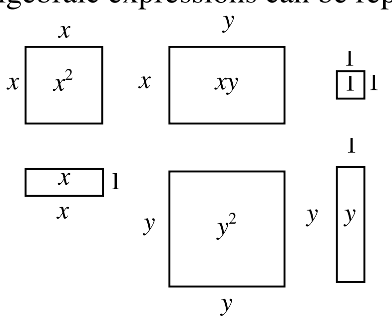
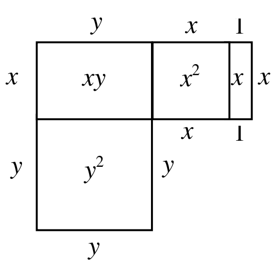
Algebraic expressions can be represented by the perimeters and areas of algebra tiles (rectangles and squares) and combinations of algebra tiles. The dimensions of each tile are shown along its sides and the area is shown on the tile itself in the figures at left. When using the tiles, **perimeter** is the distance around the exterior of a figure. The **area** of a figure is the sum of the areas of the individual pieces (length times width of each piece).



Combining terms that have the same area to write a simpler expression is called **combining like terms**. When working without tiles or pictures of the tiles, simply add or subtract the coefficients of the like terms. The Math Notes boxes on pages 49 and 57 explain the standard order of operations and their use with algebra tiles.

Using Algebra Tiles

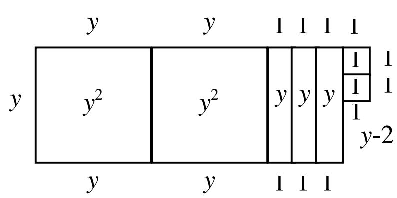
Example 1



Perimeter $= 4x + 4y + 2$

Area $= x^2 + xy + y^2 + x$

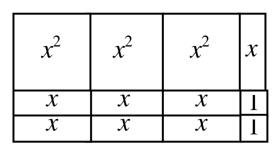
Example 2



Perimeter $= 6y + 8$

Area $= 2y^2 + 3y + 2$

Example 3



Perimeter $= 8x + 6$

Area $= 3x^2 + 7x + 2$

Combining Like Terms

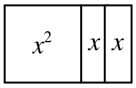
Example 4 $3x^2 + 7x + 2 + x^2 + 5 + 2x = 4x^2 + 9x + 7$

Example 5 $x^2 + xy + y^2 + 3xy + y^2 + 7 = x^2 + 4xy + 2y^2 + 7$

Example 6: $3x^2 - 4x + 3 + -x^2 + 3x - 7 = 2x^2 - x - 4$

Problems

Determine the area and the perimeter of each figure.

1.


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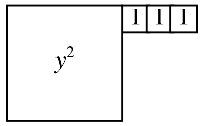

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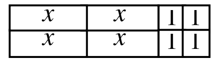

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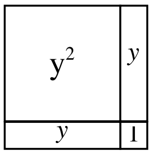

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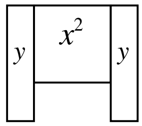

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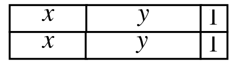

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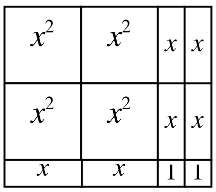

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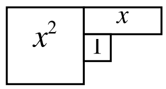

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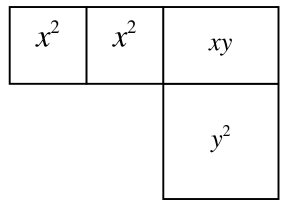


Simplify each expression by combining like terms.

