VCE MATHS METHODS UNIT 3 &4 AREA OF STUDY: EXAM PREPARATION

Functions and graphs, Algebra, Calculus, Probability, covering past exam questions and revising some techniques.

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General Guidelines regarding study:

- For effective study sessions, plan your revision program
- Study subjects in blocks and vary the subjects you study each day
- Work out when and how you study most effectively
- Ensure you take time to relax, exercise, read
- Make sure your bound book is securely bound- black masking tape down spine
- Make sure you do not have perforated edges
- Do not rewrite your bound book, but rather organise the one you have
- Bound books can have tags but they must be permanent
- Colour code your sections
- Complete trial papers under exam conditions, then mark them
- Write in any solutions you got wrong and if they are repeat mistakes, write them into the back section of your bound book
- Remember the more connections you can make between the material you have studied, the more likely you will be able to retain and apply the information
- Key: Checkpoints and trial exams
- www.itute.com (brilliant summary notes and many trial papers)

1. <u>Functions:</u>

Features of graphs such as axis intercepts, stationary points including inflection, domain/ restricted domain, range, asymptotes, symmetry and endpoints.

• graphs and identification of key features of graphs of the following functions:

– power functions (if on exam one- know general shape and plot points)

– exponential functions

– logarithmic functions, $y = \log_e(x)$ and $y = \log_{10}(x)$, (log laws!)

– circular functions, $y = \sin(x)$, $y = \cos(x)$ and $y = \tan(x)$; (can appear on Exam 1)

– modulus function, $y = |x|$

• transformation from $y = f(x)$ to $y = Af(nx + B) + C$ DRT

• graphs of polynomial functions;

• graphs of sum, difference, product (MC) and composite functions of f and g and their domains

-- know how to restrict a domain so the composite function can exist

• graphical and numerical solution of equations (exact values unless stated)

• graphs of inverse functions derived from graphs of original functions- state DOMAIN

• recognition of the general form of a graph, from a table of values

2. Algebra:

The algebra of functions: composition of functions, inverse functions and the solution of equations.

• polynomials (the remainder and factor theorems) and its use in sketching curves. Equation solving of different types of polynomials and the determination of the nature of stationary points will be expected. In addition, the use of the quadratic formula (often with m's) and the discriminant for quadratic functions

• exponential and logarithm laws (the change of base formula)

• solution of simple exponential and logarithmic and trigonometric equations (3 steps 1. domain, 2. quadrant and 3.core angle)

• one-to-one and many-to-one functions and the condition for the existence of inverse functions (1-1 function)

• finding inverses of functions

3. Calculus:

Limits, continuity and differentiability, differentiation, anti-differentiation and integration
• finding the graph of the gradient function, including its domain, from the graph of a function and visa versa
• rules for derivatives of loge(x), sin(x), cos(x), tan(x), and combinations of them
In addition, rules for antidifferentiation
• product, chain and quotient rules for differentiation and deduction of indefinite integrals in hence questions
• applications of differentiation to curve sketching (stationary points $\frac{dy}{dx} = 0$)
Equations of tangents and normals, maximum/minimum problems, rates of change, related rates of change and numerical evaluation of derivatives
• approximation formula – $\therefore f(x+h) \approx hf'(x) + f(x)$ where h is small $\delta y \approx \frac{dy}{dx}\delta x$

• informal approximation to areas under curves by left rectangles and right rectangles
• definite and indefinite integrals
• application of integration to calculating the area of a region under a curve and areas between curves.

4. Probability:

- the concept of discrete and continuous random variables
- expected value, variance and standard deviation of a random variable
- calculation and interpretation of central measures (mode, median, mean)
  The 68 – 95 – 99.7 Rule

– Bernoulli trials and two-state Markov chains, tree diagrams and Matrices (label)

• discrete random variables: graphs, tables and probability functions

– distributions for discrete random variables- mean (μ) median, mode, variance and standard deviation- formula sheet

– the binomial distribution, X Bi(*n*, *p*) and the binomial theorem expansion

– the effect of varying n and p on the graphs of PDFs

• continuous random variables:

– drawing and interpretation of probability density functions

– calculations and interpretation using CAS of mean (μ) median, mode, variance and standard deviation

– standard normal distribution Z N(0, 1), and transformed normal distributions

X N(μ, σ^2) transposition formula z= $\frac{x-\mu}{\sigma}$

- the use of inverse normal when the probability is given e.g.Invnorm(.9,100,6) (Draw a graph "CONDITIONAL PROBABILITY"

Specific tips for Maths Methods:

- Know Year 10 and 11 maths theory and skills e.g. SOHCAHTOA, **similar triangles**, Pythagoras, **trig identities**, **quadratic equation**, exact values, log laws, **area of triangles** which are not right- angled, Understand “simplify” and “state”
- Do not leave out a **Multiple Choice Q- have an educated guess**!
- Use brackets where appropriate, as this will ensure the calculator evaluates exactly what you want, otherwise it will do BODMAS
- **Sketch graphs require scales**, x & y intercepts, turning points, end points (beware of restricted domains- **watch for open and closed dots)**
- Inverses are not defined unless an domain is stated. When sketching inverses always draw in the **line y=x** (original function must be 1-1 function)
- Understand the significance of “**hence” and “ show” q’s**
- **Composite functions: the domain of these, is the domain of the inside function and to find the range sketch the composite function**
- Write mathematical working down NOT CALCULATOR SYNTAX
- $\int (x^2 + 4x)dx$ **watch this notation** –remember **+ c**
- **Get derivatives and antiderivatives once they are simplified and on the one line**
- **Rationalise all surds and state answers with only positive powers**
- Know the different ways you are asked for a derivative/ gradient function/ gradient of the curve/ gradient of the tangent at a point etc., acceptable notation:

$\frac{dy}{dx}$ or $d(\frac{2x^2+3x}{x})/d\,x$ or $f'(x)$ $f'(1)$ **label everything!**

Modulus Questions:

1. Solve |x+2|=3

$$+(x+2)=3$$
$$x=1$$

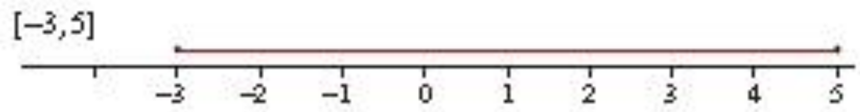
$$-(x+2)=3$$
$$-x-2=3$$
$$-x=5$$
$$x=-5$$

X=1,-5

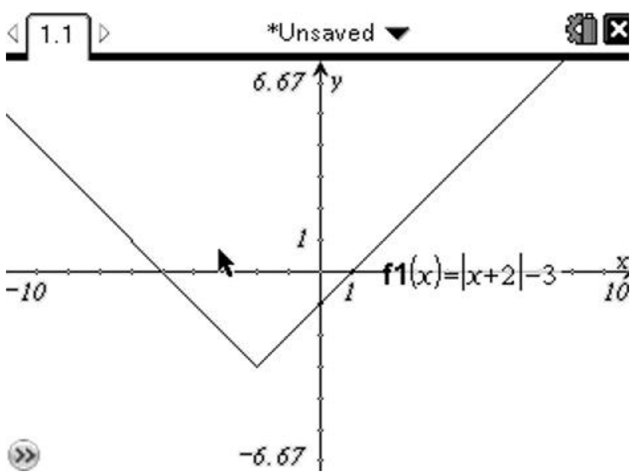
2. $\{x: abs(x-1) \leq 4\}$

$$-(x-1) \leq 4$$
$$-x+1 \leq 4$$
$$-x \leq 3$$
$$x \geq -3$$

$$\{x: abs(x-1) \leq 4\}$$
$$+(x-1) \leq 4$$
$$x-1 \leq 4$$
$$x \leq 5$$



3. Sketch $y = |x+2| - 3$



X intercepts:

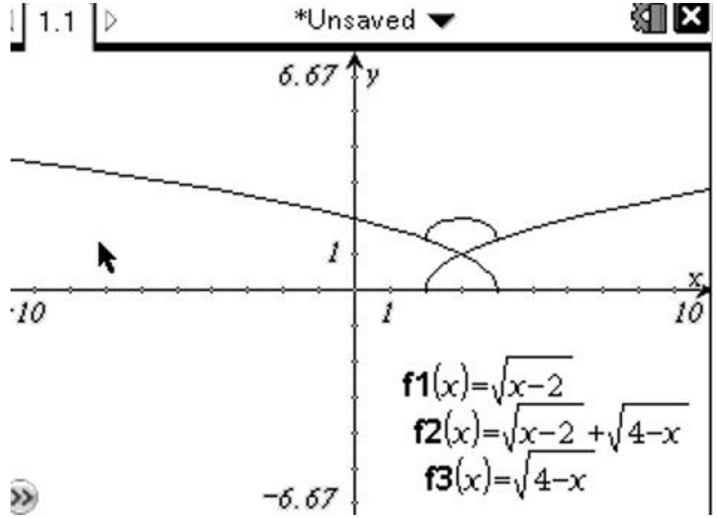
$$0 = abs(x+2) - 3$$
$$3 = +(x+2)$$
$$3 = x+2$$
$$x=1$$

$$3 = -(x+2)$$
$$3 = -x-2$$
$$x=-5$$

Composite functions:

1. If $f(x) = \sqrt{x-2}$ for all x ≥ 2 and $g(x) = \sqrt{4-x}$ for all x ≤ 4
Sketch the graph of $h(x) = \sqrt{x-2} + \sqrt{4-x}$ and state its domain

$$Dh(x) = Df(x) \cap Dg(x)$$



(f + g)(x) = f(x) + g(x) and (fg)(x) = f(x).g(x)

The domain for the composite is the intersection of the domains of f(x) and g(x)

2. If $f(x) = \sqrt{x} - 2$ for all $x \geq 2$ and $g(x) = \sqrt{4} - x$ for all $x \leq 4$, find:

a $f + g = \sqrt{x} - 2 + \sqrt{4} - x$

b $(f + g)(3) = \sqrt{3} - 2 + \sqrt{4} - 3 = 1 + 1 = 2$

c $fg = (\sqrt{x} - 2)*(\sqrt{4} - x)$

d $(fg)(3) = (\sqrt{x} - 2)*(\sqrt{4} - x) = 1*1 = 1$

Composite Functions: that need a restricted domain

For the functions $f(x) = x^2 - 1$, $x \in R$, and $g(x) = \sqrt{x}$, $x \geq 0$:

a State why $g \circ f$ is not defined. $D_f = R$ and $D_g = R^+ U\{0\}$

$R_f = [-1, \infty]$ $R_g = R^+ U\{0\}$

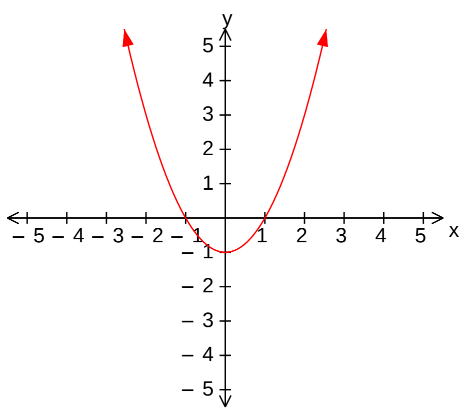
For gof to exist: Is Range of f$\subseteq$ Domain of g ? $g[f(x)]$

	Domain	Range
f	R	$[-1, \infty)$
g	$R^+ \cup \{0\}$	$R^+ \cup \{0\}$

No so we must restrict the domain

b Define a restriction f^* of f such that $g \circ f^*$ is defined. Find $g \circ f^*$.

$f(x) = x^2 - 1$



To be defined, Range of f needs to be restricted to R^+ U {0}

Restrict Domain of f^*: R\ (-1,1)

$$gof^*(x) = g(f^*(x))$$

$$= g(x^2 - 1)$$

$$= \sqrt{x^2 - 1}$$

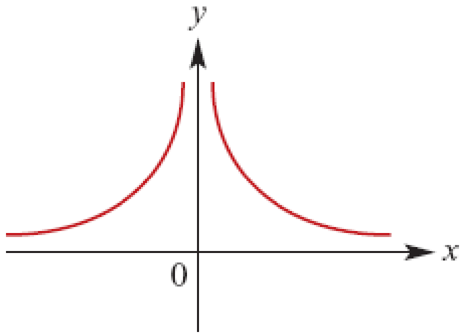
Inverses:

A function has an inverse function if and only if it is one-to-one. ie. Passes both the vertical and horizontal line test

Example: f is the function defined by $f(x) = \dfrac{1}{x^2}$ $x \in R\backslash\{0\}$.
Define a suitable restriction for f, $f*$, such that $f*^{-1}$ exists.

NB. f is not a 1-1 function

If f* has domain (-∞ , 0) f is one to one. Range: (0, ∞)



Hence $f^{-1} = -\dfrac{1}{\sqrt{x}}$
and its domain is
(-∞ , 0) An inverse is only defined if it has a domain, which ensures the function is 1-1!

$$y = \frac{1}{x^2}$$

Inverse

$$x = \frac{1}{y^2}$$

$$xy^2 = 1$$

$$y^2 = \frac{1}{x}$$

$$y = \pm\frac{1}{\sqrt{x}}$$

Matrices:

1. Dimensions: r x c, eg 3 x 2 $\begin{bmatrix} -1 & 2 \\ -3 & 4 \\ 5 & 6 \end{bmatrix}$

2. Addition: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

3. Multiplying: An n x m can multiply a m x p, giving an n x p

$$\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 2\times4+3\times1+4\times0 & 2\times0+3\times2+4\times3 \\ 5\times4+6\times1+7\times0 & 5\times0+6\times2+7\times3 \end{bmatrix} = \begin{bmatrix} 11 & 18 \\ 26 & 33 \end{bmatrix}$$

4. Inverse: $A \times A^{-1} = I$

If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\mathbf{A}^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\det(\mathbf{A}) = ad - bc$ is the determinant of matrix $\mathbf{A}$.

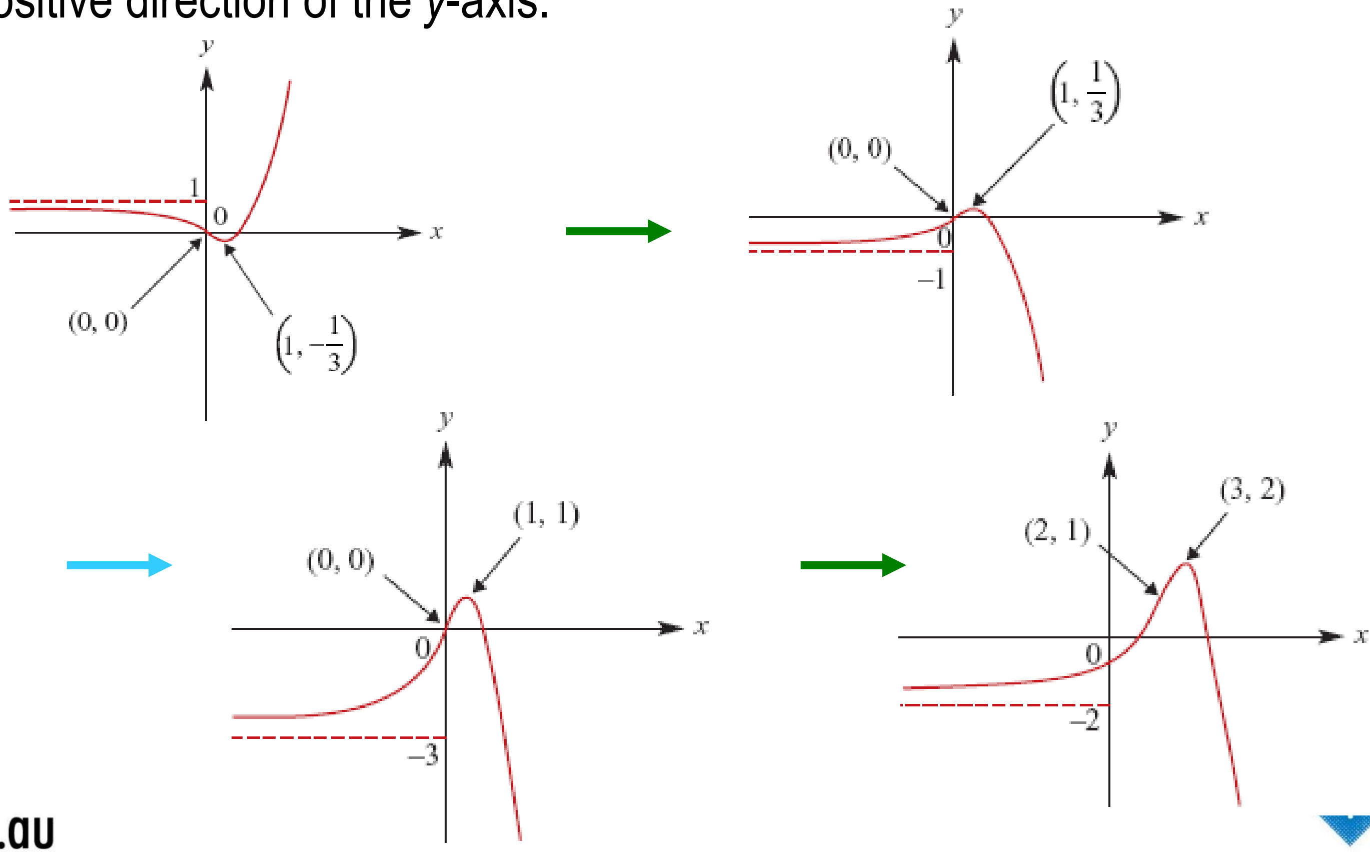
Transformations:

Sketch the graph of the image of the graph shown under the following sequence of transformations:

a reflection in the *x*-axis

a dilation of factor 3 from the *x*-axis

a translation of 2 units in the positive direction of the *x*-axis and 1 unit in the positive direction of the *y*-axis.



Transformations:

Question 15

The graph of the function $f\colon [0, \infty) \to R$ where $f(x) = 3x^{\frac{5}{2}}$ is reflected in the x-axis and then translated 3 units to the right and 4 units down.

