macroeconomic fundamentals and stock returns underlies virtually all modern asset-pricing puzzles, including the equity-premium puzzle, Hansen-Singleton (1982)-style rejection of asset-pricing models, violation of Hansen-Jagannathan (1991) bounds, and Shiller (1981)-style observations about excess stock-price volatility.

A central finding of modern empirical finance is that variation in asset returns is overwhelmingly due to variation in discount factors (see Campbell and Ammer (1993) and Cochrane (2011)). A key question is: how should we model this variation? In classic asset-pricing models, all uncertainty is loaded onto the supply side of the economy. In Lucas (1978) tree models, agents are exposed to random endowment shocks, while in production economies they are exposed to random productivity shocks. Both classes of models abstract from shocks to the demand for assets. Not surprisingly, it is very difficult for these models to simultaneously resolve the equity premium puzzle and the correlation puzzle.

We propose a simple theory of asset pricing in which demand shocks, arising from stochastic changes in agents' rate of time preference, play a central role in the determination of asset prices. These shocks amount to a parsimonious way of modeling the variation in discount rates stressed by Campbell and Ammer (1993) and Cochrane (2011). Our model implies that the law of motion for these shocks plays a first-order role in determining the equilibrium behavior of variables like the price-dividend ratio, equity returns and bond yields. So, our analysis is disciplined by the fact that the law of motion for time-preference shocks must be consistent with the time-series properties of these variables.

In our model, the representative agent has recursive preferences of the type considered by

Kreps and Porteus (1978), Weil (1989), and Epstein and Zin (1991). When the risk-aversion coefficient is equal to the inverse of the elasticity of intertemporal substitution, recursive preferences reduce to constant-relative risk aversion (CRRA) preferences. We show that, in this case, time-preference shocks have negligible effects on key asset-pricing moments such as the equity premium.

We consider two versions of our model. The benchmark model is designed to highlight the role played by time-preference shocks per se. Here consumption and dividends are modeled as random walks with conditionally homoscedastic shocks. While this model is very useful for expositional purposes, it suffers from some clear empirical shortcomings, e.g. the equity premium is constant. For this reason, we consider an extended version of the model in which the shocks to the consumption and dividend process are conditionally heteroskedastic. We find that adding these features improves the performance of the model.[1]

We estimate our model using a Generalized Method of Moments (GMM) strategy, implemented with annual data for the period 1929 to 2011. We assume that agents make decisions on a monthly basis. We then deduce the model's implications for annual data, i.e. we explicitly deal with the temporal aggregation problem.[2]

It turns out that, for a large set of parameter values, our model implies that the GMM estimators suffer from substantial small-sample bias. This bias is particularly large for moments characterizing the predictability of excess returns and the decomposition of the variance of the price-dividend ratio proposed by Cochrane (1992). In light of this fact, we modify the GMM procedure to focus on the plim of the model-implied small-sample moments rather than the plim of the moments themselves. This modification makes an important difference in assessing the model's empirical performance.

We show that time-preference shocks help explain the equity premium as long as the risk-aversion coefficient and the elasticity of intertemporal substitution are either both greater than one or both smaller than one. This condition is satisfied in the estimated benchmark and extended models.

Allowing for sampling uncertainty, our model accounts for the equity premium and the volatility of stock and bond returns, even though the estimated degree of agents' risk aversion is very moderate (roughly 1.5). Critically, the extended model also accounts for the mean,

[1] These results parallel the findings of Bansal and Yaron (2004) who show that allowing for conditional heteroskedasticity improves the performance of long-run risk models.

[2] Bansal, Kiku, and Yaron (2013) pursue a similar strategy in estimating a long-run risk model. They estimate the frequency with which agents make decisions and find that it is roughly equal to one month.

variance and persistence of the price-dividend ratio and the risk-free rate. In addition, it accounts for the correlation between stock returns and fundamentals such as consumption, output, and dividend growth at short, medium and long horizons. The model also accounts for the observed predictability of excess returns by lagged price-dividend ratios.

We define *valuation risk* as the part of the excess return to an asset that is due to the volatility of the time-preference shock. According to our estimates, valuation risk is a much more important determinant of asset returns than conventional risk. Valuation risk is an increasing function of an asset's maturity, so a natural test of our model is whether it can account for the bond term premia. We show that the model does a very good job at accounting for the level and slope of the nominal yield curve, as well as the standard deviation of nominal yields. The upward sloping nature of the nominal yield curve reflects the fact that our model predicts an upward sloping yield curve for ex-ante real yields on nominal bonds. In fact, in our model the slope of the yield curve is entirely driven by valuation risk.

The last result contrasts sharply with leading alternatives, such as the long-run risk model which generally predicts a downward-sloping real yield curve. With this distinction in mind, we discuss evidence on the slope of the real yield curve obtained using data for the U.S. and the U.K. We also show that the estimated nominal SDF for our model has the properties that Backus and Zin (1994) argue are necessary to simultaneously account for the slope of the yield curve and the persistence of bond yields.

There is a literature that models shocks to the demand for assets as arising from time-preference or taste shocks. For example, Garber and King (1983) and Campbell (1986) consider these types of shocks in early work on asset pricing. Stockman and Tesar (1995), Pavlova and Rigobon (2007), and Gabaix and Maggiori (2013) study the role of taste shocks in explaining asset prices in open economy models. In the macroeconomic literature, Eggertsson and Woodford (2003) and Eggertsson (2004), model changes in savings behavior as arising from time-preference shocks that make the zero lower bound on nominal interest rates binding.[3] Hall (2014) stresses the importance of variation in discount rates in explaining the cyclical behavior of unemployment. Bai, Rios-Rull and Storesletten (2014) stress the importance of demand shocks in generating business cycle fluctuations.

Time-preference shocks can also be thought of a simple way of capturing the notion that fluctuations in market sentiment contribute to the volatility of asset prices, as emphasized

[3] See also Huo and Rios-Rull (2013), Correia, Farhi, Nicolini, and Teles (2013), and Fernandez-Villaverde, Guerron-Quintana, Kuester, Rubio-Ramírez (2013).

by authors such as in Barberis, Shleifer, and Vishny (1998) and Dumas, Kurshev and Uppal (2009). Finally, in independent work, contemporaneous with our own, Maurer (2012) explores the impact of time-preference shocks in a calibrated continuous-time representative agent model with Duffie-Epstein (1992) preferences.[4]

Our paper is organized as follows. In Section 2 we document the correlation puzzle using U.S. data for the period 1929 to 2011 and the period 1871 to 2006. In Section 3, we present our benchmark and extended models. We discuss our estimation strategy in Section 4. In Section 5, we present our empirical results. In Section 6, we study the empirical implications of the model for bond term premia, as well as the return on stocks relative to long-term bonds. Section 7 concludes.

2. The correlation puzzle

In this section we examine the correlation between stock returns and fundamentals as measured by the growth rate of consumption, output, dividends, and earnings.

2.1. Data sources

We consider two sample periods: 1929 to 2011 and 1871 to 2006. For the first sample, we obtain nominal stock and bond returns from Kenneth French's website. We use the measure of real consumption expenditures and real Gross Domestic Product constructed by Barro and Ursúa (2011), which we update to 2011 using National Income and Product Accounts data. We compute per-capita variables using total population (POP).[5] We obtain data on real S&P500 earnings and dividends from Robert Shiller's website. We use data from Ibbotson and Associates on the nominal return to one-month Treasury bills, the nominal yield on

[4]Normandin and St-Amour (1998) study the impact of preference shocks in a model similar to ours. Unfortunately, their analysis does not take into account the fact that covariances between asset returns, consumption growth, and preferences shocks depend on the parameters governing preferences and technology. As a result, their empirical estimates imply that preference shocks reduce the equity premium. In addition, they argue that they can explain the equity premium with separable preferences and preference shocks. This claim contradicts the results in Campbell (1986) and the theorem in our Appendix B.

[5]This series is not subject to a very important source of measurement error that affects another commonly-used population measure, civilian noninstitutional population (CNP16OV). Every ten years, the CNP16OV series is adjusted using information from the decennial census. This adjustment produces large discontinuities in the CNP16OV series. The average annual growth rates implied by the two series are reasonably similar: 1.2 for POP and 1.4 for CNP16OV for the period 1952-2012. But the growth rate of CNP16OV is three times more volatile than the growth rate of POP. Part of this high volatility in the growth rate of CNP16OV is induced by large positive and negative spikes that generally occur in January. For example, in January 2000, 2004, 2008, and 2012 the annualized percentage growth rates of CNP16OV are 14.8, -1.9, -2.8, and 8.4, respectively. The corresponding annualized percentage growth rates for POP are 1.1, 0.8, 0.9, and 0.7.

intermediate-term government bonds (with approximate maturity of five years), and the nominal yield on long-term government bonds (with approximate maturity of twenty years). We convert nominal returns and yields to real returns and yields using the rate of inflation as measured by the consumer price index. We also use inflation-adjusted U.K. government bonds (index-linked gilts) with maturities 2.5, 5, 10, 15, and 25 years.

For the second sample, we use data on real stock and bond returns from Nakamura, Steinsson, Barro, and Ursúa (2013). We use the same data sources for consumption, expenditures, dividends and earnings as in the first sample.

In our estimation, we use two measures of real bond returns. Our primary measure is the ex-ante real return on nominal, one-year, five-year and twenty-year Treasury bonds taken from Luo (2014). Luo (2014) constructs alternative models of expected inflation for one, five and twenty-year horizons. He argues that the random walk model does a better job at forecasting one-year inflation than more sophisticated models like time-varying VAR methods of the sort considered by Primiceri (2005). It also does better than Bayesian vector autoregressions embodying Minnesota priors. However, he argues that the latter does best at forecasting inflation at the five- and twenty-year horizons.

We also report results for a second measure of real bond returns: the realized real return on nominal one-year, five-year and twenty-year Treasury bonds. The one-year rate is the measure of the risk-free rate used in Mehra and Prescott (1985) and the associated literature.

We compute dynamic correlations between stock returns and consumption growth for the G7 and OECD countries. The data on annual real stock returns for the G7 and the OECD countries are from Global Financial Statistics. The data on real per capita personal consumer expenditures and real per capita Gross Domestic Product (GDP) for these countries comes from Barro and Ursúa (2008) and was updated by these authors until 2009. The countries (beginning of sample period) included in our data set are: Australia (1901), Austria (1947), Belgium (1947), Canada (1900), Chile (1900), Denmark (1900), Finland (1923), France (1942), Germany (1851), Greece (1953), Italy (1900), Japan (1894), Korea (1963), Mexico (1902), Netherlands (1947), New Zealand (1947), Norway (1915), Spain (1941), Sweden (1900), Switzerland (1900), United Kingdom (1830), and United States (1869).

2.2. Empirical results

Table 1, panel A presents results for the sample period 1929 to 2011. We report correlations at the one-, five- and ten-year horizons. The five- and ten-year horizon correlations are computed using five- and ten-year overlapping observations, respectively. We report Newey-West (1987) heteroskedasticity-consistent standard errors computed with ten lags.

There are three key features of Table 1, panel A. First, consistent with Cochrane and Hansen (1992) and Campbell and Cochrane (1999), the growth rates of consumption and output are uncorrelated with stock returns at all the horizons that we consider. Second, the correlation between stock returns and dividend growth is similar to that of consumption and output growth at the one-year horizon. However, the correlation between stock returns and dividend growth is substantially higher at the five and ten-year horizons than the analogue correlations involving consumption and output growth. Third, the pattern of correlations between stock returns and dividend growth is similar to the analogue correlations involving earnings growth.

Table 1, panel B reports results for the longer sample period (1871-2006). The one-year correlation between stock returns and the growth rates of consumption and output are very similar to those obtained for the shorter sample. There is evidence in this sample of a stronger correlation between stock returns and the growth rates of consumption and output at a five-year horizon. But, at the ten-year horizon the correlations are, once again, statistically insignificant. The results for dividends and earnings are very similar across the two subsamples.

Table 2 assesses the robustness of our results for the correlation between stock returns and consumption using three different measures of consumption for the period 1929 to 2011, obtained from the National Product and Income Accounts. With one exception, the correlations in this table are statistically insignificant. The exception is the five-year correlation between stock returns and the growth rate of nondurables and services which is marginally significant.

Table 3 reports the correlation between stock returns and the growth rate of consumption and output for the G7, and the OECD. We report correlations at the one-, five- and ten-year horizons. We compute correlations for the G7 and the OECD countries pooling data across all countries. This procedure implies that countries with longer time series receive more weight in the calculations. Again, there is also a relatively weak correlation between consumption and output growth and stock returns at all the horizons we consider.

We have focused on correlations because we find them easy to interpret. One might be concerned that a different picture emerges from the pattern of covariances between stock returns and fundamentals. It does not. For example, using quarterly U.S. data for the period 1959 to 2000, Parker (2001) argues that one would require a risk aversion coefficient of 379 to account for the equity premium given his estimate of the covariance between consumption growth and stock returns.[6]

We conclude this subsection by considering the dynamic correlations between consumption growth and stock returns. Authors like Parker (2001), Campbell (2003), and Jaganathan and Wang (2007) find that in the U.S. post-war quarterly data, stock returns are correlated with future consumption growth.[7] Figure 1 reports the correlation between returns at time t and consumption growth at time $t+\tau$, where τ ranges from -10 years to $+10$ years. The dotted lines are the limits of the 95 percent confidence interval. The figure also reports the analogue correlations involving output growth. Three key features are worth noting. First, for the U.S. and Canada there does appear to be a significant correlation between returns at time t and both consumption and output growth at time $t+1$. Second, this result is not robust across the other G7 countries. Third, there is no other value of τ for which the correlation is significant for any of the countries. So, we infer that, even taking dynamic correlations into account, there is a weak correlation between stock returns and fundamentals, as measured by consumption and output growth. Nevertheless, because we estimate our structural model using U.S. data, we assess its ability to be consistent with the correlations between stock returns at t and consumption growth at $t+1$.

Viewed overall, the results in this section serve as our motivation for introducing shocks to the demand for assets. Classic representative-agent models load all uncertainty onto the supply-side of the economy. As a result, they have difficulty in simultaneously accounting for the equity premium and the correlation puzzle.[8] This difficulty is shared by the habit-formation model proposed by Campbell and Cochrane (1999) and the long-run risk models proposed by Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012). Rare-disaster models of the type proposed by Rietz (1988) and Barro (2006) also share this difficulty because all shocks, disaster or not, are to the supply side of the model. A model with a time-

[6] Parker (2001) finds that there is a larger covariance between current stock returns and the cumulative growth rate of consumption over the next 12 quarters. He shows that using this covariance measure, one would require a risk aversion coefficient of 38 to rationalize the equity premium (see also Grossman, Melino and Shiller (1987)).

[7] In related work, Backus, Routledge, and Zin (2010) show that equity returns is a leading indicator of the business-cycle components of consumption and output growth.

[8] Lynch (1996) and Gârleanu, Kogan, and Panageas (2012) provide interesting analyses of overlapping-generations models which generate an equity premium, even though the correlation between consumption growth and equity returns is low.

varying disaster probability, of the type considered by Wachter (2013) and Gourio (2012), might be able to rationalize the low correlation between consumption and stock returns as a small-sample phenomenon. The reason is that changes in the probability of disasters induce movements in stock returns without corresponding movements in actual consumption growth. This force lowers the correlation between stock returns and consumption in a sample where rare disasters are under represented. This explanation might account for the post-war correlations. But we are more skeptical that it accounts for the results in Table 1, panel B, which are based on the longer sample period, 1871 to 2006, which includes disasters such as the Great Depression and two World Wars.

Below, we focus on demand shocks as the source of the low correlation between stock returns and fundamentals, rather than the alternatives just mentioned. We model these demand shocks in the simplest possible way by introducing shocks to the time preference of the representative agent. Consistent with the references in the introduction, these shocks can be thought of as capturing changes in agents' attitudes towards savings or, more generally, investor sentiment.

3. The model

In this section, we study the properties of a representative-agent endowment economy modified to allow for time-preference shocks. The representative agent has the constant-elasticity version of Kreps-Porteus (1978) preferences used by Epstein and Zin (1991) and Weil (1989). The life-time utility of the representative agent is a function of current utility and the certainty equivalent of future utility, U^*_{t+1}:

$$U_t = \max_{C_t} \left[\lambda_t C_t^{1-1/\psi} + \delta \left(U^*_{t+1} \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}, \tag{3.1}$$

where C_t denotes consumption at time t and δ is a positive scalar. The certainty equivalent of future utility is the sure value of $t+1$ lifetime utility, U^*_{t+1} such that:

$$\left(U^*_{t+1} \right)^{1-\gamma} = E_t \left(U^{1-\gamma}_{t+1} \right).$$

The parameters ψ and γ represent the elasticity of intertemporal substitution and the coefficient of relative risk aversion, respectively. The ratio λ_{t+1}/λ_t determines how agents trade off current versus future utility. We assume that this ratio is known at time t.[9] We refer to

[9] We obtain similar results with a version of the model in which the utility function takes the form: $U_t = \left[C_t^{1-1/\psi} + \lambda_t \delta \left(U^*_{t+1} \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}$. The assumption that the agents knows λ_{t+1} at time t is made

λ_{t+1}/λ_t as the time-preference shock. Propositions 6.9 and 6.18 in Skiadas (2009) provide a set of axioms that applies to recursive utility functions with preference shocks.[10]

3.1. The benchmark model

To highlight the role of time-preference shocks, we begin with a very simple stochastic process for consumption:

$$\log(C_{t+1}/C_t) = \mu + \sigma_c \varepsilon^c_{t+1}. \tag{3.2}$$

Here, μ and σ_c are non-negative scalars and ε^c_{t+1} follows an i.i.d. standard-normal distribution.

As in Campbell and Cochrane (1999), we allow dividends, D_t, to differ from consumption. In particular, we assume that:

$$\log(D_{t+1}/D_t) = \mu + \pi_{dc}\varepsilon^c_{t+1} + \sigma_d \varepsilon^d_{t+1}. \tag{3.3}$$

Here, ε^d_{t+1} is an i.i.d. standard-normal random variable that is uncorrelated with ε^c_{t+1}. To simplify, we assume that the average growth rate of dividends and consumption is the same (μ). The parameter $\sigma_d \geq 0$ controls the volatility of dividends. The parameter π_{dc} controls the correlation between consumption and dividend shocks.[11]

The variable $\Lambda_{t+1} = \log(\lambda_{t+1}/\lambda_t)$ determines how agents trade off current versus future utility. This ratio is known at time t. We refer to Λ_{t+1} as the time-preference shock. This shock evolves according to:

$$\Lambda_{t+1} = \rho_\Lambda \Lambda_t + \sigma_\Lambda \varepsilon^\Lambda_t, \tag{3.4}$$

where, ε^Λ_t is an i.i.d. standard-normal random variable. In the spirit of the original Lucas (1978) model, we assume, for now, that ε^Λ_t is uncorrelated with ε^c_t and ε^d_t. We relax this assumption in Subsection 3.4.

to simplify the algebra and is not necessary for any of the key results.

[10] Skiadas (2009) derives a parametric SDF that satisfies the axioms in proposition 6.9 and 6.18 (see his equation 6.35). This SDF can be modified to obtain a generalized Epstein and Zin (1989) parametric utility function with stochastic risk aversion, intertemporal substitution, and time-preference shocks. We thank Soohun Kim and Ravi Jagannathan for pointing this result out to us.

[11] The stochastic process described by equations (3.2) and (3.3) implies that $\log(D_{t+1}/C_{t+1})$ follows a random walk with no drift. This implication is consistent with our data.

The CRRA case In Appendix A we solve this model analytically for the case in which $\gamma = 1/\psi$. Here preferences reduce to the CRRA form:

$$V_t = E_t \sum_{i=0}^{\infty} \delta^i \lambda_{t+i} C_{t+i}^{1-\gamma}, \tag{3.5}$$

with $V_t = U_t^{1-\gamma}$.

We denote the risk-free rate and the rate of return on a claim on consumption by $R_{f,t+1}$, and $R_{c,t+1}$, respectively. The unconditional risk-free rate depends on the persistence and volatility of time-preference shocks:

$$E\left(R_{f,t+1}\right) = \exp\left(\frac{\sigma_\Lambda^2/2}{1-\rho_\Lambda^2}\right)\delta^{-1}\exp(\gamma\mu - \gamma^2\sigma_c^2/2).$$

The unconditional equity premium implied by this model is proportional to the risk-free rate:

$$E\left(R_{c,t+1} - R_{f,t+1}\right) = E\left(R_{f,t+1}\right)\left[\exp\left(\gamma\sigma_c^2\right) - 1\right]. \tag{3.6}$$

Both the average risk-free rate and the volatility of consumption are small in the data. Moreover, the constant of proportionality in equation (3.6), $\exp\left(\gamma\sigma_c^2\right) - 1$, is independent of σ_Λ^2. So, time-preference shocks do not help to resolve the equity premium puzzle when preferences are of the CRRA form.

