Vector Geometry Review

(1) Vector Basics - Sections 12.1 and 12.2

(2) Dot and Cross Products - Sections 12.3 and 12.4

(3) Lines and Planes - Sections 12.2 and 12.5

Vectors

A **vector** is a geometric object that has magnitude (length) and direction.

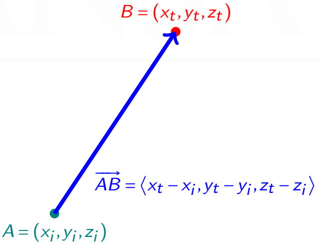
A **scalar** is a constant in $\mathbb{R}$ which has no direction, only magnitude.

Familiar examples of vectors: force, velocity, acceleration, pressure, flux

A vector can be represented geometrically by an arrow AB from A (the initial point) to B (the terminal point). Notation: $\mathbf{\vec{v}} = \vec{v} = \overrightarrow{AB}$.

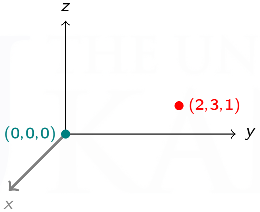
Translating a vector does **not** change it, since the magnitude and direction remain the same.

These three arrows all represent the same vector!



Cartesian Representation of Vectors

- Draw a vector $\vec{\mathbf{v}}$ with its *initial point* at the origin O.
- The **components** of $\vec{\mathbf{v}}$ are the coordinates of the *terminal point* P.



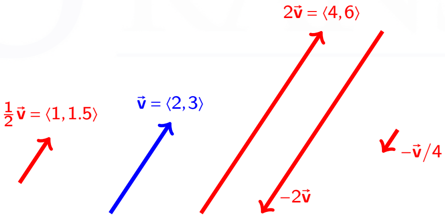
Here $\vec{\mathbf{v}} = \overrightarrow{OP} = \langle a, b, c \rangle$.

In general, if $\vec{\mathbf{v}} = \overrightarrow{AB}$ where $A = (x_1, y_1, z_1)$ and $B = (x_2, y_2, z_2)$ then

$$\vec{\mathbf{v}} = \langle x_2 - x_1\,,\, y_2 - y_1\,,\, z_2 - z_1 \rangle.$$

Scalar Multiplication

- Multiplying a vector $\vec{\mathbf{v}}$ by a positive scalar c does not change its direction, but multiplies its magnitude by c.
- If $c < 0$, the direction of $\vec{\mathbf{v}}$ is reversed and the magnitude is multiplied by $|c|$.
- Two nonzero vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ are **parallel** if they are scalar multiples of each other (there exists a scalar c such that $\vec{\mathbf{v}} = c\vec{\mathbf{w}}$).



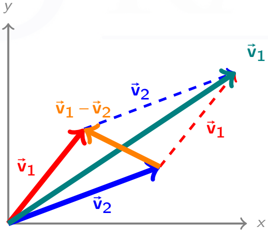
Addition and Subtraction of Vectors

- **Algebraically**, two vectors can be added or subtracted by adding or subtracting their components.

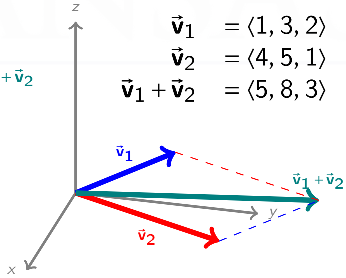
2d	3d
$\langle a, b\rangle \pm \langle c, d\rangle = \langle a \pm c, b \pm d\rangle$	$\langle a, b, c\rangle \pm \langle p, d, q\rangle = \langle a \pm p, b \pm d, c \pm q\rangle$

- **Geometrically**, adding two vectors can be visualized in terms of a parallelogram.

$$\begin{aligned} \vec{\mathbf{v}}_1 &= \langle 4, 5\rangle \\ \vec{\mathbf{v}}_2 &= \langle 8, 3\rangle \\ \vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2 &= \langle 12, 8\rangle \\ \vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2 &= \langle -4, 2\rangle \end{aligned}$$



$$\begin{aligned} \vec{\mathbf{v}}_1 &= \langle 1, 3, 2\rangle \\ \vec{\mathbf{v}}_2 &= \langle 4, 5, 1\rangle \\ \vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2 &= \langle 5, 8, 3\rangle \end{aligned}$$