The equation of the new graph is

A. $y = 3(x - 3)^{\frac{5}{2}} + 4$

B. $y = -3(x - 3)^{\frac{5}{2}} - 4$

C. $y = -3(x + 3)^{\frac{5}{2}} - 1$

D. $y = -3(x - 4)^{\frac{5}{2}} + 3$

E. $y = 3(x - 4)^{\frac{5}{2}} + 3$

Question 15 **B**

The transformation of $f(x) = 3x^{\frac{5}{2}}$

Reflection in x**-axis:** $-f(x) = -3x^{\frac{5}{2}}$

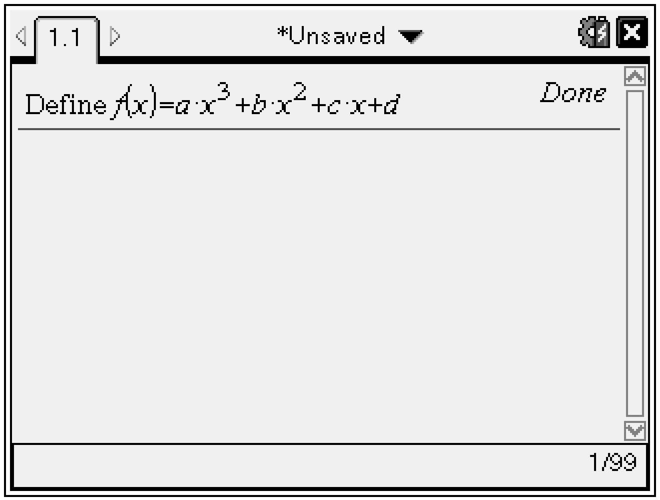
Translate 3 units right: $-f(x-3) = -3(x-3)^{\frac{5}{2}}$

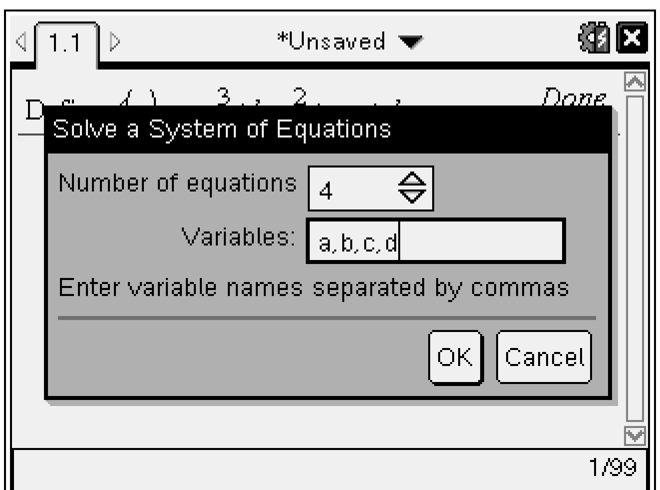
Translate 4 units down:

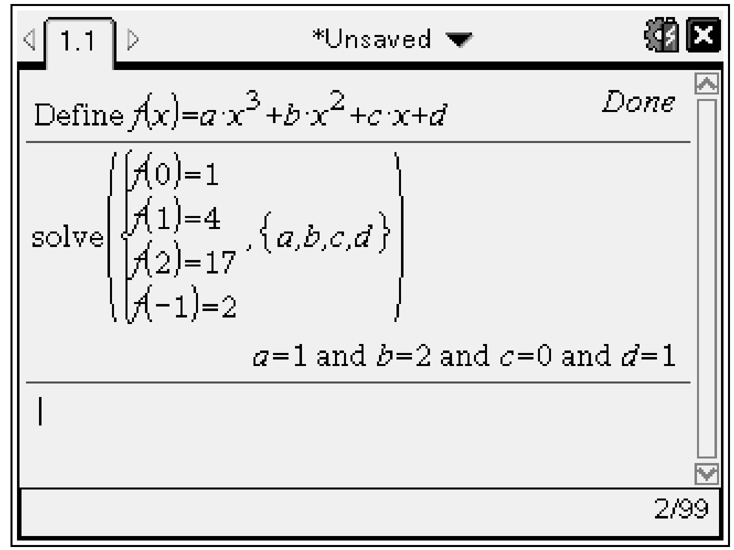
$-f(x-3) - 4 = -3(x-3)^{\frac{5}{2}} - 4$

Simultaneous Equations:

The graph of a cubic function passes through the point with coordinates (0, 1), (1, 4), (2, 17) and (−1, 2). Find the rule for this cubic function







Menu / 3: Algebra / 6: Solve system /

1: Solve system of Equations / 4 eqns a,b,c,d

QUADRATIC GRAPHS

Degree 2 polynomial $y= ax^2 + bx + c$

SKETCHES FEATURE: X-intercept, Y-intercept and TP

1. For a > 0, Cup Shape with Minimum TP

   For a < 0, Dome Shape with Maximum TP

2. Y-intercept (put x = 0) is at y = c

3. X-intercept (put y= 0) $0 = ax^2 + bx + c$

   a. Factorise

   b. Use Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. TP and Axis of Symmetry is at $x = -\frac{b}{2a}$

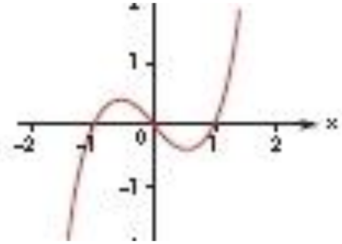
5. By completing the square, "Turning Point" form:

   $y = a(x - h)^2 + k$, has a T.P at (h, k)

6. The discriminant determines how many solutions and of what type $\Delta = b^2 - 4ac$

   Δ .>0 2 solns if p.s 2 rational solns Δ =0 1 soln , Δ <0 0 solns ie x intercepts

CUBIC GRAPHS:

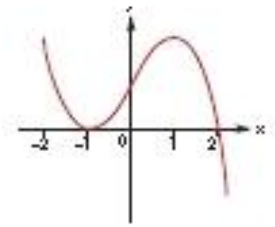


Degree 3 polynomial $y = ax^3 + bx^2 + cx + d$

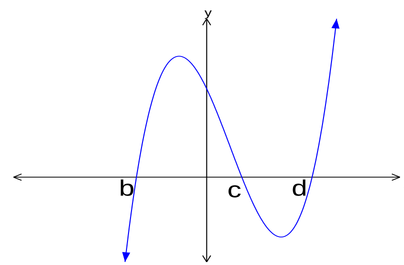
SKETCHES FEATURE: X-intercepts, Y-intercept, shape (TP & POI's)

1. For $a > 0$, Increasing Function **"positive cubic"**

   For $a < 0$, Decreasing Function **"negative cubic"**



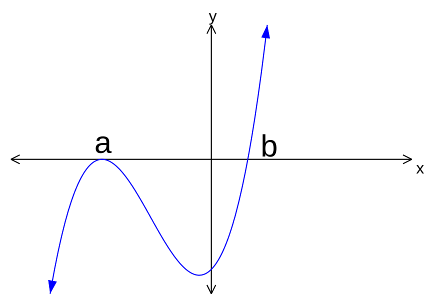
2. Y-intercept (put $x = 0$) is at $y = d$
3. X-intercept (put $y = 0$) then factorise.



4. FACTOR FORM: $y = a(x - b)(x - c)(x - d)$

REPEATED FACTOR:

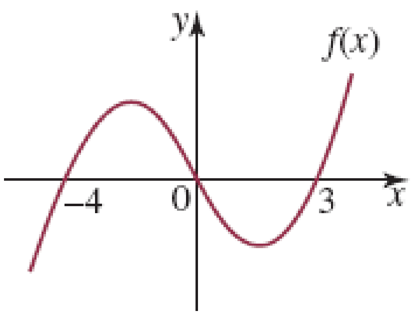
$y = (x - a)^2(x - b)$



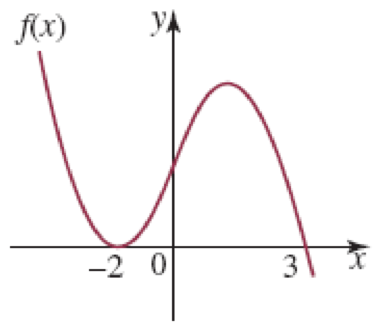
EXAMPLES:

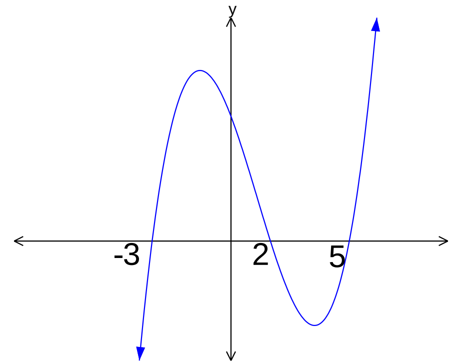
For each of the following graphs, find the rule and express it in factorised form. Assume that $a = 1$ or $a = -1$.

a



b





Increasing Fn. $a = 1$

$y = (x - -4)(x – 0)(x – 3)$

$y = x(x + 4)(x – 3)$

Decreasing Fn. $a = -1$

$y = -1(x - -2)^{2}(x – 3)$

$y = (x + 2)^{2}(3 – x)$

In factorised form: $f(x) = (x - 2)(x - 5)(x + 3)$

X- intercepts $0 = (x - 2)(x - 5)(x + 3)$

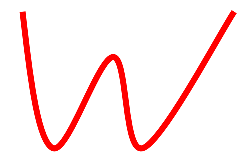
$x = 2, 5, -3$

QUARTICS:

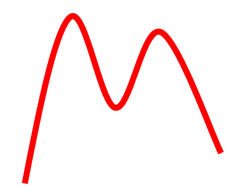
Degree 4 polynomial $y = ax^4 + bx^3 + cx^2 + dx + e$

SKETCHES FEATURE: X-intercepts, Y-intercept, Shape

1. For $a > 0$, Increasing Function **"positive quartic"**



For $a < 0$, Decreasing Function **"negative quartic"**

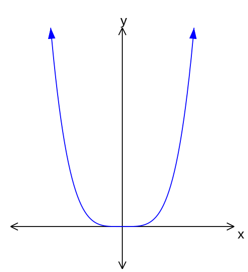


2. Y-intercept (put $x = 0$) is at $y = e$

3. X-intercept (put $y = 0$) then factorise.

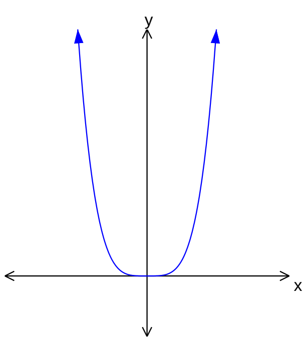
SOME BASIC QUARTICS

$y = ax^4$

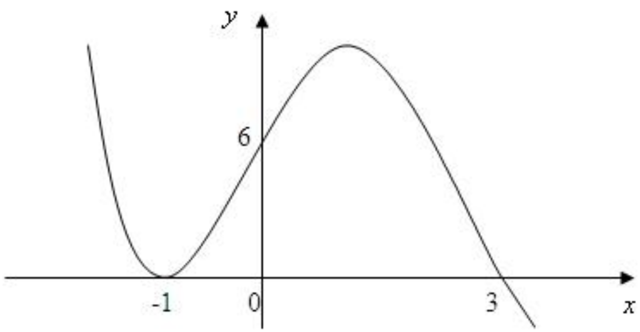


$y = ax^4 + cx^2$

$y = x^2(ax^2 + c)$



Exam Examples:



The graph of the function shown above could be

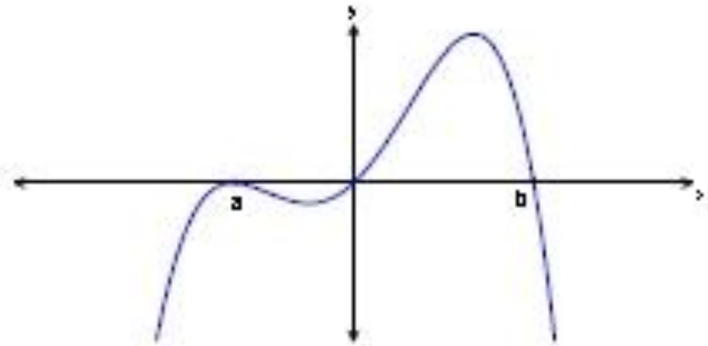
A. $y = 2(x+1)^2(3-x)$

B. $y = -2(x+1)^2(3-x)$

C. $y = -2(x-1)^2(x+3)$

D. $y = -2(x+1)(x-3)^2$

E. $y = 2(x+1)(x-3)^2$



The graph shown could be that of a function f with the equation

A. $y = -x(x+a)^2(x-b)$

B. $y = -x(x+a)(x-b)$

C. $y = x(x-a)^2(x-b)$

D. $y = -x(x+a)^2(b-x)$

E. $y = x(x-a)^2(b-x)$

Logarithms:

Change of base:

$$a^y = N$$

$$y = \log_a N$$

Find $\log_b$ of both sides

$$\log_b a^y = \log_b N$$

$$y = \frac{\log_b N}{\log_b a}$$

$$\therefore \log_a N = \frac{\log_b N}{\log_b a}$$

Some Examples:

1. $$\frac{\log_e 27}{\log_e 3}$$

$$= \log_3 27$$

$$= \log_3 3^3$$

$$= 3$$

2. $$\log_{\frac{1}{2}} 6$$

$$= \frac{\log_{10} 6}{\log_{10} .5}$$

$$\approx -2.585$$

3. If $\log_2 6 = k \log_2 3 + 1$, find the value of k.

$$\log_2 6 = \log_2 3^k + \log_2 2$$

$$\log_2 6 = \log_2 (2 \times 3^k)$$

$$6 = 2 \times 3^k$$

$$3^k = 3$$

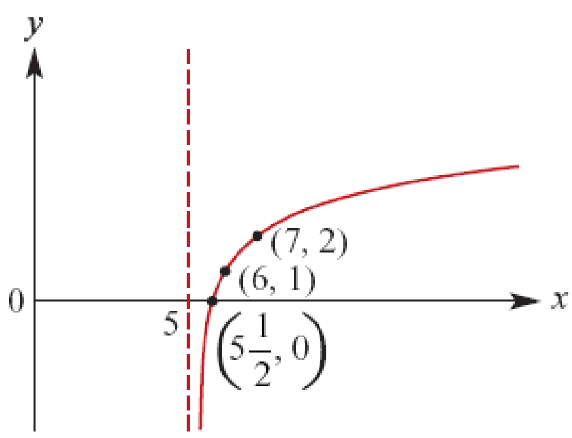
$$k = 1$$

Logarithmic Graphs:

Sketch the graph, and state the domain of each of the following:

$y = \log_2(x - 5)+1$

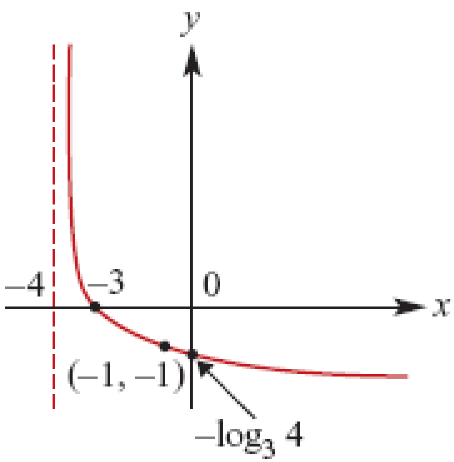
- Trans 5 in positive x
- Trans 1 in positive y
- Asymptote: x = 5
- (x-5)=1, x = 6, y = 1
- (x-5)=2, x = 7, y = 2



Domain: (5, ∞)

$y = -\log_3(x + 4)$

- Reflected in x-axis
- Trans 4 in negative x
- Asymptote: x = -4
- (x+4)=1, x = -3, y = 0
- (x+4)=3, x = -1, y = -1



Domain: (-4, ∞)

Circular Functions and Transformations:

Sketch $y = 3\sin 2(t - \frac{\pi}{4})$ for $-\pi \le t \le 2\pi$

Amplitude: 3

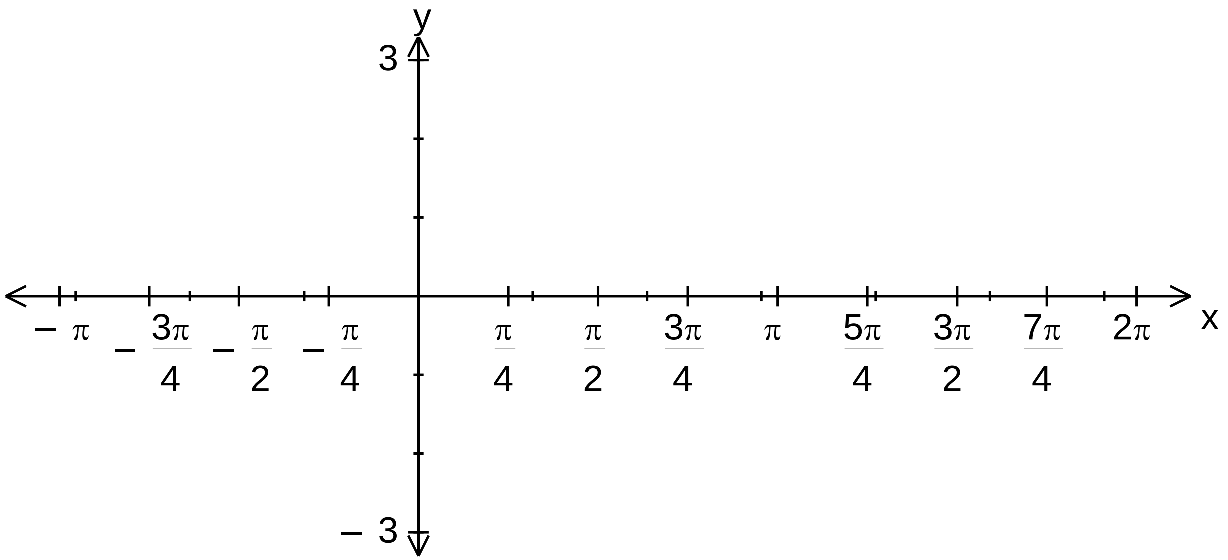
Period: $\frac{2\pi}{2} = \pi$

Translated $\frac{\pi}{4}$ in positive direction of x-axis

DRT- Dilation 3 units from the x axis(in y direction or parallel to y axis)

Dilation of 1/2 units from the y axis (in x direction or parallel to x axis)

No reflections or translations

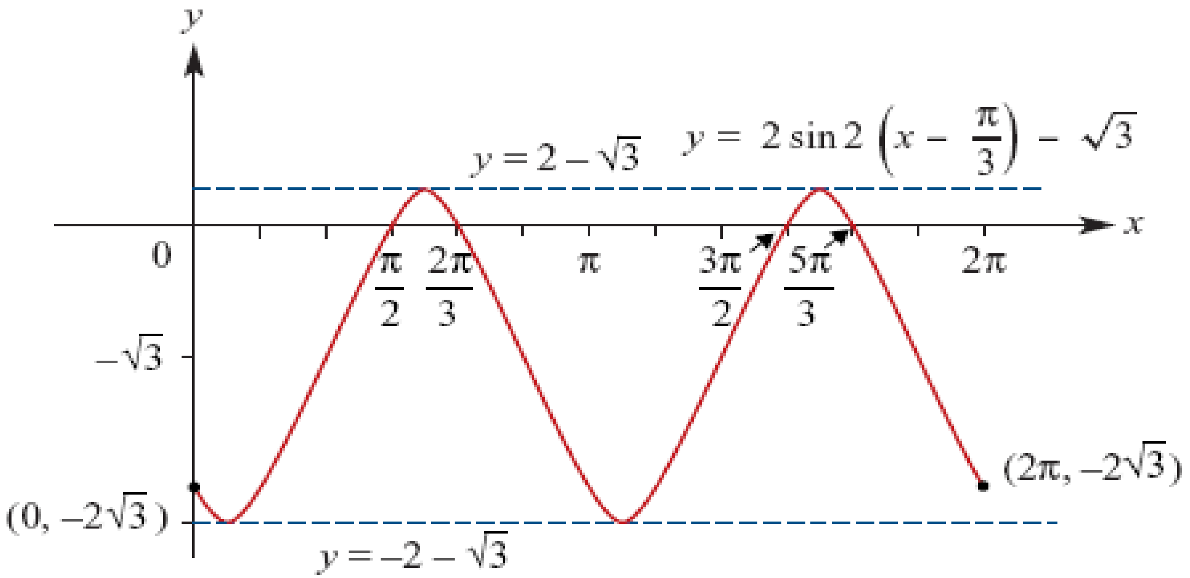


More Circular Functions:

Sketching, finding axes intercepts for $x \in [\ 0, 2\pi\ ]$

$$y = 2\sin 2(x - \frac{\pi}{3}) - \sqrt{3}$$

x-axis intercepts, y = 0
Amplitude:2
Translated √3 unit in neg y axis
Range: [-2 - √3, 2 - √3]