3.2. Solving the benchmark model

We define the return to the stock market as the return to a claim on the dividend process. The realized gross stock-market return is given by:

$$R_{d,t+1} = \frac{P_{d,t+1} + D_{t+1}}{P_{d,t}}, \tag{3.7}$$

where $P_{d,t}$ denotes the ex-dividend stock price.

It is useful to define the realized gross return to a claim on the endowment process:

$$R_{c,t+1} = \frac{P_{c,t+1} + C_{t+1}}{P_{c,t}}. \tag{3.8}$$

Here, $P_{c,t}$ denotes the price of an asset that pays a dividend equal to aggregate consumption. We use the following notation to define the logarithm of returns on the dividend and consumption claims, the logarithm of the price-dividend ratio, and the logarithm of the price-consumption ratio:

$$\begin{aligned}
r_{d,t+1} &= \log(R_{d,t+1}),\\
r_{c,t+1} &= \log(R_{c,t+1}),\\
z_{dt} &= \log(P_{d,t}/D_t),\\
z_{ct} &= \log(P_{c,t}/C_t).
\end{aligned}$$

In Appendix B we show that the logarithm of the stochastic discount factor (SDF) implied by the utility function defined in equation (3.1) is given by:

$$m_{t+1} = \theta \log(\delta) + \theta \Lambda_{t+1} - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{3.9}$$

where θ is defined as:

$$\theta = \frac{1-\gamma}{1-1/\psi}. \tag{3.10}$$

When $\gamma = 1/\psi$, the case of CRRA preferences, the value of θ is equal to one and the stochastic discount factor is independent of $r_{c,t+1}$.

We solve the model using the approximation proposed by Campbell and Shiller (1988), which involves linearizing the expressions for $r_{c,t+1}$ and $r_{d,t+1}$ and exploiting the properties of the log-normal distribution.[12]

Using a log-linear Taylor expansion we obtain:

$$r_{d,t+1} = \kappa_{d0} + \kappa_{d1} z_{dt+1} - z_{dt} + \Delta d_{t+1}, \tag{3.11}$$

$$r_{c,t+1} = \kappa_{c0} + \kappa_{c1} z_{ct+1} - z_{ct} + \Delta c_{t+1}, \tag{3.12}$$

where $\Delta c_{t+1} \equiv \log(C_{t+1}/C_t)$ and $\Delta d_{t+1} \equiv \log(D_{t+1}/D_t)$. The constants κ_{c0}, κ_{c1}, κ_{d0}, and κ_{d1} are given by:

$$\begin{aligned}
\kappa_{d0} &= \log[1+\exp(z_d)] - \kappa_{d1} z_d,\\
\kappa_{c0} &= \log[1+\exp(z_c)] - \kappa_{c1} z_c,
\end{aligned}$$

$$\kappa_{d1} = \frac{\exp(z_d)}{1+\exp(z_d)}, \quad \kappa_{c1} = \frac{\exp(z_c)}{1+\exp(z_c)},$$

where z_d and z_c are the unconditional mean values of z_{dt} and z_{ct}.

[12] See Hansen, Heaton, and Li (2008) for an alternative solution procedure.

The Euler equations associated with a claim to the stock market and a consumption claim can be written as:

$$E_t\left[\exp\left(m_{t+1}+r_{d,t+1}\right)\right]=1, \tag{3.13}$$

$$E_t\left[\exp\left(m_{t+1}+r_{c,t+1}\right)\right]=1. \tag{3.14}$$

We solve the model using the method of undetermined coefficients. First, we replace m_{t+1}, $r_{c,t+1}$ and $r_{d,t+1}$ in equations (3.13) and (3.14), using expressions (3.11), (3.12) and (3.9). Second, we guess and verify that the equilibrium solutions for z_{dt} and z_{ct} take the form:

$$z_{dt} = A_{d0}+A_{d1}\Lambda_{t+1}, \tag{3.15}$$

$$z_{ct} = A_{c0}+A_{c1}\Lambda_{t+1}. \tag{3.16}$$

This solution has the property that price-dividend ratios are constant, absent movements in Λ_{t+1}. This property results from our assumption that the logarithm of consumption and dividends follow random-walk processes. We compute A_{d0}, A_{d1}, A_{c0}, and A_{c1} using the method of undetermined coefficients.

The equilibrium solution has the property that $A_{d1}, A_{c1}>0$. We show in Appendix B that the conditional expected return to equity is given by:

$$\begin{aligned} E_t\left(r_{d,t+1}\right) &= -\log(\delta)-\Lambda_{t+1}+\mu/\psi \\ &+\left[\frac{(1-\theta)}{\theta}(1-\gamma)^2-\gamma^2\right]\sigma_c^2/2+\pi_{dc}\left(2\gamma\sigma_c-\pi_{dc}\right)/2-\sigma_d^2/2 \\ &+\left\{(1-\theta)\left(\kappa_{c1}A_{c1}\right)\left[2\left(\kappa_{d1}A_{d1}\right)-\left(\kappa_{c1}A_{c1}\right)\right]-\left(\kappa_{d1}A_{d1}\right)^2\right\}\sigma_\Lambda^2/2. \end{aligned} \tag{3.17}$$

Recall that κ_{c1} and κ_{d1} are non-linear functions of the parameters of the model.

Using the Euler equation for the risk-free rate, $r_{f,t+1}$,

$$E_t\left[\exp\left(m_{t+1}+r_{f,t+1}\right)\right]=1,$$

we obtain:

$$\begin{aligned} r_{f,t+1} &= -\log(\delta)-\Lambda_{t+1}+\mu/\psi-(1-\theta)\left(\kappa_{c1}A_{c1}\right)^2\sigma_\Lambda^2/2 \\ &+\left[\frac{(1-\theta)}{\theta}(1-\gamma)^2-\gamma^2\right]\sigma_c^2/2. \end{aligned} \tag{3.18}$$

Equations (3.17) and (3.18) imply that the risk-free rate and the conditional expectation of the return to equity are decreasing functions of Λ_{t+1}. When Λ_{t+1} rises, agents value the future more relative to the present, so they want to save more. Since risk-free bonds are in zero net supply and the number of stock shares is constant, aggregate savings cannot increase. So, in equilibrium, returns on bonds and equity must fall to induce agents to save less.

The approximate response of asset prices to shocks, emphasized by Borovička, Hansen, Hendricks, and Scheinkman (2011) and Borovička and Hansen (2011), can be directly inferred from equations (3.17) and (3.18). The response of the return to stocks and the risk-free rate to a time-preference shock corresponds to that of an AR(1) with serial correlation ρ_Λ.

Using equations (3.17) and (3.18) we can write the conditional equity premium as:

$$\begin{aligned} E_t\left(r_{d,t+1}\right) - r_{f,t+1} &= \pi_{dc}\left(2\gamma\sigma_c - \pi_{dc}\right)/2 - \sigma_d^2/2 \\ &\quad + \kappa_{d1} A_{d1}\left[2\left(1-\theta\right) A_{c1}\kappa_{c1} - \kappa_{d1} A_{d1}\right]\sigma_\Lambda^2/2. \end{aligned} \tag{3.19}$$

We define the compensation for *valuation risk* as the part of the one-period expected excess return to an asset that is due to the volatility of the time preference shock, σ_Λ^2. We refer to the part of the one-period expected excess return that is due to the volatility of consumption and dividends as the compensation for *conventional risk*.

The component of the equity premium that is due to valuation risk, v_d, is given by the last term in equation (3.19). Since the constants A_{c1}, A_{d1}, κ_{c1}, and κ_{d1} are all positive, $\theta < 1$ is a necessary condition for valuation risk to help explain the equity premium (recall that θ is defined in equation (3.10)).[13]

The intuition for why valuation risk helps account for the equity premium is as follows. Consider an investor who buys the stock at time t. At some later time, say $t+\tau$, $\tau > 0$, the investor may get a preference shock, say a decrease in $\Lambda_{t+\tau+1}$, and want to increase consumption. Since all consumers are identical, they all want to sell the stock at the same time, so the price of equity falls. Bond prices also fall because consumers try to reduce their holdings of the risk-free asset to raise their consumption in the current period. Since stocks are infinitely-lived compared to the one-period risk-free bond, they are more exposed to this source of risk. So, valuation risk, v_d, gives rise to an equity premium. In the CRRA case ($\theta = 1$) the effect of valuation risk on the equity premium is generally small (see equation 3.6)).

[13] As discussed in Epstein et al (2014), the condition $\theta < 1$ also plays a crucial role in generating a high equity premium in long-run risk models. Because long-run risks are resolved in the distant future, they are more heavily penalized than current risks. For this reason, long-run risk models can generate a large equity premium even when shocks to current consumption are small.

It is interesting to highlight the differences between time-preference shocks and conventional sources of uncertainty, which pertain to the supply-side of the economy. Suppose that there is no risk associated with the physical payoff of assets such as stocks. In this case, standard asset pricing models would imply that the equity premium is zero. In our model, there is a positive equity premium that results from the differential exposure of bonds and stocks to valuation risk. Agents are uncertain about how much they will value future dividend payments. Since Λ_{t+1} is known at time t, this valuation risk is irrelevant for one-period bonds. But, it is not irrelevant for stocks, because they have infinite maturity. In general, the longer the maturity of an asset, the higher is its exposure to time-preference shocks and the larger is the valuation risk.

We conclude by considering the case in which there are supply-side shocks to the economy but agents are risk neutral ($\gamma = 0$). In this case, the component of the equity premium that is due to valuation risk is always positive as long as $\psi < 1$. The intuition is as follows: stocks are long-lived assets whose payoffs can induce unwanted variation in the period utility of the representative agent, $\lambda_t C_t^{1-1/\psi}$. Even when agents are risk neutral, they must be compensated for the risk of this unwanted variation.

3.3. Relation to the long-run risk model

In this subsection, we briefly comment on the relation between our model and the long-run-risk model pioneered by Bansal and Yaron (2004). Both models emphasize low-frequency shocks that induce large, persistent changes in the agent's stochastic discount factor.[14] To see this point, it is convenient to re-write the representative agent's utility function, (3.1), as:

$$U_t = \left[\tilde{C}_t^{1-1/\psi} + \delta \left(U_{t+1}^*\right)^{1-1/\psi}\right]^{1/(1-1/\psi)}, \tag{3.20}$$

where $\tilde{C}_t = \lambda_t^{1/(1-1/\psi)} C_t$. Taking logarithms of this expression we obtain:

$$\log\left(\tilde{C}_t\right) = 1/\left(1 - 1/\psi\right)\log(\lambda_t) + \log\left(C_t\right).$$

Bansal and Yaron (2004) introduce a highly persistent component in the process for $\log(C_t)$, which is a source of long-run risk. In contrast, we introduce a highly persistent component into $\log(\tilde{C}_t)$ via our specification of the time-preference shock. From equation (3.9), it is clear that both specifications can induce large, persistent movements in m_{t+1}. Despite this

[14] See Dew-Becker (2014) for a version of a long-run risk model in a production economy.

similarity, the two models are *not* observationally equivalent. First, they have different implications for the correlation between observed consumption growth, $\log(C_{t+1}/C_t)$, and asset returns. Second, the two models have very different implications for the average return to long-term bonds, and the term structure of interest rates. We return to these points when we discuss our empirical results in Section 6. There we present empirical evidence for the implications of different models for the slope of the yield curve. In addition, we assess whether the SDFs of the different models have the properties that Backus and Zin (1994) argue are necessary for explaining the slope of the yield curve and the persistence of bond yields.

3.4. The extended model

The benchmark model just described is useful to highlight the role of time-preference shocks in affecting asset returns. But its simplicity leads to two important empirical shortcomings.[15] First, since consumption is a martingale, the only state variable that is relevant for asset returns is Λ_{t+1}. This property means that all asset returns are highly correlated with each other and with the price-dividend ratio. Second, and related, the model displays constant risk premia and so it cannot generate predictability in excess returns.

In this subsection, we address the shortcomings of the benchmark model by allowing for a richer model of consumption and dividend growth:

$$\log(C_{t+1}/C_t) = \mu_c + \rho_c \log(C_t/C_{t-1}) + \alpha_c \left(\sigma_{t+1}^2 - \sigma^2\right) + \pi_{c\Lambda}\varepsilon_{t+1}^{\Lambda} + \sigma_t \varepsilon_{t+1}^{c}, \tag{3.21}$$

$$\log(D_{t+1}/D_t) = \mu_d + \rho_d \log(D_{t+1}/D_t) + \alpha_d \left(\sigma_{t+1}^2 - \sigma^2\right) + \sigma_d \sigma_t \varepsilon_{t+1}^{d} + \pi_{d\Lambda}\varepsilon_{t+1}^{\Lambda} + \pi_{dc}\sigma_t \varepsilon_{t+1}^{c}, \tag{3.22}$$

and

$$\sigma_{t+1}^2 = \sigma^2 + v\left(\sigma_t^2 - \sigma^2\right) + \sigma_w w_{t+1}, \tag{3.23}$$

where ε_{t+1}^{c}, ε_{t+1}^{d}, $\varepsilon_{t+1}^{\Lambda}$, and w_{t+1} are mutually uncorrelated standard-normal variables. We restrict the unconditional average growth rate of consumption and dividends to be the same:

$$\frac{\mu_c}{1-\rho_c} = \frac{\mu_d}{1-\rho_d}.$$

Relative to the benchmark model, equations (3.21)-(3.23) incorporate three new features. First, motivated by the data, we allow for serial correlation in the growth rates of consumption and dividends. Second, as in Kandel and Stambaugh (1990) and Bansal and Yaron

[15] The shortcomings of our benchmark model are shared by other simple models like model I in Bansal and Yaron (2004), which abstract from conditional heteroskedasticity in consumption and dividends.

(2004), we allow for conditional heteroskedasticity in consumption. This feature generates time-varying risk premia: when volatility is high the stock is risky, its price is low and its expected return is high. High volatility leads to higher precautionary savings motive so that the risk-free rate falls, reinforcing the rise in the risk premium. According to our specification, increases in volatility affects the growth rate of consumption and dividends. This interaction is stressed in the DSGE models proposed by Basu and Bundick (2015), Kung (2015), and Fernandez-Villaverde, Guerron-Quintana, Kuester, and Rubio-Ramírez (2015). A common property of these models is that an increase in the volatility of shocks to the economy affects precautionary savings and reduces consumption. Third, we allow for a correlation between time-preference shocks and the growth rate of consumption and dividends. In a production economy, time-preference shocks would generally induce changes in aggregate consumption. For example, in a simple real-business-cycle model, a persistent increase in Λ_{t+1} would lead agents to reduce current consumption and raise investment in order to consume more in the future. Taken literally, an endowment economy specification does not allow for such a correlation. Importantly, only the innovation to time-preference shocks enters the law of motion for $\log(C_{t+1}/C_t)$ and $\log(D_{t+1}/D_t)$. So, equations (3.21)-(3.23) do not introduce any element of long-run risk into consumption or dividend growth.

Since the price-dividend ratio and the risk-free rate are driven by a single state variable in the benchmark model, they have the same degree of persistence. But, according to our empirical estimates, the price-dividend ratio is more persistent than the risk-free rate. A straightforward way to address this shortcoming of the benchmark model is to assume that the time-preference shock is the sum of a persistent shock and an i.i.d. shock:

$$\begin{aligned} \Lambda_{t+1} &= x_t + \sigma_\eta \eta_{t+1}, \\ x_{t+1} &= \rho_\Lambda x_t + \sigma_\Lambda \varepsilon^\Lambda_{t+1}. \end{aligned} \tag{3.24}$$

Here $\varepsilon^\Lambda_{t+1}$ and η_{t+1} are uncorrelated, i.i.d. standard normal shocks. We think of x_t as capturing low-frequency changes in the growth rate of the discount rate. In contrast, η_{t+1} can be thought of high-frequency changes in investor sentiment that affect the demand for assets (see, for example, Dumas et al. (2009)). If $\sigma_\eta = 0$ and $x_1 = \Lambda_1$, we obtain the specification of the time-preference shock used in the benchmark model. Other things equal, the larger is σ_η, the lower is the persistence of the time-preference shock.

4. Estimation methodology

We estimate the parameters of our model using the Generalized Method of Moments (GMM). Our estimator is the parameter vector $\hat{\Phi}$ that minimizes the distance between a vector of empirical moments, Ψ_D, and the corresponding model moments, $\Psi(\hat{\Phi})$. Our estimator, $\hat{\Phi}$, is given by:

$$\hat{\Phi} = \arg\min_{\Phi} \left[\Psi(\Phi) - \Psi_D\right]' \Omega_D^{-1} \left[\Psi(\Phi) - \Psi_D\right].$$

We found that, for a wide range of parameter values, the model implies that there is small-sample bias in terms of various moments, especially the predictability of excess returns. We therefore focus on the plim of the model-implied small-sample moments when constructing $\Psi(\Phi)$, rather than the plim of the moments themselves. For a given parameter vector, Φ, we create 500 synthetic time series, each of length equal to our sample size. For each sample, we calculate the sample moments of interest. The vector $\Psi(\Phi)$ that enters the criterion function is the median value of the sample moments across the synthetic time series.[16] In addition, we assume that agents make decisions at a monthly frequency and derive the model's implications for variables computed at an annual frequency. We estimate Ψ_D using a standard two-step efficient GMM estimator with a Newey-West (1987) weighting matrix that has ten lags. The latter matrix corresponds to our estimate of the variance-covariance matrix of the empirical moments, Ω_D.

When estimating the benchmark model, we include the following 19 moments in Ψ_D: the mean and standard deviation of consumption growth, the mean and standard deviation of dividend growth, the contemporaneous correlation between the growth rate of dividends and the growth rate of consumption, the mean and standard deviation of real stock returns, the mean, standard deviation and autocorrelation of the price-dividend ratio, the mean, standard deviation and autocorrelation of the real risk-free rate, the correlation between stock returns and consumption growth at the one, five and ten-year horizon, the correlation between stock returns and dividend growth at the one, five and ten-year horizon. We constrain the growth rate of dividends and consumption to be the same. In practice, when we estimate the benchmark model we find that the standard deviation of the point estimate of the risk-free rate across the 500 synthetic time series is very large. So here we report results corresponding to the case where we constrain the median risk-free rate to exactly match our point estimate.

[16] As the sample size grows, our estimator becomes equivalent to a standard GMM estimator so that the usual asymptotic results for the distribution of the estimator apply.

For the benchmark model, the vector Φ includes nine parameters: γ, the coefficient of relative risk aversion, ψ, the elasticity of intertemporal substitution, δ, the rate of time preference, σ_c, the volatility consumption growth shocks, π_{dc}, the parameter that controls the correlation between consumption and dividend growth shocks, σ_d, the volatility of dividend growth shocks, ρ_Λ, the persistence of time-preference shocks, σ_Λ, the volatility of the innovation to time-preference shocks, and μ, the mean growth rate of dividends and consumption.

In the extended model, the vector Φ includes nine additional parameters: ρ_c and ρ_d, which control the serial correlation of consumption and dividend growth, α_c and α_d, which control the effect of volatility on the growth rate of consumption and dividends, respectively, $\pi_{c\Lambda}$ and $\pi_{d\Lambda}$, which control the effect of time preference shock innovations on consumption and dividends, respectively, ν, which governs the persistence of volatility, σ_w, the volatility of innovations to volatility, and σ_η, the volatility of transitory shocks to time preference. In estimating the extended model, we add to Ψ_D the following six moments: the first-order serial correlation of consumption growth, the average yield for 5 and 20-year bonds, and the slope coefficients from the regressions of the excess-equity returns over holding periods of 1, 3 and 5 years on the lagged price-dividend ratio.

5. Empirical results

In the main text, we discuss the results that we obtain estimating the models using the ex-ante measures of real bond yields. In appendix D, we report the analogue results obtained estimating the models with ex-post measures of real bond yields.

Table 4 reports our parameter estimates along with standard errors. Several features are worth noting. First, the coefficient of risk aversion is quite low, 1.5 and 2.4, in the benchmark and extended models, respectively. We estimate this coefficient with reasonable precision. Second, for both models, the intertemporal elasticity of substitution is somewhat larger than one. Third, for both models, the point estimates satisfy the necessary condition for valuation risk to be positive, $\theta < 1$. Fourth, the parameter ρ_Λ that governs the serial correlation of the growth rate of Λ_t is estimated to be close to one in both models, 0.991 and 0.992 in the benchmark and extended model, respectively. Fifth, the parameter ν, which governs the persistence of consumption volatility in the extended model, is also quite high (0.997). The high degree of persistence in both the time-preference and the volatility shock are the root cause of the small-sample biases in our estimators.