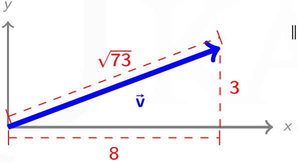
Vector Magnitude

The **magnitude** (or **length**) of a vector $\vec{\mathbf{v}}$ is the distance between its initial point and terminal point:

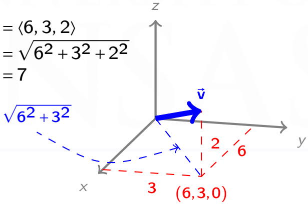
$$\vec{\mathbf{v}} = \langle a, b \rangle \qquad \|\vec{\mathbf{v}}\| = \sqrt{a^2 + b^2}$$

$$\vec{\mathbf{w}} = \langle a, b, c \rangle \qquad \|\vec{\mathbf{w}}\| = \sqrt{a^2 + b^2 + c^2}$$

$$\vec{\mathbf{v}} = \langle 8, 3 \rangle$$
$$\|\vec{\mathbf{v}}\| = \sqrt{8^2 + 3^2} = \sqrt{73}$$



$$\vec{\mathbf{v}} = \langle 6, 3, 2 \rangle$$
$$\|\vec{\mathbf{v}}\| = \sqrt{6^2 + 3^2 + 2^2} = 7$$



If $\vec{\mathbf{v}} = \overrightarrow{AB}$ with $A = (x_1, y_1, z_1)$ and $B = (x_2, y_2, z_2)$, then

$$\|\vec{\mathbf{v}}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

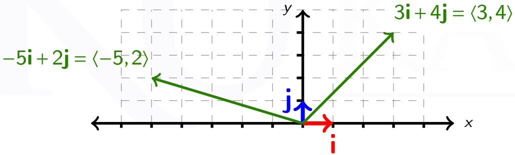
(Note: This is just the usual distance formula.)

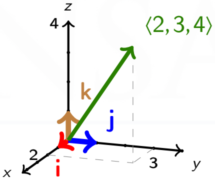
Special Vectors

- The **zero vector** is $\vec{\mathbf{0}} = \langle 0,0 \rangle$ or $\langle 0,0,0 \rangle$.

  The zero vector is the **only** vector with magnitude zero. Its direction is undefined.

- **Standard basis vectors** in $\mathbb{R}^2$: $\mathbf{i} = \langle 1,0 \rangle$ and $\mathbf{j} = \langle 0,1 \rangle$
- **Standard basis vectors** in $\mathbb{R}^3$: $\mathbf{i} = \langle 1,0,0 \rangle$, $\mathbf{j} = \langle 0,1,0 \rangle$, $\mathbf{k} = \langle 0,0,1 \rangle$





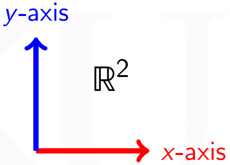
- A **unit vector** is a vector of magnitude one.

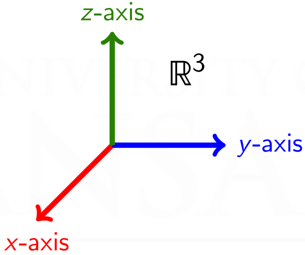
  Unit vectors useful for specifying directions without magnitudes.
  A unit vector in the direction of a given vector can be obtained by multiplying the vector by reciprocal of the magnitude. $\vec{\mathbf{u}} = \frac{1}{\|\vec{\mathbf{v}}\|}\vec{\mathbf{v}}$

  The unit vector in direction $\langle 3,4 \rangle$ is $\langle \frac{3}{5}, \frac{4}{5} \rangle$.

