Warm Up

Solve each equation.

1. $\frac{2.4}{x} = \frac{2}{9}$ 10.8

2. $1.6x = 1.8(24.8)$ 27.9

Determine whether each data set could represent a linear function.

3.

x	2	4	6	8
y	12	6	4	3

no

4.

x	–2	–1	0	1
y	–6	–2	2	6

yes

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Objective

Solve problems involving direct, inverse, joint, and combined variation.

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Vocabulary

direct variation

constant of variation

joint variation

inverse variation

combined variation

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In Chapter 2, you studied many types of linear functions. One special type of linear function is called *direct variation*. A **direct variation** is a relationship between two variables x and y that can be written in the form $y = kx$, where $k \neq 0$. In this relationship, k is the **constant of variation**. For the equation $y = kx$, y varies directly as x.

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A direct variation equation is a linear equation in the form $y = mx + b$, where $b = 0$ and the constant of variation k is the slope. Because $b = 0$, the graph of a direct variation always passes through the origin.

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Example 1: Writing and Graphing Direct Variation

Given: *y* varies directly as *x*, and *y* = 27 when *x* = 6. Write and graph the direct variation function.

$y = kx$	*y varies directly as x.*
$27 = k(6)$	*Substitute 27 for y and 6 for x.*
$k = 4.5$	*Solve for the constant of variation k.*
$y = 4.5x$	*Write the variation function by using the value of k.*

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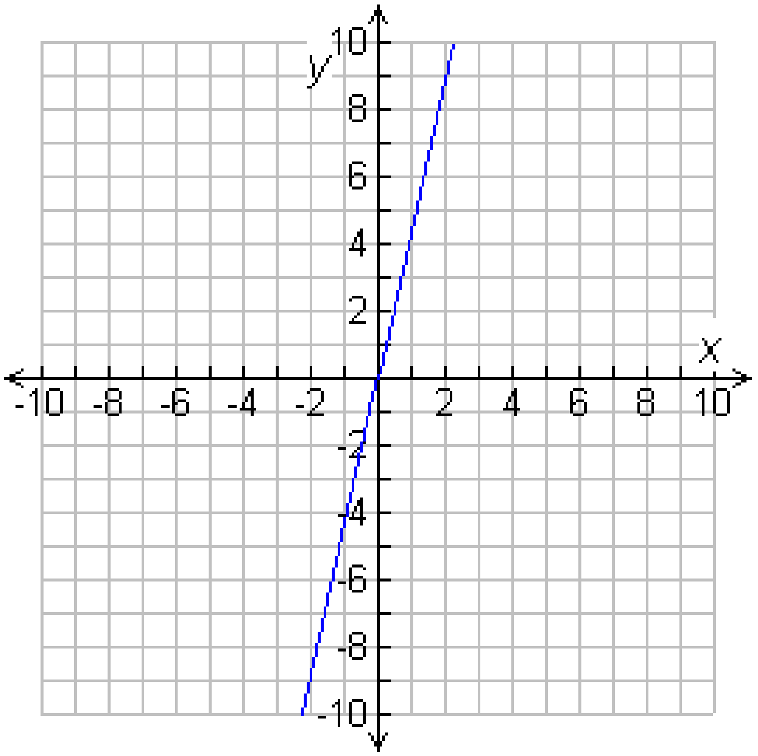
Example 1 Continued

Graph the direct variation function. The *y*-intercept is 0, and the slope is 4.5.



Check Substitute the original values of *x* and *y* into the equation.

$y = 4.5x$

27	4.5(6)
27	27 ✓

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Helpful Hint

If k is positive in a direct variation, the value of y increases as the value of x increases.

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Check It Out! Example 1

Given: *y* varies directly as *x*, and *y* = 6.5 when *x* = 13. Write and graph the direct variation function.