10. $2x^2 + x + 3 + 4x^2 + 3x + 5$

11. $y^2 + 2y + x^2 + 3y^2 + x^2$

12. $x^2 - 3x + 2 + x^2 + 4x - 7$

13. $y^2 + 2y - 3 - 4y^2 - 2y + 3$

14. $4xy + 3x + 2y - 7 + 6xy + 2x + 7$

15. $x^2 - y^2 + 2x + 3y + x^2 + y^2 + 3y$

16. $(4x^2 + 4x - 1) + (x^2 - x + 7)$

17. $(y^2 + 3xy + x^2) + (2y^2 + 4xy - x^2)$

18. $(7x^2 - 6x - 9) - (9x^2 + 3x - 4)$

19. $(3x^2 - 8x - 4) - (5x^2 + x + 1)$

Answers

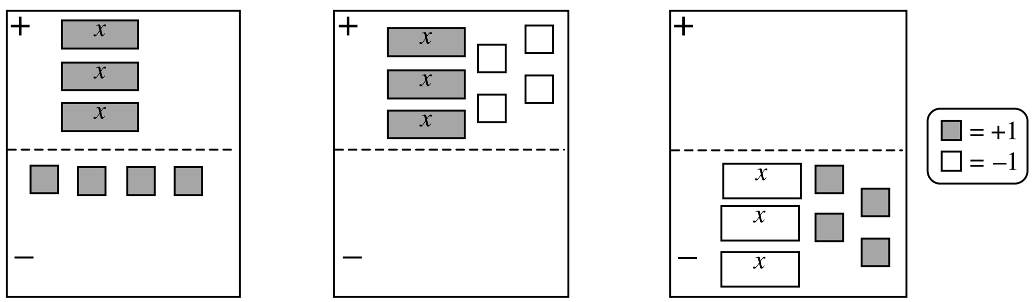
1. P $= 4x + 4$
   A $= x^2 + 2x$
2. P $= 4y + 6$
   A $= y^2 + 3$
3. P $= 4x + 8$
   A $= 4x + 4$
4. P $= 4y + 4$
   A $= y^2 + 2y + 1$
5. P $= 4y + 4$
   A $= x^2 + 2y$
6. P $= 2x + 2y + 6$
   A $= 2x + 2y + 2$
7. P $= 8x + 6$
   A $= 4x^2 + 6x + 2$
8. P $= 6x$
   A $= x^2 + x + 1$
9. P $= 6x + 4y$
   A $= 2x^2 + xy + y^2$
10. $6x^2 + 4x + 8$
11. $4y^2 + 2y + 2x^2$
12. $2x^2 + x - 5$
13. $-3y^2$
14. $10xy + 5x + 2y$
15. $2x^2 + 2x + 6y$
16. $5x^2 + 3x + 6$
17. $3y^2 + 7xy$
18. $-2x^2 - 9x - 5$
19. $-2x^2 - 9x - 5$

An expression mat is an organizing tool that is used to represent algebraic expressions. Pairs of expression mats can be modified to make an equation mat (see next section). The upper half of an expression mat is the positive region and the lower half is the negative region. Positive algebra tiles are shaded and negative tiles are blank. A matching pair of tiles with one tile shaded and the other one blank represents zero (0).

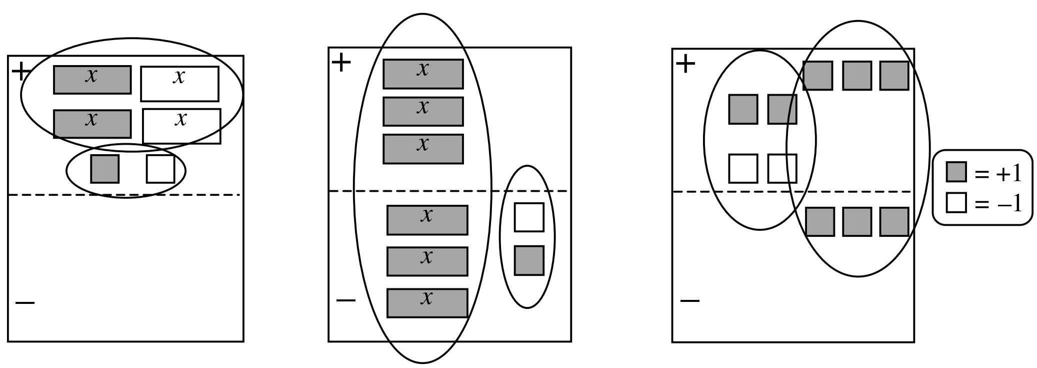
Tiles may be removed from or moved on an expression mat in one of three ways: (1) removing the same number of opposite tiles in the same region; (2) flipping a tile from one region to another. Such moves create "opposites" of the original tile, so a shaded tile becomes unshaded and an unshaded tile becomes shaded; and (3) removing an equal number of identical tiles from both the "+" and "-" regions. See the Math Notes box on page 60.

Examples

$3x - 4$ can be represented various ways.