Cartesian Coordinates in $\mathbb{R}^2$ and $\mathbb{R}^3$

Coordinates represent geometric objects in space by ordered pairs/triples of numbers, so that we can study them with algebra and calculus





- Reference point: the origin O
- Two coordinate axes
- One plane
- Four quadrants

- Reference point: the origin O
- Three coordinate axes
- Three coordinate planes
- Eight octants ▸ Link

Dot and Cross Products

In addition to vector addition and scalar multiplication, there are two other important operations on vectors.

1. The **dot product**, which takes two vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ (either both in $\mathbb{R}^2$ or both in $\mathbb{R}^3$) and produces a *scalar* $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}}$.

2. The **cross product**, which takes two vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ (both in $\mathbb{R}^3$) and produces a *vector* $\vec{\mathbf{v}} \times \vec{\mathbf{w}}$.

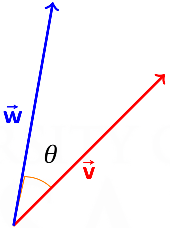
It is very important to understand the **geometry** behind the dot and cross product, not just their formulas.

The Dot Product

The **dot product** of two vectors $\vec{\mathbf{v}} = \langle a_1, b_1, c_1 \rangle$ and $\vec{\mathbf{w}} = \langle a_2, b_2, c_2 \rangle$ is the scalar

$$\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = \|\vec{\mathbf{v}}\| \, \|\vec{\mathbf{w}}\| \cos(\theta)$$

where θ is the angle between the vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$.



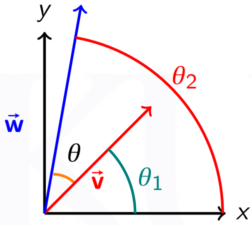
- If θ is acute $(0 \leq \theta < \frac{\pi}{2})$ then $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} > 0$.
- If $\vec{\mathbf{v}}, \vec{\mathbf{w}}$ are orthogonal $(\theta = \frac{\pi}{2})$ then $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = 0$.
- If θ is obtuse $(\frac{\pi}{2} < \theta \leq \pi)$ then $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} < 0$.

- The angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ is $\arccos\left(\dfrac{\vec{\mathbf{v}} \cdot \vec{\mathbf{w}}}{\|\vec{\mathbf{v}}\| \, \|\vec{\mathbf{w}}\|}\right)$.

The Formula for the Dot Product

Formula in $\mathbb{R}^2$: $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = a_1 a_2 + b_1 b_2$

Formula in $\mathbb{R}^3$: $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = a_1 a_2 + b_1 b_2 + c_1 c_2$



$a_1 = \|\vec{\mathbf{v}}\| \cos(\theta_1)$ $\quad$ $b_1 = \|\vec{\mathbf{v}}\| \sin(\theta_1)$

$a_2 = \|\vec{\mathbf{w}}\| \cos(\theta_2)$ $\quad$ $b_2 = \|\vec{\mathbf{w}}\| \sin(\theta_2)$

$$\begin{aligned} a_1 a_2 + b_1 b_2 &= \|\vec{\mathbf{v}}\| \, \|\vec{\mathbf{w}}\| \Big(\cos(\theta_1)\cos(\theta_2) + \sin(\theta_1)\sin(\theta_2) \Big) \\ &= \|\vec{\mathbf{v}}\| \, \|\vec{\mathbf{w}}\| \cos(\theta_2 - \theta_1) \\ &= \|\vec{\mathbf{v}}\| \, \|\vec{\mathbf{w}}\| \cos(\theta) \\ &= \vec{\mathbf{v}} \cdot \vec{\mathbf{w}}. \end{aligned}$$

The Cross Product

The **cross product** of vectors $\vec{\mathbf{v}}, \vec{\mathbf{w}}$ in $\mathbb{R}^3$ is the vector

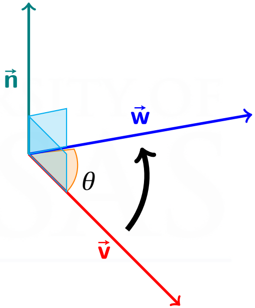
$$\vec{\mathbf{v}} \times \vec{\mathbf{w}} = \left(\|\vec{\mathbf{v}}\| \, \|\vec{\mathbf{w}}\| \sin(\theta)\right) \vec{\mathbf{n}}$$

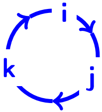
where:

(i) θ is the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$;

(ii) $\vec{\mathbf{n}}$ is the unit vector perpendicular to both $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$, given by the Right-Hand Rule.

