Vector Mechanics for Engineers: Statics

Analysis of Trusses

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Textbook: *Vector Mechanics for Engineers: Dynamics,* Beer, Johnston, Mazurek and Cornwell, McGraw-Hill, 10th edition, 2012.

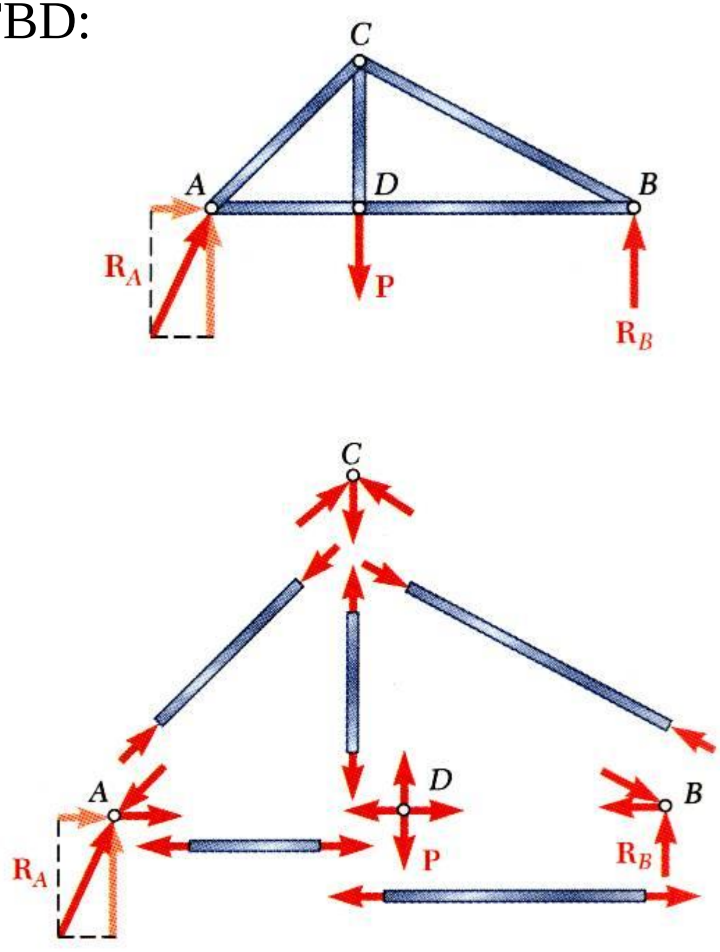
Chapter 6 –

Terms and

Analysis of Trusses, Frames, Machines and other Structures

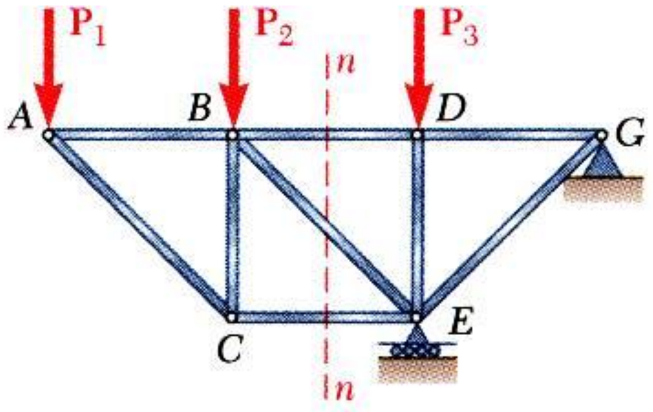
Analysis of Trusses by the Method of Joints

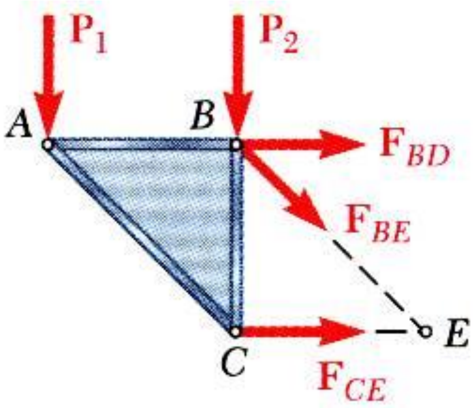
FBD:



- Dismember the truss and create a free-body diagram for each member and pin.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite.
- Conditions of equilibrium on the pins provide $2n$ equations for $2n$ unknowns. For a simple truss, $2n = m + 3$. May solve for m member forces and 3 reaction forces at the supports.
- Conditions for equilibrium for the entire truss provide 3 additional equations which are not independent of the pin equations.