Table 5 compares the small-sample moments implied by the benchmark and extended models with the estimated data moments. Recall that in estimating the model parameters we impose the restriction that the unconditional average growth rate of consumption and dividends are the same. To assess the robustness of our results to this restriction, we present two versions of the estimated data moments, with and without the restriction. With one exception, the constrained and unconstrained data moment estimates are similar, taking sampling uncertainty into account. The exception is the average growth rate of consumption, where the constrained and unconstrained estimates are statistically different.

Implications for the equity premium Table 5 shows that both the benchmark and extended model give rise to a large equity premium, 5.81 and 3.88, respectively. This result holds even though the estimated degree of risk aversion is quite moderate in both models. In contrast, long-run risk models require a high degree of risk aversion to match the equity premium.

Recall that in order for valuation risk to contribute to the equity premium, θ must be less than one. This condition is clearly satisfied by both our models: the estimated value of θ is -1.6 and -2.6 in the benchmark and extended model, respectively (Table 4). In both cases, θ is estimated quite accurately. Taking sampling uncertainty into account, the benchmark model easily accounts for the equity premium, while the extended model does so marginally. We can easily reject the null hypothesis of $\theta = 1$, which corresponds to the case of constant relative risk aversion.

The basic intuition for why our model generates a high equity premium despite a low coefficient of relative risk aversion is as follows. From the perspective of the model, stocks and bonds are different in two ways. First, the model embodies the conventional source of an equity premium, namely a differential covariance of bonds and stocks with the SDF. Since γ is relatively small, this traditional covariance effect is also small. Second, the model embodies a compensation for valuation risk that is particularly pronounced for stocks because they they have longer maturities than bonds. Recall that, given our timing assumptions, when an agent buys a bond at time t, the agent knows the value of Λ_{t+1}, so the only source of risk are movements in the marginal utility of consumption at time $t+1$. In contrast, the time-t stock price depends on the value of Λ_{t+j}, for all $j > 1$. So, agents are exposed to valuation risk, a risk that is particularly important because time-preference shocks are very persistent.

To assess the importance of valuation risk in accounting for the equity and bond premium, we set the variance of the time-preference shock in the benchmark model to zero. The equity premium falls from 5.8 percent to -0.2. So, in that model the equity premium is driven solely by valuation risk.

Next, we set the variance of various shocks to zero in the extended model. Because these shocks are correlated, the resulting decomposition is not additive. First, we set the variance of the persistent component of the preference shock (σ_Λ) to zero. As a result, the equity premium falls from 3.9 to 2.2. The bond premium, defined as the difference between yields on 20-year bonds and 1-year bonds falls from 2.2 percent to -0.6. Second, we make all the shocks in the model conditionally homoscedastic by setting σ_w to zero. The equity premium drops from 3.9 to 2.2, while the bond premium rises from 2.2 to 2.8. Finally, we set both σ_Λ and σ_w to zero. The equity premium falls from 3.9 to -0.5, while the bond premium falls from 2.2 to zero. Based on these results, we infer that both conditional volatility and valuation risk play important roles in generating an equity premium in the extended model. In contrast, the bond premium is entirely driven by valuation risk.

Implications for the risk-free rate A problem with some explanations of the equity premium is that they imply counterfactually high levels of volatility for the risk-free rate (see e.g. Boldrin, Christiano and Fisher (2001)). Table 5 shows that the volatility of the risk-free rate and stock market returns implied by our model are similar to the estimated volatilities in the data.

An empirical shortcoming of the benchmark model is its implication for the persistence of the risk-free rate. Recall that, according to equation (3.18), the risk-free rate has the same persistence as the growth rate of the time-preference shock. Table 5 shows that the AR(1) coefficient of the risk-free rate, as measured by the ex-ante real returns to one-year treasury bills, is only 0.52, with a standard error of 0.07. This estimate is substantially smaller than the analogue statistic implied by the benchmark model (0.90). The extended model does a much better job at accounting for the persistence of the risk-free rate (0.51). In this model there are both transitory and persistent shocks to the risk-free rate. The former account for roughly 44 percent of the variance of Λ_t.

Implications for the correlation puzzle Table 6 reports the model's implications for the correlation of stock returns with consumption and dividend growth.[17] Recall that in the benchmark model consumption and dividends follow a random walk. In addition, the estimated process for the growth rate of the time-preference shock is close to a random walk. So, the correlation between stock returns and consumption growth implied by the model is essentially the same across different horizons. A similar property holds for the correlation between stock returns and dividend growth.

In the extended model, persistent changes in the variance of the growth rate of consumption and dividends can induce persistent changes in the conditional mean of these variables. As a result, this model produces correlations between stock returns and fundamentals that vary across different horizons.

The benchmark model does well at matching the correlation between stock returns and consumption growth in the data, because this correlation is similar at all horizons. In contrast, the empirical correlation between stock returns and dividend growth increases with the time horizon. The estimation procedure chooses to match the long-horizon correlations and does less well at matching the yearly correlation. This choice is dictated by the fact that it is harder for the model to produce a low correlation between stock returns and dividend growth than it is to produce a low correlation between stock returns and consumption growth. This property reflects the fact that the dividend growth rate enters directly into the equation for stock returns (see equation (3.11)).

The extended model does better at capturing the fact that the correlations between equity returns and dividend growth rises with the horizon for two reasons. When volatility is high, the returns to equity are high. Since $\alpha_d < 0$, the growth rate of dividends is low. As a result, the one-year correlation between dividend growth and equity returns is negative. The variance of the shock to the dividend growth rate is mean reverting. So, the effect of a negative value of α_d becomes weaker as the horizon extends. The direct association between equity returns and dividend growth (see equation (3.11)), which induces a positive correlation, eventually dominates as the horizon gets longer.

An additional force that allows the extended model to generate a lower short-term correlation between equity returns and dividend growth, is that the estimated value of $\pi_{d\Lambda}$ is

[17] The moments reported in this table are slightly different from those reported in Table 1. The reason is that the moments in the two tables were computed using different samples. In Table 6, we compute a leaded and a lagged correlation, so we lose one observation in the beginning of the sample and another in the end of the sample.

negative. The estimation algorithm chooses parameters that allow the model to do reasonably well in matching the one- and five-year correlation, at the cost of doing less well at matching the ten-year correlation. Presumably, this choice reflects the greater precision with which the one-year and five-year correlations are estimated relative to the ten-year correlation. Taking sampling uncertainty into account, the extended model matches the correlation between stock returns and consumption growth at different horizons. Interestingly, the correlation between stock returns and consumption growth increases with the horizon.

We conclude by highlighting an important difference between our model and two leading alternatives. The first is the external-habit model proposed by Campbell and Cochrane (1999).[18] Working with their parameter values, we find that the correlation between stock returns and consumption growth are equal to 0.53, 0.69, and 0.62 at the one-, five- and ten-year horizon, respectively.

The second alternative is the long-run risk model proposed by Bansal, Kiku, and Yaron (2012). Working with their parameter values, we find that the correlation between stock returns and consumption growth are equal to 0.34, 0.49, and 0.54 at the one-, five- and ten-year horizon, respectively. Their model also implies correlations between stock returns and dividend growth equal to 0.64, 0.90, and 0.93 at the one-, five- and ten-year horizon, respectively.

Our estimates reported in Table 1 imply that, for both models, the correlations between fundamentals and returns are counterfactually high. The source of this empirical shortcoming is that all the uncertainty in the long-run risk model stems from the endowment process.

Implications for the price-dividend ratio In Table 5 we see that both the benchmark and the extended models match the average price-dividend ratio very well. The benchmark model somewhat underpredicts the persistence and volatility of the price-dividend ratio. The extended model does much better at matching those moments. The moments implied by this model are within two standard errors of their sample counterparts.

Table 7 presents evidence reproducing the well-known finding that excess returns are predictable based on lagged price-dividend ratios. We report the results of regressing excess-equity returns over holding periods of 1, 3 and 5 years on the lagged price-dividend ratio. The slope coefficients are -0.10, -0.27, and -0.42, respectively, while the R-squares are

[18] As in Wachter (2005), we consider a version of the model in which equities are a claim to consumption. We thank Jessica Wacher for sharing her program for solving the Campbell-Cochrane model with us.

0.07, 0.14, and 0.26, respectively.

The analogue results for the benchmark model are shown in the top panel of Table 7. In this model, consumption is a martingale with conditionally homoscedastic innovations. So, by construction, excess returns are unpredictable in population at a monthly frequency. Since we aggregate the model to annual frequency, temporal aggregation produces a small amount of predictability (see column titled "Model (plim)").

Stambaugh (1999) and Boudoukh et al. (2008) argue that the predictability of excess returns may be an artifact of small-sample bias and persistence in the price-dividend ratio. Our results are consistent with this hypothesis. The column labeled "Model (median)" reports the plim of the small moments implied by our model. The slope coefficients for the 1, 5, and 10 year horizons, are -0.05, -0.14 and -0.21, respectively. In each case, the median Monte Carlo point estimates are contained within a two-standard deviation band of the respective data estimates.

Table 7 also presents results for the extended model. Because of conditional heteroskedasticity in consumption, periods of high volatility in consumption growth are periods of high expected equity returns and low equity prices. So, in principle, the model is able to generate predictability in population. At our estimated parameter values this predictability is quite small. But, once we allow for the effects of small-sample bias, the extended model does quite well at accounting for the regression slope coefficients. Finally, we note that the benchmark model understates the regression R-squares. Taking sampling uncertainty into account, the extended model does better on this dimension.

Cochrane (1992) proposes a decomposition of the variation of the price-dividend ratio into three components: excess returns, dividend growth, and the risk-free rate.[19] While this decomposition is not additive, authors like Bansal and Yaron (2004) use it to compare the importance of these three components in the model and data.

In our sample, the point estimate for the percentage of the variation in price–dividend ratio due to excess return fluctuations is 102.2 percent, with a standard error of 30 percent. Dividend growth accounts for -14.5 with a standard error of 13 percent. Finally, the risk-free rate accounts for -20.4 percent with a standard error of 14.8 percent. These results are similar to those in Cochrane (1992) and Bansal and Yaron (2004).

[19] The fraction of the volatility of the price-dividend ratio attributed to variable x is given by $\sum_{j=1}^{15} \Omega^j cov(z_{dt}, x_{t+j})/var(z_{dt})$, where $\Omega = 1/(1+E(R_{d,t+1}))$. See Cochrane (1992) for details.

Based on the small-sample moments implied by the benchmark model, the fraction of the variance of the price-dividend ratio accounted for by excess returns, dividend growth, and the risk-free rate is 34.6, −2.8, and 54.2, respectively. So, this model clearly overstates the importance of the risk-free rate and understates the importance of excess returns in accounting for the variance of the price-dividend ratio.

The extended model does substantially better than the benchmark model. Based on the small-sample moments implied by the extended model, the fraction of the variance of the price-dividend ratio accounted for by excess returns, dividend growth, and the risk-free rate is 51.2, −38.5, and 5.6, respectively. The fraction attributed to excess returns is just within two standard errors of the point estimate. The fraction attributed to dividend growth is well within two standard errors of the point estimate.

It is interesting to contrast these results to those in Bansal and Yaron (2004). Their model also attributes a large fraction of the variance of the price-dividend ratio to excess returns. At the same time, their model substantially overstates the role of dividend growth in accounting for the variance of the price-dividend ratio.

Correlation between the price-dividend ratio and the risk-free rate We now discuss the implications of our model for the correlation between the price-dividend ratio and the risk-free rate. Table 5 reports that our estimate of this correlation is 0.27 with a standard error of 0.11. The benchmark model predicts that this correlation should be sharply negative (−0.95). The intuition for this result is clear. When there is a shock to the rate of time preference, agents want to save more, both in the form of bonds and stocks. So, the price-dividend ratio must rise to clear the equity market and the risk-free rate must fall to clear the bond market. The extended model does much better along this dimension of the data (−0.20). Recall that a positive shock to ε_t^{Λ} drives the risk-free rate down. Since $\pi_{d\Lambda}$ is negative, the same shock lowers the growth rate of dividends which causes the price dividend ratio to fall. So, the extended model embodies a force that generates a positive correlation between the risk-free rate and the price-dividend ratio. This forces helps to substantially lower the sharp negative correlation between these variables in the benchmark model.

It is interesting to compare our model with Bansal, Kiku and Yaron (2012). The long-run risk shock in that model induces a counterfactually sharp positive correlation: 0.65. The intuition is straightforward. A shock that induces a persistent change in the growth rate of dividends causes stock prices to go up since equity now represents a claim to a higher flow of dividends. Dividends do not change much in the short run, so the price-dividend ratio goes up. At the same time, the shock causes agents to want to shift away from bond holdings and into equity. So, the risk-free rate must rise to clear the bond market. Other things equal, the long-run risk shock induces a positive correlation between the price-dividend ratio and the risk-free rate. In contrast to our model, the Bansal, Kiku and Yaron (2012) does not embody any forces that would lead to a negative correlation between the price-dividend ratio and the risk-free rate. The net result is that their model overstates this correlation.

We are hesitant to use this correlation as a litmus test to choose between our model and Bansal, Kiku and Yaron's for two reasons. First, the point estimate of this correlation is sensitive to sample period selection. The correlation is positive (0.39) before 1983 but negative (-0.42) after 1983. Second, the sign of this correlation varies substantially across the G7 countries and is again sensitive to breaking the sample in 1983 (see Table 8).

6. Bond term premia

As we emphasize above, valuation risk plays an important role in generating an equity premium in the benchmark and extended models. Since the valuation premium increases with the maturity of an asset, a natural way to assess the plausibility of our models is to evaluate their implications for nominal and real bond yields.

This section is organized as follows. First, we price nominal bonds using the model-implied nominal SDF. Second, we estimate a stochastic process for inflation and modify the endowment process to allow consumption growth to be correlated with inflation. We then show that the extended model does a good job at accounting for the nominal yield curve. Third, we derive the ex-ante expected real return associated with nominal bonds of different maturities. We argue that, in our model, the upward sloping nature of the nominal yield curve reflects the upward sloping nature of the ex-ante real yield curve. Fourth, we evaluate whether the estimated nominal SDF in our extended model has the properties that Backus and Zin (1994) argue are necessary to simultaneously account for the slope of the yield curve and the persistence of bond yields.

Model implications Let $m^{\$}_{t+1}$ denote the logarithm of the nominal SDF:

$$m^{\$}_{t+1} = \theta \log(\delta) + \theta \log\left(\frac{\lambda_{t+1}}{\lambda_t}\right) - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1} - \iota_{t+1}.$$

Here, ι_{t+1} denotes the rate of inflation computed as $\log(P_{t+1}/P_t)$ where P_t is the economy-wide nominal price level.

Motivated by the specification in Stock and Watson (2007), we assume that inflation evolves according to the following stochastic process:[20]

$$\begin{aligned}
\iota_{t+1} &= \mu_\iota + \chi_t + \sigma_\iota \varepsilon_{t+1}^\iota, \\
\chi_{t+1} &= \rho_\chi \chi_t + \sigma_{\chi t} \varepsilon_{t+1}^\chi, \\
\sigma_{\chi t+1}^2 &= (1 - v_\chi)\, \sigma_\chi^2 + v_\chi \sigma_{\chi t}^2 + \sigma_{\chi e} e_{t+1}.
\end{aligned}$$

Here, ε_{t+1}^ι and ε_{t+1}^χ are uncorrelated, i.i.d. standard normal shocks. We modify the endowment process in the extended model to allow inflation to influence the growth rate of consumption:

$$\Delta c_{t+1} = \mu_c + \rho_c \cdot \Delta c_t + \alpha_c \left(\sigma_{t+1}^2 - \sigma^2\right) + \pi_{c\lambda} \varepsilon_{t+1}^\lambda + \sigma_t \varepsilon_{t+1}^c + \pi_{c\chi} \sigma_{\chi t} \varepsilon_{t+1}^\chi.$$

The only new element relative to our extended-model specification is the term $\pi_{c\chi} \sigma_{\chi t} \varepsilon_{t+1}^\chi$.

Let Θ_π denote the vector of parameters related to the inflation process as well as $\pi_{c\chi}$:

$$\Theta_\pi = \left[\mu_\iota,\ \sigma_\iota^2,\ \rho_\chi,\ v_\chi,\ \sigma_\chi^2,\ \sigma_{\chi e}^2,\ \pi_{c\chi}\right].$$

Our procedure for estimating Θ_π is as follows. Because we are interested in evaluating the implications of the extended model for the nominal yield curve, we hold fixed all the parameters that we estimated for that model. We estimate Θ_π using the simulated method of moments. Given a value for Θ_π, we simulate π_t and Δc_t for $10,000 \times 12$ periods. We time-aggregate the monthly series and compute the model-implied annual moments, $\Psi_\pi\left(\Theta_\pi\right)$. Our estimator is the parameter vector $\hat{\Theta}_\pi$ that minimizes the distance between a vector of empirical moments, Ψ_π, and the corresponding model moments, $\Psi_\pi(\hat{\Theta}_\pi)$:

$$\hat{\Theta}_\pi = \underset{\Theta_\pi}{\arg\min} \left[\Psi_\pi(\Theta_\pi) - \Psi_\pi\right]' \Omega_\pi^{-1} \left[\Psi_\pi(\Theta_\pi) - \Psi_\pi\right].$$

Here Ω_π is the variance-covariance matrix of Ψ_π. The vector Ψ_π is given by:

$$\Psi_\pi = \left[E\left(\pi_t\right), \sigma\left(\pi_t\right), \mathrm{corr}\left(\pi_t,\ \pi_{t-j}\right), \mathrm{corr}\left(z_t,\ z_{t-j}\right),\ \left\{E\left(\left[\pi_t - E\left(\pi_t\right)\right]^4\right)\right\}^{\frac{1}{4}} / \sigma\left(y_t\right), \mathrm{cov}\left(\pi_t,\ \Delta c_t\right)\right].$$

[20]The main differences between our specification and Stock and Watson's (2007) are as follows. First, we assume that shocks to the transitory component of inflation, $\sigma_\iota \varepsilon_{t+1}^\iota$, are homoscedastic. This simplification is motivated by Stock and Watson's finding that there is little variation in the volatility of the transitory component of inflation. Second, we assume that χ_t and $\sigma_{\chi t+1}^2$ follow AR(1) processes instead of random walks. Finally, Stock and Watson (2007) work with a quarterly specification while we work with a monthly specification.

Here, z_t is the squared deviation of inflation from its mean: $z_t = [\pi_t - E(\pi_t)]^2$ and $j =$ 1,2,3. In practice, we found that it is difficult to simultaneously estimate μ_ι and the other parameters of the model. So, we set μ_ι equal to the average monthly rate of inflation in our data.

Table 9 reports our results. The parameters ρ_χ and v_χ are precisely estimated and exceed 0.9, so the level and volatility of inflation are quite persistent. Interestingly, the spillover parameter between inflation and consumption growth is negative (-0.50). However, it is imprecisely estimated with a standard error of 0.39. This imprecision is consistent with the large standard error (1.09) of our estimate of the covariance between consumption growth and inflation (0.68). Clearly, we cannot reject the hypothesis that this covariance is zero.

We reach a similar conclusion when we focus on the correlation between consumption growth and inflation. While the point estimate of this correlation is 0.41, the standard error is 0.20 (see Table 11). So, we cannot reject the hypothesis that this correlation is close to zero.

The correlation for the sample period considered by Piazzesi and Schneider (2007), 1952-2006, is -0.27 with a standard error of 0.14. The analogue numbers for the sample period considered by Wachter (2006), 1952-2004, is -0.27 with a standard error of 0.14. So, for their sample periods, we cannot rule out the hypothesis that this correlation is zero.

6.1. The model's implications for the nominal yield curve

Table 10 reports the mean and standard deviation of nominal yields on short-term (one year) Treasury Bills, intermediate-term government bonds (with approximate maturity of five years), and long-term government bonds (with approximate maturity of twenty years). This table also reports the implication of the estimated extended model for those moments of the data. With one exception, the model does a very good at accounting for the level and slope of the nominal yield curve as well as the standard deviation of nominal yields. The exception is that the model somewhat understates the standard deviation of the 20-year nominal yield.