Features of f(x) = atan(n x):

a is a dilation factor from the x-axis (not important)

x = (2k +1)π/2 for k ∈ Z

Asymptotes : π/2, 3π/2, 5 π /2…

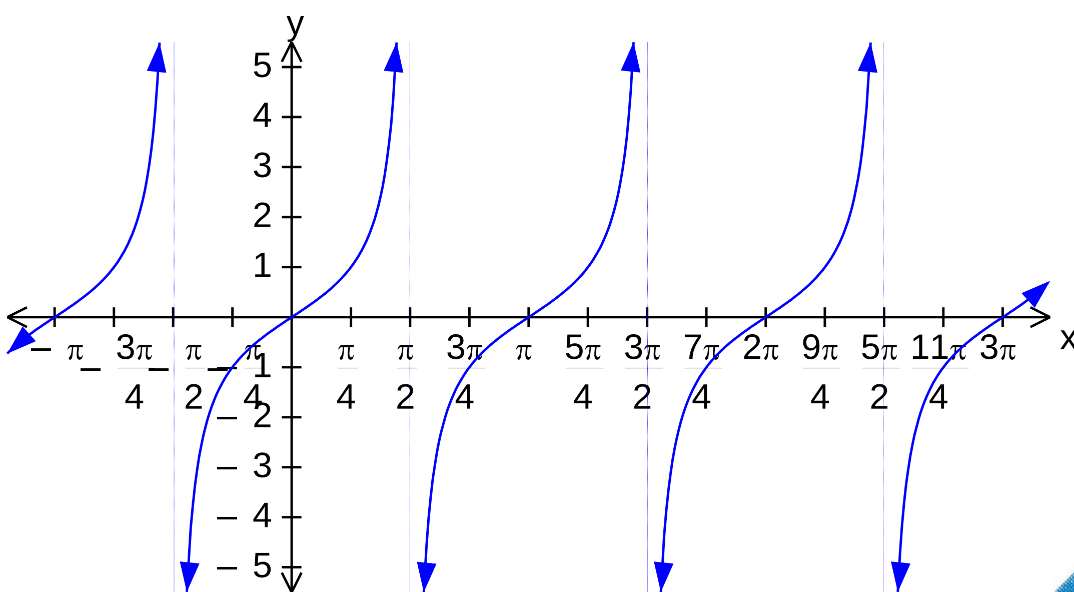
x = kπ/n for k ∈ Z

x-intercepts

x = kπ for k ∈ Z

Dilation factor of 1/n from the y-axis

π/n is the period hence Period: π



Question 4

For the system of equations

$z = 1$

$y - x = 2$

$x + y = 5$

An equivalent matrix representation is

A.
$$\begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

B.
$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

C.
$$\begin{bmatrix} 0 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

D.
$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

E.
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

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The answer is A

Write out equations in full

$$0x + 0y + 1z = 1$$
$$-1x + 1y + 0z = 2$$
$$1x + 1y + 0z = 5$$

Difficult sketch graphs

Sketch $y = 3\tan(2x - \frac{\pi}{3}) + \sqrt{3}$ for $\frac{\pi}{6} \le x \le \frac{13\pi}{6}$

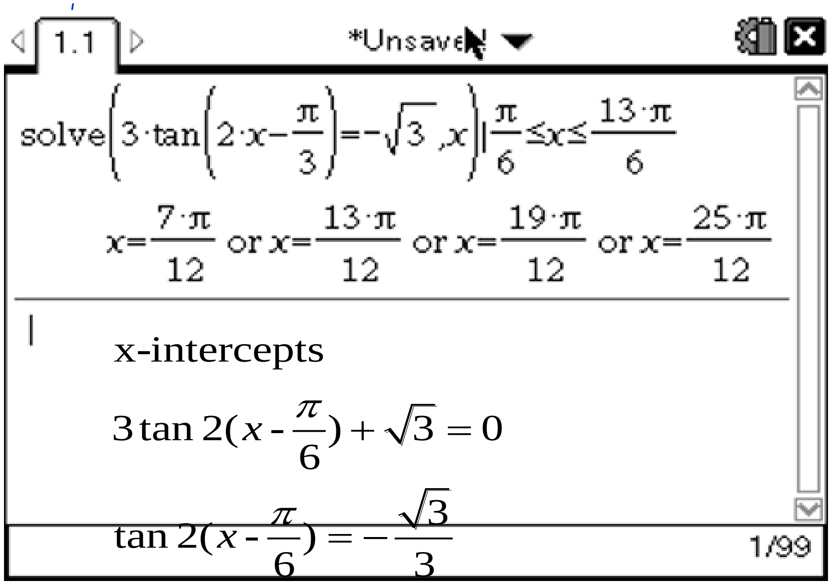
Must factorise bracket.. $y = 3\tan 2(x - \frac{\pi}{6}) + \sqrt{3}$

$$Asymptotes \quad x = \frac{(2k+1)\pi}{4} + \frac{\pi}{6}$$

$$= \frac{\pi}{4} + \frac{\pi}{6}, \frac{3\pi}{4} + \frac{\pi}{6}, \frac{5\pi}{4} + \frac{\pi}{6} \ldots$$

$$= \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12} \ldots$$

Period = π/2 Dilation of 3 from x-axis Translated π/6 in positive x,



x-intercepts

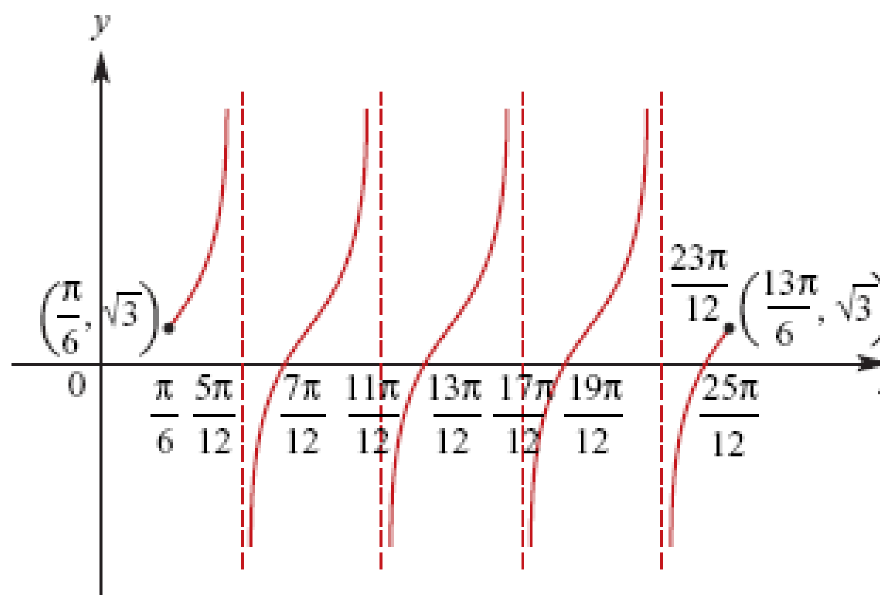
$$3\tan 2(x - \frac{\pi}{6}) + \sqrt{3} = 0$$

$$\tan 2(x - \frac{\pi}{6}) = -\frac{\sqrt{3}}{3}$$

$$2(x - \frac{\pi}{6}) = \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{17\pi}{6}, \frac{23\pi}{6}$$

$$x - \frac{\pi}{6} = \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}$$

$$x = \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}, \frac{25\pi}{12}$$



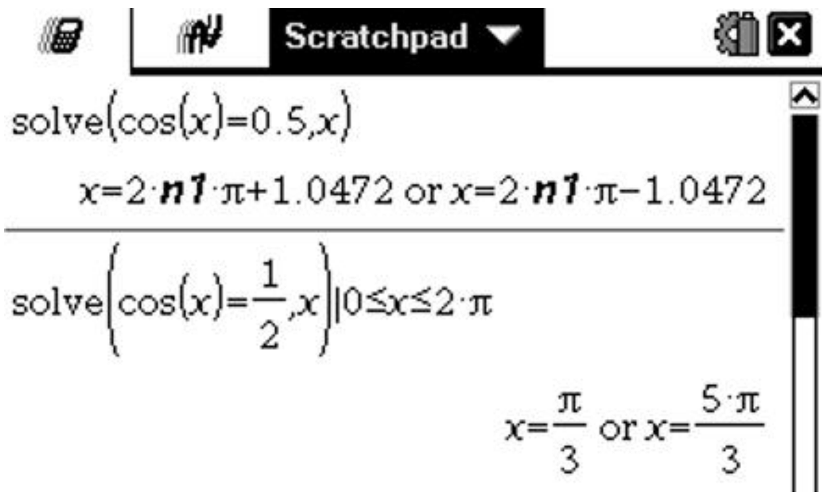
Solving Circular Function Equations: Using CAS:

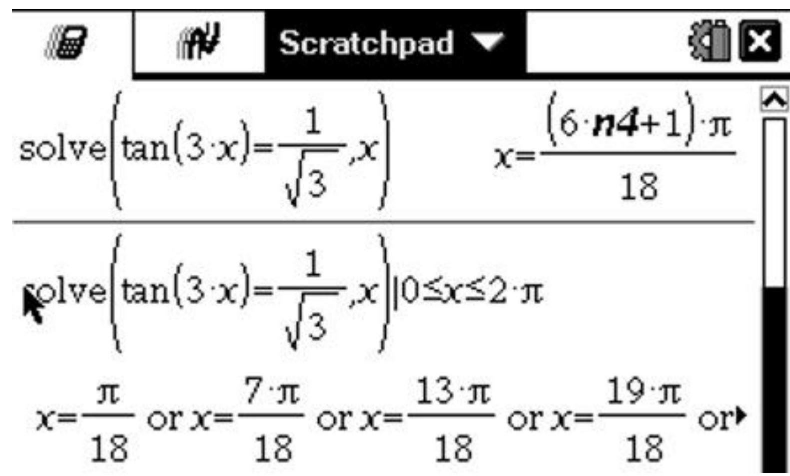
Find the general solutions and then find the first three positive solutions:

a cos (x) = 0.5 **b** $\sqrt{3}$ tan (3x) = 1

Find the specific solutions of:

c cos (x)= ½ $x \in [0,2\pi]$ **d** $\sqrt{3}$ tan (3x) = 1 $x \in [0,2\pi]$





Solving Circular Function Equations: by hand

Find without the use of a calculator, the following between $x \in [0, \pi]$

$$2\sin(2x) = \sqrt{2}$$

$$\sin(2x) = \frac{\sqrt{2}}{2}$$

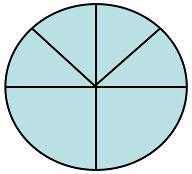
1. Domain $2x \in [0, 2\pi]$

2. Core Angle

$$x = \sin^{-1}\frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}$$

3. Quadrants 1 and 2 (sin positive)



$$2x = \frac{\pi}{4}, \pi - \frac{\pi}{4}, 2\pi + \frac{\pi}{4}, 3\pi - \frac{\pi}{4}$$

$$2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$$

$$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$$

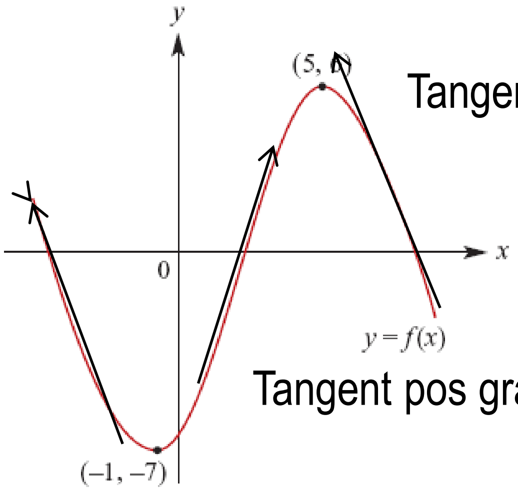
Differentiation:

For the graph of $f : R \rightarrow R$ find:

a $\{x: f'(x) > 0\}$

b $\{x: f'(x) < 0\}$

c $\{x: f'(x) = 0\}$

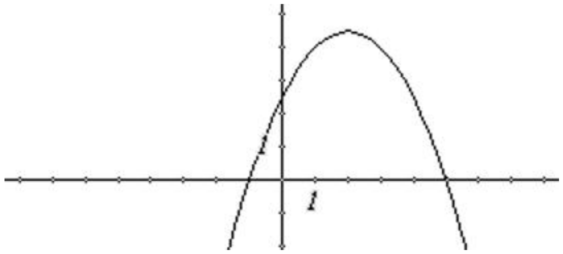


$$\{x : f'(x) > 0\} = \{-1 < x < 5\} \text{ or } (-1, 5)$$

$$\{x : f'(x) < 0\} = \{x < -1\} \cup \{x > 5\} \text{ or } (-\infty, -1) \cup (5, \infty)$$

$$\{x : f'(x) = 0\} = \{-1, 5\}$$

d. Sketch the gradient function



Differentiation:

Find the derivative of $x^4 - 2x^{-3} + x^{-1} + 2,\ x \neq 0$

$$f(x) = x^4 - 2x^{-3} + x^{-1} + 2, x \neq 0$$

$$f'(x) = 4x^3 - 2(-3x^{-4}) + (-1)x^{-2} + (0)2$$

$$= 4x^3 + 6x^{-4} - x^{-2}$$

1.Chain Rule: a function inside another function

Differentiate: $y = (4x^3 - 5x)^{-2}$

Let $u = 4x^3 - 5x$ $\quad y = (u)^{-2}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$= -2\,(u)^{-3} \times (12x^2 - 5)$

$= -2\,(4x^3 - 5x)^{-3} \times (12x^2 - 5)$

$$= \frac{-2(12x^2 - 5)}{(4x^3 - 5x)^3}$$

e.g3

$$y = \frac{16}{3x^2 + 1}$$

.

$$y = 16(3x^2 + 1)^{-1}$$

2.Product Rule: one function is multiplied by another

Differentiate with respect to x: $x^3(3x-5)^4$

$$y = x^3(3x-5)^4$$

$u = x^3 \qquad v = (3x-5)^4$

$u' = 3x^2 \qquad v' = 4(3)(3x-5)^3$

$$\frac{dy}{dx} = u \times \frac{dv}{dx} + v \times \frac{du}{dx}$$

$$= x^3 \times 4 \times (3)(3x-5)^3 + 3x^2(3x-5)^4$$

$$= 12x^3(3x-5)^3 + 3x^2(3x-5)^4$$

$$= 3x^2(3x-5)^3\left[4x + (3x-5)\right]$$

$$= 3x^2(3x-5)^3(7x-5)$$

Differentiation: cont

Quotient Rule: one function divided by the other

e.g. $y = \dfrac{x-2}{x^2+4x+1}$

$u = x - 2$
$u' = 1$

$v = (x^2 + 4x + 1)$
$v' = 2x + 4$

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2} = \frac{(x^2+4x+1)(1)-(x-2)(2x+4)}{(x^2+4x+1)^2}$$

$$= \frac{x^2+4x+1-(2x^2-8)}{(x^2+4x+1)^2}$$

$$= \frac{-x^2+4x+9}{(x^2+4x+1)^2}$$

Differentiation applications:

Find the equations of the tangent and normal to $y = x^3 + {}^1/_2x^2$ at the point where $x = 1$

1. Find dy/dx $\quad dy/dx = 3x^2 + x$

2. Put the x value into dy/dx $\quad$ At $x = 1$

$dy/dx = 3 + 1 = 4$

3. At $x = 1$
$y - y_1 = m_T(x - x_1)$ with the point, find the equation of the tangent

(x_1, y_1) becomes $(1, {}^3/_2)$

$y - {}^3/_2 = 4(x - 1)$

$2y - 3 = 8x - 8$

$2y = 8x - 5$

4. At $x = 1$
$y - y_1 = m_n(x - x_1)$ with the point, find the equation of the normal

(x_1, y_1) becomes $(1, {}^3/_2)$

$y - {}^3/_2 = -1/4(x - 1)$

$4y - 6 = -x + 1$

$0 = x + 4y - 5$

Linear Approximation:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad f'(x) \cong \frac{f(x+h) - f(x)}{h}$$

Assuming h is small

Find the approximate value of $\sqrt{27}$ without using a calculator

$\sqrt{(25+2)} = \sqrt{(x+h)}$

Let $f(x) = \sqrt{x}$

So $f'(x) = \dfrac{1}{2} x^{-\frac{1}{2}}$

$= \dfrac{1}{2\sqrt{x}}$

$$f(x+h) \cong f(x) + hf'(x)$$

$= \sqrt{x} + 2 * \dfrac{1}{2\sqrt{x}}$

=5+1/5

=5 1/5

Stationary Points:

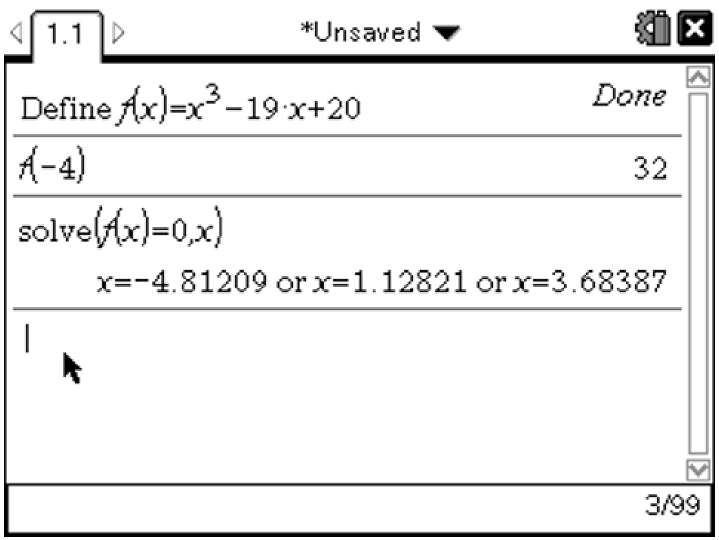
Plot the graph of $y = x^3 - 19x + 20$ and determine:

a the value of y when $x = -4$

b the values of x when $y = 0$

c the value of $\dfrac{dy}{dx}$ when $x = -1$

d the coordinates of the local maximum

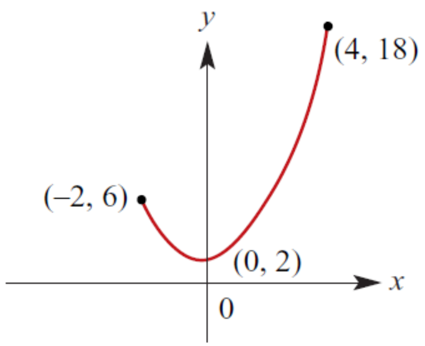


Absolute Max and Min Points and Max and Min Problem:

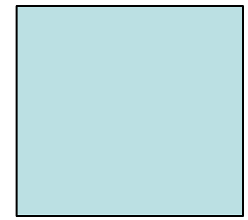
Let $f : [-2, 4] \rightarrow R$, $f(x) = x^2 + 2$. Find the absolute maximum and the absolute minimum value of the function.

From graph, maximum occurs at x = 4, and is 18

From graph, minimum occurs at x = 0, and is 2



$$V = x(2-2x)(2-2x)$$
$$V = x(2x-2)(2x-2)$$
$$V = x(4x^2-8x+4)$$
$$V = 4x^3-8x^2+4x$$



e.g.From a square piece of metal of side length 2 m, four squares are removed as shown in the figure opposite. The metal is then folded about the lines to give an open box with sides of height *x* m. Find the domain and then sketch the graph of *V* against *x*.