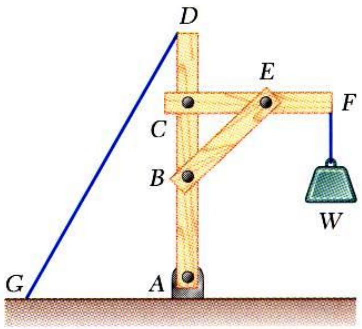
Analysis of Trusses by the Method of Sections

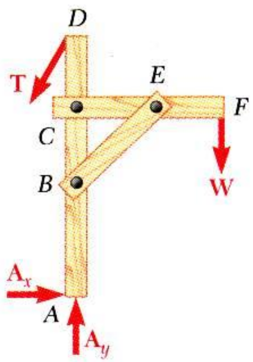


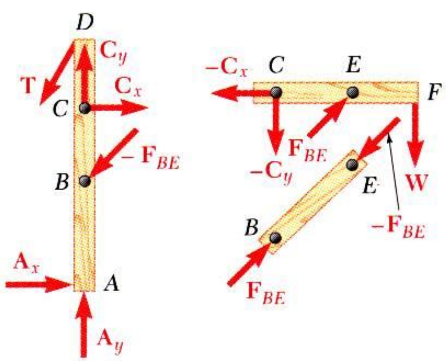


- When the force in only one member or the forces in a very few members are desired, the *method of sections* works well.
- To determine the force in member *BD, pass a section* through the truss as shown and create a free body diagram for the left side.
- With only three members cut by the section, the equations for static equilibrium may be applied to determine the unknown member forces, including F_{BD}.
- Similarly, for F_{CE} write moment about B.
- What about F_{BE} ?

Analysis of Frames

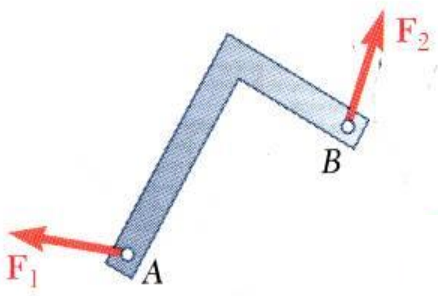


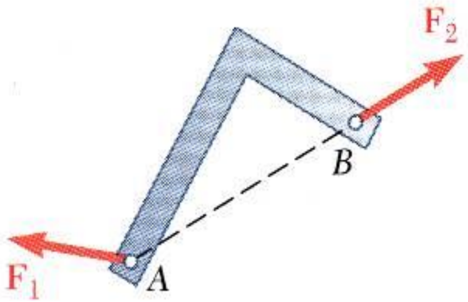


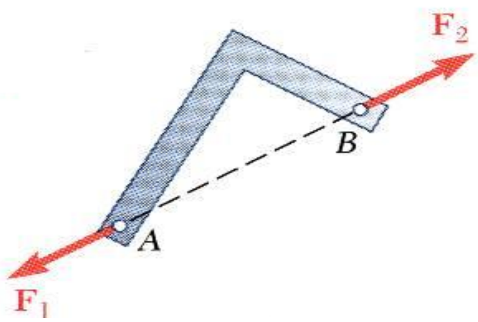


- *Frames* and *machines* are structures with at least one *multiforce* member. Frames are designed to support loads and are usually stationary. Machines contain moving parts and are designed to transmit and modify forces.
- A free body diagram of the complete frame is used to determine the external forces acting on the frame.
- Internal forces are determined by dismembering the frame and creating free-body diagrams for each component.
- Forces on two force members have known lines of action but unknown magnitude and sense.
- Forces on multiforce members have unknown magnitude and line of action. They must be represented with two unknown components.
- Forces between connected components are equal, have the same line of action, and opposite sense.