The upward sloping nature of the nominal yield curve reflects the fact that in our model the ex-ante real yield curve is upward sloping (see Table 13). We derive these ex-ante real yields using the expected rate of inflation implied by the inflation process discussed above.

According to our model, long-term bonds command a positive risk premium that increases with the maturity of the bond because longer maturity assets are more exposed to valuation risk. The one-year, five-year and 20-year ex-ante expected real return on the corresponding nominal bonds are 0.58, 1.46, and 2.81, respectively. The slope of nominal yield curve is virtually identical to the slope of the ex-ante expected real yield curve.

The implications of our model do not depend sensitively on the value of $\pi_{c\chi}$. Suppose we set this parameter to zero, so that there is no interaction between inflation and consumption growth. As Table 10 shows, the implications of the model for the nominal yield curve are virtually unchanged.

It is worth contrasting the predictions of our model for yield curves with those of leading competitors. Long-run risk models of the type pioneered by Bansal and Yaron (2004) can generate upward sloping nominal yield curves (see, e.g. Bansal and Shaliastovich (2013)). But their ability to do so depends critically on the presence of a negative covariance between consumption growth and inflation.

To understand this claim, recall that, as stressed by Piazzesi and Schneider (2007) and Beeler and Campbell (2012), long-run risk models generally imply negative long-term bond yields and a negative bond term premium. Indeed, Beeler and Campbell (2012) find that the return on a 20-year real bond in the Bansal, Kiku and Yaron (2012) model is -0.88. The intuition for this result is that in a long-run risk model agents are concerned that consumption growth may be dramatically lower in some future state of the world. Since real bonds promise a certain payout in all states of the world, they offer insurance against this possibility. The longer the maturity of the real bond, the more insurance it offers and the higher is its price. So, the real term premium is downward sloping. For similar reasons, standard rare-disaster models also imply a downward sloping term structure for real bonds and a negative real yield on long-term bonds (see, for example the benchmark model in Nakamura et al. (2013)).[21]

For these models to generate an upward sloping nominal yield curve there must be forces at work to counteract the impact of a downward-sloping real yield curve. Consider for example Bansal and Shaliastovich (2013) who generate an upward-sloping nominal yield curve in a long-run risk model. The key to their result is that they work with a sample period in which there is a negative correlation between the persistent components of consumption growth and inflation (see their table 4).[22] Given their specification, a shock that produces

[21]According to Nakamura et al. (2013), these implications can be reversed by introducing the possibility of default on bonds and assuming that probability of partial default is increasing in the maturity of the bond.

[22]In a similar vein, Wachter (2006) generates an upward sloping nominal yield curve in an external habits

persistent increases in inflation generates both low real yields on long-term bonds and persistently low consumption growth. So, long-term nominal bonds are particularly risky and the nominal yield curve is upward sloping.

A key question is: how robust is the sign of the correlation between inflation and consumption growth? Table 11 displays that correlation, for different horizons, over the sample periods 1929-2011 and 1952-2011. For the full sample, the point estimates for the one, five, 10 and 20 year correlations are positive. We cannot reject that the one-year correlation is close to zero. In all cases the correlations are not statistically different from zero. For the post-war period the point estimates are negative for the one and five-year correlation but positive for the 10 and 20 year correlation. But, again, none of the correlations are statistically different from zero. We conclude that the strong negative correlation between the growth rate of consumption and inflation is a weak foundation on which to base an upward sloping nominal yield curve.

We now discuss other evidence regarding the slope of the real yield curve. Consider first results based on inflation-index U.K. gilts. Table 12 reports the difference between the yield of a bond with 2.5 years maturity and yields of bonds with 5, 10, 15, and 20 years of maturity. Our benchmark sample period is 1985 to 2015. In every case, the longer duration bond has, on average, a higher yield than the bond with duration 2.5 years.[23] One might be concerned about the effect of the recent financial crisis on these results. As Table 12 indicates, the qualitative results are unaffected if we use a sample period from 1985 to 2006.[24]

Table 12 also reports the slope of the real yield curve for the U.S. based on monthly TIPS data for the period from July 2004 to November 2015. Consistent with Alvarez and Jermann (2005), we find that the term structure of real yields is upward sloping. Also, like Campbell, Shiller and Viceira (2009) we find that the average real yield on long-term TIPS is positive. Over our sample this average is equal to 1.54 with a standard error of 0.21.

We conclude with some suggestive calculations that are premised on our model-based result that the covariance of consumption growth with inflation is not a very important determinant of bond yields. Table 13 reports the mean and standard deviation of ex-ante real

model where innovations to consumption and inflation growth are negatively correlated.

[23] The statistical significance of these average differences depends on whether we use zero or 12 lags in computing Newey-West standard errors.

[24] Evans (1998) and Piazzesi and Schneider (2006) finds that the U.K. real yield curve is downward sloping for the periods January 1983-November 1995 and December 1995-March 2006. Our results indicate that their findings depend on the sample period that they work with.

yields on short-term (one year) Treasury Bills, intermediate-term government bonds (with approximate maturity of five years), and long-term government bonds (with approximate maturity of twenty years). The ex-ante yields on five (twenty) year bonds are computed as the difference between the five (twenty) year nominal yield and the ex-ante five (twenty) year inflation rate (yields are expressed on an annualized basis). In all cases, we compute expected inflation using the inflation process discussed above. The results in Table 13 are consistent with our other evidence: the ex-ante real yield curve is upward sloping and the long-term real yield is positive. This table also shows that our model does a good job at accounting for the level and slope of this measure of the real yield curve.

Accounting for the slope of the yield curve and the persistence of bond yields Backus and Zin (1994) investigate the properties of the log of the nominal SDF that are consistent with two key features of nominal yields: the yield curve is upward sloping and yields are very persistent. These properties can be summarized as follows. First, the log of the nominal SDF must have negative serial correlation to account for the positive slope of the nominal yield curve. Second, the log of the nominal SDF must be close to i.i.d. but still have a small predictable component.[25] The log of the nominal SDF in both our benchmark and extended model satisfy these two properties.

Backus and Zin's (1994) estimate various ARMA representations for the log of the nominal SDF using data on the mean and autocovariance of bond yields.[26] Their best statistical fit for the nominal SDF is an ARMA(2,3). For such a representation, the properties that they stress are most easily seen in the impulse response function of the extended model's log nominal SDF.

To deduce the implications of our model for the ARMA representation of the nominal SDF we proceed as follows. First, we generate 1000 synthetic time series for the log of the nominal SDF, each of length equal to our sample size. For each of the 1000 synthetic time series, we estimate an ARMA(2,3) for the log of the nominal SDF and compute the impulse response function to an innovation. Figure 2 displays the average impulse response functions of the log of the nominal SDF along with a 95 percent confidence interval computed across the 1000 synthetic time series.

[25] See Ljungqvist and Sargent (2000) for a detailed exposition of these two properties.

[26] Bansal and Viswanathan (1993) follow a similar approach but use a semi-nonparametric approximation to the SDF.

The estimated SDF for our extended model has the two characteristics that Backus and Zin (1994) argue to be required to explain an upward sloping nominal yield curve and the persistence of bond yields. We already showed that our model generates an upward sloping nominal yield curve. So, we conclude by briefly discussing the serial correlation of the nominal yields. The first-order autocorrelation of the one, five and 20 year yields are: 0.91 (0.05), 0.95 (0.04) and 0.97 (0.04), where the standard errors are indicated in parenthesis. The analogue model statistics are 0.81, 0.92, and 0.94. So, our model clearly generates empirically plausible persistence in nominal bond yields.

Long-term equity premium The benchmark and extended models imply that the difference between stock returns and 20-year ex-ante real bond yield is roughly 1 percent. In the data, the difference between stock returns and the ex-ante real 20-year bond yields is roughly 4.16 percent with a standard error of 2.39. So, taking sampling uncertainty into account, both the benchmark and extended models are consistent with the data.

In our model, the positive premium that equity commands over long-term bonds reflects the difference between an asset of infinite and twenty-year maturity. Consistent with this perspective, Binsbergen, Hueskes, Koijen, and Vrugt (2011) estimate that 90 (80) percent of the value of the S&P 500 index corresponds to dividends that accrue after the first five (ten) years.

It is important to emphasize that the equity premium in our model is not solely driven by the term premium. One way to see this property is to consider the results of regressing the equity premium on two alternative measures of excess bond yields. The first measure is the difference between yields on bonds of 20-year and 1-year maturities. The second measure is the difference between yields on bonds of 5-year and 1-year maturities. Table 14 reports our results. Not surprisingly, the benchmark model does poorly since the expected equity premium and the term premia are constant. The extended model does better in the sense that the model slopes are within one standard error of the slope point estimates. Also, both models are consistent with the fact that the R^2 in these regressions are quite low.

We conclude with an interesting observation made by Binsbergen, Brandt, and Koijen (2012). Using data over the period 1996 to 2009, these authors decompose the S&P500 index into portfolios of short-term and long-term dividend strips. The first portfolio entitles the holder to the realized dividends of the index for a period of up to three years. The second portfolio is a claim on the remaining dividends. Binsbergen et al (2012) find that the short-term dividend portfolio has a higher risk premium than the long-term dividend portfolio, i.e. there is a negative stock term premium. They argue that this observation is inconsistent with habit-formation, long-run risk models and standard of rare-disaster models. Our model, too, has difficulty in accounting for the Binsbergen et al (2012) negative stock term premium.[27] Of course, our sample is very different from theirs and their negative stock term premium result is heavily influenced by the recent financial crisis (see Binsbergen et al (2011)). Also, Boguth, Carlson, Fisher, and Simutin (2012) argue that the Binsbergen et al (2012) results may be significantly biased because of the impact of small pricing frictions.

7. Conclusion

In this paper we argue that allowing for demand shocks substantially improves the performance of representative-agent, asset-pricing models. Specifically, it allows the model to account for the equity premium, bond term premia, and the correlation puzzle with low degrees of estimated risk aversion. According to our estimates, valuation risk is by far the most important determinant of the equity premium and the bond term premia.

While our model is consistent with many features of stock and bond returns, it does have a number of empirical shortcomings. For example, it is difficult for the model to generate the non-negative correlation observed between the price-dividend ratio and the risk-free rate. As discussed above, long-run risk models generate correlations that are much larger than observed in the data. This result suggests that incorporating long-run risk into our model would improve the model's performance. Similarly, our model understates the positive correlation between stock returns at time t and consumption growth at time $t+1$. We suspect that more sophisticated information structures in which agents receive news about future movements in consumption could help resolve this problem.

[27] Recently, Nakamura et al (2013) show that a time-vaying rare disaster model in which the component of consumption growth due to a rare disaster follows an AR(1) process is consistent with the Binsbergen et al (2012) results. Belo et al. (2013) show that the Binsbergen et al. (2012) result can be reconciled in a variety of models if the dividend process is replaced with processes that generate stationary leverage ratios.

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Table 1

Correlation Between Stock Returns and Per Capita
Growth Rates of Fundamentals

Panel A, 1929-2011

Horizon	Consumption	Output	Dividends	Earnings
1 year	-0.05	0.05	0.05	0.10
	(0.12)	(0.10)	(0.11)	(0.10)
5 years	0.00	0.00	0.30	0.20
	(0.14)	(0.12)	(0.13)	(0.13)
10 years	-0.11	-0.09	0.59	0.30
	(0.20)	(0.14)	(0.14)	(0.11)

Panel B, 1871-2006

Horizon	Consumption	Output	Dividends	Earnings
1 year	0.09	0.14	-0.04	0.13
	(0.09)	(0.10)	(0.10)	(0.10)
5 years	0.40	0.25	0.38	0.44
	(0.18)	(0.14)	(0.15)	(0.18)
10 years	0.25	0.00	0.64	0.41
	(0.18)	(0.11)	(0.17)	(0.13)

Table 2

Correlation Between Stock Returns and Per Capita
Growth Rates of Fundamentals

NIPA measures of consumption, 1929-2011

Horizon	Durables	Non-durables	Non-durables and services
1 year	-0.04	0.05	0.27
	(0.13)	(0.14)	(0.14)
5 years	0.07	-0.08	0.18
	(0.11)	(0.09)	(0.08)
10 years	0.21	-0.20	0.06
	(0.15)	(0.13)	(0.14)

Table 3

Correlation Between Stock Returns and Per Capita
Growth Rates of Fundamentals

Horizon	Consumption	Output	Horizon	Consumption	Output
	G7			OECD	
1 year	0.013	0.178	1 year	0.038	0.125
	(0.061)	(0.079)		(0.033)	(0.045)
5 years	0.189	0.345	5 years	0.130	0.272
	(0.105)	(0.091)		(0.067)	(0.067)
10 years	0.280	0.397	10 years	0.151	0.262
	(0.130)	(0.117)		(0.103)	(0.104)

Table 4

Parameter Estimates and Standard Errors
Models estimated with ex-ante real bond yields

Parameter	Benchmark model	Extended model	Parameter	Benchmark model	Extended model
γ	1.516	2.3961	ρ	0.99132	0.99168
	(0.029616)	(0.11244)		(0.00056369)	(0.00078787)
ψ	1.4567	2.2107	σ_{λ}	0.00058631	0.00038594
	(0.041753)	(0.17437)		(2.8373e-05)	(1.8036e-05)
δ	0.99795	0.99796	Implied value of θ	-1.6459	-2.5492
	(0.0017074)	(8.0205e-05)		(0.19603)	(0.29743)
σ_c	0.0069004	0.0046033	ν	0.00	0.99717
	(0.00014956)	(0.00013664)			(0.00030196)
μ	0.0015644	0.0010428	σ_{ω}	0.00	1.68E-06
	(4.4976e-05)	(7.8579e-05)			(4.9166e-08)
$\pi_{c\lambda}$	0.00	-0.0029185	α_c	0.00	-95.3767
		(0.00023329)			(4.1769)
σ_d	0.017103	0.025274	α_d	0.00	-151.8814
	(0.00039626)	(0.19243)			(22.7162)
π_{dc}	0.00049127	-1.3205	ρ_c	0.00	0.094708
	(0.00044218)	(0.2236)			(0.045391)
$\pi_{d\lambda}$	0.00	-0.010907	ρ_d	0.00	0.33427
		(0.0015436)			(0.11204)
σ_{η}	0.00	0.0077828			
		(0.00024356)			

Table 5

Model estimated with ex-ante real bond yields

Moments	Data (constrained)	Data (unconstrained)	Benchmark model	Extended model
Average growth rate of of consumption	1.4941 (0.30)	2.2197 (0.22413)	1.87	1.38
Average growth rate of dividends	1.4941 (0.30395)	0.12538 (0.65732)	1.87	1.38
Standard deviation of the growth rate of consumption	2.1865 (0.34494)	2.1624 (0.31345)	2.38	2.65
Standard deviation of the growth rate of dividends	6.914 (1.0409)	6.5181 (1.1219)	5.85	7.25
Contemporaneous correlation between consumption growth and dividend growth	0.2215 (0.1006)	0.23722 (0.10649)	0.03	0.29
First-order serial correlation of consumption	0.20014 (0.1475)	0.1584 (0.1240)	-0.01	0.30
Correlation between stock returns and one-period lagged consumption growth	-0.1555 (0.1070)	-0.26065 (0.0999)	0.00	-0.03
Correlation between stock returns and one-period lead consumption growth	0.49504 (0.0670)	0.50929 (0.0490)	0.01	0.17
Average return to equities	7.8252 (1.7396)	6.3948 (1.8804)	5.94	4.00
Standard deviation of return to equities	17.2537 (1.3783)	17.6831 (1.461)	17.37	19.23
Average risk-free rate	0.12613 (0.8088)	0.12087 (0.8088)	0.13	0.13
Standard deviation of the risk-free rate	3.5615 (0.81818)	3.4989 (0.83447)	4.30	4.06
First-order serial correlation of the risk-free rate	0.51802 (0.068326)	0.52154 (0.070816)	0.90	0.51
Equity premium	7.6991 (1.8141)	6.2739 (1.9485)	5.81	3.88
Average price-dividend ratio	3.3761 (0.14887)	3.3775 (0.14887)	3.22	3.32
Standard deviaton of price-dividend ratio	0.47324 (0.074827)	0.44502 (0.081116)	0.31	0.46
First-order serial correlation of price dividend ratio	0.95041 (0.034499)	0.93257 (0.042679)	0.84	0.89
Contemporaneous correlation between risk free rate and price-dividend ratio	0.2736, (0.11356)	0.30705 (0.11909)	-0.95	-0.20

Table 6

Correlation Between Stock Returns and Per Capita Growth Rates of Consumption and Dividends

Model estimated with ex-ante real bond yields

Moments	Data (constrained)	Data (unconstrained)	Benchmark model	Extended model
Consumption growth				
1-year correlation between equity returns and consumption growth	-0.07 (0.11)	-0.05 (0.12)	0.02	-0.19
5-year correlation between equity returns and consumption growth	-0.01 (0.16)	-0.05 (0.13)	0.02	-0.05
10-year correlation between equity returns and consumption growth	-0.08 (0.30)	-0.21 (0.21)	0.03	0.11
Dividend growth				
1-year correlation between equity returns and dividend growth	0.08 (0.11)	0.05 (0.11)	0.34	-0.10
5-year correlation between equity returns and dividend growth	0.22 (0.14)	0.29 (0.14)	0.32	0.03
10-year correlation between equity returns and dividend growth	0.51 (0.22)	0.59 (0.15)	0.38	0.20

Table 7

Predictability of Excess Returns by Price-dividend Ratio at Various Horizons

Model estimated with ex-ante real bond yields

Benchmark model

	Data	Model (median)	Model (plim)	Data	Model (median)	Model (plim)
	Slope coefficient			R-square	(% of values larger than R-square in data)	
1 year	-0.10 (0.03)	-0.05	0.008	0.07	0.01 (0.03)	0.0002
3 years	-0.27 (0.06)	-0.14	0.027	0.14	0.03 (0.08)	0.001
5 years	-0.42 (0.12)	-0.21	0.034	0.26	0.04 (0.05)	0.001

Extended model

	Data	Model (median)	Model (plim)	Data	Model (median)	Model (plim)
1 year	-0.10 (0.03)	-0.07	-0.04	0.07	0.03 (0.13)	0.01
3 years	-0.27 (0.06)	-0.19	-0.10	0.14	0.08 (0.23)	0.04
5 years	-0.42 (0.12)	-0.30	-0.16	0.26	0.12 (0.15)	0.06

Table 8

Correlation of price-dividend ratio and ex-ante risk-free rate

	Sample	Full-sample	Pre 1983	Post 1983
Canada	1952-2014	0.0223	0.288	-0.213
		(0.139)	(0.136)	(0.239)
France	1932-2014	-0.484	-0.576	0.132
		(0.152)	(0.124)	(0.179)
Germany	1968-2014	-0.115	-0.382	-0.117
		(0.105)	(0.189)	(0.277)
Italy	1940-2014	-0.588	-0.725	0.344
		(0.166)	(0.0823)	(0.2)
Japan	1960-2014	0.342	-0.128	0.682
		(0.176)	(0.16)	(0.109)
U.K.	1934-2014	0.246	0.147	-0.253
		(0.122)	(0.0819)	(0.216)
US	1939-2011	0.30705	0.39	-0.42
		(0.11909)	(0.113)	(0.227)

Table 9

Parameter Estimates and Standard Errors
Inflation Process

Parameter		Parameter	
μ_i	0.003	σ_χ^2	2.08E-07
			(1.09E-07)
σ_i^2	4.45E-07	$\sigma_{\chi e}^2$	3.38E-14
	(5.21E-07)		(7.45E-16)
ρ_χ	0.9111	$\pi_{c\chi}$	-0.50433
	(0.0027)		(0.3767)
ν_χ	0.99625		
	(0.00048)		

Table 10

Term Structure of Nominal Bond Yields

Moments	Data	Extended model	Extended model, $\pi_{c\chi} = 0$
	Average yield		
Long-term bonds	5.17 (0.89)	6.31	6.30
Intermediate-term bond	4.59 (0.95)	4.96	4.95
Short-term bond	3.53 (0.93)	4.18	4.16
	Standard deviation of yield		
Long-term bonds	2.63 (0.49)	1.48	1.48
Intermediate-term bond	2.87 (0.52)	2.52	2.52
Short-term bond	2.63 (0.49)	3.56	3.55

Table 11

Correlation between inflation and real consumption growth

Horizon	Sample: 1929-2011	Sample: 1952-2011
1 year	0.41	-0.17
	(0.20)	(0.14)
5 years	0.26	-0.08
	(0.21)	(0.16)
10 years	0.26	0.01
	(0.19)	(0.15)
20 years	0.23	0.23
	(0.19)	(0.28)