Domain

$x > 0$

$2 - 2x > 0$

$2 > 2x$

$x < 1$

$\therefore 0 < x < 1$

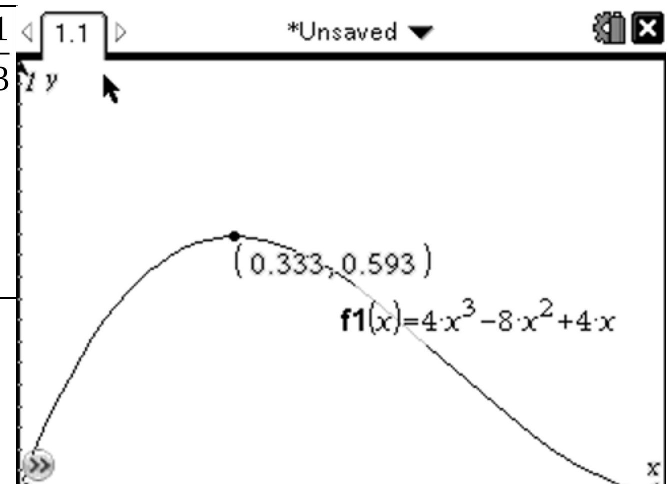
$$\frac{dV}{dx} = 12x^2 - 16x + 4$$

Local max occurs when $\frac{dV}{dx} = 0$

$$0 = 4(3x^2 - 4x + 1)$$
$$0 = 4(3x-1)(x-1)$$
$$x = \frac{1}{3}, 1$$

Local Max occurs at $x = \frac{1}{3}$

$$V = 4\left(\frac{1}{27}\right) - 8\left(\frac{1}{9}\right) + 4\left(\frac{1}{3}\right)$$
$$= \frac{16}{27}$$



Derivatives:

f(x) = $\log_e x$, then f'(x) = $\frac{1}{x}$

f(x) = $\log_e kx$, then f'(x) = $\frac{1}{x}$

h(x) = $\log_e f(x)$, then h'(x) = $\frac{f'(x)}{f(x)}$

$$f(x) = \log_e |5x+3|,\ x \neq \frac{-3}{5}$$

$$f'(x) = \frac{5}{5x+3}$$

e.g $\quad y = x^2 \log_e x$

$$\boldsymbol{u = x^2 \qquad v = log e x}$$
$$\boldsymbol{u' = 2x \qquad v' = 1/x}$$

$$y = x^2 \log_e x,\ x > 0$$

Product rule

$$\frac{dy}{dx} = x^2 \times \frac{1}{x} + 2x \times \log_e x$$
$$= x + 2x \log_e x$$

e.g $\quad y = \frac{\log_e x}{x^2}$

$$\boldsymbol{v = log e x \qquad u = x^2}$$
$$\boldsymbol{v' = 1/x \qquad u' = 2x}$$

$$y = \frac{\log_e x}{x^2} . x > 0$$

Quotient rule

$$\frac{dy}{dx} = \frac{x^2 \times \frac{1}{x} - 2x \times \log_e x}{(x^2)^2}$$
$$= \frac{x - 2x \log_e x}{x^4}$$
$$= \frac{1 - 2\log_e x}{x^3}$$

Derivatives of Circular functions:

f(x)	f'(x)
Sin kx	kCos kx
Cos kx	- kSin kx
Tan kx	$\frac{k}{\cos^2 kx} = k\sec^2 kx$

Find the derivative of each of the following with respect to θ :

e.g. $\sin^2(2\theta + 1)$

$(\sin(2\theta+1))^2$

Chain rule

$$\frac{dy}{d\theta} = 2\sin(2\theta+1) \times 2\cos(2\theta+1)$$

$$= 4\sin(2\theta+1)\cos(2\theta+1)$$

e. g. $\tan(3\theta^2 + 1)$

$(\tan(3\theta^2+1))$

Chain rule

$$\frac{dy}{d\theta} = \sec^2(3\theta^2+1) \times 6\theta$$

$$= 6\theta\sec^2(3\theta^2+1)$$

e.g. $e^{2x}\mathrm{Sin}(2x + 1)$ Product rule

$$\frac{dy}{dx} = e^{2x} \times 2\cos(2x+1) + 2e^{2x} \times \sin(2x+1)$$

$$= 2e^{2x}[\cos(2x+1) + \sin(2x+1)]$$

Antiderivative:

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

Find the general antiderivative of:

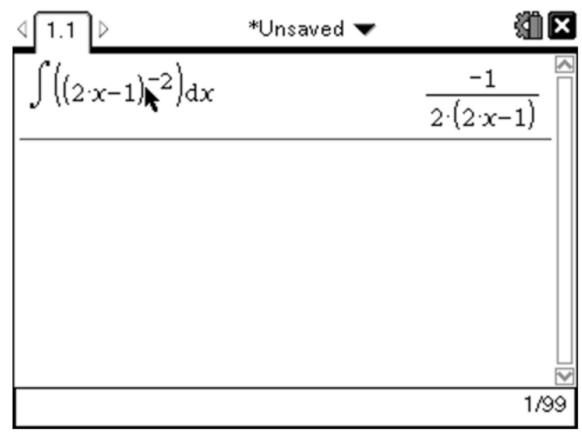
a $(3x + 1)^5$

$$\int (3x+1)^5\;dx$$

$$= \frac{(3x+1)^6}{3\times 6} + c$$

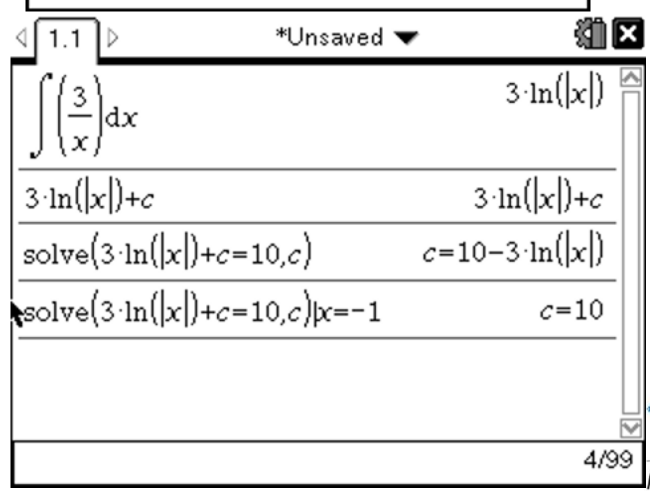
$$= \frac{1}{18}(3x+1)^6 + c$$

b $(2x - 1)^{-2}$



$$\int \frac{1}{x}\,dx = \log_e |x| + c$$

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\log_e |ax+b| + c$$



Further Antiderivatives:

$$\int e^{kx}dx = \frac{1}{k}e^{kx} + c$$

Find the general antiderivative of each of the following:

$$\int e^{3x} + 2\ dx$$

$$= \frac{1}{3}e^{3x} + 2x + c$$

If the gradient at a point (x, y) on a curve is given by $5e^{2x}$ and the curve passes through (0, 7.5), find the equation of the curve.

$$\frac{dy}{dx} = 5e^{2x}$$

$$y = \int 5e^{2x}dx$$

$$= \frac{5}{2}e^{2x} + c$$

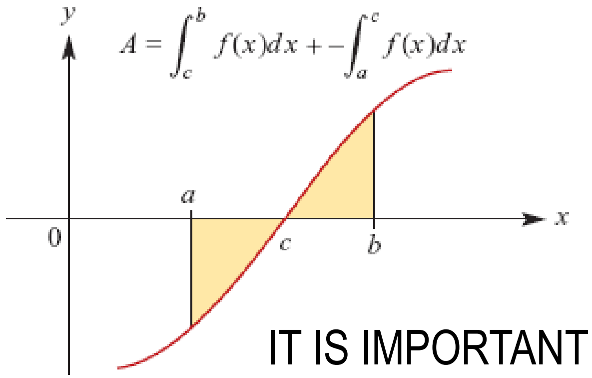
$$\text{At } (0, 7.5)$$

$$7.5 = \frac{5}{2}e^{0} + c$$

$$c = 5$$

$$y = \frac{5}{2}e^{2x} + 5$$

<u>**Area under Curves**</u>: COMBINED AREAS



$$\int_a^b f(x)dx = \int_c^b f(x)dx - \int_a^c f(x)dx$$

$$or = \int_c^b f(x)dx + \int_c^a f(x)dx$$

IT IS IMPORTANT TO SKETCH THE FUNCTION TO UNDERSTAND THE NATURE OF THE AREA

e.g.Find the exact area of the shaded region

$$A = \int_2^4 x^2 - 4 \text{ dx} - \int_1^2 x^2 - 4 \text{ dx}$$

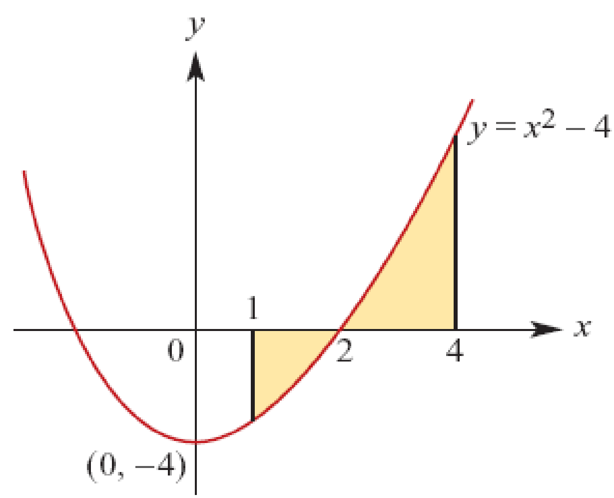
$$= \left[\frac{x^3}{3} - 4x\right]_2^4 + \left[\frac{x^3}{3} - 4x\right]_2^1$$

$$= \left((\frac{64}{3} - 16) - (\frac{8}{3} - 8)\right) + \left((\frac{1}{3} - 4) - (\frac{8}{3} - 8)\right)$$

$$= \frac{64 - 8 + 1 - 8}{3} - 16 + 8 - 4 + 8$$

$$= \frac{49}{3} - 4$$

$$= \frac{37}{3} \text{ sq units}$$



Antidifferentiation by Recognition

a. **Find f'(x)** $f(x) = x\log_e(kx)$

$$f'(x) = \log_e(kx) + x * \frac{1}{x} = \log_e(kx) + 1$$

$$Know \int \log_e(kx) + 1dx = x\log_e(kx) + c$$

$$so \int \log_e(kx)dx + \int 1dx = x\log_e(kx) + c$$

$$\int \log_e(kx)dx = -\int 1dx + x\log_e(kx) + c$$

$$\int \log_e(kx)dx = -x + x\log_e(kx) + c$$

b. Hence find $\int \boldsymbol{log_e(kx)dx} =$

a. **Find f'(x)** $f(x) = x^2\log_e(kx)$ **b. Hence find** $\int \boldsymbol{xlog_e(kx)dx} =$

$$f'(x) = 2x\log_e(kx) + x^2 * \frac{1}{x} = 2x\log_e(kx) + x$$

$$Know \int 2x\log_e(kx) + xdx = x^2\log_e(kx) + c$$

$$so \int 2x\log_e(kx)dx + \int xdx = x^2\log_e(kx) + c$$

$$\int 2x\log_e(kx)dx = -\int xdx + x^2\log_e(kx) + c$$

$$\int 2x\log_e(kx)dx = -1/2\,x^2 + x^2\log_e(kx) + c$$

$$\int x\log_e(kx)dx = 1/4\,x^2\log_e(kx) - \frac{x^2}{4} + c$$

Linear Equations:

Question 1

Find the values of p and q for which the equations

$$-3x + py = q$$

have

$$4x - 5y = 20$$

i. a unique solution

ii. no solution

iii. infinitely many solutions.

$$\Delta = \begin{vmatrix} -3 & p \\ 4 & -5 \end{vmatrix} = 15 - 4p$$

$(1)\ \text{x}4 \Rightarrow -12x + 4py = 4q$

$(2)\ \text{x}3 \Rightarrow \quad 12x - 15y = 60$ M1

i. for a unique solution $\Delta \neq 0 \Rightarrow p \neq \frac{15}{4}$ and $q \in R$ A1

ii. for no solution $\Delta = 0 \Rightarrow p = \frac{15}{4}$ and $q \neq -15$ A1

iii. for infinitely many solutions $\Delta = 0 \Rightarrow p = \frac{15}{4}$ and $q = -15$ A1

EXPECTED VALUE OF DRVs:

Expected Value = Mean = Average = μ

$$E(x) = \sum_x x \times \Pr(X = x)$$

$$= x_1 \times p(x_1) + x_2 \times p(x_2) + x_3 \times p(x_3) + \ldots + x_n \times p(x_n)$$

e.g.1

x	1	2	3	4	5
$\Pr(X = x)$	$\frac{2}{5}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

E(x) = 1 x $^{2}/_{5}$ + 2 x $^{1}/_{10}$ + 3 x $^{3}/_{10}$ + 4 x $^{1}/_{10}$ + 5 x $^{1}/_{10}$

E(x) = 2 $^{2}/_{5}$

e.g.2

x	0	1	2	3
$\Pr(X = x)$	0.064	0.288	0.432	0.216

$$E(x) = \sum x.p(x)$$

$$= 0 \times .064 + 1 \times .288 + 2 \times .432 + 3 \times .216$$

$$= 0 + .288 + .864 + .648$$

$$= 1.8$$

Discrete Random Variables:

Within a Probability Distribution, the expectation of another function can be considered… Eg. Biased coin "Three up": Costs \$2 to play, returns as shown. What is the expected profit /loss?

x	0	1	2	3
$\Pr(X = x)$	0.064	0.288	0.432	0.216
Return r(y)	\$6	\$1	\$0	\$5
Gain g(y)	\$4	-\$1	-\$2	\$3

$$E(R) = \sum r(y) \times p(x)$$

$$= 6 \times 0.064 + 1 \times 0.288 + 0 \times 0.432 + 5 \times 0.216$$

Expected profit = 1.752 – 2 = -0.248

$$= 1.752$$

g(y) = r(y) -2

$$E(G) = \sum (r(y) - 2) \times p(x)$$

$$= (6-2)0.064 + (1-2)0.288 + (0-2)0.432 + (5-2)0.216$$

$$= -0.248$$

Binomial Distributions:

Mean: $E(x) = \mu = np$ Variance: $Var(x) = \sigma^2 = npq$ Standard Deviation: $SD(x) = \sigma = \sqrt{(npq)}$

Eg 1. A fair die is rolled 15 times. Find:

a. the expected number of 3s rolled

b. the standard deviation

a. $E(x) = np$

$= 15 \times \frac{1}{6}$

$= 2.5$

b. $Var(x) = npq$

$= 15 \times \frac{1}{6} \times \frac{5}{6}$

$= 2.08$

$SD(x) = \sqrt{(npq)}$

$= \sqrt{(2.08)}$

$= 1.44$

c. 95% lies between

$2.5+2\times1.44=6.48$

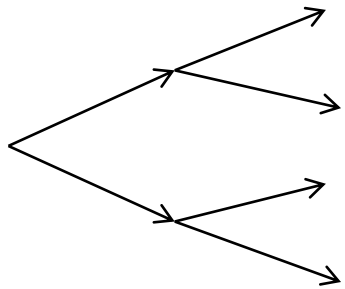
$2.5-2\times1.44=.72$

NB. These formulae are not on the formula sheet

Markov Chains:

Suppose that the probability that a bus is late, given that it is late the previous day, is 0.15, while the probability that it is on time if it is on time the previous day is 0.90.

a Find the transition matrix that can be used to represent this information.



$$T = \begin{bmatrix} & \\ & \end{bmatrix}$$

condition

b What is the probability that the bus will be late on day 3:

i if it is on time on day 0?

$$S_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} 0.15 & 0.1 \\ 0.85 & 0.9 \end{bmatrix}^3 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.1053 \\ 0.8948 \end{bmatrix}$$

ii if it is late on day 0?

$$S_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} 0.15 & 0.1 \\ 0.85 & 0.9 \end{bmatrix}^3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.1054 \\ 0.8946 \end{bmatrix}$$

Steady State:

Example: 2010 INSIGHT Exam 1:

Ciara is a good netballer. She plays goal shooter in her team and knows that from past experience her probability of scoring a goal depends on the success of her previous attempt. She knows that if she scored a goal previously then her probability of scoring a goal on the next attempt is 0.7. If she is unsuccessful on the previous attempt, her probability of being unsuccessful on the next attempt is 0.8.

a. If she has scored a goal, what is the chance of her not scoring on her next attempt?

$$\begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix}$$

b. She has five shots at goal. Her attempt is a goal. What is the probability that it takes her until her final shot to score another goal?

$$\Pr(GMMMG) = 1 \times 0.3 \times 0.8 \times 0.8 \times 0.2 = 0.0384$$

c. Find the probability of Ciara scoring a goal in the long term.

$$T = \begin{bmatrix} 1-a & b \\ a & 1-b \end{bmatrix}$$

$$T^n = \begin{bmatrix} \frac{b}{a+b} & \frac{b}{a+b} \\ \frac{a}{a+b} & \frac{a}{a+b} \end{bmatrix}$$

This involves calculating the steady state probabilities

using the general matrix $\begin{bmatrix} 1-a & b \\ a & 1-b \end{bmatrix}$

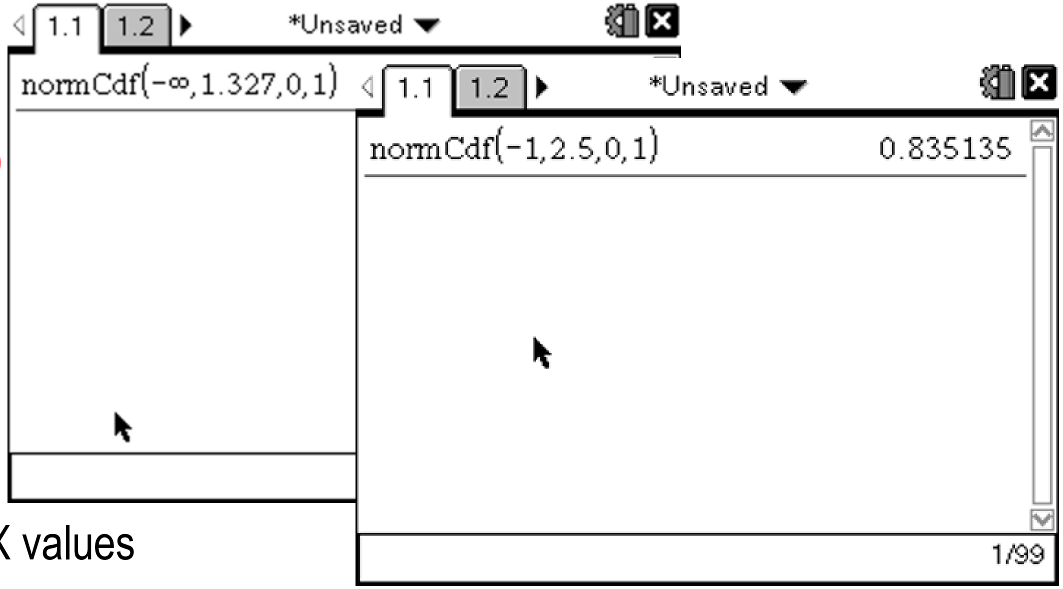
the $\Pr(G) = \dfrac{b}{a+b} = \dfrac{0.2}{0.2+0.3} = \dfrac{0.2}{0.5} = 0.4$

Normal Distribution:

Using Ti-nspire: NormCdf(lower, upper,0,1) for Z values

Pr(Z ≤ 1.327)

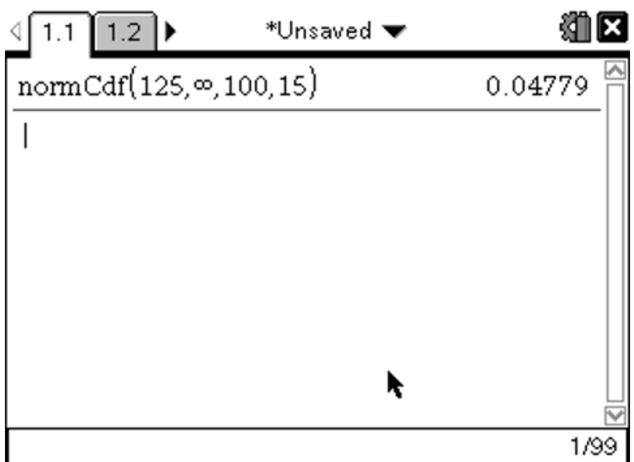
Pr(-1 ≤ Z ≤ 2.5)



Using CAS: X values

If X is the IQ test, normally distributed with μ = 100 and σ = 15
What proportion of the population have an IQ greater than 125?