$y = kx$	*y varies directly as x.*
$6.5 = k(13)$	*Substitute 6.5 for y and 13 for x.*
$k = 0.5$	*Solve for the constant of variation k.*
$y = 0.5x$	*Write the variation function by using the value of k.*

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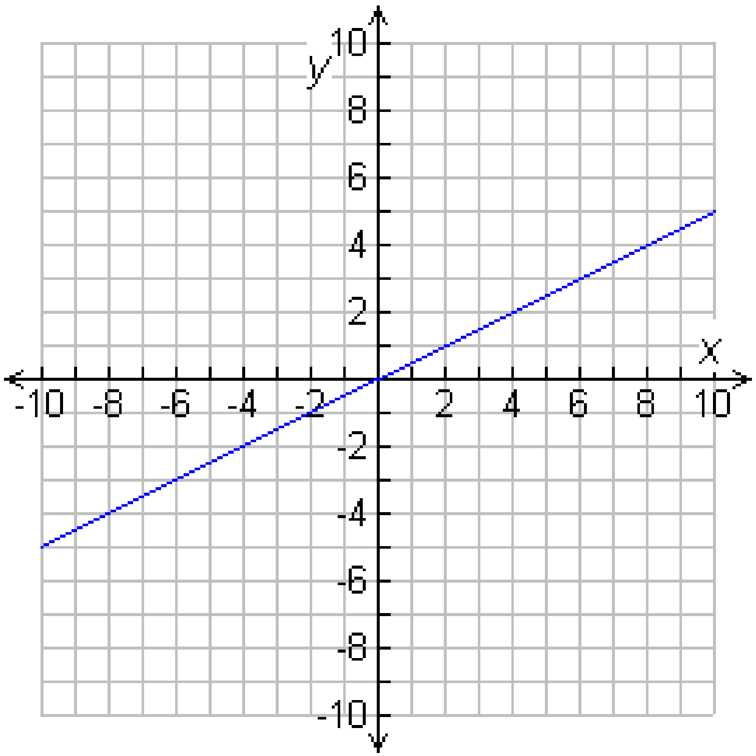
Check It Out! Example 1 Continued

Graph the direct variation function. The y-intercept is 0, and the slope is 0.5.



Check Substitute the original values of x and y into the equation.

y	$= 0.5x$
6.5	0.5(13)
6.5	6.5 ✓

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When you want to find specific values in a direct variation problem, you can solve for *k* and then use substitution or you can use the proportion derived below.

$$y_1 = kx_1 \rightarrow \frac{y_1}{x_1} = k \quad \text{and} \quad y_2 = kx_2 \rightarrow \frac{y_2}{x_2} = k \quad \text{so,} \quad \frac{y_1}{x_1} = \frac{y_2}{x_2}.$$

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Example 2: Solving Direct Variation Problems

The cost of an item in euros *e* varies directly as the cost of the item in dollars *d*, and *e* = 3.85 euros when *d* = $5.00. Find *d* when *e* = 10.00 euros.

Method 1 Find k.

$$e = kd$$

$$3.85 = k(5.00)$$ *Substitute.*

$$0.77 = k$$ *Solve for k.*

Write the variation function.

$$e = 0.77d$$ *Use 0.77 for k.*

$$10.00 = 0.77d$$ *Substitute 10.00 for e.*

$$12.99 \approx d$$ *Solve for d.*

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Example 2 Continued

Method 2 Use a proportion.

$$\frac{e_1}{d_1} = \frac{e_2}{d_2}$$

$$\frac{3.85}{5.00} = \frac{10.00}{d}$$ *Substitute.*

$$3.85d = 50.00$$ *Find the cross products.*

$$12.99 \approx d$$ *Solve for d.*

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Check It Out! Example 2

The perimeter P of a regular dodecagon varies directly as the side length s, and $P = 18$ in. when $s = 1.5$ in. Find s when $P = 75$ in.

Method 1 Find k.

$P = ks$

$18 = k(1.5)$ *Substitute.*

$12 = k$ *Solve for k.*

Write the variation function.

$P = 12s$ *Use 12 for k.*

$75 = 12s$ *Substitute 75 for P.*

$6.25 \approx s$ *Solve for s.*

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Check It Out! Example 2 Continued

Method 2 Use a proportion.

$$\frac{P_1}{s_1} = \frac{P_2}{s_2}$$

$$\frac{18}{1.5} = \frac{75}{s}$$ *Substitute.*

$18s = 112.5$ *Find the cross products.*

$6.25 = s$ *Solve for s.*

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A **joint variation** is a relationship among three variables that can be written in the form $y = kxz$, where k is the constant of variation. For the equation $y = kxz$, y varies jointly as x and z.

Reading Math

The phrases "y varies directly as x" and "y is directly proportional to x" have the same meaning.