Equilibrium of a Two-Force Body

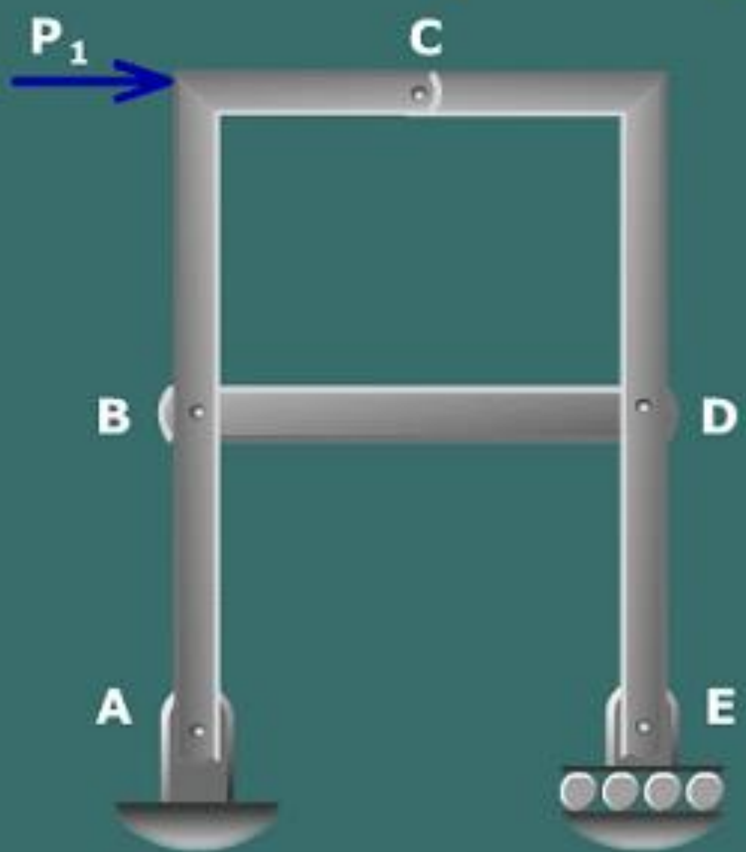


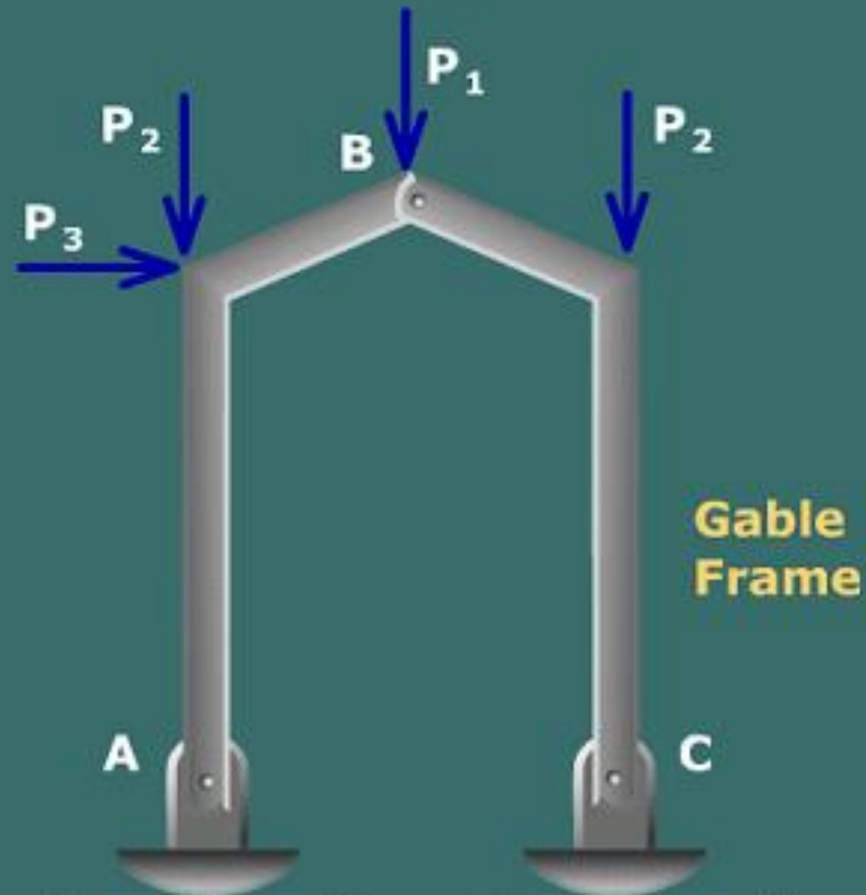




- Consider a plate subjected to two forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$
- For static equilibrium, the sum of moments about *A* must be zero. The moment of $\boldsymbol{F_2}$ must be zero. It follows that the line of action of $\boldsymbol{F_2}$ must pass through *A*.
- Similarly, the line of action of $\boldsymbol{F_1}$ must pass through *B* for the sum of moments about *B* to be zero.
- Requiring that the sum of forces in any direction be zero leads to the conclusion that $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ must have equal magnitude but opposite sense.

Frames and machines are structures that contain pin-connected multi-force members (member with three or more forces or two forces and a couple or couples acting on it). Frames (sign frame, building frame, etc.) are designed to support the system of loads while remaining stationary:

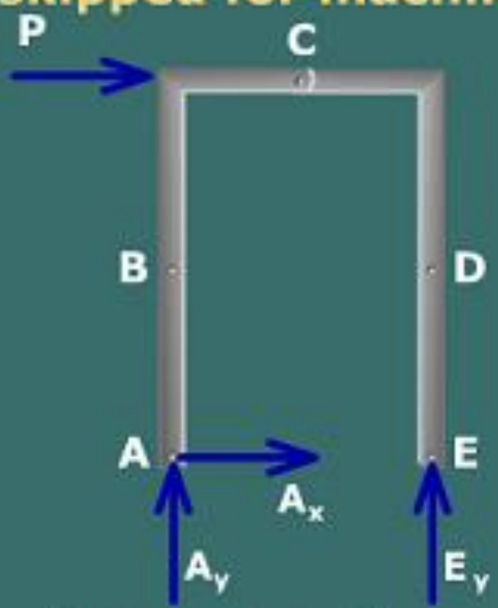


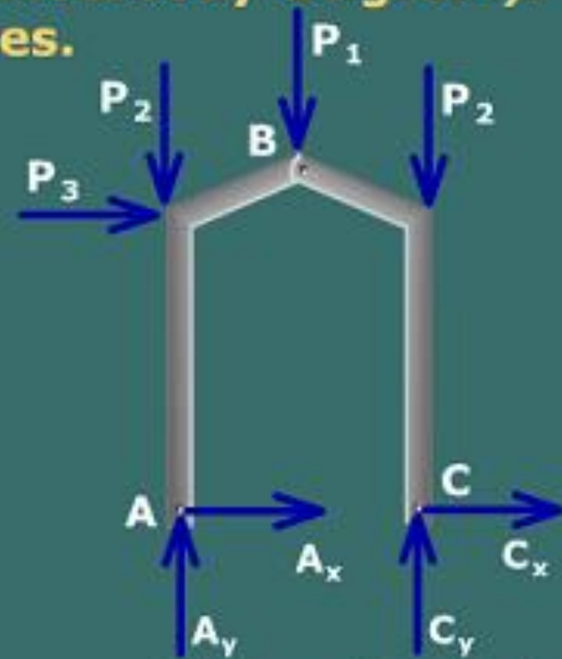


Frames and machines

Analysis Procedure:

Step one in the analysis of statically determinate frames generally begins with consideration of equilibrium of the overall frame to determine support reactions (not always able to find all external reactions from this free-body diagram). This step is normally skipped for machines.