Pr (x > 125)

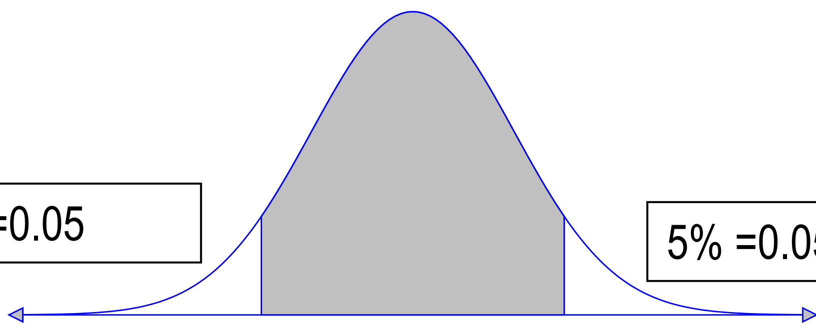


Inverse Normal: Probability given

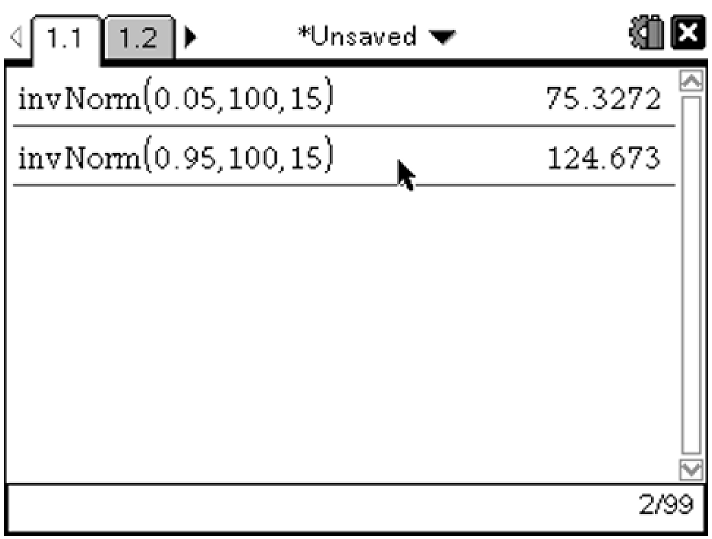
If X is the IQ test, normally distributed with $\mu = 100$ and $\sigma = 15$
Between what IQ values does 90% of the population lie?

90% =0.9

5% =0.05



5% =0.05



Between 75.3 and 124.7

General Comments 2011 Exam 2

- Marks ranged from 2 to 79 out of a possible score of 80.
- Mean- 44 compared with 40 in 2007. Median- 45 marks. Of the whole cohort, 12% of students scored 85% or more of the available marks, and 26% scored 75% or more of the available marks.
- The mean score for the multiple-choice section was 13 (out of 22)
- As stated in the instructions, students must show appropriate working for questions worth more than one mark
- Ensure you: read questions carefully, give answers to the required accuracy, transcribe the correct equation, provide answers to all parts of the question, answer the question that is being asked and after completing a question students, should reread the question.
- Exact answers were required for the intercepts

New content to the course:

- General solutions to trigonometric equations
- Average value of a function
- Functional Equations
- Matrices – transformations, transition matrices and steady state
- Solving simultaneous equations

General Comments

- Correct mathematical notation is expected and should always be used
- Students must use the units that are given in the question
- Use the units that are given in the question- do not convert units unless the question asks for the answer to be in a specific unit.
- From 2010, it will be assumed that students will provide **exact answers** to questions unless specified otherwise
- Many students gave the exact answer and then wrote down an approximate answer. The final answer will be considered the final answer!
- Students should retain **sufficient decimal place** accuracy in computation to ensure that they can provide numerical answers to a specified accuracy
- Students should take care when sketching graphs. If they are subsequently required to sketch over part of a graph, they need to make the relevant parts visible
- More care needs to be taken with closed and open circles for **end points** (gradient graphs)

2009 VCAA EXAM 2-Multiple Choice

49% B

Question 1

The simultaneous linear equations

$$kx - 3y = 0$$
$$5x - (k + 2)y = 0$$

where k is a real constant, have a unique solution provided

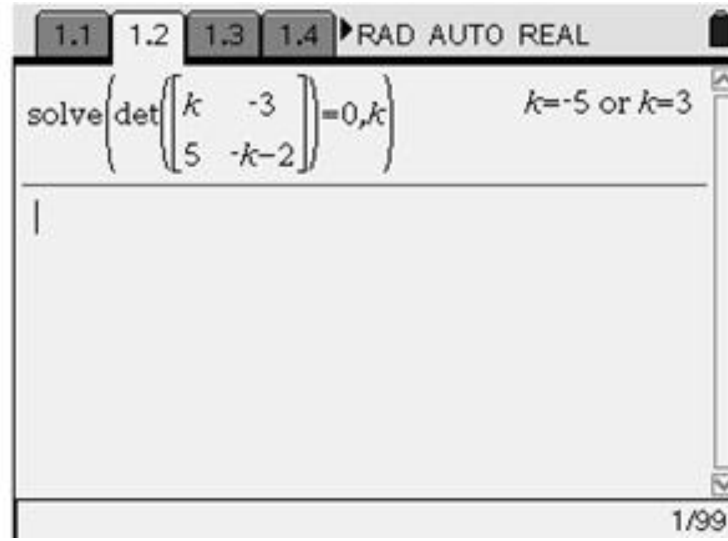
A. $k \in \{-5, 3\}$

B. $k \in R \setminus \{-5, 3\}$

C. $k \in \{-3, 5\}$

D. $k \in R \setminus \{-3, 5\}$

E. $k \in R \setminus \{0\}$



If det=0 there is no inverse and hence no solution. Therefore

$$k \in R \setminus \{-5, 3\}$$

Question 2

At the point (1, 1) on the graph of the function with rule $y = (x - 1)^3 + 1$

A. there is a local maximum.

B. there is a local minimum.

C. there is a stationary point of inflection.

D. the gradient is not defined.

E. there is a point of discontinuity.

95% C

Sketch on CAS

Notice the cubic is in the inflexion point form-

POI=(1,1)

$$\frac{dy}{dx} = 3(x-1)^2$$

$$\frac{dy}{dx} = 0$$

$$x = 1$$

Question 3

The maximal domain D of the function $f: D \to R$ with rule $f(x) = \log_e(2x+1)$ is

86% B

A. $R \setminus \left\{-\frac{1}{2}\right\}$

B. $\left(-\frac{1}{2}, \infty\right)$

C. R

D. $(0, \infty)$

E. $\left(-\infty, -\frac{1}{2}\right)$

The maximum domain of $f(x) = \log_e(2x+1)$ occurs when

$$f(x) = \log_e(2x+1)$$
$$2x+1 > 0$$
$$x > -\frac{1}{2}$$

Question 4

55% A

The general solution to the equation $\sin(2x) = -1$ is

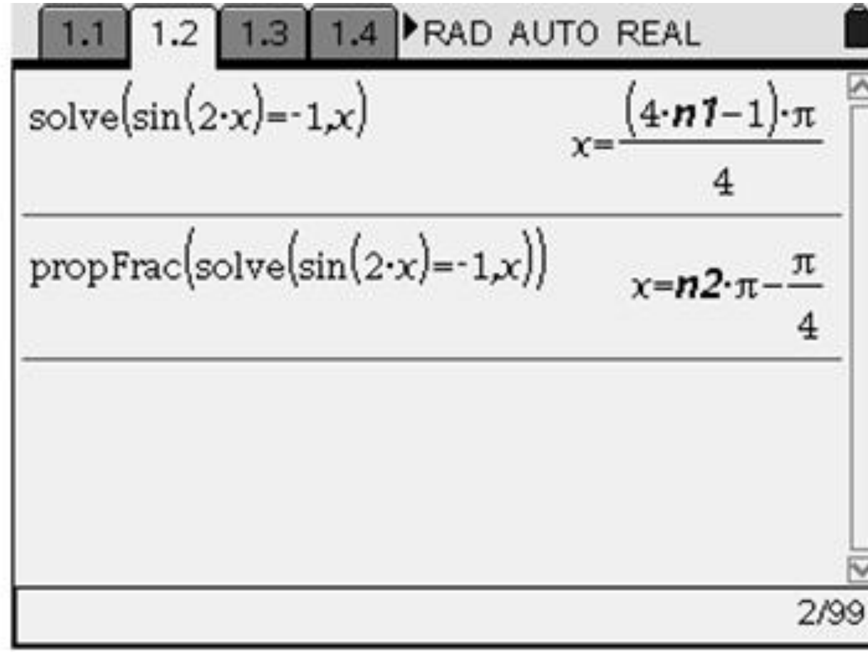
A. $x = n\pi - \dfrac{\pi}{4},\ n \in Z$

B. $x = 2n\pi + \dfrac{\pi}{4}$ or $x = 2n\pi - \dfrac{\pi}{4},\ n \in Z$

C. $x = \dfrac{n\pi}{2} + (-1)^n \dfrac{\pi}{2},\ n \in Z$

D. $x = \dfrac{n\pi}{2} + (-1)^n \dfrac{\pi}{4},\ n \in Z$

E. $x = n\pi + \dfrac{\pi}{4}$ or $x = 2n\pi + \dfrac{\pi}{4},\ n \in Z$



Question 5

Let $f: R \to R$, $f(x) = x^2$.

Which one of the following is **not** true?

A. $f(xy) = f(x)f(y)$

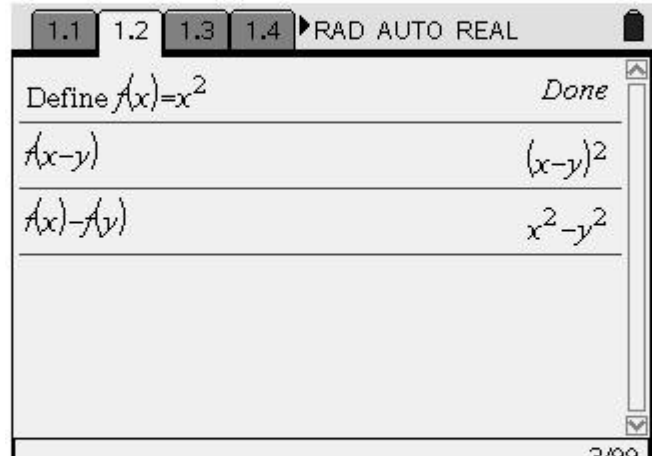
B. $f(x) - f(-x) = 0$

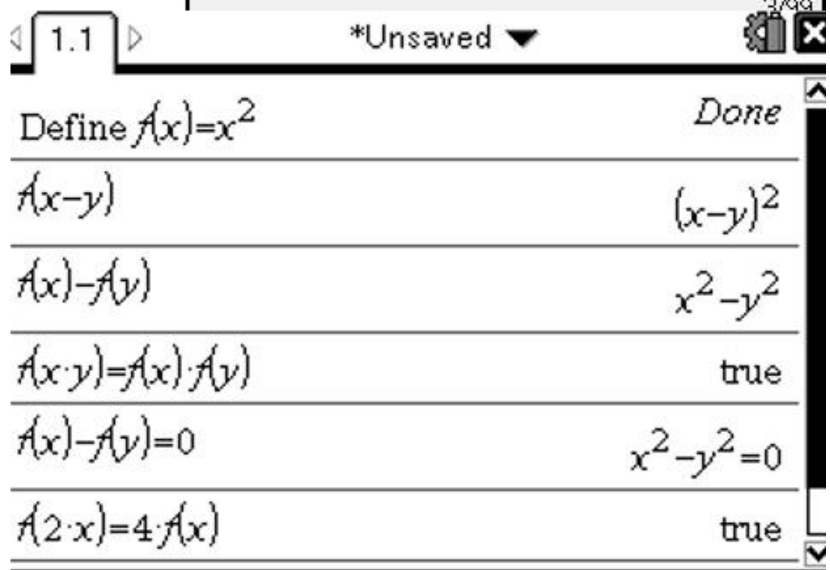
C. $f(2x) = 4f(x)$

D. $f(x-y) = f(x) - f(y)$

E. $f(x+y) + f(x-y) = 2(f(x) + f(y))$

53% D





For a functional equation, a CAS output of ‘true’ indicates that it is *always* true. An output of ‘false’ indicates that is *never* true. Any other output indicates that it is *sometimes* true but not always

Question 6

The continuous random variable X has a normal distribution with mean 14 and standard deviation 2.

If the random variable Z has the standard normal distribution, then the probability that X is greater than 17 is equal to

A. $\Pr(Z > 3)$

B. $\Pr(Z < 2)$

C. $\Pr(Z < 1.5)$

D. $\Pr(Z < -1.5)$

E. $\Pr(Z > 2)$

64% D

$$X \sim N(14, 4)$$

$$\begin{aligned}\Pr(X > 17) &= \Pr\left(Z > \frac{17-14}{2}\right)\\ &= \Pr\left(Z > \frac{3}{2}\right)\\ &= \Pr\left(Z < -\frac{3}{2}\right)\end{aligned}$$

Question 7

For $y = e^{2x}\cos(3x)$ the rate of change of y with respect to x when $x = 0$ is

84% B

A. 0

B. 2

C. 3

D. -6

E. -1

1.1 *Unsaved

$$\frac{d}{dx}\left(e^{2\cdot x}\cdot\cos(3\cdot x)\right)|x=0 \qquad 2\cdot\ln(e)$$

Using CAS, evaluate when x = 0.

$$y = e^{2x}\cos(3x)$$

$$\frac{dy}{dx} = 2e^{2x}\cos(3x) - 3e^{2x}\sin(3x)$$

When x = 0

$$\frac{dy}{dx} = 2e^{0}\cos(0) - 3e^{0}\sin(0) = 2$$

Question 8

For the function $f: R \to R, f(x) = (x+5)^2(x-1)$, the subset of R for which the gradient of f is negative is

A. $(-\infty, 1)$

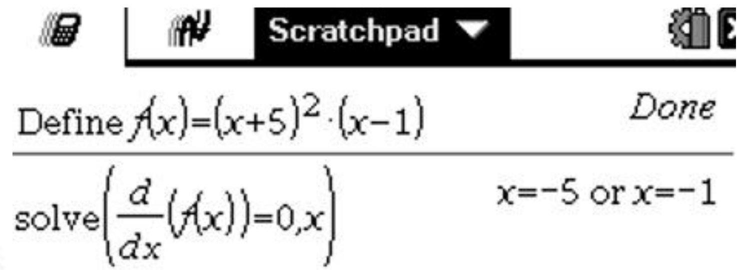
B. $(-5, 1)$

C. $(-5, -1)$

D. $(-\infty, -5)$

E. $(-5, 0)$

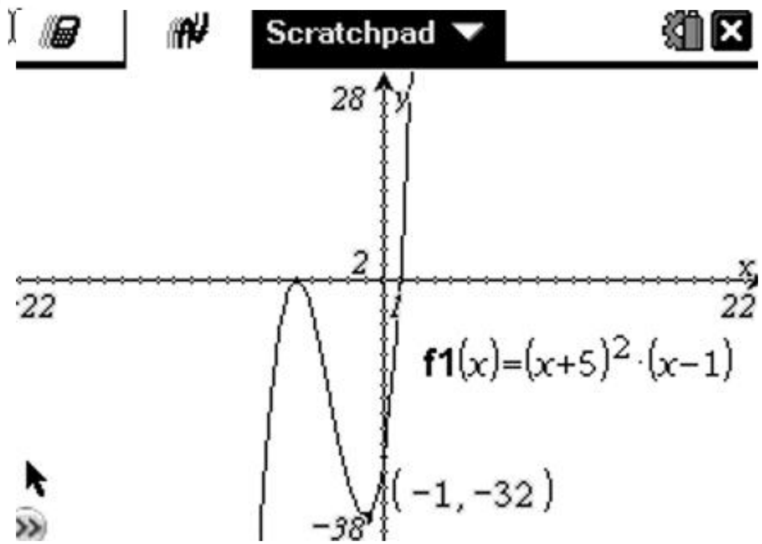
79% C



$f(x) = (x+5)^2(x-1)$

There are turning points at $x = -5$ and $x = -1$.
The coefficient of x^3 is positive. So the shape of the curve is

The gradient is negative for $x \in (-5, -1)$.



Question 9

The tangent at the point (1, 5) on the graph of the curve $y = f(x)$ has equation $y = 3 + 2x$.

The tangent at the point (3, 8) on the curve $y = f(x - 2) + 3$ has equation

A. $y = 2x - 4$

B. $y = x + 5$

C. $y = -2x + 14$

D. $y = 2x + 4$

E. $y = 2x + 2$

49% E

The curve of $y = f(x)$ has been translated 2 units to the right and 3 units up. The image of the point (1, 5) is (3, 8). Hence the gradient of $y = f(x-2)+3$ at the point (3, 8) will be 2.

$y - 8 = 2(x - 3)$

Hence, the equation of the tangent at (3, 8) is $y = 2x + 2$.

A very clever Q!

Question 10

72% B

The discrete random variable X has a probability distribution as shown.

x	**0**	**1**	**2**	**3**
$\Pr(X = x)$	0.4	0.2	0.3	0.1

The median of X is

A. 0

B. 1

C. 1.1

D. 1.2

E. 2

The median is the 50th percentile which is $x = 1$.

Question 11

61% D

The continuous random variable X has a probability density function given by

$$f(x)=\begin{cases}\pi\sin(2\pi x) & \text{if } \quad 0\le x\le \frac{1}{2}\\ 0 & \text{elsewhere}\end{cases}$$

The value of a such that $\Pr(X>a)=0.2$ is closest to

A. 0.26

B. 0.30

C. 0.32

D. 0.35

E. 0.40

$$\Pr(X>a)=\int_a^{\frac{1}{2}}(\pi\sin(2\pi x))dx=0.2$$

Using CAS, $a\approx 0.35$

$$\text{solve}\left(\int_a^{\frac{1}{2}}(f(x))\,dx=0.2,a\right)\Big|0\le a\le\frac{1}{2}$$

$$a=0.352416$$

Question 12

A transformation $T: R^2 \rightarrow R^2$ that maps the curve with equation $y = \sin(x)$ onto the curve with equation $y = 1 - 3\sin(2x + \pi)$ is given by

A. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \pi \\ 1 \end{bmatrix}$

B. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \frac{\pi}{2} \\ 1 \end{bmatrix}$

C. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} \pi \\ 1 \end{bmatrix}$

D. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\frac{\pi}{2} \\ 1 \end{bmatrix}$

E. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\frac{\pi}{2} \\ -1 \end{bmatrix}$

61% D (17% A)
Another clever Q!

$$y = 1 - 3\sin(2x + \pi) = 1 - 3\sin\left(2\left(x + \frac{\pi}{2}\right)\right)$$

from the curve with equation $y = \sin(x)$ involves

- a reflection in the x-axis
- a dilation of a factor of $\frac{1}{2}$ from the y-axis
- a dilation of a factor of 3 from the x-axis
- a translation of $\frac{\pi}{2}$ units to the left and
- a translation of 1 unit up.

Hence $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -\frac{\pi}{2} \\ 1 \end{bmatrix}$

Question 13

71% C

A fair coin is tossed twelve times.

The probability (correct to four decimal places) that at most 4 heads are obtained is

A. 0.0730

B. 0.1209

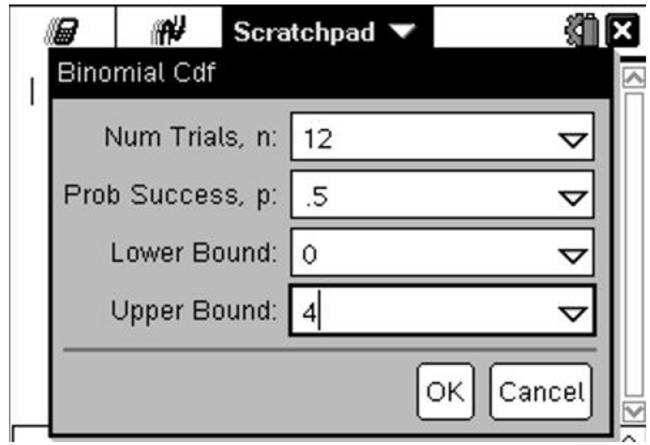
C. 0.1938

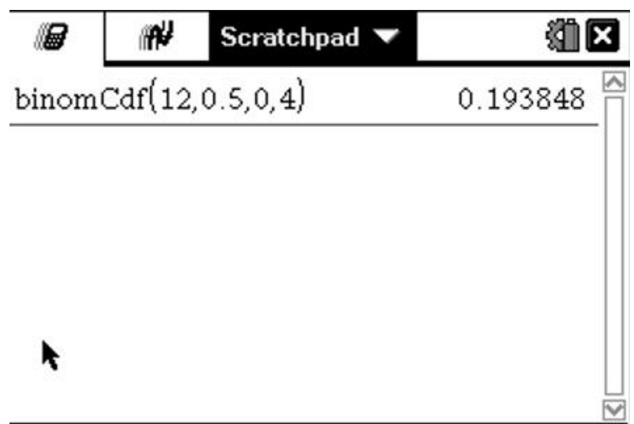
D. 0.8062

E. 0.9270

$X \sim Bi(12, 0.5)$

$\Pr(X \le 4) =$





Question 14

Which one of the following is **not** true for the function with rule $f(x) = \left| x^{\frac{3}{5}} \right| + 2$?