Table 12

Slope of the yield curve on U.K. gilts and U.S. TIPS

	Sample	Mean	Newey-West standard errors: Zero lags in weighting matrix	Newey-West standard errors: 12 lags in weighting matrix
Difference between long-term yields and 2.5 year yields, inflation-indexed U.K. gilts				
5 year minus 2.5 year	1985-2015	0.19	0.03	0.10
	1985-2006.Q3	0.09	0.03	0.11
10 year minus 2.5 year	1985-2015	0.38	0.05	0.17
	1985-2006.Q3	0.22	0.05	0.17
15 year minus 2.5 year	1985-2015	0.42	0.06	0.20
	1985-2006.Q3	0.26	0.06	0.19
20 year minus 2.5 year	1985-2015	0.42	0.08	0.26
	1985-2006.Q3	0.24	0.08	0.27
Difference between long-term yields and 5-year yields, U.S. TIPS				
10 year minus 5 year	2004-2015	0.54	0.03	0.09
20 year minus 5 year	2004-2015	0.95	0.05	0.15

Table 13

Term Structure of Ex-ante Real Bond Yields

Moments	Data (ex-ante real yields)	Extended model
Average yield		
Long-term bonds	2.90 (0.84)	2.81
Intermediate-term bond	1.93 (0.99)	1.46
Short-term bond	0.46 (0.78)	0.58
Return to equity minus long-term bond yield	2.54 (2.09)	1.13
Standard deviation of yield		
Long-term bonds	2.59 (0.52)	1.38
Intermediate-term bond	3.14 (0.57)	2.40
Short-term bond	3.85 (0.77)	2.89
Return to equity minus long-term bond yield	20.09 (1.96)	18.55

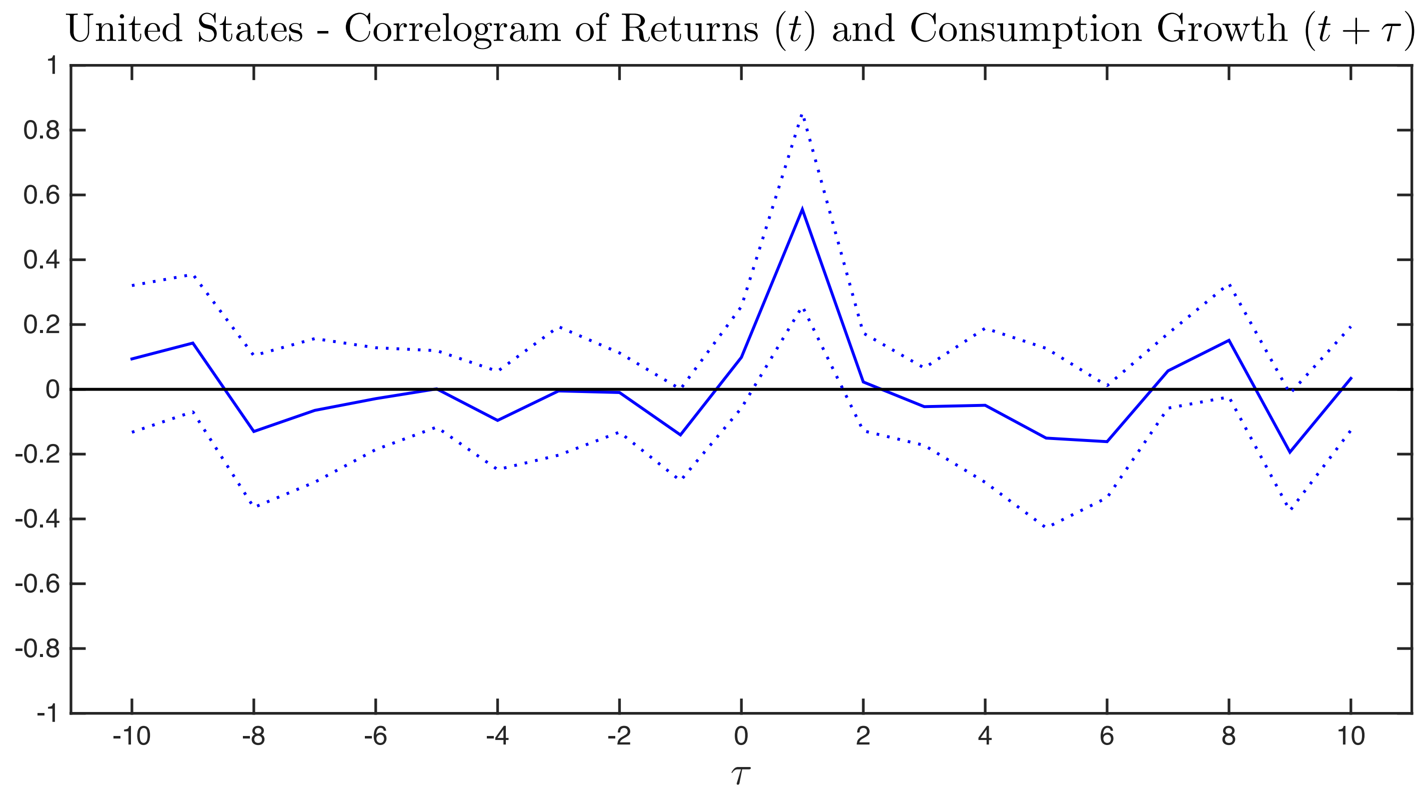
Table 14

Regressions of Excess Stock Returns on Long-term Bond Yields in Excess of Short Rate

	Data 1929-2011	Extended model
	Long-term government bonds (20 years)	
R-square	0.03	0.03
Slope	0.91 (0.46)	0.98
	Intermediate term government bonds (5 years)	
R-square	0.02	0.02
Slope	0.72 (0.51)	1.00

Figure 1: Correlograms of stock returns and fundamentals

Panel A



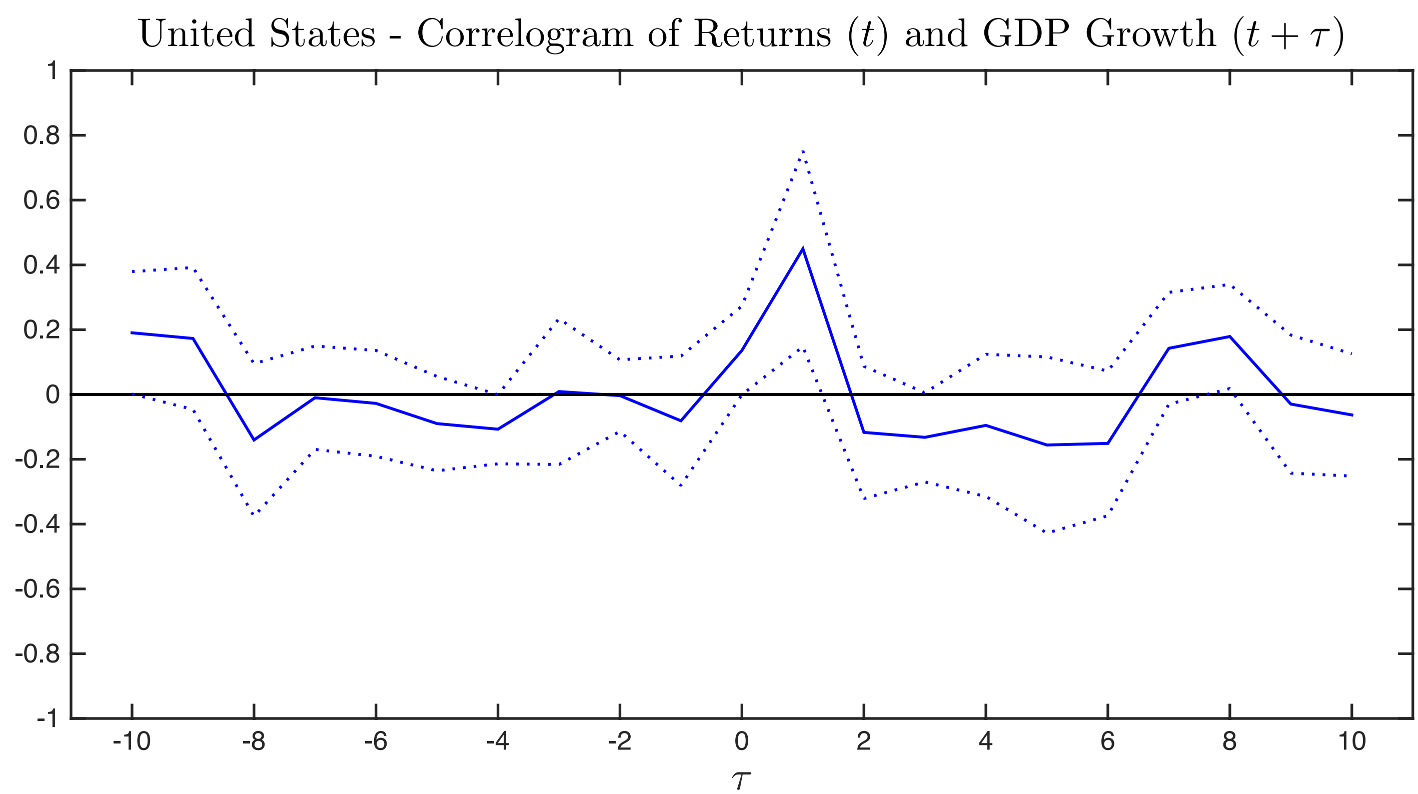


Figure 1: Correlograms of stock returns and fundamentals

Panel B

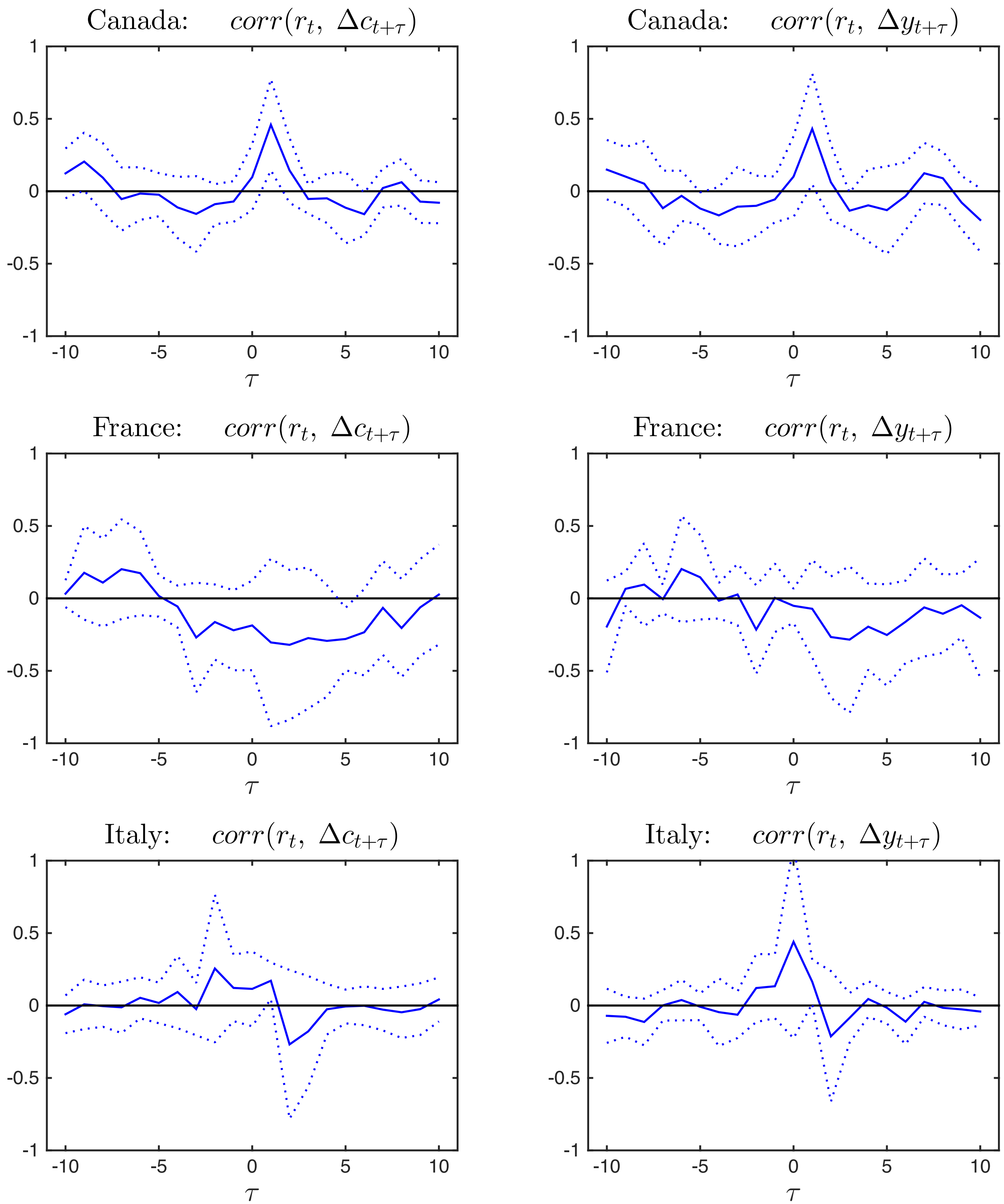


Figure 1: Correlograms of stock returns and fundamentals

Panel C

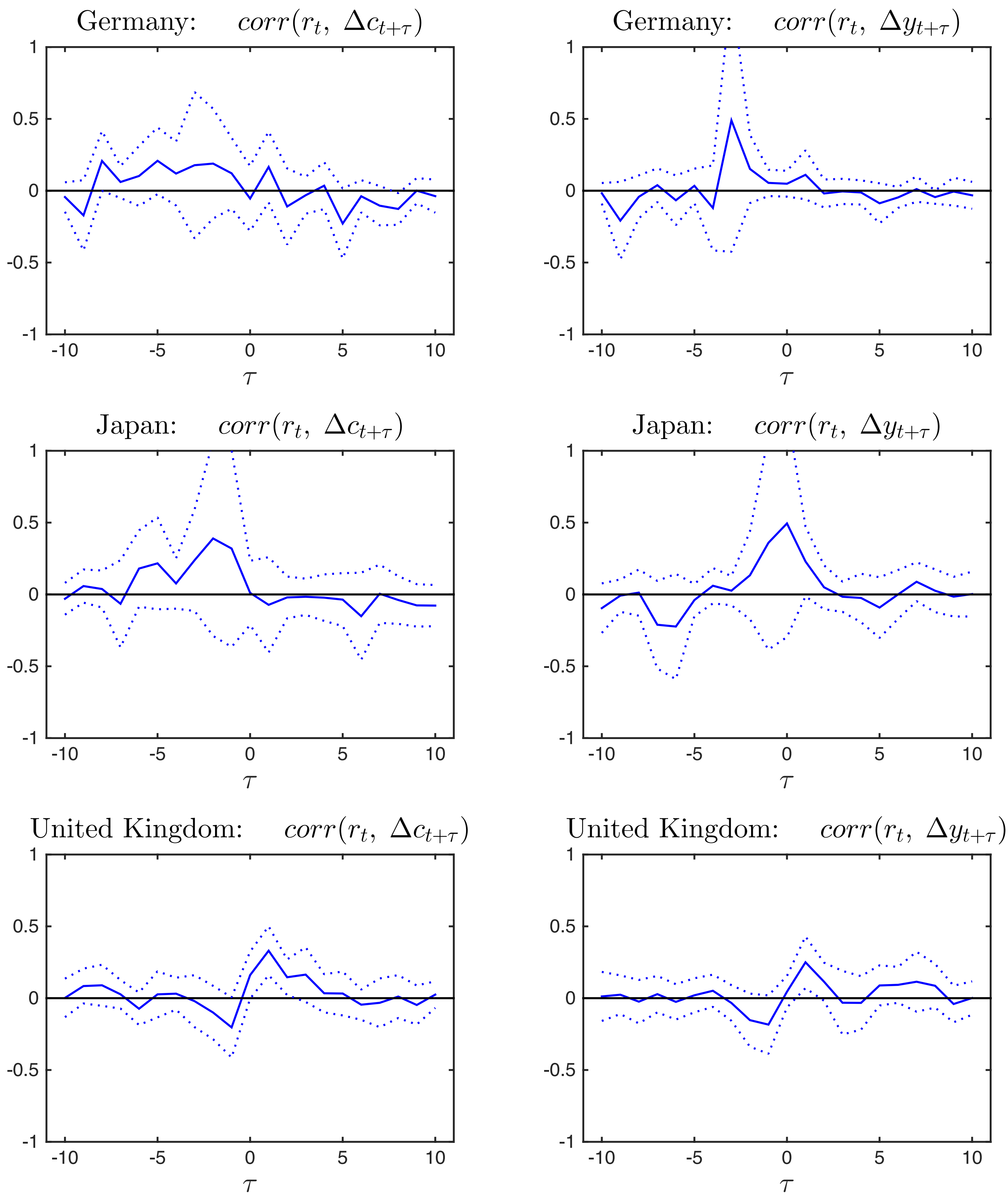
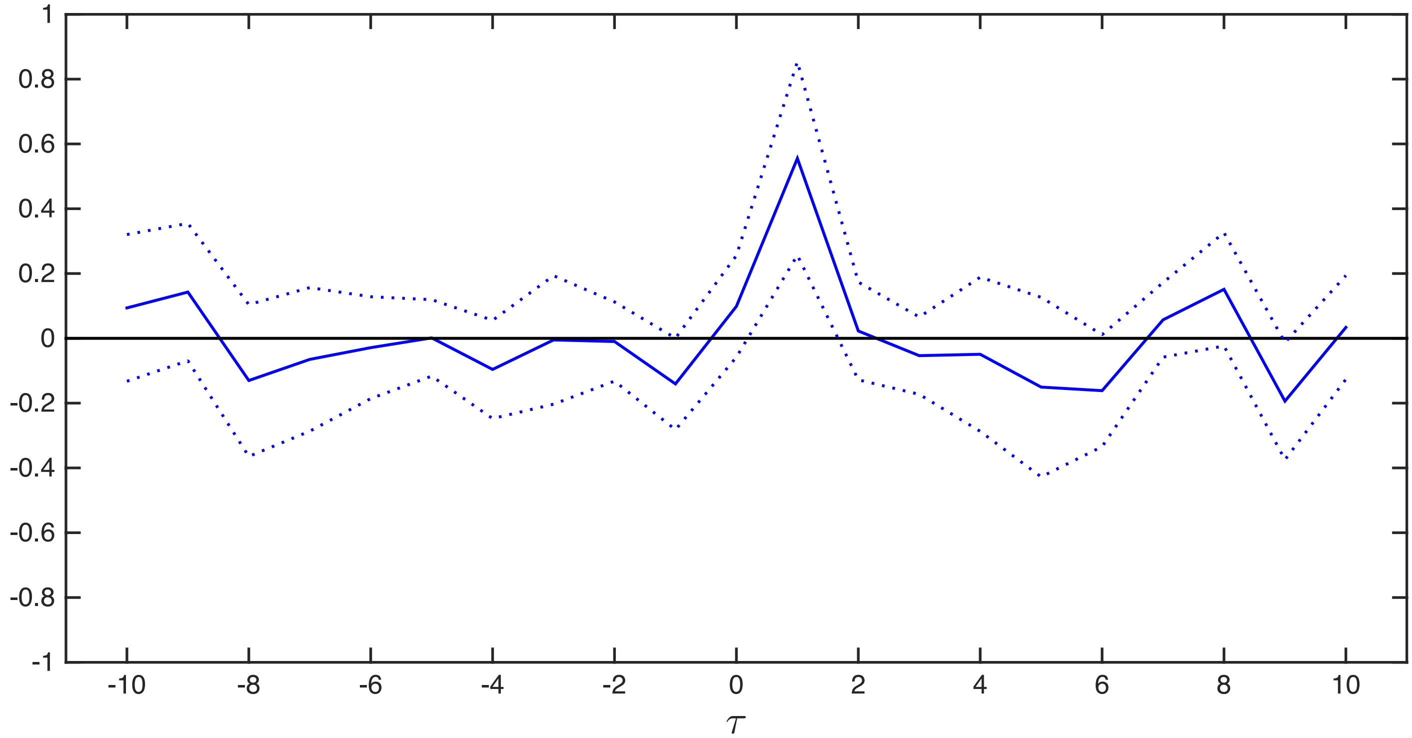