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Example 3: Solving Joint Variation Problems

The volume *V* of a cone varies jointly as the area of the base *B* and the height *h*, and $V = 12\pi$ ft³ when $B = 9\pi$ ft³ and $h = 4$ ft. Find *b* when $V = 24\pi$ ft³ and $h = 9$ ft.

Step 1 Find *k*.

$V = kBh$

$12\pi = k(9\pi)(4)$ *Substitute.*

$\frac{1}{3} = k$ *Solve for k.*

Step 2 Use the variation function.

$V = \frac{1}{3}Bh$ *Use $\frac{1}{3}$ for k.*

$24\pi = \frac{1}{3}B(9)$ *Substitute.*

$8\pi = B$ *Solve for B.*

The base is 8π ft².

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Check It Out! Example 3

The lateral surface area L of a cone varies jointly as the area of the base radius r and the slant height l, and $L = 63\pi$ m² when $r = 3.5$ m and $l = 18$ m. Find r to the nearest tenth when $L = 8\pi$ m² and $l = 5$ m.

Step 1 Find k.

$L = krl$

$63\pi = k(3.5)(18)$ *Substitute.*

$\pi = k$ *Solve for k.*

Step 2 Use the variation function.

$L = \pi rl$ *Use π for k.*

$8\pi = \pi r(5)$ *Substitute.*

$1.6 = r$ *Solve for r.*

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A third type of variation describes a situation in which one quantity increases and the other decreases. For example, the table shows that the time needed to drive 600 miles decreases as speed increases.

Speed (mi/h)	Time (h)	Distance (mi)
30	20	600
40	15	600
50	12	600

This type of variation is an inverse variation. An **inverse variation** is a relationship between two variables x and y that can be written in the form $y = \frac{k}{x}$, where $k \neq 0$. For the equation $y = \frac{k}{x}$, y varies inversely as x.

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Example 4: Writing and Graphing Inverse Variation

Given: *y* varies inversely as *x*, and *y* = 4 when *x* = 5. Write and graph the inverse variation function.

$y = \frac{k}{x}$ — *y varies inversely as x.*

$4 = \frac{k}{5}$ — *Substitute 4 for y and 5 for x.*

$k = 20$ — *Solve for k.*

$y = \frac{20}{x}$ — *Write the variation formula.*

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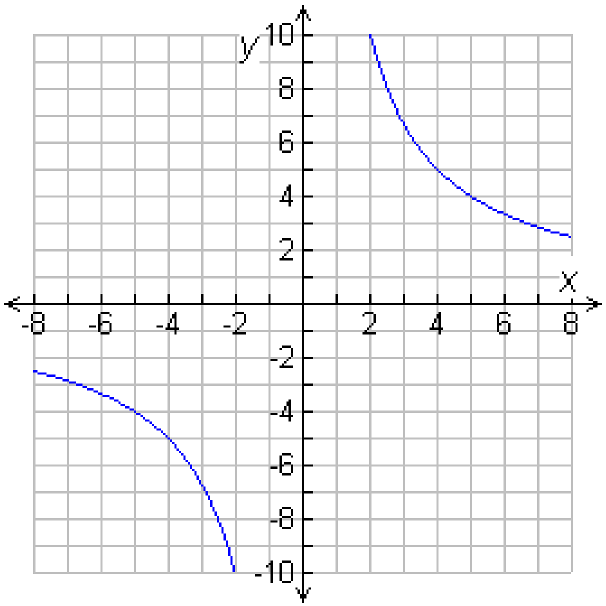
Example 4 Continued

To graph, make a table of values for both positive and negative values of *x*. Plot the points, and connect them with two smooth curves. Because division by 0 is undefined, the function is undefined when *x* = 0.

x	*y*
–2	–10
–4	–5
–6	$-\frac{10}{3}$
–8	$-\frac{5}{2}$

x	*y*
2	10
4	5
6	$\frac{10}{3}$
8	$\frac{5}{2}$



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Check It Out! Example 4

Given: *y* varies inversely as *x*, and *y* = 4 when *x* = 10. Write and graph the inverse variation function.