A. $f(32) = 10$.

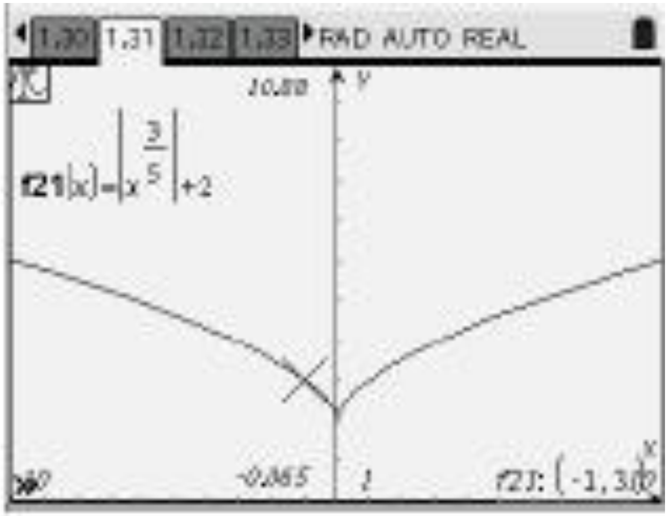
B. The gradient of the function at the point (0, 2) is not defined.

C. $f(x) \geq 2$ for all real values of x.

D. There is a stationary point where $x = 0$.

E. $f(-1) = 3$.

59% D



Question 15

74% E An Excellent Q

For $y=\sqrt{1-f(x)}$, $\dfrac{dy}{dx}$ is equal to

A. $\dfrac{2f'(x)}{\sqrt{1-f(x)}}$

B. $\dfrac{-1}{2\sqrt{1-f'(x)}}$

C. $\dfrac{1}{2}\sqrt{1-f'(x)}$

D. $\dfrac{3}{2(1-f'(x))}$

E. $\dfrac{-f'(x)}{2\sqrt{1-f(x)}}$

$$y=\sqrt{1-f(x)}=(1-f(x))^{\frac{1}{2}}$$

$$\frac{dy}{dx}=\frac{1}{2}(1-f(x))^{-\frac{1}{2}}\times -f'(x)$$

$$=\frac{-f'(x)}{2\sqrt{1-f(x)}}$$

Question 16

74% C

The inverse of the function $f: R^+ \to R,\ f(x) = e^{2x+3}$ is

A. $f^{-1}: R^+ \to R \qquad f^{-1}(x) = e^{-2x-3}$

B. $f^{-1}: R^+ \to R \qquad f^{-1}(x) = e^{\frac{x-3}{2}}$

C. $f^{-1}: (e^3, \infty) \to R \qquad f^{-1}(x) = \log_e\left(\sqrt{x}\right) - \frac{3}{2}$

D. $f^{-1}: (e^3, \infty) \to R \qquad f^{-1}(x) = e^{\frac{x-3}{2}}$

E. $f^{-1}: (e^3, \infty) \to R \qquad f^{-1}(x) = -\log_e(2x-3)$

$f(x) = e^{2x+3}$

Let $y = e^{2x+3}$

Inverse swap x and y.

$x = e^{2y+3}$

$2y + 3 = \log_e(x)$

$2y = \log_e(x) - 3$

$y = \frac{1}{2}\log_e(x) - \frac{3}{2} = \log_e\left(\sqrt{x}\right) - \frac{3}{2}$

The range of f is (e^3, ∞).

Hence

$f^{-1}: (e^3, \infty) \to R$, where $f^{-1}(x) = \log_e\left(\sqrt{x}\right) - \frac{3}{2}$

Question 17

The sample space when a fair twelve-sided die is rolled is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. Each ou
is equally likely.

31% A 43% B

For which one of the following pairs of events are the events independent?

A. $\{1, 3, 5, 7, 9, 11\}$ and $\{1, 4, 7, 10\}$

B. $\{1, 3, 5, 7, 9, 11\}$ and $\{2, 4, 6, 8, 10, 12\}$

C. $\{4, 8, 12\}$ and $\{6, 12\}$

D. $\{6, 12\}$ and $\{1, 12\}$

E. $\{2, 4, 6, 8, 10, 12\}$ and $\{1, 2, 3\}$

For independent events

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

For option **A**

Let $A = \{1, 3, 5, 7, 9, 11\}$ and $B = \{1, 4, 7, 10\}$

$$\Pr(A) = \frac{1}{2} \text{ and } \Pr(B) = \frac{1}{3}$$

$$\Pr(A \cap B) = \frac{1}{6}$$

$$\Pr(A) \times \Pr(B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Hence $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$

Note the events in option **B** are mutually exclusive.

Question 18

The average value of the function $f: R \setminus \left\{-\frac{1}{2}\right\} \to R,\ f(x) = \frac{1}{2x+1}$ over the interval $[0, k]$ is $\frac{1}{6}\log_e(7)$.
The value of k is

52% B

A. $\dfrac{-6}{\log_e(7)} - \dfrac{1}{2}$

B. 3

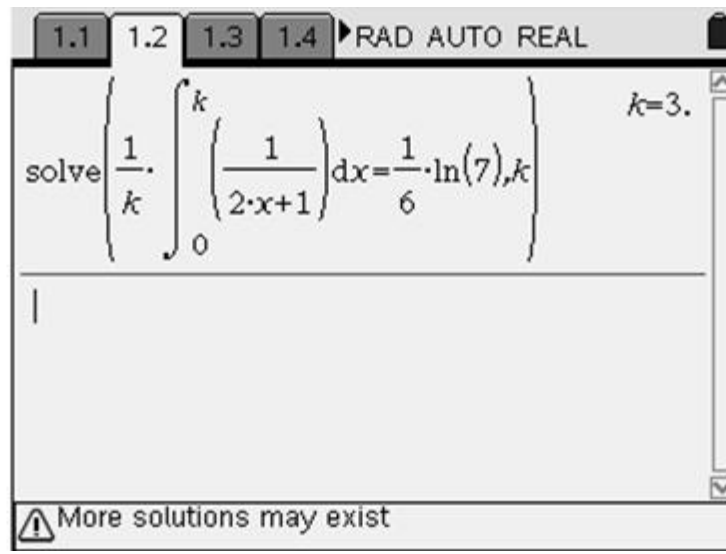
C. e^3

D. $\dfrac{-\log_e(7)}{2(\log_e(7)+6)}$

E. 171

Solve $\dfrac{1}{k-0}\displaystyle\int_0^k \left(\frac{1}{2x+1}\right)dx = \frac{1}{6}\log_e(7)$ for k.

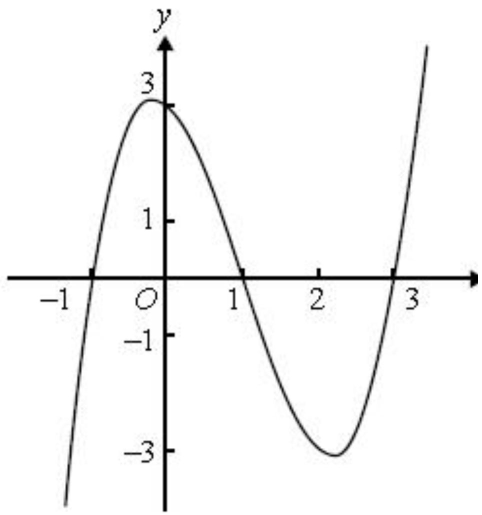
$k = 3.$



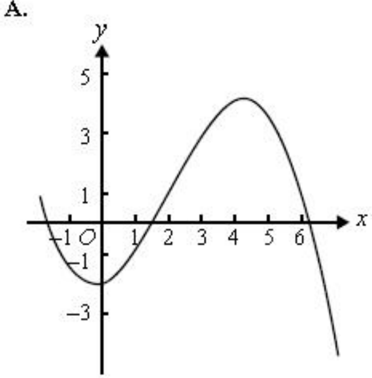
Question 19

66% E

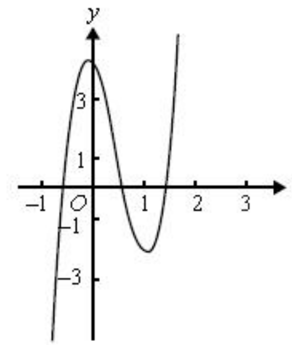
The graph of a function f, with domain R, is as shown.



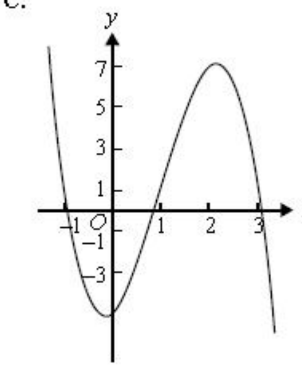
A.



B.

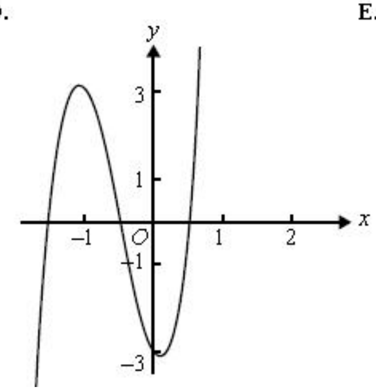


C.

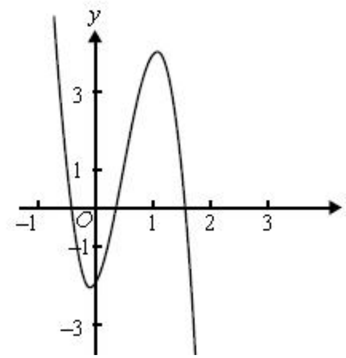


The graph which best represents $1-f(2x)$ is

D.



E.



The graph of f has been reflected in the x-axis, dilated by a factor of $\frac{1}{2}$ from the y-axis and then translated 1 unit up.

Question 20

The number of solutions for x of the equation $|a\cos(2x)| = |a|$, where $x \in [-2\pi, 2\pi]$ and a is a non-zero constant, is

A. 3

B. 4

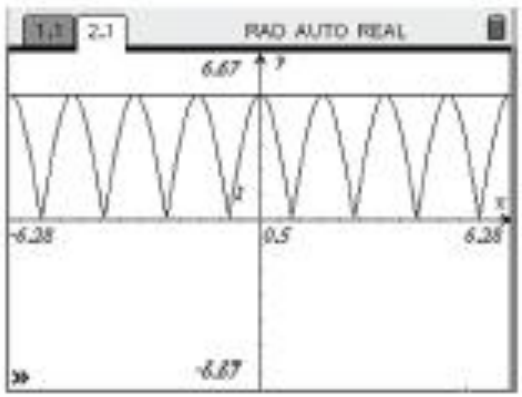
C. 5

D. 7

E. 9

34% E 28% B

The number of solutions to $|a\cos(x)| = |a|$ where $x \in [-2\pi, 2\pi]$ is 9. The line $y = |a|$ will touch the curve $y = |a\cos(x)|$ at the two ends points and the seven turning points.



Question 21

A cubic function has the rule $y = f(x)$. The graph of the derivative function f' crosses the x-axis at (2, 0) and (–3, 0). The maximum value of the derivative function is 10.

The value of x for which the graph of $y = f(x)$ has a local maximum is

43% B 24%A

A. –2

B. 2

C. –3

D. 3

E. $-\frac{1}{2}$

$f'(x) > 0$ for $x \in (-3, 2)$, $f'(x) = 0$ at $x = 2$ and $f'(x) < 0$ for $x > 2$. Hence the graph of f has a local maximum at $x = 2$. The derivative is always positive and a maximum at $x = 0$.

Question 22

Consider the region bounded by the x-axis, the y-axis, the line with equation $y = 3$ and the curve with equation $y = \log_e(x - 1)$.

The exact value of the area of this region is

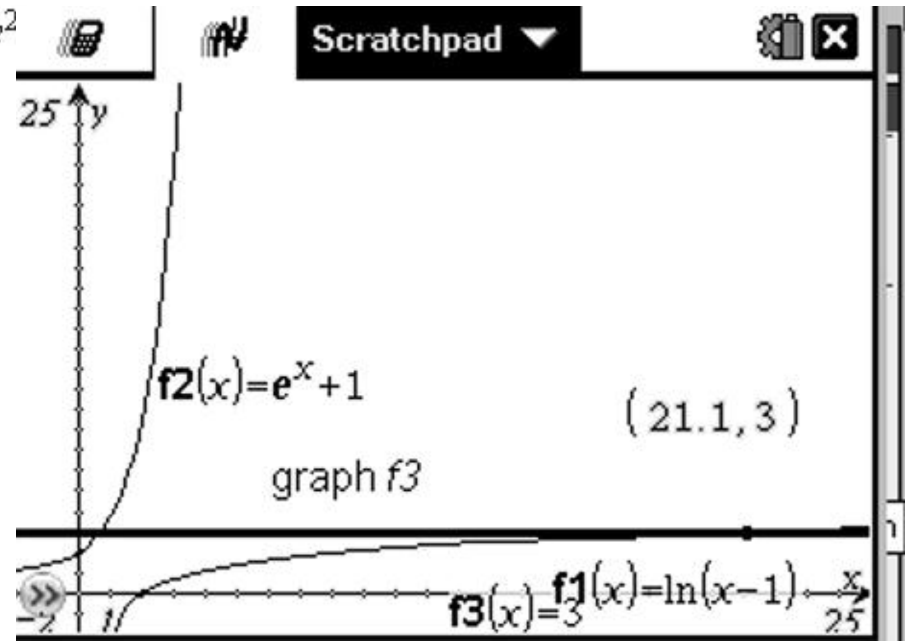
A. $e^{-3} - 1$

B. $16 + 3\log_e(2)$

C. $3e^3 - e^{-3} + 2$

D. $e^3 + 2$

E. $3e^2$



45% D 21% C

Solve $3 = \log_e(x-1)$ for x.

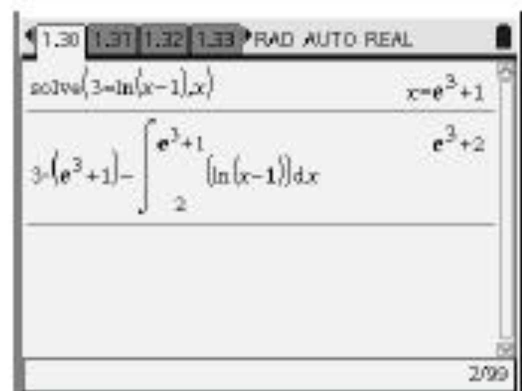
$x = e^3 + 1$

The area bounded by the line $y = 3$, the x-axis, the y-axis and the curve with equation $f(x) = \log_e(x-1)$

$$= \text{area of the rectangle} - \int_2^{e^3+1} (\log_e(x-1))\,dx$$

$$= 3\times(e^3+1) - \int_2^{e^3+1} (\log_e(x-1))\,dx$$

$$= e^3 + 2$$



General Comments 2011

- 15,494
- Students who were pedantic about the use of maths conventions and notation were less likely to lose marks for careless errors
- Time management was an issue- use the reading time
- If a*b=0 then a=0 or b=0 or both can be 0
- G(x)*F(x)=F(x)*k(x)
- Show questions- can’t use the information given
- Integrate Q’s must have dx
- Inequalities- draw a graph y> (x+2)(x-4)
- Coordinates
- Ensure you: read questions carefully, give answers to the required accuracy, transcribe the correct equation, provide answers to all parts of the question, answer the question that is being asked and after completing a question students, should reread the question.
- Exact answers were required for the intercepts

2008 VCAA EXAM 2:

73% C

Question 1

The area under the curve $y = \sin(x)$ between $x = 0$ and $x = \frac{\pi}{2}$ is approximated by two rectangles as shown. This approximation to the area is

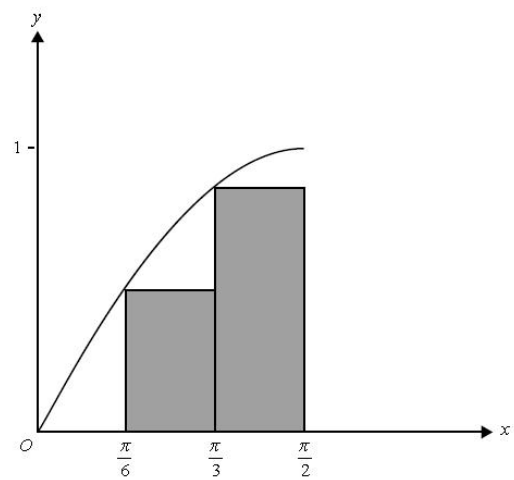
A. 1

B. $\frac{\pi}{2}$

C. $\frac{(\sqrt{3}+1)\pi}{12}$

D. 0.5

E. $\frac{(\sqrt{3}+1)\pi}{6}$



The area of a rectangle = length × width

$$\text{Area} = f\left(\frac{\pi}{6}\right) \times \left(\frac{\pi}{3} - \frac{\pi}{6}\right) + f\left(\frac{\pi}{3}\right) \times \left(\frac{\pi}{2} - \frac{\pi}{3}\right)$$

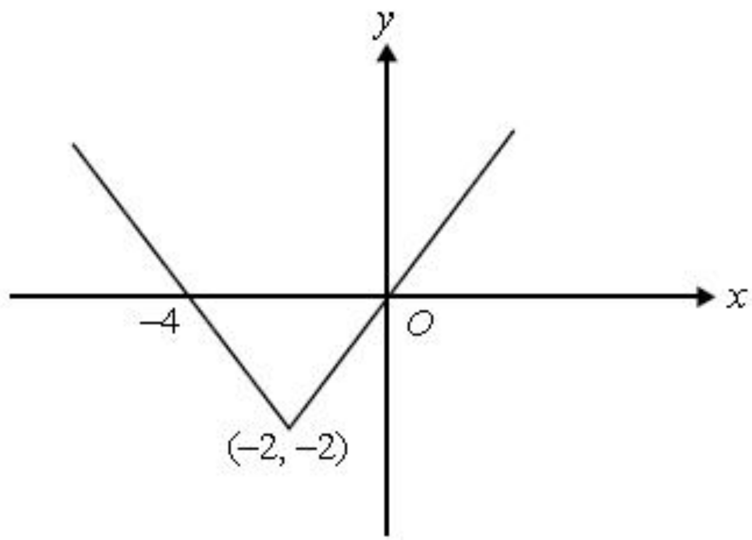
$$= \sin\left(\frac{\pi}{6}\right) \times \left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{3}\right) \times \left(\frac{\pi}{6}\right)$$

$$= \frac{\pi}{12} + \frac{\sqrt{3}\pi}{12}$$

$$= \frac{(\sqrt{3}+1)\pi}{12}$$

Question 2

93% C



The rule of the function whose graph is shown is

A. $y = |x| - 4$

B. $y = |x - 2| + 2$

C. $y = |x + 2| - 2$

D. $y = |2 - x| - 2$

E. $y = |2 + x| + 2$

This is the graph of $y = |x|$ translated 2 units to the left and 2 units down. The coordinates of the vertex are $(-2, -2)$ and the x-intercepts are $(0, -4)$ and $(0, 0)$.