(Point the fingers of your right hand toward $\vec{\mathbf{v}}$ and then curl them toward $\vec{\mathbf{w}}$. Your thumb will point in the direction of $\vec{\mathbf{n}}$.)

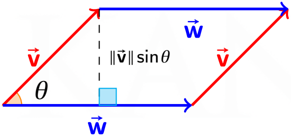




$\mathbf{i} \times \mathbf{j} = \mathbf{k}$ $\quad$ $\mathbf{j} \times \mathbf{k} = \mathbf{i}$ $\quad$ $\mathbf{k} \times \mathbf{i} = \mathbf{j}$

Properties of the Cross Product

- If $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ are parallel, then $\vec{\mathbf{v}} \times \vec{\mathbf{w}} = \vec{\mathbf{0}}$.
- $(\vec{\mathbf{v}} \times \vec{\mathbf{w}}) \perp \vec{\mathbf{v}}$ and $(\vec{\mathbf{v}} \times \vec{\mathbf{w}}) \perp \vec{\mathbf{w}}$.
- $\vec{\mathbf{v}} \times \vec{\mathbf{w}} = -\vec{\mathbf{w}} \times \vec{\mathbf{v}}$.
- $\|\vec{\mathbf{v}} \times \vec{\mathbf{w}}\|$ is the area of the parallelogram with sides $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$.



- To calculate the cross product of two vectors in $\mathbb{R}^2$, treat them as vectors in $\mathbb{R}^3$:

$$\vec{\mathbf{v}} = \langle v_1, v_2 \rangle = \langle v_1, v_2, 0 \rangle \qquad \vec{\mathbf{w}} = \langle w_1, w_2 \rangle = \langle w_1, w_2, 0 \rangle$$

In this case $\vec{\mathbf{v}} \times \vec{\mathbf{w}}$ will always be a multiple of $\mathbf{k} = \langle 1, 0, 0 \rangle$.

Calculating Cross Products with Determinants

The determinant of a 2×2 matrix is $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

The determinant of a 3×3 matrix can be calculated by decomposing into a linear combination of 2×2 matrices.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

Cross Product Formula: $\vec{\mathbf{v}} \times \vec{\mathbf{w}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

Lines in 2-Space (Review)

A line in $\mathbb{R}^2$ is the set of points satisfying a linear equation in x and y.

Point-slope form: The line through (x_0, y_0) with slope m is defined by

$$y - y_0 = m(x - x_0).$$

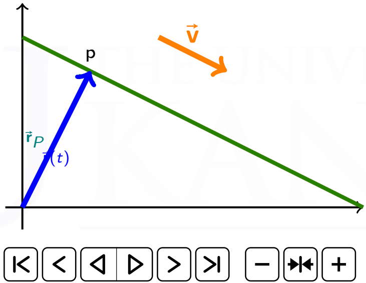
Slope-intercept form: The line with slope m and y-intercept b is defined by

$$y = mx + b.$$

(Exception: A vertical line has undefined slope and cannot be written in either of these forms; its equation is $x = a$.)

Lines in 2-Space: Vector Forms

A line can also be represented using a **direction vector**.
The idea: specify a **point on the line** and a direction to move in.



- The line $y = -\frac{x}{2} + 5$ has slope $m = -\frac{1}{2}$.
- When the x-value changes by $+\mathbf{2}$, the y-value changes by $-\mathbf{1}$.
- That is, the line is parallel to the vector $\vec{\mathbf{v}} = \langle 2, -1 \rangle$.

Lines in 2-Space: Parametrization

Every line L in $\mathbb{R}^2$ has a **direction vector** $\vec{\mathbf{v}}$:

- For any two points P, Q on L, the vector $\overrightarrow{PQ}$ is parallel to $\vec{\mathbf{v}}$.
- That is, there is a scalar t such that $\overrightarrow{PQ} = t\vec{\mathbf{v}}$.
- Every nonzero multiple of $\vec{\mathbf{v}}$ is also a direction vector for L.