$\Sigma M_A = 0$: Determine E_y

$\Sigma F_x = 0$: Determine A_x

$\Sigma F_y = 0$: Determine A_y

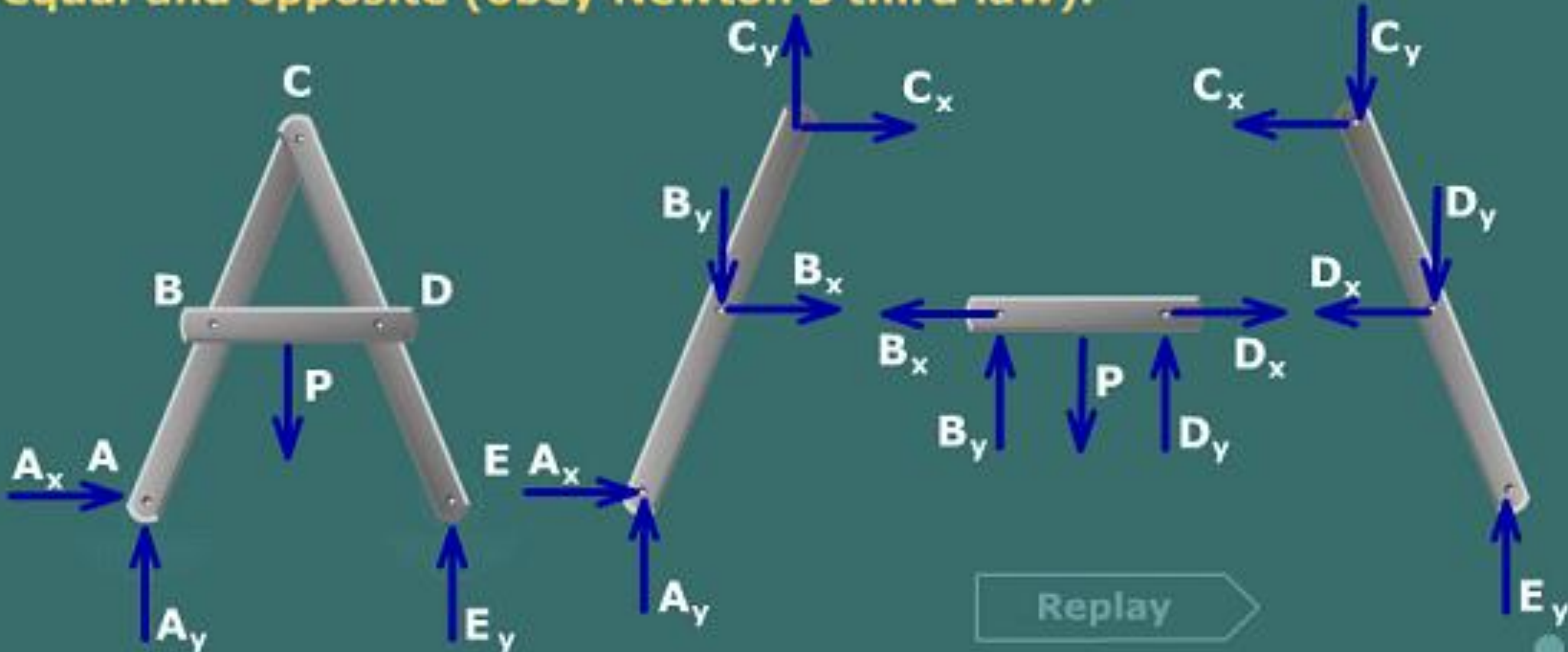
$\Sigma M_A = 0$: Determine C_y

$\Sigma F_y = 0$: Determine A_y

$\Sigma F_x = 0$: Relate A_x and C_x

Step two is to dismember the frame or machine drawing free-body diagrams of the individual members and/or connecting pins. Several cases are possible and need to be fully understood. Consider the three member A-frame below.

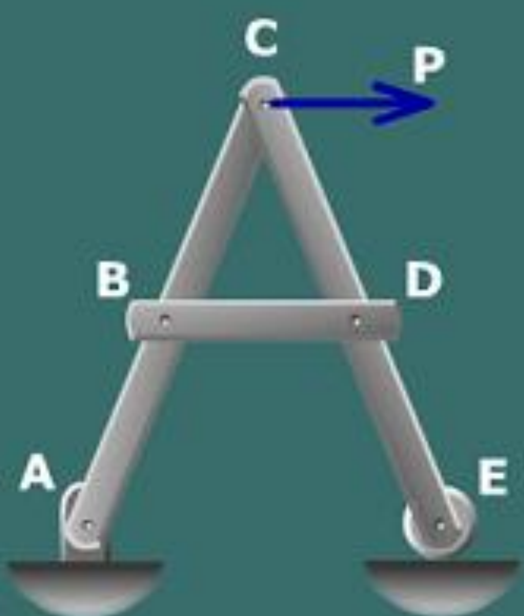
Case 1 - Two members joined at a pin without an external load when separated must have the pin forces on each member shown equal and opposite (obey Newton's third law).