Figure 1: Correlograms of stock returns and fundamentals

Panel A

United States - Correlogram of Returns (t) and Consumption Growth $(t+\tau)$



United States - Correlogram of Returns (t) and GDP Growth $(t+\tau)$

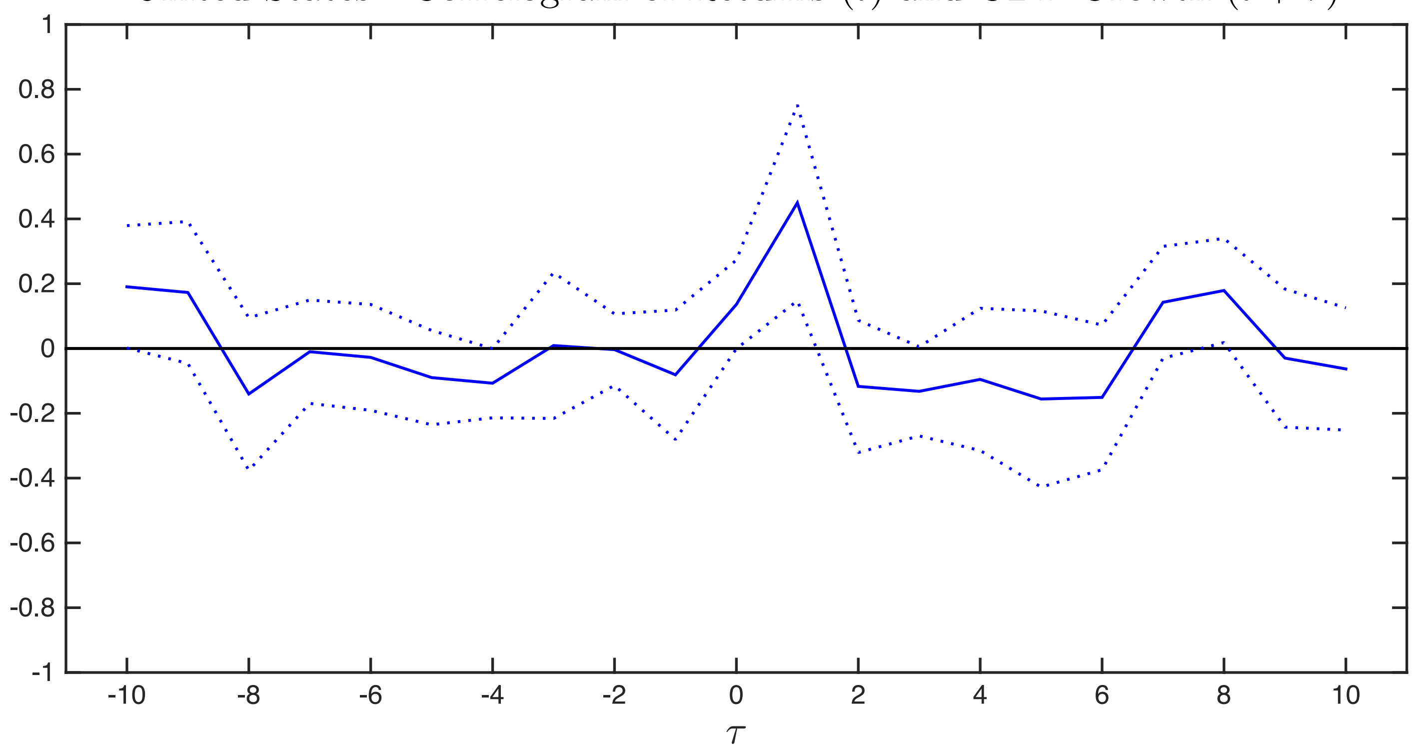


Figure 1: Correlograms of stock returns and fundamentals

Panel B

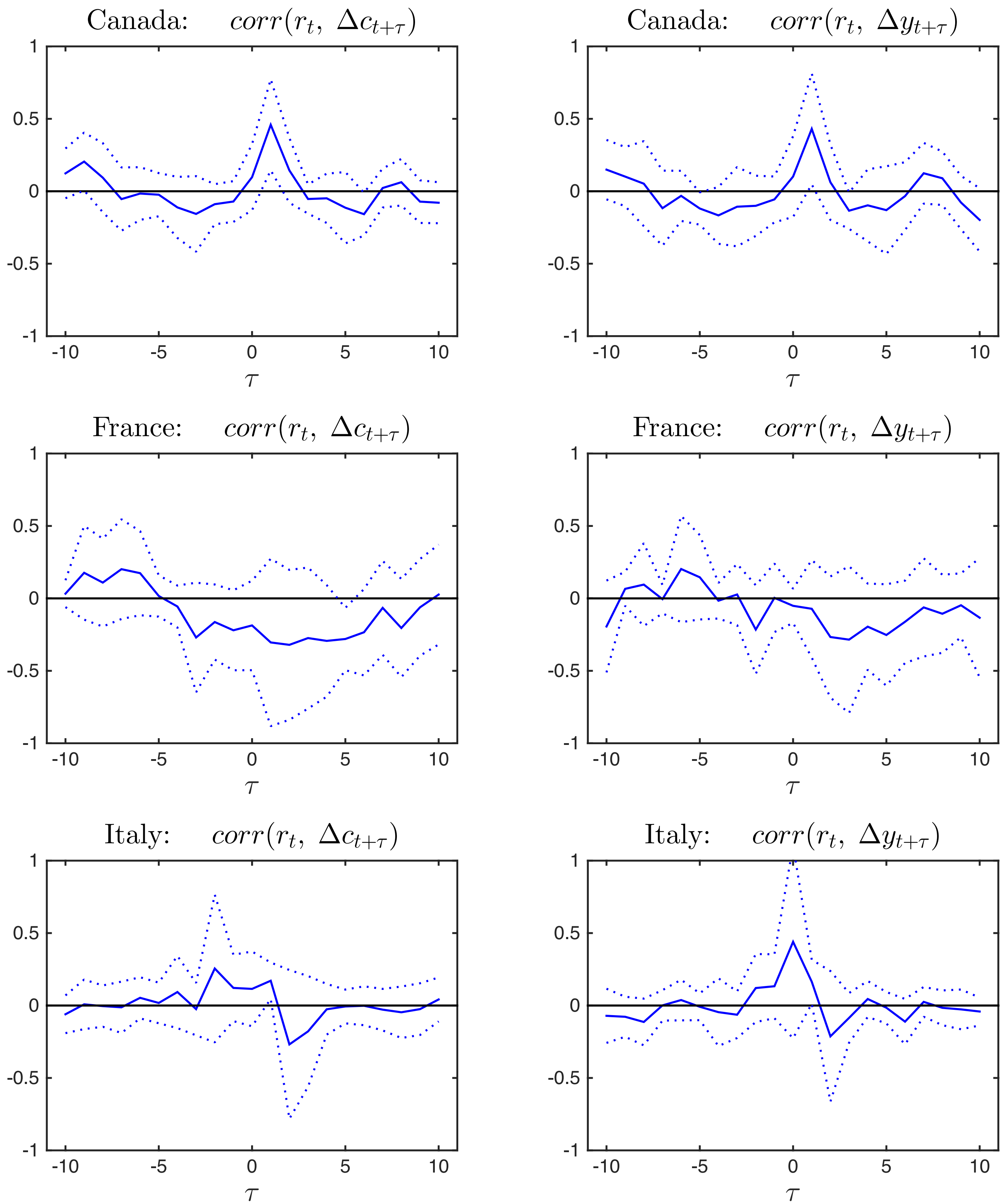


Figure 1: Correlograms of stock returns and fundamentals

Panel C

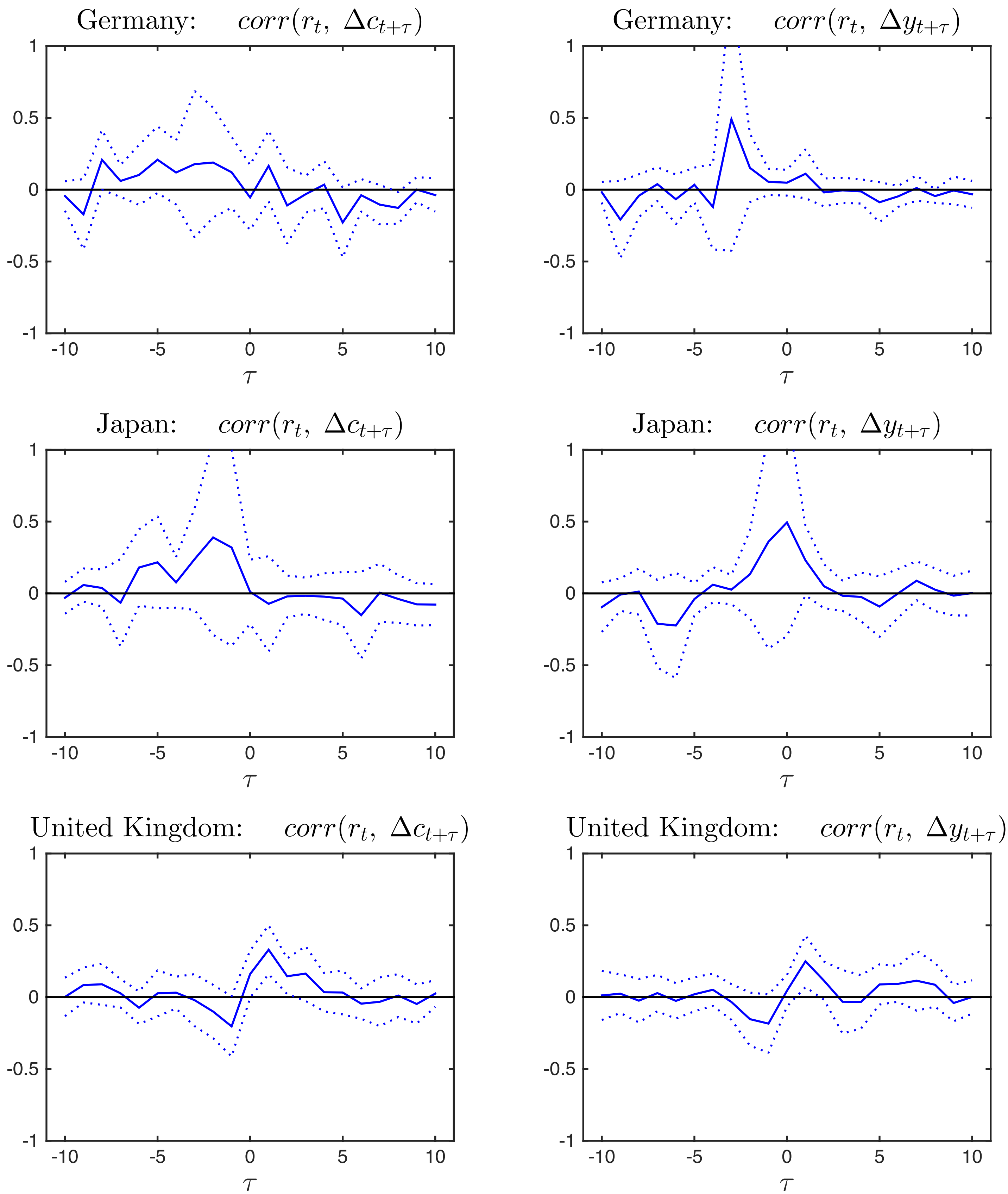
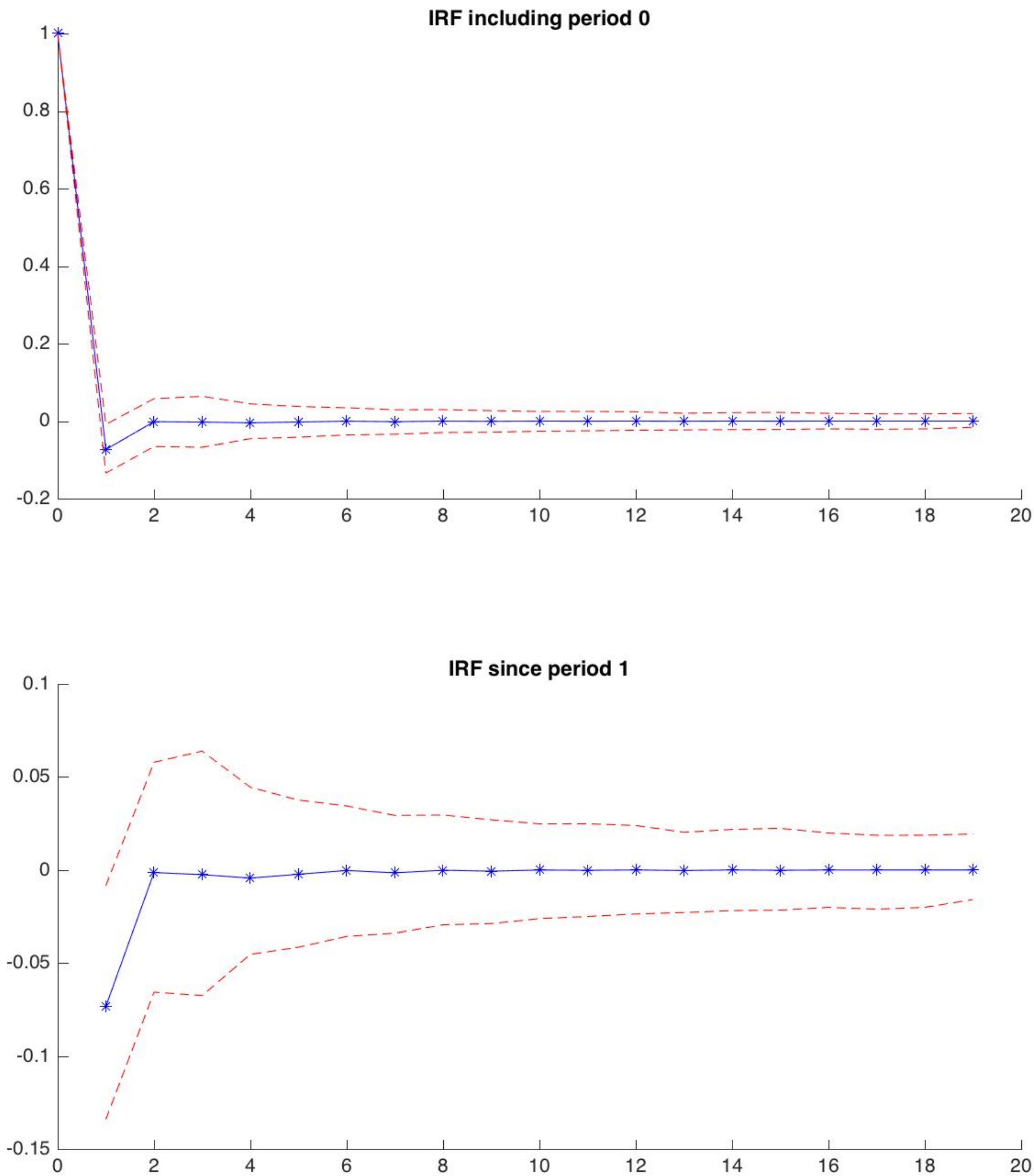


Figure 2: Impulse Response Function of ARMA(2,3) model
estimated from synthetic log SDF series

Panel B: nominal log SDF



8. Appendix

8.1. Appendix A

In this appendix, we solve the model in Section 3 analytically for the case of CRRA utility. Let $C_{a,t}$ denote the consumption of the representative agent at time t. The representative agent solves the following problem:

$$U_t = \max E_t \sum_{i=0}^{\infty} \delta^i \lambda_{t+i} \frac{C_{a,t+i}^{1-\gamma}}{1-\gamma},$$

subject to the flow budget constraints

$$W_{a,i+1} = R_{c,i+1} \left(W_{a,i} - C_{a,i} \right),$$

for all $i \geq t$. The variable $R_{c,i+1}$ denotes the gross return to a claim that pays the aggregate consumption as in equation (3.8), financial wealth is $W_{a,i} = (P_{c,i} + C_i) S_{a,i}$, and $S_{a,i}$ is the number of shares on the claim to aggregate consumption held by the representative agent. The first-order condition for $S_{a,t+i+1}$ is:

$$\delta^i \lambda_{t+i} C_{a,t+i}^{-\gamma} = E_t \left(\delta^{i+1} \lambda_{t+i+1} C_{a,t+i+1}^{-\gamma} R_{c,i+1} \right).$$

In equilibrium, $C_{a,t} = C_t$, $S_{a,t} = 1$. The equilibrium value of the intertemporal marginal rate of substitution is:

$$M_{t+1} = \delta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}. \tag{8.1}$$

The Euler equation for stock returns is the familiar,

$$E_t \left[M_{t+1} R_{c,t+1} \right] = 1.$$

We now solve for $P_{c,t}$. It is useful to write $R_{c,t+1}$ as

$$R_{c,t+1} = \frac{\left(P_{c,t+1}/C_{t+1} + 1 \right)}{P_{c,t}/C_t} \left(\frac{C_{t+1}}{C_t} \right).$$

In equilibrium:

$$E_t \left[M_{t+1} \left(\frac{P_{c,t+1}}{C_{t+1}} + 1 \right) \left(\frac{C_{t+1}}{C_t} \right) \right] = \frac{P_{c,t}}{C_t}. \tag{8.2}$$

Replacing the value of M_{t+1} in equation (8.2):

$$E_t \left[\delta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{P_{c,t+1}}{C_{t+1}} + 1 \right) \left(\frac{C_{t+1}}{C_t} \right) \right] = \frac{P_{c,t}}{C_t}.$$

Using the fact that λ_{t+1}/λ_t is known as of time t we obtain:

$$\delta\frac{\lambda_{t+1}}{\lambda_t}E_t\left[\exp\left(\mu+\sigma_c\varepsilon_{t+1}^c\right)^{1-\gamma}\left(\frac{P_{ct+1}}{C_{t+1}}+1\right)\right]=\frac{P_{ct}}{C_t}.$$

We guess and verify that P_{ct+1}/C_{t+1} is independent of ε_{t+1}^c. This guess is based on the fact that the model's price-consumption ratio is constant absent time-preference shocks. Therefore,

$$\delta\frac{\lambda_{t+1}}{\lambda_t}\exp\left[(1-\gamma)\mu+(1-\gamma)^2\sigma_c^2/2\right]E_t\left(\frac{P_{c,t+1}}{C_{t+1}}+1\right)=\frac{P_{c,t}}{C_t}. \tag{8.3}$$

We now guess that there are constants k_0, k_1,..., such that

$$\frac{P_{c,t}}{C_t}=k_0+k_1\left(\lambda_{t+1}/\lambda_t\right)+k_2\left(\lambda_{t+1}/\lambda_t\right)^{1+\rho}+k_3\left(\lambda_{t+1}/\lambda_t\right)^{1+\rho+\rho^2}+\ \dots \tag{8.4}$$

Using this guess,

$$\begin{aligned}&E_t\left(\frac{P_{c,t+1}}{C_{t+1}}+1\right)\\ =\ &E_t\left(k_0+k_1\left(\left(\lambda_{t+1}/\lambda_t\right)^{\rho}\exp\left(\sigma_\lambda\varepsilon_{t+1}^\lambda\right)\right)+k_2\left(\left(\lambda_{t+1}/\lambda_t\right)^{\rho}\exp\left(\sigma_\lambda\varepsilon_{t+1}^\lambda\right)\right)^{1+\rho}+\dots+1\right)\\ =\ &k_0+k_1\left(\lambda_{t+1}/\lambda_t\right)^{\rho}\exp\left(\sigma_\lambda^2/2\right)+k_2\left(\lambda_{t+1}/\lambda_t\right)^{\rho(1+\rho)}\exp\left((1+\rho)^2\sigma_\lambda^2/2\right)+\dots+1.\end{aligned} \tag{8.5}$$

Substituting equations (8.4) and (8.5) into equation (8.3) and equating coefficients leads to the following solution for the constants k_i:

$$\begin{aligned}k_0&=0,\\ k_1&=\delta\exp\left[(1-\gamma)\mu+(1-\gamma)^2\sigma_c^2/2\right],\end{aligned}$$

and for $n\geq 2$

$$k_n=k_1^n\exp\left\{\left[1+(1+\rho)^2+\left(1+\rho+\rho^2\right)^2+\dots+\left(1+\dots+\rho^{n-2}\right)^2\right]\sigma_\lambda^2/2\right\}.$$

We assume that the series $\{k_n\}$ converges, so that the equilibrium price-consumption ratio is given by equation (8.4). Hence, the realized return on the consumption claim is

$$R_{c,t+1}=\frac{C_{t+1}}{C_t}\frac{k_1\left(\lambda_{t+2}/\lambda_{t+1}\right)+k_2\left(\lambda_{t+2}/\lambda_{t+1}\right)^{1+\rho}+\dots+1}{k_1\left(\lambda_{t+1}/\lambda_t\right)+k_2\left(\lambda_{t+1}/\lambda_t\right)^{1+\rho}+\dots}. \tag{8.6}$$