- If P is a point on L, then the line can be described by the function

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_P + t\vec{\mathbf{v}}.$$

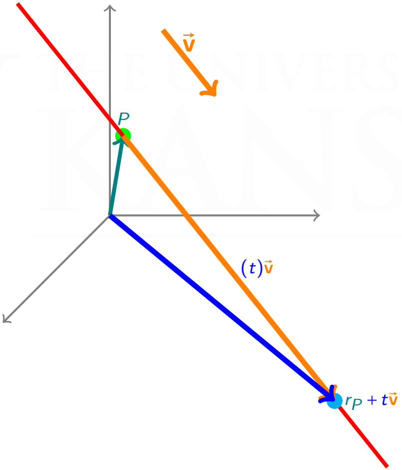
(“Start at P, and then change your position by $t\vec{\mathbf{v}}$.”)

- L has many parametrizations, depending on the choices of P and $\vec{\mathbf{v}}$.
  (P is the starting point, t is time, $\vec{\mathbf{v}}$ is velocity.)

Lines in 3-Space

Lines in $\mathbb{R}^3$ can be parametrized exactly the same as lines in $\mathbb{R}^2$.

In $\mathbb{R}^3$, a line is still determined by a point and a direction.



Equations of a Line in 3-Space

Let L be a line in $\mathbb{R}^3$, with direction vector $\vec{\mathbf{v}} = \langle v_1, v_2, v_3 \rangle$, containing a point $P_0 = (x_0, y_0, z_0)$.

Vector form $\vec{\mathbf{r}} - \vec{\mathbf{r}}_0 = t\vec{\mathbf{v}}$ for all t

$\vec{\mathbf{r}}(t) = \langle x_0 + tv_1,\ y_0 + tv_2,\ z_0 + tv_3 \rangle$

Parametric form $x = x_0 + tv_1,\ y = y_0 + tv_2,\ z = z_0 + tv_3$

These two forms are more or less the same.
The name of the parameter t does not matter.

Symmetric form $$\frac{x - x_0}{v_1} = \frac{y - y_0}{v_2} = \frac{z - z_0}{v_3}$$ (provided $v_1, v_2, v_3 \neq 0$)

This form consists of <u>two</u> equations on x, y, z, with no parameter.

Lines in $\mathbb{R}^3$: Examples

Example 1: Find equations for the line through point $P = (2, 3, 4)$ parallel to $\vec{\mathbf{v}} = \langle 5, 6, 7 \rangle$.

Solution:

Vector form $\vec{\mathbf{r}}(t) = \langle 2 + 5t, 3 + 6t, 4 + 7t \rangle$

Parametric form $x = 2 + 5t \quad y = 3 + 6t \quad z = 4 + 7t$

Symmetric form $\dfrac{x-2}{5} = \dfrac{y-3}{6} = \dfrac{z-4}{7}$

Lines in $\mathbb{R}^3$: Examples

Example 2: Find a vector form of the line through $P = (2, 3, 5)$ and $Q = (4, 2, 1)$.

Solution: The first step is to find a direction vector. Use $\overrightarrow{PQ}$.

$$\overrightarrow{PQ} = \langle 4-2, 2-3, 1-5 \rangle = \langle 2, -1, -4 \rangle.$$

Therefore, a vector form of the line is

$$\vec{\mathbf{r}}(t) = \langle 2+2t, 3-t, 5-4t \rangle.$$

Using the direction vector $\overrightarrow{QP} = \langle -2, 1, 4 \rangle$ and the point P would give

$$\vec{\mathbf{s}}(t) = \langle 2-2t, 3+t, 5+4t \rangle$$

and starting at Q instead of P would give

$$\vec{\mathbf{q}}(t) = \langle 4-2t, 2+t, 1+4t \rangle.$$

Relative Position of Two Lines in Space

- Two lines can be parallel. Direction vectors for parallel lines are scalar multiples of each other.
- Two non-parallel lines can intersect at a point.
- Two lines can be skew. Skew lines are not parallel and do not intersect.