$y = \frac{k}{x}$ *y varies inversely as x.*

$4 = \frac{k}{10}$ *Substitute 4 for y and 10 for x.*

$k = 40$ *Solve for k.*

$y = \frac{40}{x}$ *Write the variation formula.*

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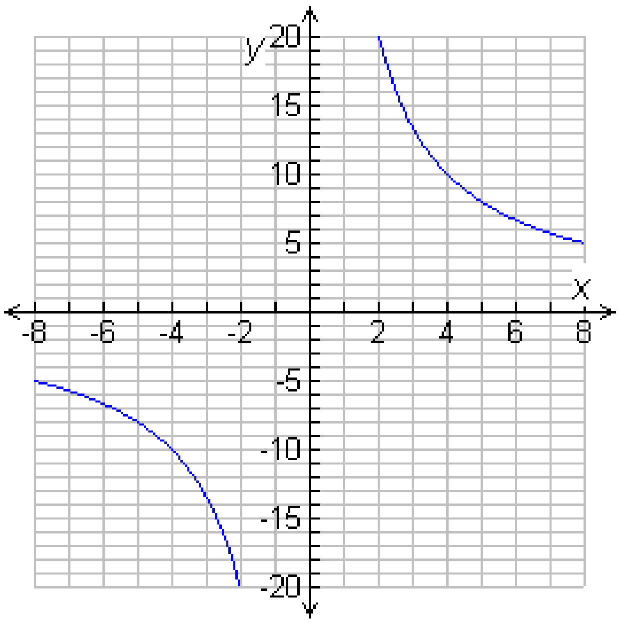
Check It Out! Example 4 Continued

To graph, make a table of values for both positive and negative values of *x*. Plot the points, and connect them with two smooth curves. Because division by 0 is undefined, the function is undefined when *x* = 0.

x	*y*
–2	–20
–4	–10
–6	$-\frac{20}{3}$
–8	–5

x	*y*
2	20
4	10
6	$\frac{20}{3}$
8	5



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When you want to find specific values in an inverse variation problem, you can solve for k and then use substitution or you can use the equation derived below.

$$y_1 = \frac{k}{x_1} \rightarrow y_1x_1 = k \quad \text{and} \quad y_2 = \frac{k}{x_2} \rightarrow y_2x_2 = k \quad \text{so,} \quad y_1x_1 = y_2x_2.$$

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Example 5: Sports Application

The time *t* needed to complete a certain race varies inversely as the runner's average speed *s*. If a runner with an average speed of 8.82 mi/h completes the race in 2.97 h, what is the average speed of a runner who completes the race in 3.5 h?

Method 1 Find *k*.

$$t = \frac{k}{s}$$

$$2.97 = \frac{k}{8.82} \qquad \textit{Substitute.}$$

$$k = 26.1954 \qquad \textit{Solve for k.}$$

$$t = \frac{26.1954}{s} \qquad \textit{Use 26.1954 for k.}$$

$$3.5 = \frac{26.1954}{s} \qquad \textit{Substitute 3.5 for t.}$$

$$s \approx 7.48 \qquad \textit{Solve for s.}$$

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Example 5 Continued

Method Use $t_1s_1 = t_2s_2$.

$$t_1s_1 = t_2s_2$$

$$(2.97)(8.82) = 3.5s$$ *Substitute.*

$$26.1954 = 3.5s$$ *Simplify.*

$$7.48 \approx s$$ *Solve for s.*

So the average speed of a runner who completes the race in 3.5 h is approximately 7.48 mi/h.

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Check It Out! Example 5

The time *t* that it takes for a group of volunteers to construct a house varies inversely as the number of volunteers *v*. If 20 volunteers can build a house in 62.5 working hours, how many working hours would it take 15 volunteers to build a house?

Method 1 Find *k*.

$$t = \frac{k}{v}$$

$$62.5 = \frac{k}{20}$$ *Substitute.*

$$k = 1250$$ *Solve for k.*

$$t = \frac{1250}{v}$$ *Use 1250 for k.*

$$t = \frac{1250}{15}$$ *Substitute 15 for v.*

$$t \approx 83\frac{1}{3}$$ *Solve for t.*

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Check It Out! Example 5 Continued

Method 2 Use $t_1v_1 = t_2v_2$.

$$t_1v_1 = t_2v_2$$

$$(62.5)(20) = 15t \qquad \textit{Substitute.}$$

$$1250 = 15t \qquad \textit{Simplify.}$$

$$83\frac{1}{3} \approx t \qquad \textit{Solve for t.}$$

So the number of working hours it would take 15 volunteers to build a house is approximately $83\frac{1}{3}$ hours.