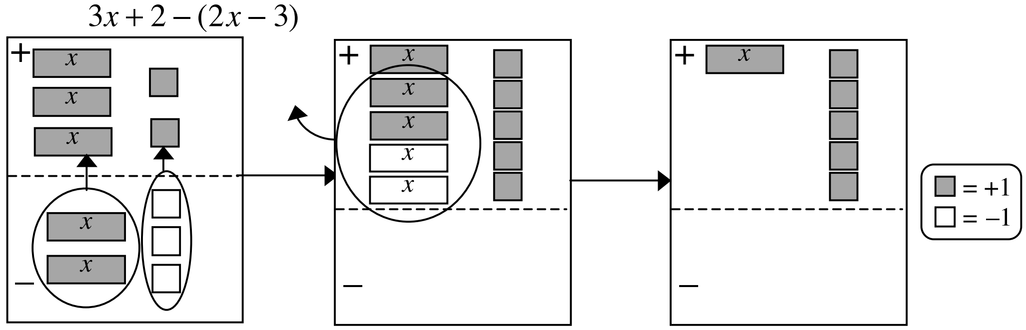
The expression mats at right all represent zero.



Example 1

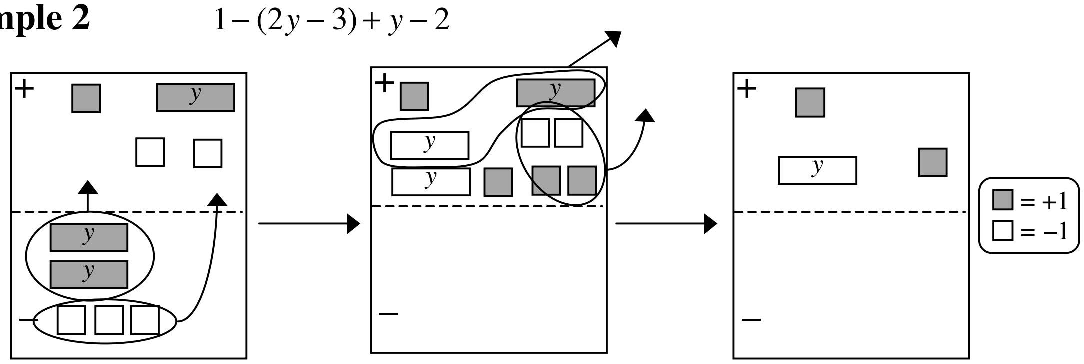
$3x + 2 - (2x - 3)$

Expressions can be simplified by moving tiles to the top (change the sign) and looking for zeros.



$3x + 2 - (2x - 3)$ $\quad$ $3x + 2 - 2x + 3$ $\quad$ $x + 5$

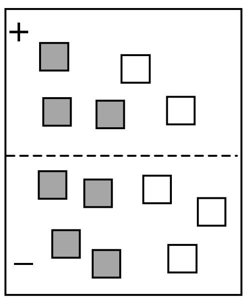
Example 2 $1-(2y-3)+y-2$

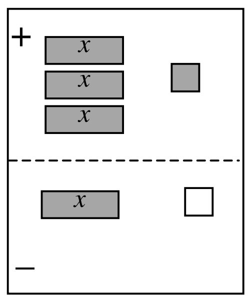


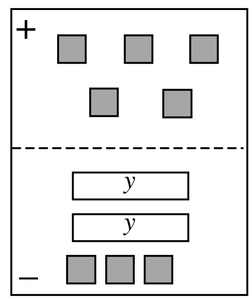
$1-(2y-3)+y-2$ $\quad$ $1-2y+3+y-2$ $\quad$ $-y+2$

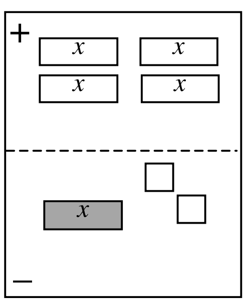
Problems

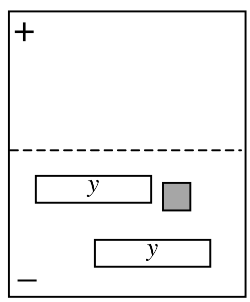
Simplify each expression.

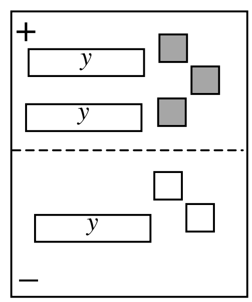
1. 

2. 

3. 

4. 

5. 

6. 