Question 3

59% D

The average value of the function with rule $f(x) = \log_e(3x+1)$ over the interval $[0, 2]$ is

A. $\dfrac{\log_e(7)}{2}$

B. $\log_e(7)$

C. $\dfrac{7\log_e(7)}{3} - 2$

D. $\dfrac{7\log_e(7) - 6}{6}$

E. $\dfrac{35\log_e(7) - 12}{18}$

$$\textbf{Average value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{2-0}\int_0^2 \left(\log_e(3x+1)\right)dx$$

$$= \frac{7\log_e(7) - 6}{6}$$

22% of the students found the **average rate of change and not average value**

Question 5

Let X be a discrete random variable with a binomial distribution. The mean of X is 1.2 and the variance of X is 0.72.

The values of n (the number of independent trials) and p (the probability of success in each trial) are

A. $n = 4, \quad p = 0.3$

B. $n = 3, \quad p = 0.6$

C. $n = 2, \quad p = 0.6$

D. $n = 2, \quad p = 0.4$

E. $n = 3, \quad p = 0.4$

68% E Simult eqns involving Bi Distribution- Clever Q

$\mu = np = 1.2 \ldots\ldots\ldots(1)$

$\sigma^2 = npq = 0.72 \ldots(2)$

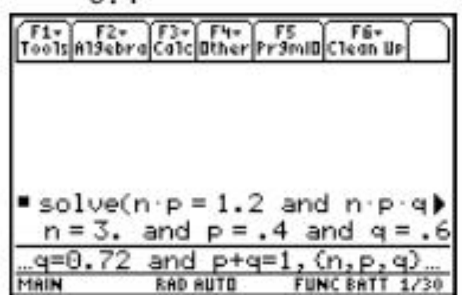
Substitute (1) into (2)

$1.2q = 0.72$

$q = 0.6$

$p = 1 - q = 0.4$

$$n = \frac{1.2}{0.4} = 3$$



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Question 8

The graph of the function $f: D \rightarrow R,\ f(x) = \dfrac{x-3}{2-x}$, where D is the maximal domain has asymptotes

A. $x = 3, \quad y = 2$

B. $x = -2, \quad y = 1$

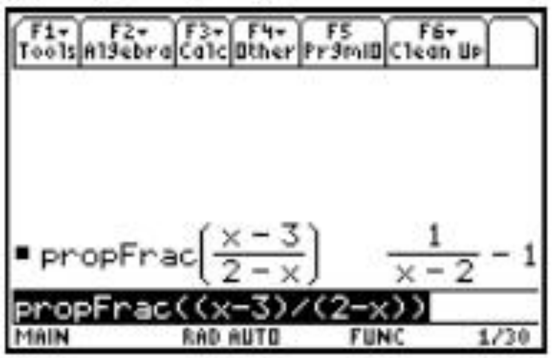
C. $x = 1, \quad y = -1$

D. $x = 2, \quad y = -1$

E. $x = 2, \quad y = 1$

86% D

$$\frac{x-3}{2-x} = \frac{1}{x-2} - 1$$



Question 10

The range of the function $f:\left[\frac{\pi}{8}, \frac{\pi}{3}\right) \to R,\ f(x) = 2\sin(2x)$ is

A. $\left(\sqrt{2}, \sqrt{3}\right]$

B. $\left[\sqrt{2}, 2\right)$

C. $\left[\sqrt{2}, 2\right]$

D. $\left(\sqrt{2}, \sqrt{3}\right)$

E. $\left[\sqrt{2}, \sqrt{3}\right)$

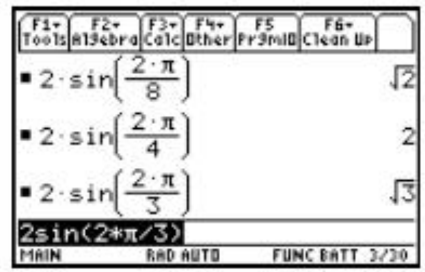
45% C

The period is π. Hence the maximum occurs at the turning point where $x = \frac{\pi}{4}$.

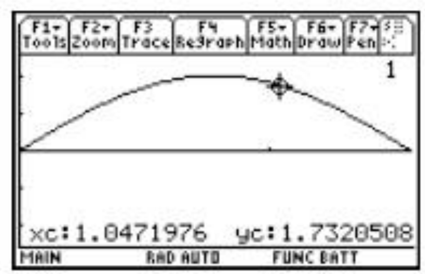
$$f\left(\frac{\pi}{8}\right) = 2\sin\left(\frac{\pi}{4}\right) = \sqrt{2}$$

$$f\left(\frac{\pi}{4}\right) = 2\sin\left(\frac{\pi}{2}\right) = 2$$

Hence the range is $\left[\sqrt{2},\ 2\right]$ **C**



Screen dump showing the point $\left(\frac{\pi}{3}, \sqrt{3}\right)$.



Question 11

The probability density function for the continuous random variable X is given by

76% D

$$f(x)=\begin{cases}|1-x| & \text{if} \quad 0\le x\le 2\\ 0 & \text{elsewhere}\end{cases}$$

The probability that $X<1.5$ is equal to

A. 0.125

B. 0.375

C. 0.5

D. 0.625

E. 0.75

$$\Pr(X<1.5)=\int_{-\infty}^{1.5} f(x)dx$$

$$\int_{0}^{1.5}|1-x|dx=0.625$$

Question 12

Let $f: R \to R$, $f(x) = e^x + e^{-x}$.

For all $u \in R$, $f(2u)$ is equal to

A. $f(u) + f(-u)$

B. $2f(u)$

C. $\left(f(u)\right)^2 - 2$

D. $\left(f(u)\right)^2$

E. $\left(f(u)\right)^2 + 2$

44% C

$$f(2u) = e^{2u} + e^{-2u}$$

$$\left(f(u)\right)^2 = (e^u + e^{-u})^2 = e^{2u} + 2 + e^{-2u}$$

Hence, $f(2u) = (f(u))^2 - 2$.

Question 14

The minimum number of times that a fair coin can be tossed so that the probability of obtaining a head on each trial is less than 0.0005 is

A. 8

B. 9

C. 10

D. 11

E. 12

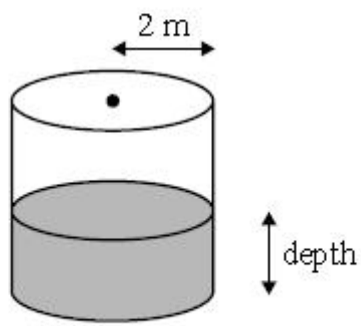
60% D

Solve $\left(\frac{1}{2}\right)^n < 0.0005$ **for** n.

$n = \mathbf{11}$

Question 16

Water is being poured into a long cylindrical storage tank of radius 2 metres, with its circular base on the ground, at a rate of 2 cubic metres per second.



52% C

The rate of change of the depth of the water, in metres per second, in the tank is

A. $\frac{1}{8\pi}$

B. $\frac{1}{4\pi}$

C. $\frac{1}{2\pi}$

D. 2π

E. 8π

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

$$\frac{dV}{dt} = 2 \text{ m}^3\text{s}^{-1}$$

$$V = \pi r^2 h = 4\pi h$$

$$\frac{dV}{dh} = 4\pi$$

$$\frac{dh}{dt} = \frac{1}{4\pi} \times 2 = \frac{1}{2\pi} \text{ms}^{-1}$$

Question 17

The graph of the function $f(x) = e^{2x} - 2$ intersects the graph of $g(x) = e^x$ where

89% B

A. $x = -1$

B. $x = \log_e(2)$

C. $x = 2$

D. $x = \dfrac{1+\sqrt{7}}{2}$

E. $x = \log_e\left(\dfrac{1+\sqrt{7}}{2}\right)$

Solve $e^{2x} - 2 = e^x$ **for** x.

$x = \log_e(2)$

Question 18

Let $f: \left[0, \frac{\pi}{2}\right] \to R$, $f(x) = \sin(4x) + 1$. The graph of f is transformed by a reflection in the x-axis followed by a dilation of factor 4 from the y-axis.

The resulting graph is defined by

A. $g: \left[0, \frac{\pi}{2}\right] \to R \qquad g(x) = -1 - 4\sin(4x)$

B. $g: [0, 2\pi] \to R \qquad g(x) = -1 - \sin(16x)$

C. $g: \left[0, \frac{\pi}{2}\right] \to R \qquad g(x) = 1 - \sin(x)$

D. $g: [0, 2\pi] \to R \qquad g(x) = 1 - \sin(4x)$

E. $g: [0, 2\pi] \to R \qquad g(x) = -1 - \sin(x)$

36% E

$f(x) = \sin(4x) + 1$

Reflection in the x-axis

$h(x) = -\sin(4x) - 1$

Dilation by a factor of 4 from the y-axis

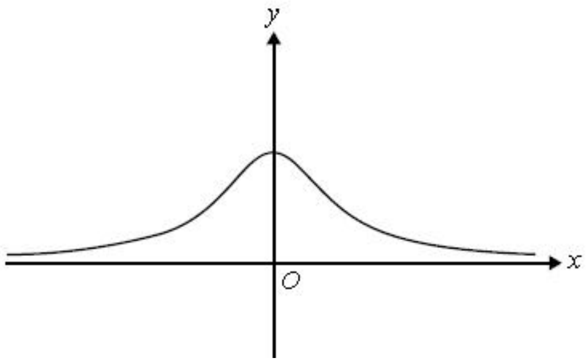
$g(x) = -\sin(x) - 1$

Note the dilation by a factor of 4 from the y-axis affects the domain,

$\left[0 \times 4, \frac{\pi}{2} \times 4\right] = [0, 2\pi]$

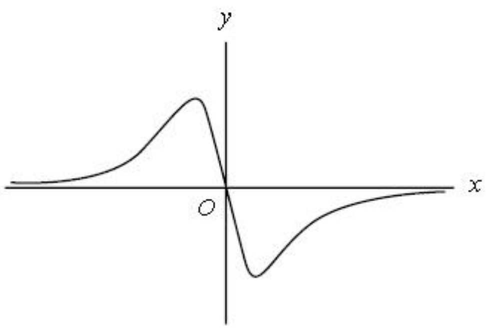
Question 19

The graph of a function f is shown below.

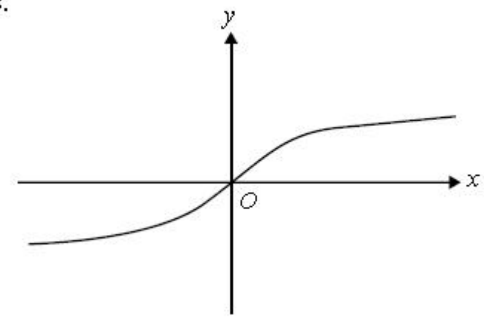


The graph of an **antiderivative** of f could be

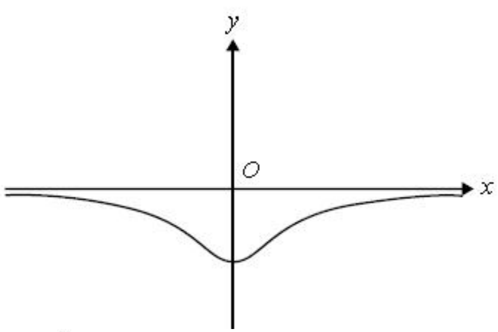
A.



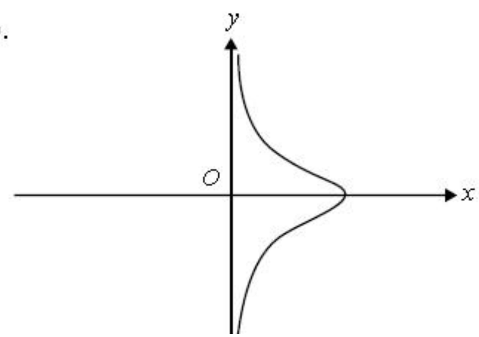
B.



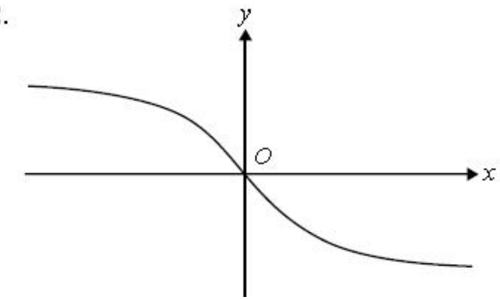
C.



D.



E.



50% B

The graph shown in the next column is the *derivative* of the graph that is required. The derivative is always positive and a maximum at $x = 0$.

Question 20

The function $f: B \to R$ with rule $f(x) = 4x^3 + 3x^2 + 1$ will have an inverse function for

50% B

A. $B = R$

B. $B = \left(\frac{1}{2}, \infty\right)$

C. $B = \left(-\infty, \frac{1}{2}\right]$

D. $B = \left(-\infty, \frac{1}{2}\right)$

E. $B = \left[-\frac{1}{2}, \infty\right)$

For f to have an inverse function it must be a one to one function.

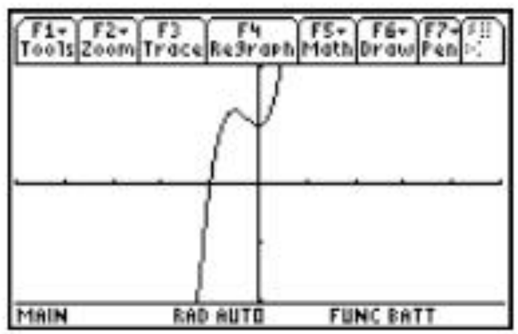
The turning points occur at

$$12x^2 + 6x = 0$$

$$6x(2x+1) = 0$$

$$x = -\frac{1}{2} \text{ or } x = 0$$

Hence f is one to one when $B = \left(\frac{1}{2}, \infty\right)$. B



Question 21

75% D

The graph of $y = x^3 - 12x$ has turning points where $x = 2$ and $x = -2$

The graph of $y = |x^3 - 12x|$ has a positive gradient for

A. $x \in R$

B. $x \in \{x : x < -2\} \cup \{x : x > 2\}$

C. $x \in \left\{x : x < -2\sqrt{3}\right\} \cup \left\{x : x > 2\sqrt{3}\right\}$

D. $x \in \left\{x : -2\sqrt{3} < x < -2\right\} \cup \{x : 0 < x < 2\} \cup \left\{x : x > 2\sqrt{3}\right\}$

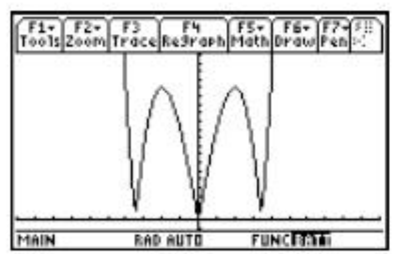
E. $x \in \{x : -2 < x < 0\} \cup \left\{x : 2 < x < 2\sqrt{3}\right\} \cup \left\{x : x > 2\sqrt{3}\right\}$

Question 21

The graph of $y = x^3 - 12x$ has turning points are at $x = 2$ and $x = -2$ and x-intercepts at $x^3 - 12x = 0$, $x(x^2 - 12) = 0$, $x = -2\sqrt{3}$ or $x = 2\sqrt{3}$.

Hence, the graph of $y = \left|x^3 - 12x\right|$ has a positive gradient for

$x \in \{x : -2\sqrt{3} < x < -2\} \cup \{x : 0 < x < 2\} \cup \{x : x > 2\sqrt{3}\}$

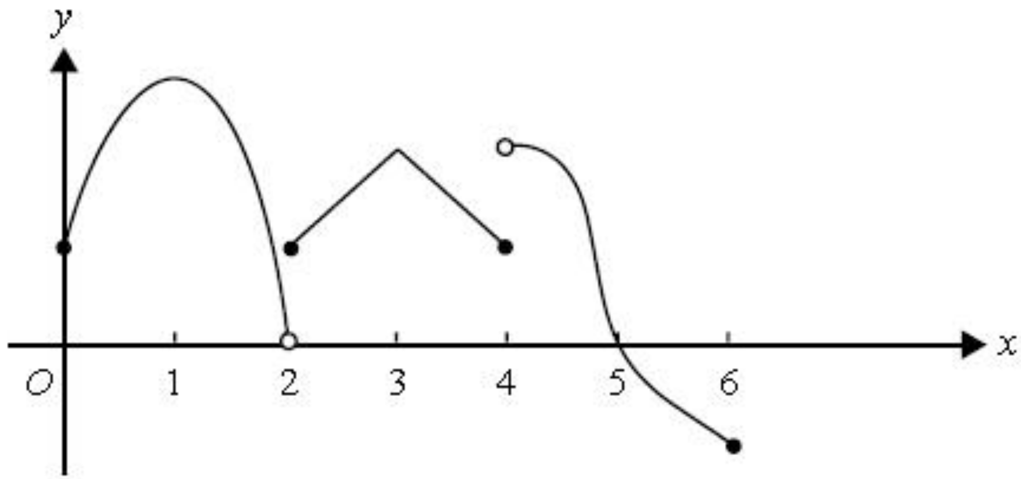


D

Question 22

53% C

The graph of the function f with domain $[0, 6]$ is shown below.



Which one of the following is **not** true?

A. The function is not continuous at $x = 2$ and $x = 4$.

B. The function exists for all values of x between 0 and 6.

C. $f(x) = 0$ for $x = 2$ and $x = 5$.

D. The function is positive for $x \in [0, 5)$.

E. The gradient of the function is not defined at $x = 4$.

$f(x) = 0$ for x = 5 but is discontinuous at $x = -2$. Hence C is false.

VCAA 2010

60% D

Question 1

The function with rule $f(x) = 4\tan\left(\dfrac{x}{3}\right)$ has period

A. $\dfrac{\pi}{3}$

B. 6π

C. 3

D. 3π

E. $\dfrac{2\pi}{3}$

$$\textbf{Period} = \frac{\pi}{\frac{1}{3}} = 3\pi$$

Question 2

69% E

For $f(x) = x^3 + 2x$, the average rate of change with respect to x for the interval [1, 5] is

A. 18

B. 20.5

C. 24

D. 32.5

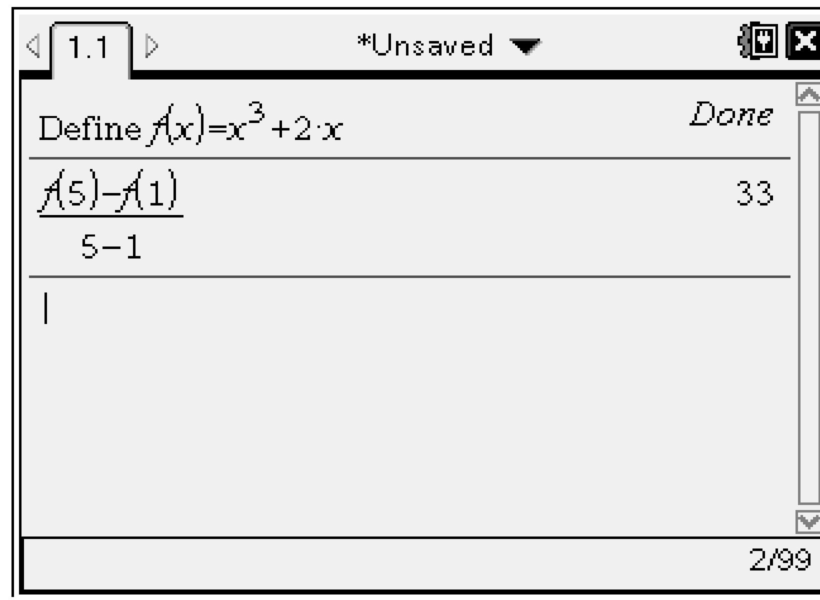
E. 33

$$f(x) = x^3 + 2x$$

$$\textbf{Average rate of change} = \frac{f(5) - f(1)}{5-1}$$

$$= \frac{(125+10) - (1+2)}{4}$$

$$= 33$$



Question 4

83% D

If $f(x) = \frac{1}{2}e^{3x}$ and $g(x) = \log_e(2x) + 3$ then $g(f(x))$ is equal to

A. $2x^3 + 3$

B. $e^{3x} + 3$

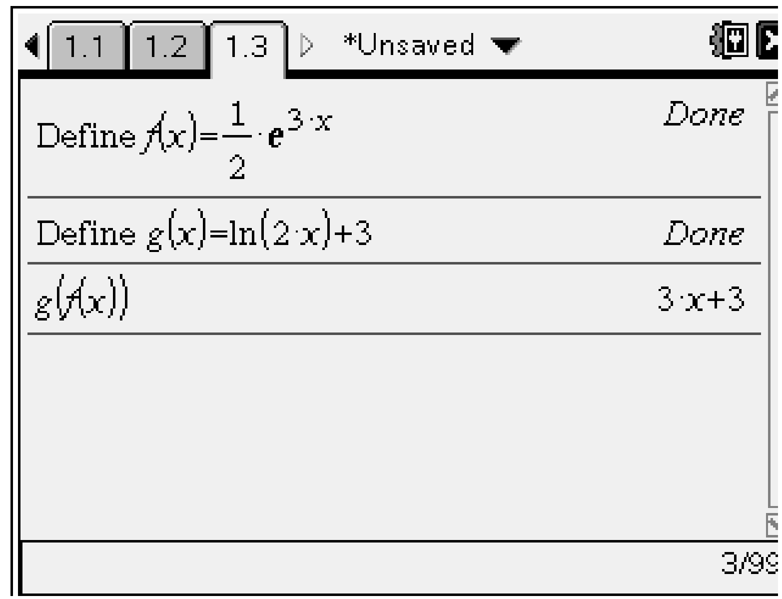
C. e^{8x+9}

D. $3(x+1)$

E. $\log_e(3x) + 3$

$$f(x) = \frac{1}{2}e^{3x} \text{ and } g(x) = \log_e(2x) + 3$$

$$\begin{aligned} g(f(x)) &= \log_e\left(e^{3x}\right) + 3 \\ &= 3x + 3 \end{aligned}$$



Question 5

For the system of simultaneous linear equations

$$x = 5$$
$$z + y = 10$$
$$z - y = 6$$

an equivalent matrix equation is

A. $\begin{bmatrix} x \\ y \\ z \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 6 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 6 \end{bmatrix}$

C. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 2\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & -1 \\ 2 & 0 & 0 \end{bmatrix}\begin{bmatrix} 5 \\ 6 \\ 10 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 6 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 10 \end{bmatrix}$

80% B

$x = 5$, $z + y = 10$ **and** $z - y = 6$

Rearranging the variables

$x = 5$, $y + z = 10$ **and** $-y + z = 6$

The matrix equation is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 6 \end{bmatrix}$$

Question 13

The continuous random variable X has a normal distribution with mean 20 and standard deviation 6. The continuous random variable Z has the standard normal distribution.