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You can use algebra to rewrite variation functions in terms of *k*.

Direct Variation

$$y = kx \rightarrow k = \underbrace{\frac{y}{x}}_{\text{Constant ratio}}$$

Inverse Variation

$$y = \frac{k}{x} \rightarrow k = \underbrace{xy}_{\text{Constant product}}$$

Notice that in direct variation, the *ratio* of the two quantities is constant. In inverse variation, the *product* of the two quantities is constant.

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Example 6: Identifying Direct and Inverse Variation

Determine whether each data set represents a direct variation, an inverse variation, or neither.

A.

x	6.5	13	104
y	8	4	0.5

In each case $xy = 52$. The product is constant, so this represents an inverse variation.

B.

x	5	8	12
y	30	48	72

In each case $\frac{y}{x} = 6$. The ratio is constant, so this represents a direct variation.

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Example 6: Identifying Direct and Inverse Variation

Determine whether each data set represents a direct variation, an inverse variation, or neither.

C.

x	3	6	8
y	5	14	21

Since xy and $\frac{y}{x}$ are not constant, this is neither a direct variation nor an inverse variation.

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Check It Out! Example 6

Determine whether each data set represents a direct variation, an inverse variation, or neither.

6a.

x	3.75	15	5
y	12	3	9

In each case $xy = 45$. The ratio is constant, so this represents an inverse variation.

6b.

x	1	40	26
y	0.2	8	5.2

In each case $\frac{y}{x} = 0.2$. The ratio is constant, so this represents a direct variation.

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A **combined variation** is a relationship that contains both direct and inverse variation. Quantities that vary directly appear in the numerator, and quantities that vary inversely appear in the denominator.

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Example 7: Chemistry Application

The change in temperature of an aluminum wire varies inversely as its mass *m* and directly as the amount of heat energy *E* transferred. The temperature of an aluminum wire with a mass of 0.1 kg rises 5°C when 450 joules (J) of heat energy are transferred to it. How much heat energy must be transferred to an aluminum wire with a mass of 0.2 kg raise its temperature 20°C?

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Example 7 Continued

Step 1 Find k.

$\Delta T = \dfrac{kE}{m}$ Combined variation

$5 = \dfrac{k(450)}{0.1}$ Substitute.

$\dfrac{1}{900} = k$ Solve for k.

Step 2 Use the variation function.

$\Delta T = \dfrac{E}{900m}$ Use $\frac{1}{900}$ for k.

$20 = \dfrac{E}{900(0.2)}$ Substitute.

$3600 = E$ Solve for E.

The amount of heat energy that must be transferred is 3600 joules (J).

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Check It Out! Example 7

The volume *V* of a gas varies inversely as the pressure *P* and directly as the temperature *T*. A certain gas has a volume of 10 liters (L), a temperature of 300 kelvins (K), and a pressure of 1.5 atmospheres (atm). If the gas is heated to 400K, and has a pressure of 1 atm, what is its volume?

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Check It Out! Example 7

Step 1 Find k.

$$V = \frac{kT}{P}$$ *Combined variation*

$$10 = \frac{k(300)}{1.5}$$ *Substitute.*

$$0.05 = k$$ *Solve for k.*

Step 2 Use the variation function.

$$V = \frac{0.05T}{P}$$ *Use 0.05 for k.*

$$V = \frac{0.05(400)}{(1)}$$ *Substitute.*

$$V = 20$$ *Solve for V.*

The new volume will be 20 L.

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Lesson Quiz: Part I

1. The volume V of a pyramid varies jointly as the area of the base B and the height h, and V = 24 ft^3 when B = 12 ft^2 and h = 6 ft. Find B when V = 54 ft^3 and h = 9 ft.

18 ft^2

2. The cost per person c of chartering a tour bus varies inversely as the number of passengers n. If it costs $22.50 per person to charter a bus for 20 passengers, how much will it cost per person to charter a bus for 36 passengers?

$12.50

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Lesson Quiz: Part II

3. The pressure *P* of a gas varies inversely as its volume *V* and directly as the temperature *T*. A certain gas has a pressure of 2.7 atm, a volume of 3.6 L, and a temperature of 324 K. If the volume of the gas is kept constant and the temperature is increased to 396 K, what will the new pressure be?

3.3 atm

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