The equation that prices the one-period risk-free asset is:

$$E_t\left[M_{t+1}R_{f,t+1}\right]=1.$$

Taking logarithms of both sides of this equation and noting that $R_{f,t+1}$ is known at time t, we obtain:

$$r_{f,t+1} = -\log E_t \left(M_{t+1}\right).$$

Using equation (8.1),

$$\begin{aligned} E_t \left(M_{t+1}\right) &= \delta \frac{\lambda_{t+1}}{\lambda_t} E_t \left[\exp\left(-\gamma\left(\mu + \sigma_c \varepsilon_{t+1}^c\right)\right)\right] \\ &= \delta \frac{\lambda_{t+1}}{\lambda_t} \exp\left(-\gamma\mu + \gamma^2\sigma_c^2/2\right). \end{aligned}$$

Therefore,

$$r_{f,t+1} = -\log\left(\delta\right) - \log\left(\lambda_{t+1}/\lambda_t\right) + \gamma\mu - \gamma^2\sigma_c^2/2.$$

Using equation (3.4), we obtain

$$E\left[\left(\lambda_{t+1}/\lambda_t\right)^{-1}\right] = \exp\left(\frac{\sigma_\lambda^2/2}{1-\rho^2}\right).$$

We can then write the unconditional risk-free rate as:

$$E\left(R_{f,t+1}\right) = \exp\left(\frac{\sigma_\lambda^2/2}{1-\rho^2}\right)\delta^{-1}\exp(\gamma\mu - \gamma^2\sigma_c^2/2).$$

Thus, the equity premium is given by:

$$E\left[\left(R_{c,t+1}\right) - R_{f,t+1}\right] = \exp\left(\frac{\sigma_\lambda^2/2}{1-\rho^2}\right)\delta^{-1}\exp(\gamma\mu - \gamma^2\sigma_c^2/2)\left[\exp\left(\gamma\sigma_c^2\right) - 1\right],$$

which can be written as:

$$E\left[\left(R_{c,t+1}\right) - R_{f,t+1}\right] = E\left(R_{f,t+1}\right)\left[\exp\left(\gamma\sigma_c^2\right) - 1\right].$$

8.2. Appendix B

This appendix provides a detailed derivation of the equilibrium of the model economy where the representative agent has Epstein-Zin preferences and faces time-preference shocks. The agent solves the following problem:

$$U\left(W_t\right) = \max_{C_t}\left[\lambda_t C_t^{1-1/\psi} + \delta\left(U_{t+1}^*\right)^{1-1/\psi}\right]^{1/(1-1/\psi)}, \tag{8.7}$$

where $U_{t+1}^* = \left[E_t\left(U\left(W_{t+1}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)}$. The optimization is subject to the following budget constraint:

$$W_{t+1} = R_{c,t+1}\left(W_t - C_t\right).$$

The agent takes as given the stochastic processes for the return on the consumption claim $R_{c,t+1}$ and the preference shock λ_{t+1}. For simplicity, we omit the dependence of life-time utility on the processes for λ_{t+1} and $R_{c,t+1}$.

The first-order condition with respect to consumption is,

$$\lambda_t C_t^{-1/\psi} = \delta \left(U_{t+1}^*\right)^{-1/\psi} \left[E_t \left(U\left(W_{t+1}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)-1} E_t \left(U\left(W_{t+1}\right)^{-\gamma} U'\left(W_{t+1}\right) R_{c,t+1}\right),$$

and the envelope condition is:

$$U'\left(W_t\right) = U\left(W_t\right)^{1/\psi} \delta \left(U_{t+1}^*\right)^{-1/\psi} \left[E_t \left(U\left(W_{t+1}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)-1} E_t \left(U\left(W_{t+1}\right)^{-\gamma} U'\left(W_{t+1}\right) R_{c,t+1}\right).$$

Combining the first-order condition and the envelope condition we obtain:

$$U'\left(W_t\right) = U\left(W_t\right)^{1/\psi} \lambda_t C_t^{-1/\psi}. \tag{8.8}$$

This equation can be used to replace the value of $U'\left(W_{t+1}\right)$ in the first order condition:

$$\lambda_t C_t^{-1/\psi} = \delta \left(U_{t+1}^*\right)^{-1/\psi} \left[E_t \left(U\left(W_{t+1}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)-1} E_t \left(U\left(W_{t+1}\right)^{1/\psi-\gamma} \lambda_{t+1} C_{t+1}^{-1/\psi} R_{c,t+1}\right).$$

Using the expression for U_{t+1}^*, this last equation can be written after some algebra as,

$$1 = E_t \left(M_{t+1} R_{c,t+1}\right). \tag{8.9}$$

Here, M_{t+1} is the stochastic discount factor, or intertemporal marginal rate of substitution, which is given by:

$$M_{t+1} = \delta \frac{\lambda_{t+1}}{\lambda_t} \frac{U\left(W_{t+1}\right)^{1/\psi-\gamma}}{\left(U_{t+1}^*\right)^{1/\psi-\gamma}} \frac{C_{t+1}^{-1/\psi}}{C_t^{-1/\psi}}.$$

We guess and verify the policy function for consumption and the form of the utility function. As in Weil (1989) and Epstein and Zin (1991), we guess that:

$$\begin{aligned} U\left(W_t\right) &= a_t W_t, \\ C_t &= b_t W_t. \end{aligned}$$

Replacing these guesses in equation (8.8) and simplifying yields:

$$a_t^{1-1/\psi} = \lambda_t b_t^{-1/\psi}. \tag{8.10}$$

Using the same guesses in the Hamilton-Jacobi-Bellman equation (8.7) and simplifying, we obtain:

$$a_t = \left[\lambda_t b_t^{1-1/\psi} + \delta \left(\left[E_t \left(\left(a_{t+1} \frac{W_{t+1}}{W_t}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)}\right)^{1-1/\psi}\right]^{1/(1-1/\psi)}.$$

Finally, using the budget constraint to replace W_{t+1}/W_t we obtain:

$$a_t = \left[\lambda_t b_t^{1-1/\psi} + \delta\left(\left[E_t\left(\left(a_{t+1}\left(1-b_t\right)R_{c,t+1}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)}\right)^{1-1/\psi}\right]^{1/(1-1/\psi)}. \tag{8.11}$$

Equations (8.10) and (8.11) give a solution to a_t and b_t.

Combining equations (8.10) and (8.11) we obtain:

$$\lambda_t b_t^{-1/\psi}\left(1-b_t\right) = \delta\left(\left[E_t\left(\left(a_{t+1}\left(1-b_t\right)R_{c,t+1}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)}\right)^{1-1/\psi},$$

which we can replace in the expression for the stochastic discount factor together with (8.10) to obtain:

$$M_{t+1} = \left(\delta\frac{\lambda_{t+1}}{\lambda_t}\right)^{(1-\gamma)/(1-1/\psi)}\left(\frac{b_{t+1}}{b_t}\left(1-b_t\right)\right)^{-((1/\psi-\gamma)/\psi)/(1-1/\psi)}\left(\frac{C_{t+1}}{C_t}\right)^{-1/\psi}\left(R_{c,t+1}\right)^{1/\psi-\gamma}.$$

Now note that $\theta = (1-\gamma)/(1-1/\psi)$, and that

$$\frac{\frac{C_{t+1}}{C_t}}{R_{c,t+1}} = \frac{b_{t+1}R_{c,t+1}\left(W_t - C_t\right)/\left(b_t W_t\right)}{R_{c,t+1}} = \frac{b_{t+1}\left(1-b_t\right)}{b_t},$$

to finally get,

$$M_{t+1} = \left(\delta\frac{\lambda_{t+1}}{\lambda_t}\right)^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}}\left(R_{c,t+1}\right)^{\theta-1}.$$

Taking logarithms of both sides and equating the consumption of the representative agent to aggregate consumption yields equation (3.9).

The nominal SDF is

$$M_{t+1}^{\$} = M_{t+1}\frac{P_t}{P_{t+1}},$$

and that the log-nominal SDF is $m_{t+1}^{\$} = m_{t+1} - \iota_{t+1}$, where ι_{t+1} is the inflation rate.

The rest of the equilibrium derivation involves solving for the real return on the consumption claim, $r_{c,t+1}$, the real return on the dividend claim, $r_{d,t+1}$, and $r_{f,t+1}$ and $r_{f,t+1}^{\$}$, the short term real and nominal interest rates, respectively. Until this point, we did not need to specify the process for the time-preference, consumption growth, and dividend growth shocks, or inflation. We solve the rest of the model assuming the most general processes used in the paper, the inflation process and consumption growth of Section 6 and the processes in equations (3.22) through (3.24) of Subsection 3.4. Recovering the equilibrium values for the benchmark model is done by setting $\pi_{c\lambda} = \pi_{d\lambda} = \sigma_\eta = 0$, $\nu = \sigma_w = \alpha_c = \alpha_d = 0$, and $\pi_{c\chi} = \mu_\iota = \sigma_\iota = \rho_\chi = \nu_\chi = \sigma_\chi = \sigma_{\chi e} = 0$.

To price a real claim on aggregate consumption, we must solve the pricing condition:

$$E_t \left[\exp \left(m_{t+1} + r_{c,t+1} \right) \right] = 1.$$

We guess that the logarithm of the price consumption ratio, $z_{ct} \equiv \log \left(P_{c,t}/C_t \right)$, is

$$z_{ct} = A_{c0} + A_{c1} x_t + A_{c2} \eta_{t+1} + A_{c3} \sigma_t^2 + A_{c4} \Delta c_t + A_{c5} \sigma_{\chi t}^2,$$

and approximate

$$r_{c,t+1} = \kappa_{c0} + \kappa_{c1} z_{ct+1} - z_{ct} + \Delta c_{t+1}. \tag{8.12}$$

Replacing the approximation for $r_{c,t+1}$ on the pricing condition gives

$$E_t \left[\exp \left(\theta \log \left(\delta \right) + \theta \log \left(\lambda_{t+1}/\lambda_t \right) + \left(1 - \gamma \right) \Delta c_{t+1} + \theta \kappa_{c0} + \theta \kappa_{c1} z_{ct+1} - \theta z_{ct} \right) \right] = 1.$$

Computing this expectation requires some algebra and yields the equation

$$\begin{aligned}
0 \;=\; & \theta \log \left(\delta \right) + \left(1 - \gamma + \theta \kappa_{c1} A_{c4} \right) \mu_c - \left(1 - \gamma + \theta \kappa_{c1} A_{c4} \right) \alpha_c \sigma^2 + \theta \kappa_{c0} + \theta \kappa_{c1} A_{c0} - \theta A_{c0} + \theta \kappa_{c1} A_{c5} \left(1 - v_\chi \right) \\
& + \left(\left(1 - \gamma + \theta \kappa_{c1} A_{c4} \right) \alpha_c + \theta \kappa_{c1} A_{c3} \right) \left(1 - \nu \right) \sigma^2 + \left(\left(1 - \gamma + \theta \kappa_{c1} A_{c4} \right) \pi_{c\lambda} + \theta \kappa_{c1} A_{c1} \sigma_\lambda \right)^2 /2 \\
& + \left(\theta \kappa_{c1} A_{c2} \right)^2 /2 + \left(\left(1 - \gamma + \theta \kappa_{c1} A_{c4} \right) \alpha_c + \theta \kappa_{c1} A_{c3} \right)^2 \sigma_w^2 /2 + \left(\theta \kappa_{c1} A_{c5} \sigma_{\chi e} \right)^2 /2 \\
& + \theta x_t - \theta A_{c1} x_t + \theta \kappa_{c1} A_{c1} \rho x_t + \theta \sigma_\eta \eta_{t+1} - \theta A_{c2} \eta_{t+1} - \theta A_{c4} \Delta c_t + \left(1 - \gamma + \theta \kappa_{c1} A_{c4} \right) \rho_c \Delta c_t \\
& - \theta A_{c3} \sigma_t^2 + \left(\left(1 - \gamma + \theta \kappa_{c1} A_{c4} \right) \alpha_c + \theta \kappa_{c1} A_{c3} \right) v \sigma_t^2 + \left(1 - \gamma + \theta \kappa_{c1} A_{c4} \right)^2 \sigma_t^2 /2 \\
& - \theta A_{c5} \sigma_{\chi t}^2 + \left(1 - \gamma + \theta \kappa_{c1} A_{c4} \right)^2 \pi_{c\chi}^2 \sigma_{\chi t}^2 /2 + \theta \kappa_{c1} A_{c5} v_\chi \sigma_{\chi t}^2.
\end{aligned}$$

In equilibrium, this equation must hold in all possible states resulting in the restrictions:

$$A_{c1} = \frac{1}{1 - \kappa_{c1} \rho},$$

$$A_{c2} = \sigma_\eta,$$

$$A_{c3} = \frac{\alpha_c v + \frac{1-\gamma}{1-\kappa_{c1}\rho_c}/2}{1 - \kappa_{c1} v} \frac{1 - 1/\psi}{1 - \kappa_1 \rho_c},$$

$$A_{c4} = \frac{\left(1 - 1/\psi \right) \rho_c}{1 - \kappa_{c1} \rho_c},$$

$$A_{c5} = \frac{\left(1 - \gamma \right) \left(1 - 1/\psi \right) \pi_{c\chi}^2}{2 \left(1 - \kappa_{c1} v_\chi \right) \left(1 - \kappa_{c1} \rho_c \right)^2}.$$

The sign of A_{c5} is the same as that of θ and is independent of the sign of $\pi_{c\chi}$. Finally,

$$
\begin{aligned}
A_{c0} &= \frac{\log(\delta) + (1 - 1/\psi + \kappa_{c1}A_{c4})\mu_c - (1 - 1/\psi + \kappa_{c1}A_{c4})\alpha_c\sigma^2 + \kappa_{c0} + ((1 - 1/\psi + \kappa_{c1}A_{c4})\alpha_c + \kappa_{c1}A_{c}}{1 - \kappa_{c1}} \\
&+ \frac{\theta((1 - 1/\psi + \kappa_{c1}A_{c4})\pi_{c\lambda} + \kappa_{c1}A_{c1}\sigma_\lambda)^2/2 + \theta(\kappa_{c1}A_{c2})^2/2 + \theta((1 - 1/\psi + \kappa_{c1}A_{c4})\alpha_c + \kappa_{c1}A_{c3})^2}{1 - \kappa_{c1}} \\
&+ \frac{\kappa_{c1}A_{c5}(1 - v_\chi)\sigma_\chi^2 + \theta(\kappa_{c1}A_{c5}\sigma_{\chi e})^2/2}{1 - \kappa_{c1}}
\end{aligned}
$$

To solve for the one period real risk-free rate that pays one unit of consumption next period we again use the real SDF. In logarithms, the Euler equation is:

$$
\begin{aligned}
r_{f,t+1} &= -\log(E_t(\exp(m_{t+1}))) \\
&= -\log\left(E_t\left(\exp\left(\theta\log(\delta) + \theta\log(\lambda_{t+1}/\lambda_t) - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)r_{c,t+1}\right)\right)\right).
\end{aligned}
$$

Using equation (8.12), we get:

$$
r_{f,t+1} = -\log(E_t(\exp(\theta\log(\delta) + \theta\log(\lambda_{t+1}/\lambda_t) - \gamma\Delta c_{t+1} + (\theta - 1)(\kappa_{c0} + \kappa_{c1}z_{ct+1} - z_{ct})))).
$$

Substituting in the consumption process and the solution for the price-consumption ratio, and after much algebra, we obtain,

$$
\begin{aligned}
r_t^f &= -\theta\log(\delta) + \widetilde{\gamma}\mu_c - \widetilde{\gamma}\alpha_c\sigma^2 - ((\theta - 1)\kappa_{c1}A_{c2})^2/2 - ((\theta - 1)\kappa_{c1}A_{c1}\sigma_\lambda - \widetilde{\gamma}\pi_{c\lambda})^2/2 \\
&\quad - (\theta - 1)\kappa_{c0} - (\theta - 1)(\kappa_{c1} - 1)A_{c0} - ((\theta - 1)\kappa_{c1}A_{c3} - \widetilde{\gamma}\alpha_c)(1 - \nu)\sigma^2 \\
&\quad - ((\theta - 1)\kappa_{c1}A_{c5}\sigma_{\chi e})^2/2 - (\theta - 1)\kappa_{c1}A_{c5}(1 - v_\chi)\sigma_\chi^2 \\
&\quad - ((\theta - 1)\kappa_{c1}A_{c3} - \widetilde{\gamma}\alpha_c)^2\sigma_w^2/2 - \log(\lambda_{t+1}/\lambda_t) + (1/\psi)\rho_c\Delta c_t \\
&\quad - ((\theta - 1)\kappa_{c1}A_{c3} - \widetilde{\gamma}\alpha_c)v\sigma_t^2 + (\theta - 1)A_{c3}\sigma_t^2 - \widetilde{\gamma}^2\sigma_t^2/2 - (\widetilde{\gamma}\pi_{c\chi})^2\sigma_{\chi t}^2/2 - (\kappa_{c1}v_\chi - 1)(\theta - 1)A_{c5}\sigma_{\chi t}^2,
\end{aligned}
$$

where

$$
\widetilde{\gamma} = \gamma - (\theta - 1)\kappa_{c1}A_{c4}.
$$

Setting the relevant parameters to zero as detailed above, we get the benchmark-model value of the risk-free rate (3.18).

Next, we price a claim that pays a real dividend. Again, we assume the price-dividend ratio is given by

$$
z_{dt} = A_{d0} + A_{d1}x_t + A_{d2}\eta_{t+1} + A_{d3}\sigma_t^2 + A_{d4}\Delta c_t + A_{d5}\Delta d_t + A_{d6}\sigma_{\chi t}^2,
$$

and approximate the log-linearized return to the claim to the dividend:

$$r_{d,t+1} = \kappa_{d0} + \kappa_{d1} z_{dt+1} - z_{dt} + \Delta d_{t+1}. \tag{8.13}$$

The pricing condition is

$$E_t \left[\exp \left(m_{t+1} + r_{d,t+1} \right) \right] = 1.$$

Using the expressions for m_{t+1}, $r_{c,t+1}$ and $r_{d,t+1}$,

$$\begin{aligned} 1 &= E_t \Big[\exp \theta \log (\delta) + \theta \log (\lambda_{t+1}/\lambda_t) - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1)(\kappa_{c0} + \kappa_{c1} z_{ct+1} - z_{ct} + \Delta c_{t+1}) \\ &\quad + \kappa_{d0} + \kappa_{d1} z_{dt+1} - z_{dt} + \Delta d_{t+1} \Big]. \end{aligned}$$

Replacing the consumption and dividend growth processes and of the price-consumption and price-dividend ratios, and repeating similar algebra as above, we obtain:

$$A_{d1} = \frac{1}{1 - \kappa_{d1} \rho},$$

$$A_{d2} = \sigma_\eta,$$

$$A_{d3} = \frac{(1 + \kappa_{d1} A_{d5}) \alpha_d \upsilon - \widehat{\gamma} \alpha_c \upsilon - \left(\alpha_c \upsilon + \frac{1-\gamma}{1-\kappa_{c1}\rho_c}/2 \right) \frac{1/\psi - \gamma}{1-\kappa_{c1}\rho_c} + (1 + \kappa_{d1} A_{d5})^2 \sigma_d^2/2 + ((1 + \kappa_{d1} A_{d5}) \pi_{dc} - \widehat{\gamma})^2}{1 - \kappa_{d1} \upsilon}$$

$$A_{d4} = \frac{-(1/\psi) \rho_c}{1 - \kappa_{d1} \rho_c},$$

$$A_{d5} = \frac{\rho_d}{1 - \kappa_{d1} \rho_d},$$

$$A_{d6} = \frac{((1 + \kappa_{d1} A_{d5}) \pi_{d\chi} - \widehat{\gamma} \pi_{c\chi})^2 /2 + (\theta - 1)(\kappa_{c1} \upsilon_\chi - 1) A_{c5}}{1 - \kappa_{d1} \upsilon_\chi},$$

and

$$\begin{aligned} A_{d0}(1 - \kappa_{d1}) &= \theta \log (\delta) - \widehat{\gamma} \mu_c + (1 + \kappa_{d1} A_{d5}) \mu_d - ((1 + \kappa_{d1} A_{d5}) \alpha_d - \widehat{\gamma} \alpha_c) \sigma^2 + (\theta - 1) \kappa_{c0} + (\theta - 1)(\kappa_c \\ &\quad + ((1 + \kappa_{d1} A_{d5}) \alpha_d - \widehat{\gamma} \alpha_c + (\theta - 1) \kappa_{c1} A_{c3} + \kappa_{d1} A_{d3})(1 - \upsilon) \sigma^2 + ((\theta - 1) \kappa_{c1} A_{c2} + \kappa_{d1} A_d \\ &\quad + ((1 + \kappa_{d1} A_{d5}) \pi_{d\lambda} - \widehat{\gamma} \pi_{c\lambda} + ((\theta - 1) \kappa_{c1} A_{c1} + \kappa_{d1} A_{d1}) \sigma_\lambda)^2 /2 \\ &\quad + ((1 + \kappa_{d1} A_{d5}) \alpha_d - \widehat{\gamma} \alpha_c + (\theta - 1) \kappa_{c1} A_{c3} + \kappa_{d1} A_{d3})^2 \sigma_w^2/2 \\ &\quad + ((\theta - 1) \kappa_{c1} A_{c5} + \kappa_{d1} A_{d6})(1 - \upsilon_\chi) \sigma_\chi^2 + ((\theta - 1) \kappa_{c1} A_{c5} + \kappa_{d1} A_{d6})^2 \sigma_{\chi e}^2/2, \end{aligned}$$

$$\widehat{\gamma} = (\gamma - (\theta - 1) \kappa_{c1} A_{c4} - \kappa_{d1} A_{d4}).$$

To complete the derivation of the equilibrium we must solve for the steady state values of z_c and z_d:

$$z_c = A_{c0} + A_{c3}\sigma^2 + A_{c4}\frac{\mu_c}{1-\rho_c} + A_{c5}\sigma_\chi^2,$$

and

$$z_d = A_{d0} + A_{d3}\sigma^2 + A_{d4}\frac{\mu_c}{1-\rho_c} + A_{d6}\sigma_\chi^2.$$

Having solved for these constants, we can compute the expected return on the dividend claim, $E_t(r_{d,t+1})$. Setting the relevant parameters to zero, we obtain the benchmark-model value of $E_t(r_{d,t+1})$ given in equation (3.17). We now derive the expression for the conditional real risk premium in the benchmark model:

$$E_t(r_{d,t+1}) - r_{f,t+1} = E_t(\kappa_{d0} + \kappa_{d1} z_{dt+1} - z_{dt} + \Delta d_{t+1}) - r_{f,t+1}.$$

Replacing the values of z_{dt} and Δd_{t+1} and computing expectations,

$$E_t(r_{d,t+1}) - r_{f,t+1} = \kappa_{d0} + (\kappa_{d1} - 1) A_{d0} - x_t + \mu - r_{f,t+1}.$$

Replacing the values of A_{d0} and $r_{f,t+1}$ and simplifying, we obtain expression (3.19).