The probability that Z is between -2 and 1 is equal to

51% B

A. $\Pr(18 < X < 21)$

B. $\Pr(14 < X < 32)$

C. $\Pr(14 < X < 26)$

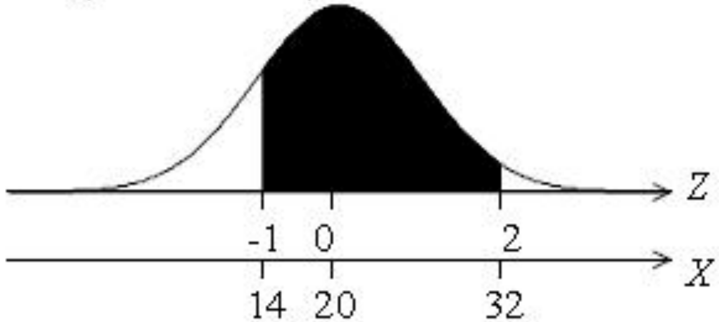
D. $\Pr(8 < X < 32)$

E. $\Pr(X > 14) + \Pr(X < 26)$

$X \sim \mathrm{N}(20, 36)$

Two standard deviations to the left of the mean and one standard deviation to the right of the mean is the same as two standard deviations to the right of the mean and one to the left.

$$\Pr(-2 < Z < 1)$$
$$= \Pr(-1 < Z < 2)$$
$$= \Pr(20 - 1\times 6 < X < 20 + 2\times 6)$$
$$= \Pr(14 < X < 32)$$



Question 14

A bag contains four white balls and six black balls. Three balls are drawn from the bag without replacement. The probability that they are all black is

81% A

A. $\frac{1}{6}$

B. $\frac{27}{125}$

C. $\frac{24}{29}$

D. $\frac{3}{500}$

E. $\frac{8}{125}$

4 white balls and 6 black balls

$$\Pr(BBB) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{1}{6}$$

Question 15

83% C

The discrete random variable X has the following probability distribution.

X	0	1	2
$\Pr(X = x)$	a	b	0.4

If the mean of X is 1 then

A. $a = 0.3$ and $b = 0.1$

B. $a = 0.2$ and $b = 0.2$

C. $a = 0.4$ and $b = 0.2$

D. $a = 0.1$ and $b = 0.5$

E. $a = 0.1$ and $b = 0.3$

$$E(X) = 0 \times a + 1 \times b + 2 \times 0.4 = 1$$

$$b = 0.2$$

Question 17

The function f is differentiable for all $x \in R$ and satisfies the following conditions. 60% B

- $f'(x) < 0$ where $x < 2$
- $f'(x) = 0$ where $x = 2$
- $f'(x) = 0$ where $x = 4$
- $f'(x) > 0$ where $2 < x < 4$
- $f'(x) > 0$ where $x > 4$

Which one of the following is true?

A. The graph of f has a local maximum point where $x = 4$.

B. The graph of f has a stationary point of inflection where $x = 4$.

C. The graph of f has a local maximum point where $x = 2$.

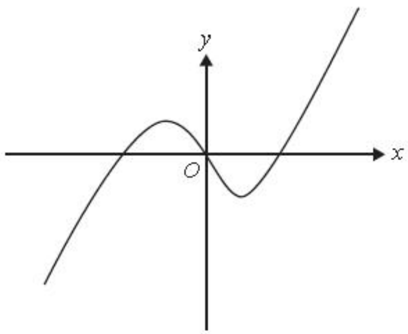
D. The graph of f has a local minimum point where $x = 4$.

E. The graph of f has a stationary point of inflection where $x = 2$.

The graph has a stationary point of inflection at $x = 4$ because $f'(4) = 0$ and $f'(x) > 0$ where $2 < x < 4$ and where $x > 4$.

Question 19

The graph of the gradient function $y = f'(x)$ is shown below.

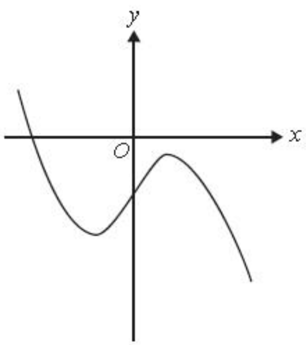


77% D

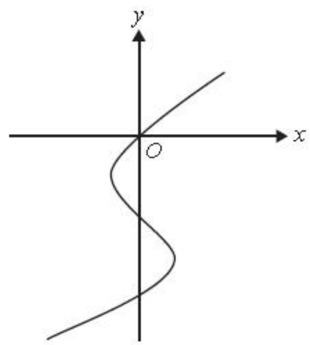
The gradient function looks like a cubic polynomial function in the form $f'(x) = ax(x-b)(x-c)$ where $a > 0$ and a, b and c are real constants. Hence f could be a quartic polynomial where the coefficient of x^4 is positive.

Which of the following could represent the graph of the function $f(x)$?

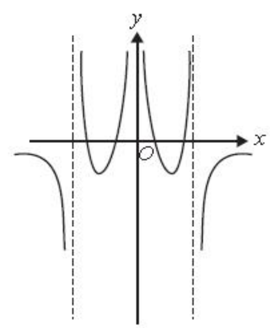
A.



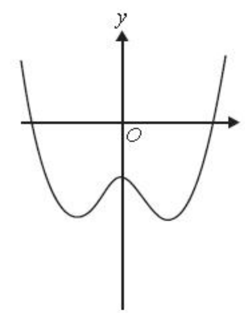
B.



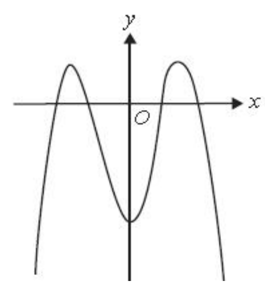
C.



D.



E.



Question 20

Let f be a differentiable function defined for all real x, where $f(x) \geq 0$ for all $x \in [0, a]$.

If $\int_0^a f(x)dx = a$, then $2\int_0^{5a}\left(f\left(\frac{x}{5}\right)+3\right)dx$ is equal to

25% D

A. $2a + 6$

B. $10a + 6$

C. $20a$

D. $40a$

E. $50a$

$$\int_0^a f(x)dx = a$$

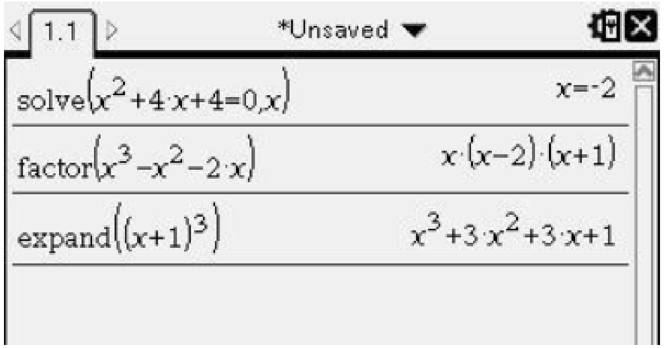
$$2\int_0^{5a}\left(f\left(\frac{x}{5}\right)+3\right)dx$$

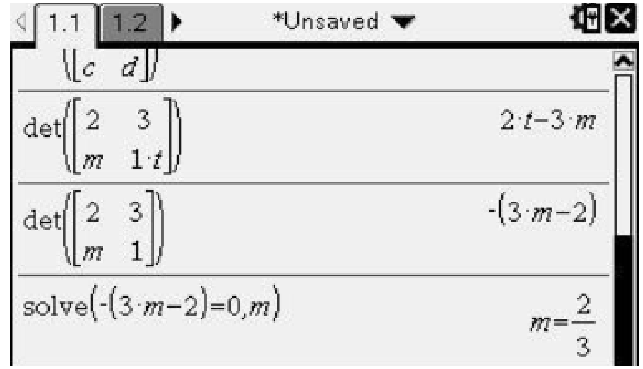
$$= 2\times 5a + 2\int_0^{5a}(3)dx$$

$$= 10a + 30a$$

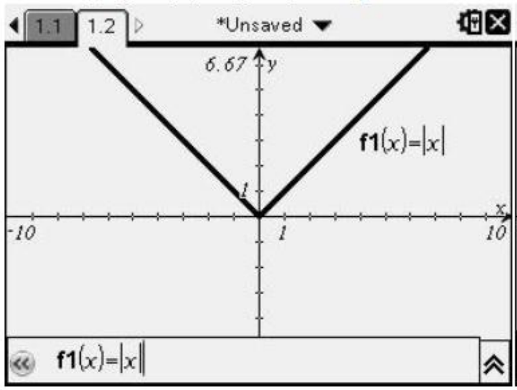
$$= 40a$$

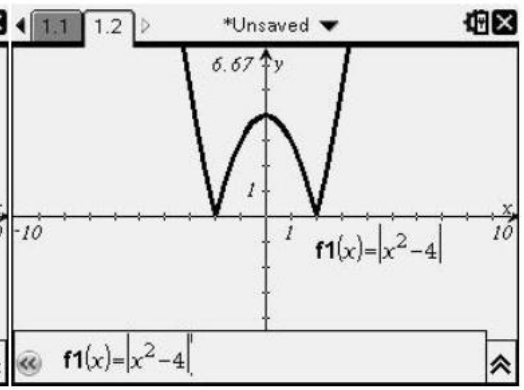
Short Cuts:





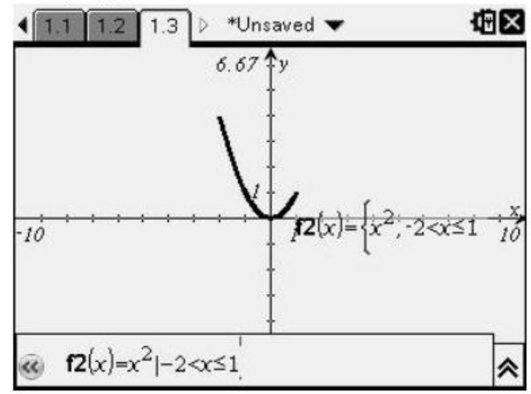
For example $y = |x|$ and $y = |x^2 - 4|$





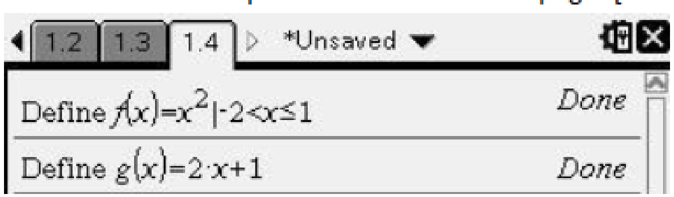
e.g. Graph $y = x^2$ for $x \in (-2,1]$

Enter $f2(x) = x^2| -2 < x \leq 1$ into the graphs bar



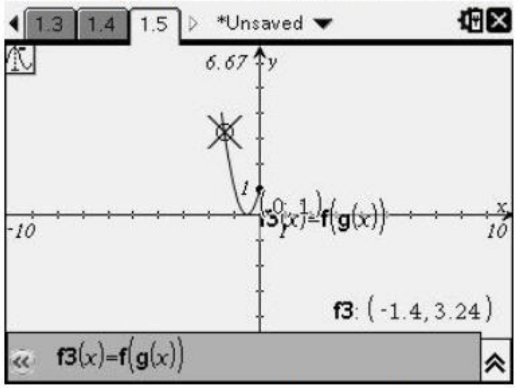
E.g. Find the equation of $fog(x)$ when $f(x) = x^2,\ x \in (-2,1]$ and $g(x) = 2x + 1, x \in R$

1. Define the two equations in the Calulate page. [Menu] [1] [1]



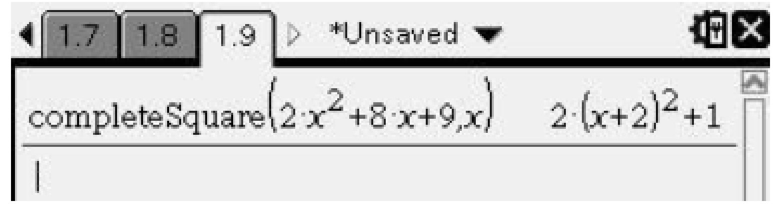
Short Cuts: cont.

2. Open a graph page and type, f(g(x)) into the graph bar



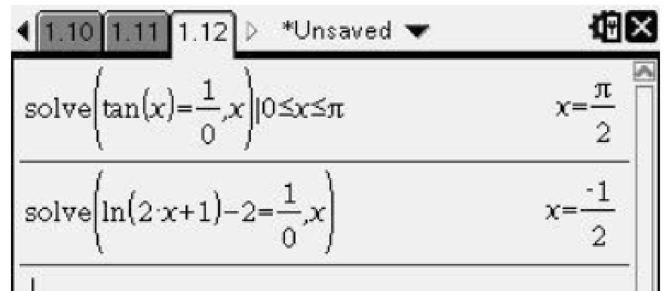
The trace feature can be used to find out the range and domain. Trace: [Menu] [5] [1]
Here $fog(x) = (2x + 1)^2$ where the Domain = (-1.5,1] and Range =[0,4)

e.g. Find the turning point of $y = 2x^2 + 8x + 9$

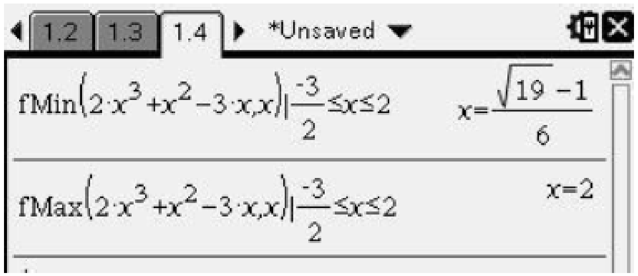


So from that the turning point will be at (-2,1)

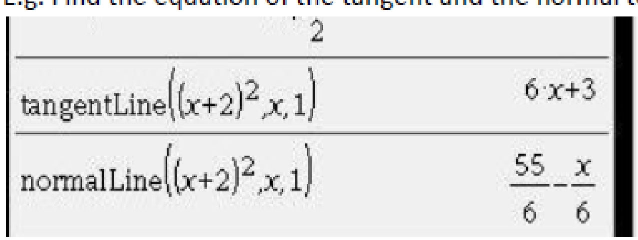
Finding Vertical Asymptotes



E.g. Find the values of x for which $y = 2x^3 + x^2 - 3x$ has a maxmimum and a minimum for $x \in [-\frac{3}{2}, 2]$



Tangents at a point: [Menu] [4] [9] – (terms, variable, point)
Normals at a point: [Menu] [4] [A] - (terms, variable, point)
E.g. Find the equation of the tangent and the normal to the curve $y = (x + 2)^2$ when $x = 1$.



More short cuts:

The x-y Function Test

Every now and then you will come across this kind of question in a multiple choice :

If $f(x) + f(y) = f(xy)$, which of the following is true?

A. $f(x) = x^2$

B. $f(x) = \ln(x)$

C. $f(x) = \frac{1}{x}$

D. $f(x) = x$

E. $f(x) = (x + 2)^2$

condition for x, then test whether the option is true..

1.13 | 1.14 | 1.15 | *Unsaved

Define $f(x)=x^2$ — Done

solve$(f(x)+f(y)=f(x\cdot y),x)$

$x=\frac{-y}{\sqrt{y^2-1}}$ and $\frac{y^2}{y^2-1}\geq 0$ or $x=\frac{y}{\sqrt{y^2-1}}$ and ▸

Define $f(x)=\ln(x)$ — Done

solve$(f(x)+f(y)=f(x\cdot y),x)$ — true

Define $f(x)=2\cdot x^3+3\cdot x^2-4\cdot x+2+\frac{1}{x}$ — Done

Define $df(x)=\frac{d}{dx}(f(x))$ — Done

$df(x)$ — $6\cdot x^2+6\cdot x-\frac{1}{x^2}-4$

The same thing can be done for the double derivativ

$df(x)$ — $6\cdot x^2+6\cdot x-\frac{1}{x^2}-4$

Define $ddf(x)=\frac{d}{dx}(df(x))$ — Done

$ddf(x)$ — $12\cdot x+\frac{2}{x^3}+6$

1.1 | *Unsaved

Define $f(x)=a\cdot x^3+b\cdot x^2+c\cdot x+d$ — Done

Define $df(x)=\frac{d}{dx}(f(x))$ — Done

$df(x)$ — $3\cdot a\cdot x^2+2\cdot b\cdot x+c$

solve$(f(-2)=0$ and $f(2)=0$ and $df(-1)=0$ and $f(0$▸

$a=\frac{1}{2}$ and $b=\frac{-1}{4}$ and $c=-2$ and $d=1$

Using tCollect to simplify awkward expressions

Sometimes the calculator won't simplify something the way we want it to. tCollect simplifies expressions that involves trigonometric powers higher than 1 or lower than -1 to linear trigonometric expressions.

1.1 | 1.2 | 1.3 | *Unsaved

tCollect$((\cos(x))^2)$ — $\frac{\cos(2\cdot x)+1}{2}$

tCollect$(|2\cdot(\sin(2\cdot x))^2+1|)$ — $2-\cos(4\cdot x)$

1.5 | 1.6 | 1.7 | *Unsaved

solve$\left(\int_0^{10}(k\cdot x^2+1)dx=1,k\right)$ — $k=\frac{-27}{1000}$

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Sources:

- VCCA website (assessment reports and past papers)
- INSIGHT solutions
- Xavier College Maths Methods Power Points- P.Burke
- Essential Mathematics Text

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