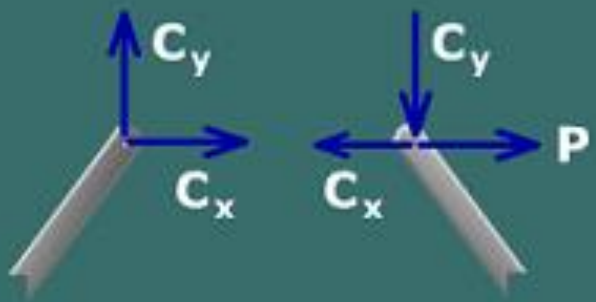


Replay

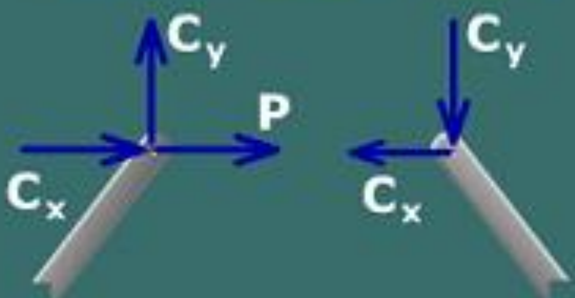
Case 2 - Two members joined at a pin with an external load at the pin may be separated several ways.

The pin (with load) may be considered to remain in either member with the interaction forces between the pin and other member shown. Consider pin C of the A-frame.