Finally, we price a one period nominal risk free bond:

$$r^{\$}_{f,t+1} = -\log\left(E_t\left(\exp\left(m_{t+1} - \iota_{t+1}\right)\right)\right).$$

It can be shown that the difference between real and nominal interest rates is:

$$r^{\$}_{f,t+1} - r_{f,t+1} = -\sigma_\iota^2/2 + \mu_\iota + \chi_t.$$

8.3. Appendix C

In this appendix we solve for the prices of real and nominal zero-coupon bonds of different maturities. Let $P_t^{(n)}$ be the time-t price of a bond that pays one unit of consumption at $t+n$, with $n \geq 1$. The Euler equation for the one-period risk-free bond price $P_t^{(1)} = 1/R_{f,t+1}$ is

$$P_t^{(1)} = E_t(M_{t+1}).$$

The price for a risk-free bond maturing in $n > 1$ periods can be written recursively as:

$$P_t^{(n)} = E_t\left(M_{t+1} P_{t+1}^{(n-1)}\right).$$

In Appendix B we derive the value of the risk-free rate:

$$r_{f,t+1} = -\ln\left[P_t^{(1)}\right].$$

It is useful to write the risk-free rate as:

$$r_{f,t+1} = -\log\left(\frac{\lambda_{t+1}}{\lambda_t}\right) + (1/\psi)\,\rho_c \Delta c_t - p^1 - q_1\sigma_t^2 - \zeta_1 \sigma_{\chi t}^2,$$

where

$$\begin{aligned}
p^1 \quad = \quad & \theta \log(\delta) - \gamma_1\mu_c + \gamma_1\alpha_c\sigma^2 + (\theta-1)\,\kappa_{c0} + (\theta-1)\,(\kappa_{c1}-1)\,A_{c0} \\
& + ((\theta-1)\,\kappa_{c1}A_{c3} - \gamma_1\alpha_c)\,(1-\nu)\,\sigma^2 \\
& + ((\theta-1)\,\kappa_{c1}A_{c2})^2/2 + ((\theta-1)\,\kappa_{c1}A_{c1}\sigma_\lambda - \gamma_1\pi_{c\lambda})^2/2 \\
& + ((\theta-1)\,\kappa_{c1}A_{c3} - \gamma_1\alpha_c)^2\,\sigma_w^2/2 \\
& + ((\theta-1)\,\kappa_{c1}A_{c5})^2\,\sigma_{\chi e}^2/2 + (\theta-1)\,\kappa_{c1}A_{c5}\,(1-v_\chi)\,\sigma_\chi^2,
\end{aligned}$$

$$q_1 = \gamma_1^2/2 + ((\theta-1)\,\kappa_{c1}A_{c3} - \gamma_1\alpha_c)\,v - (\theta-1)\,A_{c3},$$

$$\zeta_1 = \gamma_1^2\pi_{c\chi}^2/2 + (\kappa_{c1}v_\chi - 1)\,(\theta-1)\,A_{c5},$$

and

$$\gamma_1 = \widetilde{\gamma} = \gamma - (\theta-1)\,\kappa_{c1}A_{c4}.$$

Let $p_t^{(n)} = \ln\left[P_t^{(n)}\right]$. Therefore, $p_t^{(1)} = -r_{f,t+1}$. The price of a risk-free bond that pays one unit of consumption in n periods:

$$p_t^{(n)} = \ln E_t\left[\exp\left(m_{t+1} + p_{t+1}^{(n-1)}\right)\right].$$

After much algebra,

$$p_t^{(n)} = p^n + \left(1+\rho+...+\rho^{n-1}\right)x_t + \sigma_\eta\eta_{t+1} + q_n\sigma_t^2 - (\gamma_n\rho_c + (\theta-1)\,A_{c4})\,\Delta c_t + \zeta_n\sigma_{\chi t}^2,$$

with

$$q_n = \gamma_n^2/2 + ((\theta-1)\,\kappa_{c1}A_{c3} - \gamma_n\alpha_c)\,v - (\theta-1)\,A_{c3} + vq_{n-1},$$

$$\zeta_n = (\gamma_n\pi_{c\chi})^2/2 + (\kappa_{c1}v_\chi - 1)\,(\theta-1)\,A_{c5} + v_\chi\zeta_{n-1},$$

$$\begin{aligned}
p^n \quad = \quad & \theta\log(\delta) - \gamma_n\mu_c + \gamma_n\alpha_c\sigma^2 + (\theta-1)\,\kappa_{c0} + (\theta-1)\,(\kappa_{c1}-1)\,A_0 \\
& + ((\theta-1)\,\kappa_{c1}A_{c3} + q_{n-1} - \gamma_n\alpha_c)\,(1-\nu)\,\sigma^2 \\
& + ((\theta-1)\,\kappa_{c1}+1)^2\,\sigma_\eta^2/2 + \left(\left((\theta-1)\,\kappa_{c1}A_{c1} + \left(1+\rho+...+\rho^{n-2}\right)\right)\sigma_\lambda - \gamma_n\pi_{c\lambda}\right)^2/2 \\
& + ((\theta-1)\,\kappa_{c1}A_{c3} + q_{n-1} - \gamma_n\alpha_c)^2\,\sigma_w^2/2 \\
& + \left((\theta-1)\,\kappa_{c1}A_{c5} + \zeta_{n-1}\right)(1-v_\chi)\,\sigma_\chi^2 + \left((\theta-1)\,\kappa_{c1}A_{c5} + \zeta_{n-1}\right)^2\sigma_{\chi e}^2/2 \\
& + p^{(n-1)},
\end{aligned}$$

and the sequence of γ_n is given by

$$\begin{aligned}
\gamma_n &= \gamma_1 \left(1+\rho_c+...+\rho_c^{n-2}\right) + (1/\psi)\,\rho_c^{n-1} + \left(1+\rho_c+...+\rho_c^{n-3}\right)(\theta-1)\,A_{c4} \\
&\quad ... \\
\gamma_3 &= \gamma_1 (1+\rho_c) + (1/\psi)\,\rho_c^2 + (\theta-1)\,A_{c4} \\
\gamma_2 &= \gamma_1 + (1/\psi)\,\rho_c \\
\gamma_1 &= \gamma - (\theta-1)\,\kappa_1 A_{c4}.
\end{aligned}$$

Finally, we define the yield on an n-period real zero-coupon bond as $y_t^{(n)} = -\frac{1}{n} p_t^{(n)}$.

The price a one-period nominal bond is $p_t^{(1)\$} = -r_{f,t+1}^{\$}$, with

$$r_{f,t+1}^{\$} = -\log\left(\frac{\lambda_{t+1}}{\lambda_t}\right) + (1/\psi)\,\rho_c \Delta c_t - p^{1\$} - q_1\sigma_t^2 - \zeta_1^{\$}\sigma_{\chi t}^2 + \chi_t,$$

with

$$\begin{aligned}
p^{1\$} &= p^1 - \mu_\iota + \sigma_\iota^2/2, \\
\zeta_1^{\$} &= \zeta_1.
\end{aligned}$$

The price of a bond that pays one unit of numeraire in n periods:

$$p_t^{(n)\$} = \ln E_t\left(\exp\left(m_{t+1}^{\$} + p_{t+1}^{(n-1)\$}\right)\right).$$

This can be solved to obtain:

$$p_t^{(n)\$} = p^{n\$} + \left(1+\rho+...+\rho^{n-1}\right)x_t + \sigma_\eta \eta_{t+1} + q_n \sigma_t^2 - \left(\gamma_n \rho_c + (\theta-1)\,A_{c4}\right)\Delta c_t + \zeta_n^{\$}\sigma_{\chi t}^2 - \left(1+\rho_\chi+...+\rho_\chi^{n-1}\right)\chi_t$$

with γ_n and q_n as defined above and

$$\zeta_n^{\$} = \left(1+\rho_\chi+...+\rho_X^{n-2} + \gamma_n \pi_{c\chi}\right)^2/2 + (\kappa_{c1}v_\chi - 1)(\theta-1)\,A_{c5} + v_\chi \zeta_{n-1},$$

$$\begin{aligned}
p^{n\$} &= \theta\log(\delta) - \mu_\iota - \gamma_n\mu_c + \gamma_n\alpha_c\sigma^2 + (\theta-1)\,\kappa_{c0} + (\theta-1)(\kappa_{c1}-1)\,A_0 \\
&\quad + \left((\theta-1)\,\kappa_{c1}A_{c3} + q_{n-1} - \gamma_n\alpha_c\right)(1-\nu)\,\sigma^2 \\
&\quad + \left((\theta-1)\,\kappa_{c1}+1\right)^2\sigma_\eta^2/2 + \left(\left((\theta-1)\,\kappa_{c1}A_{c1} + \left(1+\rho+...+\rho^{n-2}\right)\right)\sigma_\lambda - \gamma_n\pi_{c\lambda}\right)^2/2 \\
&\quad + \left((\theta-1)\,\kappa_{c1}A_{c3} + q_{n-1} - \gamma_n\alpha_c\right)^2\sigma_w^2/2 \\
&\quad + \left((\theta-1)\,\kappa_{c1}A_{c5} + \zeta_{n-1}^{\$}\right)(1-v_\chi)\,\sigma_\chi^2 + \left((\theta-1)\,\kappa_{c1}A_{c5} + \zeta_{n-1}^{\$}\right)^2\sigma_{\chi e}^2/2 + \sigma_\iota^2/2 \\
&\quad + p^{(n-1)\$}.
\end{aligned}$$

We define the yield on an n-period nominal zero-coupon bond as $y_t^{(n)\$} = -\frac{1}{n}p_t^{(n)\$}$.

8.4. Appendix D

This Appendix contains a set of tables that provide versions of the results discussed in the main text that use ex-post, instead of ex-ante versions of the real interest rate.

Table 4 (appendix)

Parameter Estimates and Standard Errors
Models estimated with ex-post real bond yields

Parameter	Benchmark model	Extended model	Parameter	Benchmark model	Extended model
γ	1.7005	2.2228	ρ	0.99143	0.99108
	(0.017871)	(0.22671)		(0.00047649)	(0.00059552)
ψ	1.4623	2.3876	σ_{λ}	0.00054228	0.00040323
	(0.043679)	(0.62451)		(2.0143e-05)	(2.6299e-05)
δ	0.99784	0.9978	Implied value of θ	-2.2158	-2.1041
	(0.0021963)	(0.00013075)		(0.099733)	(0.50544)
σ_c	0.0068554	0.0046005	ν	0.00	0.99673
	(0.00015674)	(0.00046367)			(0.00057417)
μ	0.0015234	0.0009117	σ_{ω}	0.00	1.70E-06
	(5.2956e-05)	(5.7211e-05)			(2.6272e-07)
$\pi_{c\lambda}$	0.00	-0.0018895	α_c	0.00	-119.3912
		(0.00029618)			(28.7094)
σ_d	0.017153	0.025564	α_d	0.00	-145.5293
	(0.00041145)	(0.52335)			(68.6766)
π_{dc}	0.0015658	-1.3161	ρ_c	0.00	0.090348
	(0.00041998)	(0.80412)			(0.045439)
$\pi_{d\lambda}$	0.00	-0.010369	ρ_d	0.00	0.33438
		(0.0054912)			(0.39606)
σ_{η}	0.00	0.0082606			
		(0.00043981)			

Table 5 (appendix)

Models estimated with ex-post real bond yields

Moments Matched in Estimation	Data (constrained, ex-post bond yields)	Data (unconstrained, ex-post bond yields)	Benchmark model	Extended model
Average growth rate of of consumption	1.4941 (0.30395)	2.2197 (0.22413)	1.84	1.20
Average growth rate of dividends	1.4941 (0.30395)	0.12538 (0.65732)	1.84	1.20
Standard deviation of the growth rate of consumption	2.1865 (0.34494)	2.1624 (0.31345)	2.36	2.72
Standard deviation of the growth rate of dividends	6.914 (1.0409)	6.5181 (1.1219)	5.89	6.95
Contemporaneous correlation between consumption growth and dividend growth	0.2215 (0.1006)	0.23722 (0.10649)	0.09	0.21
First-order serial correlation of consumption	0.2001 (0.1475)	**0.1584** (0.1240)	-0.01	0.43
Correlation between stock returns and one-period lagged consumptio	-0.1555 (0.1070)	-0.2607 (0.0999)	0.00	-0.04
Correlation between stock returns and one-period lead consumption g	0.4950 (0.0670)	0.5093 (0.0490)	0.01	0.22
Average return to equities	7.8252 (1.74)	6.3948 (1.8804)	6.25	3.87
Standard deviation of return to equities	17.2537 (1.3783)	17.6831 (1.461)	15.93	16.67
Average risk-free rate	0.19347 (0.84341)	0.10476 (0.84457)	0.19	0.19
Standard deviation of the risk-free rate	3.4656 (0.82769)	3.5054 (0.83489)	3.99	4.23
First-order serial correlation of the risk-free rate	0.59242 (0.082348)	0.59736 (0.078284)	0.90	0.49
Equity premium	7.6318 (1.758)	6.29 (1.8812)	6.06	3.68
Average price-dividend ratio	3.3761 (0.14887)	3.3775 (0.14887)	3.12	3.30
Standard deviaton of price-dividend ratio	0.47324 (0.074827)	0.44502 (0.081116)	0.28	0.40
First-order serial correlation of price dividend ratio	0.95041 (0.034499)	0.93257 (0.042679)	0.84	0.89
Contemporaneous correlation between risk free rate and price dividend ratio	**0.20139** (0.12648)	0.21274 (0.12982)	-0.95	-0.24

Table 6 (appendix)

Correlation Between Stock Returns and Per Capita
Growth Rates of Consumption and Dividends

Model estimated with ex-post bond returns

Moments	Data (constrained)	Data (unconstrained)	Benchmark model	Extended model
Consumption growth				
1-year correlation between equity returns and consumption growth	-0.07 (0.11)	-0.05 (0.12)	0.04	-0.14
5-year correlation between equity returns and consumption growth	-0.01 (0.16)	-0.05 (0.13)	0.05	0.01
10-year correlation between equity returns and consumption growth	-0.08 (0.30)	-0.21 (0.21)	0.05	0.18
Dividend growth				
1-year correlation between equity returns and dividend growth	0.08 (0.11)	0.05 (0.11)	0.37	-0.11
5-year correlation between equity returns and dividend growth	0.22 (0.14)	0.29 (0.14)	0.35	0.07
10-year correlation between equity returns and dividend growth	0.51 (0.22)	0.59 (0.15)	0.41	0.26

Table 7 (apendix)

Predictability of Excess Returns by Price-dividend Ratio at Various Horizons

Model estimated with ex-post real bond yields

	Data	Model (median)	Model (plim)	Data	Model (median)	Model (plim)
	Slope coefficient			R-square	(% of values larger than R-square in data)	
Benchmark model						
1 year	-0.10 (0.03)	-0.05	0.008	0.08	0.01 (0.02)	0.0003
3 years	-0.27 (0.07)	-0.14	0.027	0.14	0.03 (0.08)	0.001
5 years	-0.41 (0.12)	-0.21	0.034	0.26	0.04 (0.04)	0.001
Extended model						
1 year	-0.10 (0.03)	-0.06	-0.03	0.08	0.02 (0.05)	0.01
3 years	-0.27 (0.07)	-0.18	-0.08	0.14	0.06 (0.19)	0.02
5 years	-0.41 (0.12)	-0.26	-0.13	0.26	0.09 (0.12)	0.04

Table 8 (appendix)

Correlation of price-dividend ratio and ex-post risk-free rate

	Sample	Full-sample	Pre 1983	Post 1983
Canada	1952-2014	0.02	0.55	-0.31
		(0.14)	(0.06)	(0.25)
France	1932-2014	-0.56	-0.67	0.07
		(0.17)	(0.13)	(0.17)
Germany	1968-2014	-0.11	-0.10	-0.20
		(0.08)	(0.13)	(0.20)
Italy	1940-2014	-0.54	-0.63	0.35
		(0.14)	(0.08)	(0.21)
Japan	1960-2014	0.39	-0.01	0.66
		(0.16)	(0.20)	(0.08)
U.K.	1934-2014	0.32	0.27	-0.25
		(0.11)	(0.06)	(0.22)
US	1939-2011	0.21274	0.172	-0.44
		(0.12982)	(0.148)	(0.21)

Table 9 (appendix)

Term Structure of Bond Yields

Model estimated with ex-post yields

Moments	Data (ex-post real yields)	Data (ex-ante real yields)	Benchmark model	Extended model
		Average yield		
Long-term bonds (sample 1929-1992)	1.32 (1.10)	2.90 (0.84)	5.30	2.53
Intermediate-term bond (sample 1929-2007)	1.39 (0.91)	1.93 (0.99)	2.25	1.53
Short-term bond (sample 1929-2011)	0.42 (0.80)	0.46 (0.78)	0.19	0.19
Return to equity minus long-term bond yield (sample 1929-1992)	4.16 (2.39)	2.54 (2.09)	0.95	1.34
		Standard deviation of yield		
Long-term bonds (sample 1929-1992)	3.02 (0.65)	2.59 (0.52)	1.73	1.39
Intermediate-term bond (sample 1929-2007)	3.29 (0.53)	3.14 (0.57)	3.19	2.49
Short-term bond (sample 1929-2011)	3.88 (0.77)	3.85 (0.77)	3.99	4.23
Return to equity minus long-term bond yield (sample 1929-1992)	20.2 (2.47)	20.09 (1.96)	15.59	16.53

Table 11 (appendix)

Regressions of Excess Stock Returns on Long-term Bond Yields in Excess of Short Rate

Model estimated with ex-post real yields

	Data 1929-2011	Benchmark model	Extended model
Long-term government bonds (20 years)			
R-square	0.02	0.12	0.04
Slope	0.60 (0.96)	3.40	0.97
Intermediate term government bonds (5 years)			
R-square	0.002	0.07	0.03
Slope	-0.30 (1.10)	3.30	0.97