7.	$3+5x-4-7x$	8.	$-x-4x-7$	9.	$-(-x+3)$
10.	$4x-(x+2)$	11.	$5x-(-3x+2)$	12.	$x-5-(2-x)$
13.	$1-2y-2y$	14.	$-3x+5+5x-1$	15.	$3-(y+5)$
16.	$-(x+y)+4x+2y$	17.	$3x-7-(3x-7)$	18.	$-(x+2y+3)-3x+y$

Answers

1.	0	2.	$2x+2$	3.	$2y+2$
4.	$-5x+2$	5.	$2y-1$	6.	$-y+5$
7.	$-2x-1$	8.	$-5x-7$	9.	$x-3$
10.	$3x-2$	11.	$8x-2$	12.	$2x-7$
13.	$-4y+1$	14.	$2x+4$	15.	$-y-2$
16.	$3x+y$	17.	0	18.	$-4x-y-3$

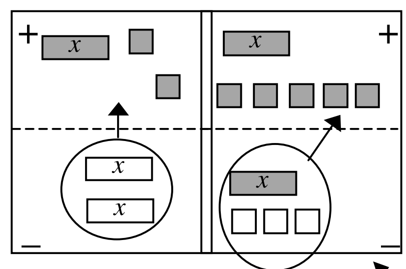
Combining two expression mats into an equation mat creates a concrete model for solving equations. Practicing solving equations using the model will help students transition to solving equations abstractly with better accuracy and understanding.

In general, and as shown in the first example below, the negative in front of the parenthesis causes everything inside to "flip" from the top to the bottom or the bottom to the top of an expression mat, that is, all terms in the expression change signs. After simplifying the parentheses, simplify each expression mat. Next, isolate the variables on one side of the equation mat and the non-variables on the other side by removing matching tiles from both sides. Then determine the value of the variable. Students should be able to explain their steps. See the Math Notes box on page 69.

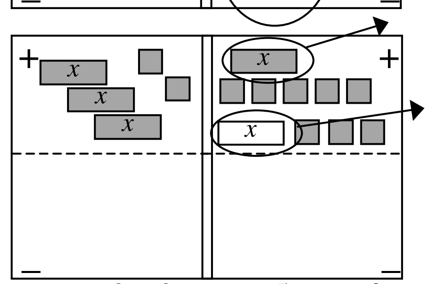
Procedure and Example

Solve $x + 2 - (-2x) = x + 5 - (x - 3)$.

First build the equation on the equation mat.

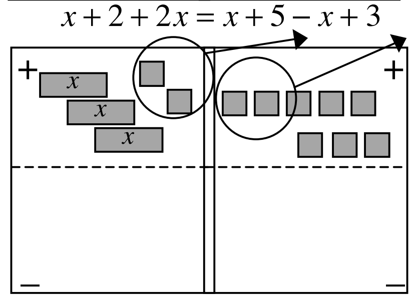


Second, simplify each side using legal moves on each expression mat, that is, on each side of the equation mat.



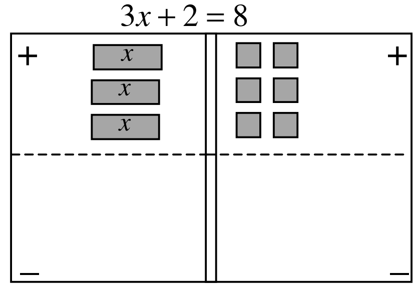
$$x + 2 + 2x = x + 5 - x + 3$$

Isolate x-terms on one side and non-x-terms on the other by removing matching tiles from both sides of the equation mat.



$$3x + 2 = 8$$

Finally, since both sides of the equation are equal, determine the value of x.



$$3x = 6$$
$$x = 2$$

Once students understand how to solve equations using an equation mat, they may use the visual experience of moving tiles to solve equations with variables and numbers. The procedures for moving variables and numbers in the solving process follow the same rules. Note: When the process of solving an equation ends with different numbers on each side of the equal sign (for example, 2 = 4), there is no solution to the problem. When the result is the same expression or number on each side of the equation (for example, $x+2=x+2$) it means that all numbers are solutions. See Section 3.2.2-3.2.4 in this guide.