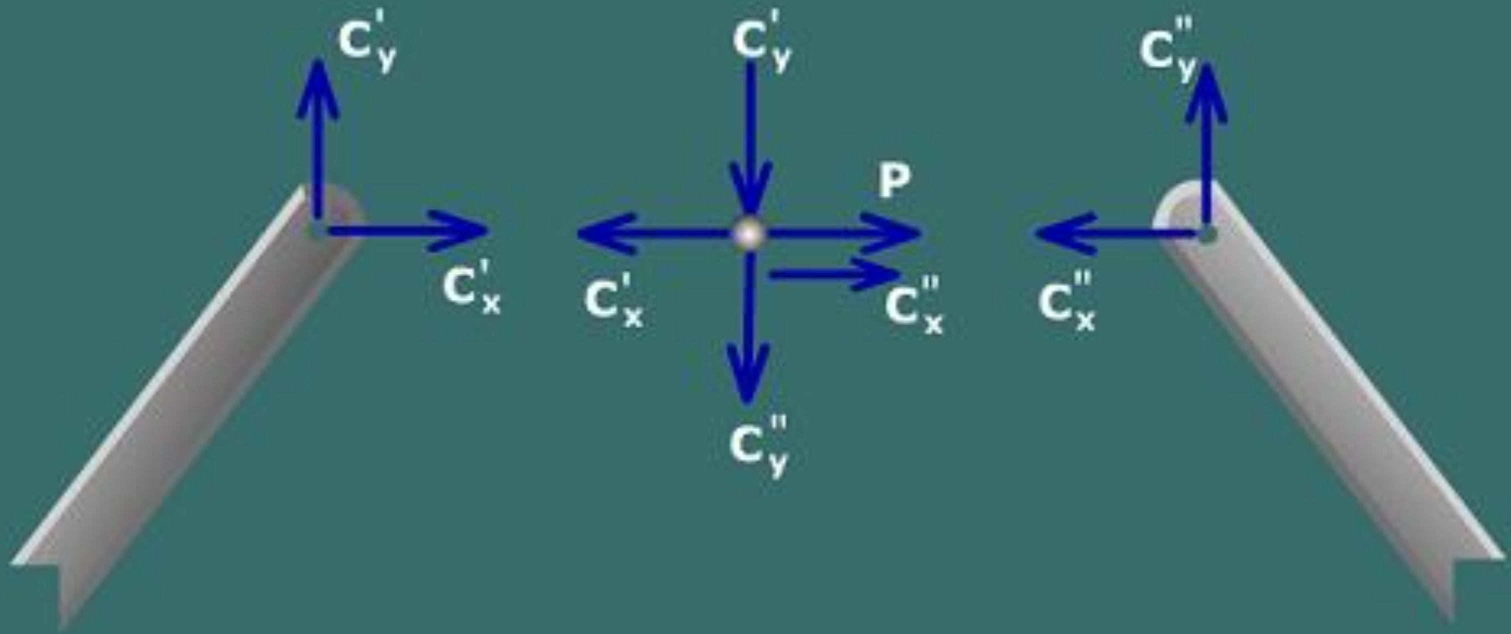


Pin in member CDE



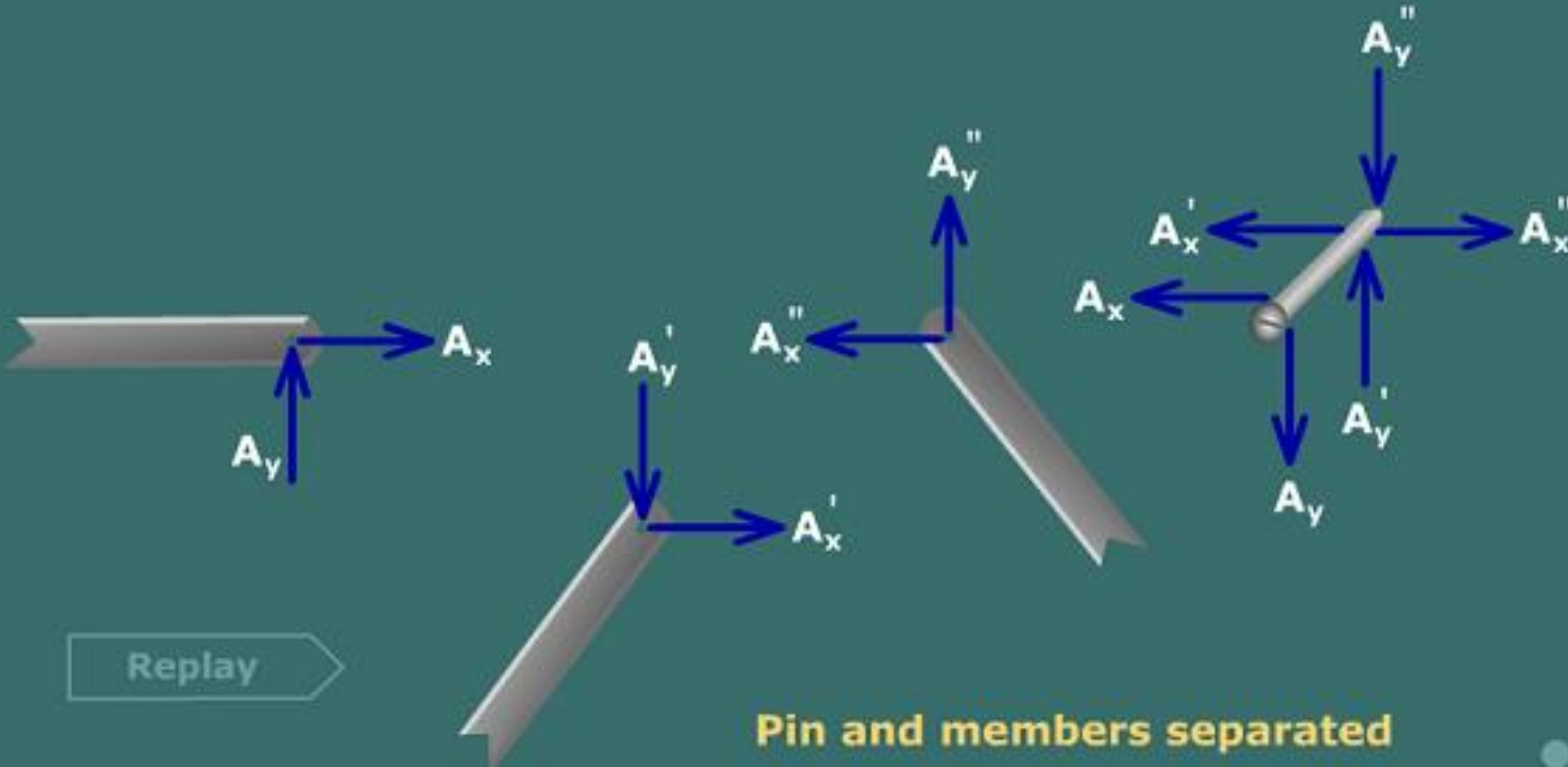
Pin in member ABC

The pin (with load) may be separated from the members and free-body diagrams drawn for each with interaction forces between the members and pin shown. Generally one of the previous possibilities is easier.



Pin and members separated

Case 3 - Three or more members joined at a pin connection with or without a pin load may be handled in a similiar manner. The pin may be left in a member or separated from the members as shown below.



Replay

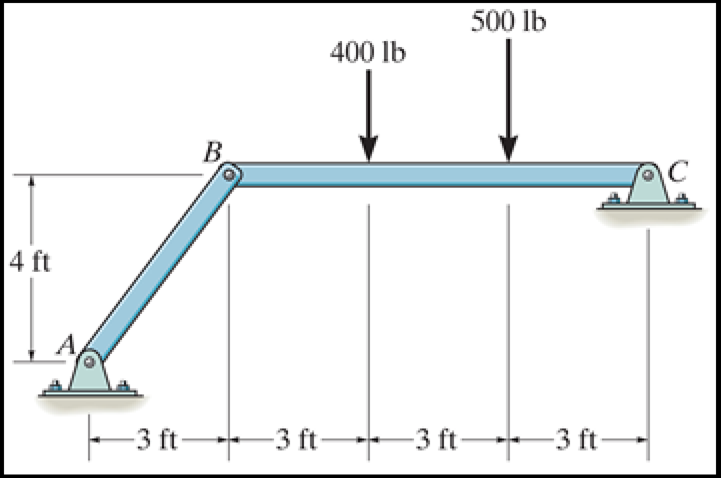
Pin and members separated

ATTENTION QUIZ

1. When determining the reactions at joints A, B and C, what is the minimum number of unknowns in solving this problem?

   A) 6 B) 5

   C) 4 D) 3



2. For the above problem, imagine that you have drawn a FBD of member BC. What will be the easiest way to write an equation involving unknowns at B?