Link

Example 3: The two lines L_1 and L_2 given by the equations

$$\begin{array}{llll} L_1: & x = 3 - 2t & y = 1 + t & z = 4 - 3t \\ L_2: & x = -5 + t & y = 4 - t & z = 1 + 6t \end{array}$$

have direction vectors $\vec{\mathbf{v}}_1 = \langle -2, 1, -3 \rangle$ and $\vec{\mathbf{v}}_2 = \langle 1, -1, 6 \rangle$, which are not scalar multiples — so L_1 and L_2 are not parallel. **Do they intersect?**

Relative Position of Two Lines in Space

Example 3 (continued):

$$\begin{aligned} L_1: \quad & \vec{\mathbf{r}}_1(t) = \langle 3,1,4\rangle + t\langle -2,1,-3\rangle \\ L_2: \quad & \vec{\mathbf{r}}_2(t) = \langle -5,4,1\rangle + t\langle 1,-1,6\rangle \end{aligned}$$

To check if they intersect, solve the system of equations $\vec{\mathbf{r}}_1(t) = \vec{\mathbf{r}}_2(s)$:

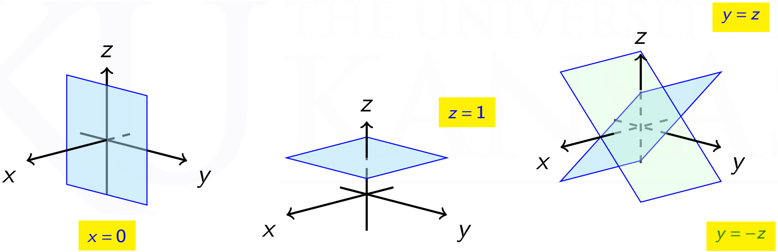
$$\begin{cases} 3-2t &= -5+s \\ 1+t &= 4-s \\ 4-3t &= 1+6s \end{cases}$$

(Be sure to change the name of one of the parameters, since they refer to different lines!)

- Solution: $t = 5$, $s = -2$.
- Lines L_1 and L_2 intersect at $\vec{\mathbf{r}}_1(5) = \vec{\mathbf{r}}_2(-2) = (-7, 6, -11)$.
- If the system has no solution, then the lines are skew.

Planes in Space

If a line in $\mathbb{R}^3$ is defined by two linear equations (in its symmetric form), what kind of set is defined by one linear equation? **A plane.**

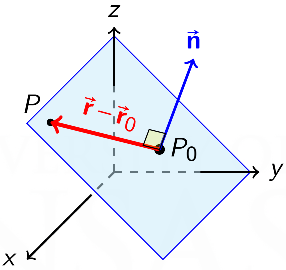


Question: How do we translate between the algebraic equation of a plane and its geometric properties?

Equations for Planes

$P_0(x_0, y_0, z_0)$: point in $\mathbb{R}^3$
$\vec{\mathbf{r}}_0 = \langle x_0, y_0, z_0 \rangle$
$\vec{\mathbf{n}} = \langle n_1, n_2, n_3 \rangle$: nonzero vector

Then there is a unique plane F that passes through P_0 and is orthogonal to $\vec{\mathbf{n}}$.



Let $P(x, y, z)$ be a general point on the plane F and let $\vec{\mathbf{r}} = \langle x, y, z \rangle$.

Vector equation of F $\quad \vec{\mathbf{n}} \cdot (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0) = 0$
Scalar equation of F $\quad n_1(x - x_0) + n_2(y - y_0) + n_3(z - z_0) = 0$

The vector $\vec{\mathbf{n}}$ is called a **normal vector** to F.
Any nonzero multiple of $\vec{\mathbf{n}}$ is also a normal vector to F.

Equations for Planes: Examples

Example 4: Find equations for the plane containing the point $(7, -8, 5)$ with normal vector (i) $\vec{\mathbf{n}} = \langle -2, 1, 4\rangle$; (ii) $\vec{\mathbf{n}} = \langle -2, 0, 4\rangle$; (iii) $\vec{\mathbf{n}} = \langle 0, 0, 3\rangle$.