Example 1 Solve $3x+3x-1=4x+9$

Solution

$3x+3x-1=4x+9$	problem
$6x-1=4x+9$	simplify
$2x=10$	add 1, subtract 4*x* on each side
$x=5$	divide

Example 2 Solve $-2x+1-(-3x+3)=-4+(-x-2)$

Solution

$-2x+1-(-3x+3)=-4+(-x-2)$	problem
$-2x+1+3x-3=-4-x-2$	remove parenthesis (flip)
$x-2=-x-6$	simplify
$2x=-4$	add *x*, add 2 to each side
$x=-2$	divide

Problems

Solve each equation.

1. $2x-3=-x+3$
2. $1+3x-x=x-4+2x$
3. $4-3x=2x-6$
4. $3+3x-(x-2)=3x+4$
5. $-(x+3)=2x-6$
6. $-4+3x-1=2x+1+2x$
7. $-x+3=10$
8. $5x-3+2x=x+7+6x$
9. $4y-8-2y=4$
10. $9-(1-3y)=4+y-(3-y)$
11. $2x-7=-x-1$
12. $-2-3x=x-2-4x$
13. $-3x+7=x-1$
14. $1+2x-4=-3-(-x)$
15. $2x-1-1=x-3-(-5+x)$
16. $-4x-3=x-1-5x$
17. $10=x+6+2x$
18. $-(x-2)=x-5-3x$
19. $6-x-3=4x-8$
20. $0.5x-(-x+3)=x-5$

Answers

1.	2	2.	5	3.	2	4.	1	5.	1
6.	−6	7.	−7	8.	no answer	9.	6	10.	−7
11.	2	12.	all numbers	13.	2	14.	0	15.	2
16.	no answer	17.	$1\frac{1}{3}$	18.	−7	19.	$2\frac{1}{5}$	20.	−4

Students solve proportional reasoning (ratio) problems in a variety of ways. They may find the number or cost per one unit and then multiply by the number of units. They may also organize work in a table. Later in the course students will use ratio equations or proportions. See Section 5.2.1-5.2.2 for this topic.

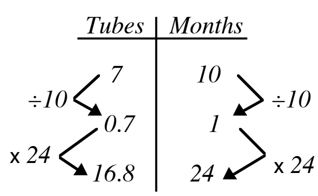
Example 1

Mrs. Salem's student assistant can correct 18 homework papers in 10 minutes. At this rate how many papers can she correct during a 55 minute class?

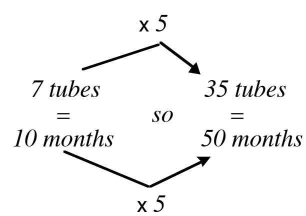
18÷10 = 1.8 papers per minute
1.8 x 55 = 99 papers

Example 2

Toby uses 7 tubes of toothpaste every 10 months. How many tubes will he use in 24 months? How long will 35 tubes last him?



or



about 17 tubes

50 months

Problems

Solve and explain your reasoning

1. Alice knows six cups of rice will make enough Spanish rice to feed 15 people. How much rice is needed to feed 75 people?
2. Elaine can plant 16 flowers in 10 minutes. How many can she plant in 25 minutes?
3. Ivanna needs to buy 360 cherries for a large salad. She can buy nine cherries for $0.57. How much will 360 cherries cost?
4. A plane travels 3400 miles in eight hours. How far would it travel in six hours at the same rate?
5. Leslie can write a 500-word essay in a hour. If she writes an essay in 20 minutes, approximately how many words should the essay contain?
6. About eight out of every 100 people in the state have red hair. If a typical classroom in the state has 25 students, how many would you expect to have red hair? If the typical middle school has 650 students, how many would you expect to have red hair?
7. When Carlos rides his bike to school, it takes 15 minutes to go 8 blocks. If he rides at the same speed, how long should it take him to travel 30 blocks?
8. Simba the cat is on a diet. Ten pounds of special low-fat food costs $22.50. How much would 30 pounds cost? How much would 36 pounds cost?
9. Elizabeth came to bat 110 times in 20 games. How many times should she expect to bat in 62 games?
10. Ly can deliver 32 newspapers in 25 minutes on his bike. Next week he needs to deliver 80 newspapers in the same neighborhood. How long should it take him if he works at the same rate as he did for 32 newspapers?

Answers

1. 30 cups	2. 40 flowers	3. $22.80	4. 2550 miles	5. ≈167 words
6. 2 & 52 people	7. 56.25 min.	8. $67.50; $81.00	9. 341 at-bats	10. 62.5 min.