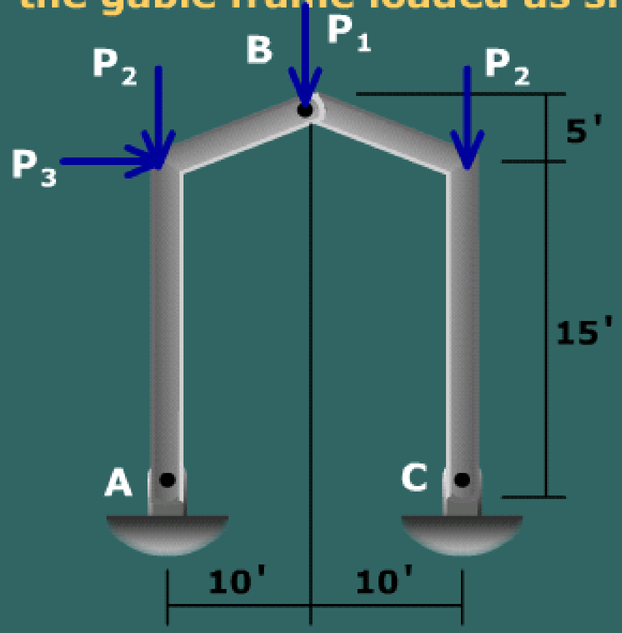
   A) $\sum M_C = 0$ B) $\sum M_B = 0$

   C) $\sum M_A = 0$ D) $\sum F_Y = 0$

Example Problem

Frames - Table frame

Determine the horizontal and vertical components of force at all pins for the gable frame loaded as shown.



Let $P_1 = 2000$ lb

$P_2 = 1000$ lb

$P_3 = 500$ lb

Solving for all pin forces will require that the frame be dismembered to show forces that exist at pin B connecting the two members. In addition a free-body diagram of the overall frame can be used to advantage.

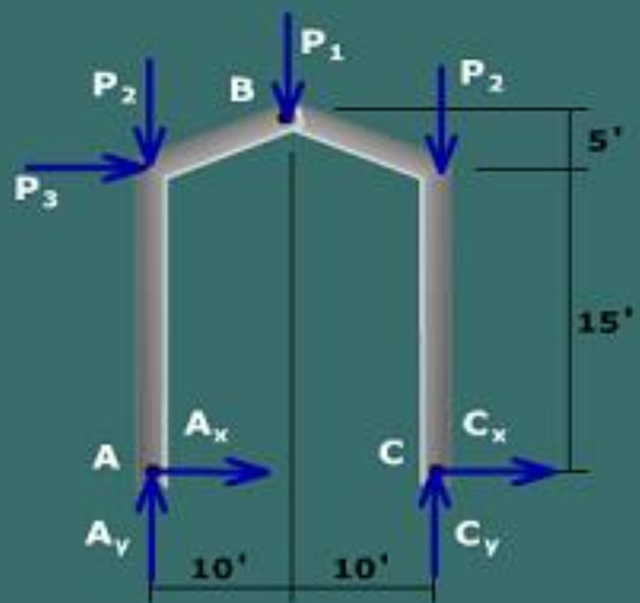
Frames - Table frame

Apply the equilibrium equations and solve for the vertical reaction forces. The horizontal components may be related.