Solution:

(i)
$$\begin{aligned} & \langle -2,1,4\rangle \cdot \langle x-7,\, y+8,\, z-5\rangle = 0 \\ \text{or} \quad & -2(x-7)+(y+8)+4(z-5) = 0 \\ \text{or} \quad & -2x+y+4z = -2 \end{aligned}$$

(ii)
$$\begin{aligned} & \langle -2,0,4\rangle \cdot \langle x-7,\, y+8,\, z-5\rangle = 0 \\ \text{or} \quad & -2(x-7)+4(z-5) = 0 \\ \text{or} \quad & -2x+4z = 6 \end{aligned}$$

(iii)
$$\begin{aligned} & \langle 0,0,3\rangle \cdot \langle x-7,\, y+8,\, z-5\rangle = 0 \\ \text{or} \quad & 3(z-5) = 0 \\ \text{or} \quad & z = 5 \end{aligned}$$

Equations for Planes: Examples

Example 5: Find an equation through the plane F containing the three points $A(1,-2,0)$, $B(3,1,4)$, $C(2,1,-2)$.

Solution: Geometrically, three points certainly determine a plane. So we need a normal vector.

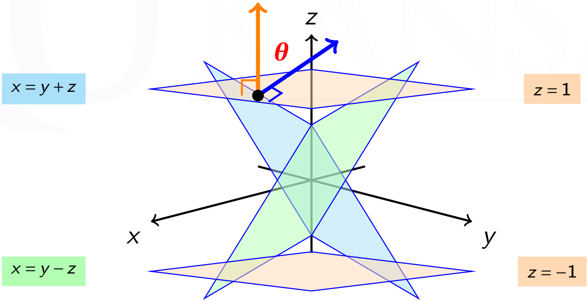
- The vectors $\overrightarrow{AB} = \langle 2,3,4 \rangle$ and $\overrightarrow{AC} = \langle 1,3,-2 \rangle$ both lie **in** F.
- The normal vector $\vec{\mathbf{n}}$ needs to be orthogonal to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$.
- Thus, we can use the **cross product** $\overrightarrow{AB} \times \overrightarrow{AC} = \langle -18, 8, 3 \rangle$ for $\vec{\mathbf{n}}$.

One solution: $\boxed{-18(x-1)+8(y+2)+3z=0.}$

There are other possibilities: $-18(x-3)+8(y-1)+3(z-4)=0$, etc.

Relative Position of Two Planes in Space

- Two planes are <u>parallel</u> exactly when their normal vectors are scalar multiples of one another.
- If two planes are not parallel, then they <u>intersect</u>.
  - When two planes intersect, their intersection is a line.
  - The angle θ between two planes is the angle between their normal vectors (at most $\pi/2$). If $\theta = 0$ then the planes are parallel.



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Example 6: Determine the line L of intersection of the planes F_1 and F_2 whose equations are

$$F_1: \quad 2x - 3y + 5z = 1, \qquad F_2: \quad 3x - 4y = 7.$$

Solution: Normal vectors for the planes: $\vec{\mathbf{n}}_1 = \langle 2, -3, 5 \rangle$, $\vec{\mathbf{n}}_2 = \langle 3, -4, 0 \rangle$.

Since L lies in both planes, its direction $\vec{\mathbf{v}}$ is orthogonal to both $\vec{\mathbf{n}}_1$ and $\vec{\mathbf{n}}_2$:

$$\vec{\mathbf{v}} = \vec{\mathbf{n}}_1 \times \vec{\mathbf{n}}_2 = \langle 20, 15, 1 \rangle.$$

Solve the system $2x - 3y + 5z = 1$, $3x - 4y = 7$ to get a point on L. There are many solutions; one is $(17, 11, 0)$.

Answer: $\boxed{\vec{\mathbf{r}}(t) = \langle 17 + 20t,\ 11 + 15t,\ t \rangle.}$