$$\Sigma M_A = 0 : \quad 20\,C_y - 10\,P_1 - 20\,P_2 - 15\,P_3 = 0$$

$$\Sigma F_y = 0 : \quad A_y + C_y - P_1 - 2\,P_2 = 0$$

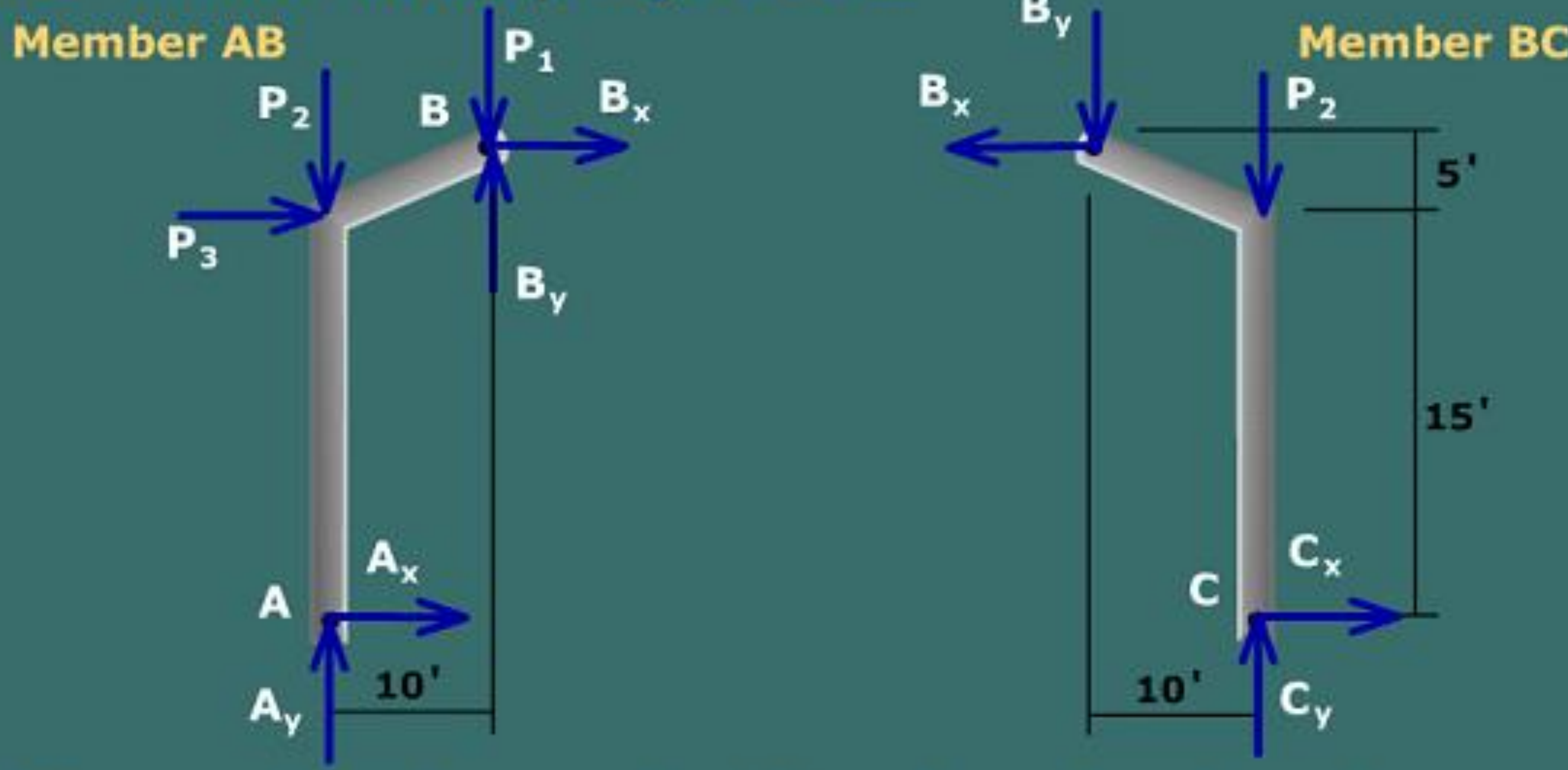
$$\Sigma F_x = 0 : \quad A_x + C_x + P_3 = 0$$



The first two equations give: $C_y = 2375$ lb $\quad A_y = 1625$ lb

FBD for each member separately

Separate the members leaving the pin with load P_1 in member AB. The two free-body diagrams are:



Either member AB or member BC can be used to determine the pin forces at B and the horizontal pin reaction at A and C. Consider member AB.

Frames - Table frame

Apply the equilibrium equations to member AB.

$$\Sigma M_B = 0: \quad 10\,P_2 + 5\,P_3 + 20\,\underline{A_x} - 10\,A_y = 0$$

$$\Sigma F_x = 0: \quad A_x + P_3 + \underline{B_x} = 0$$

$$\Sigma F_y = 0: \quad A_y + \underline{B_y} - P_2 - P_1 = 0$$

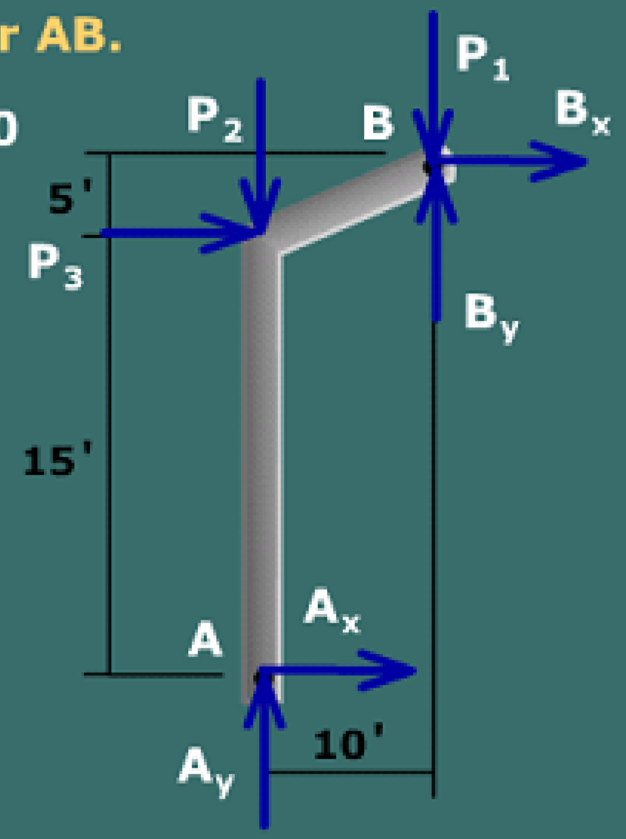
Solving:

$A_x = 187.5\ \text{lb}$;

$B_x = -687.5\ \text{lb}$;

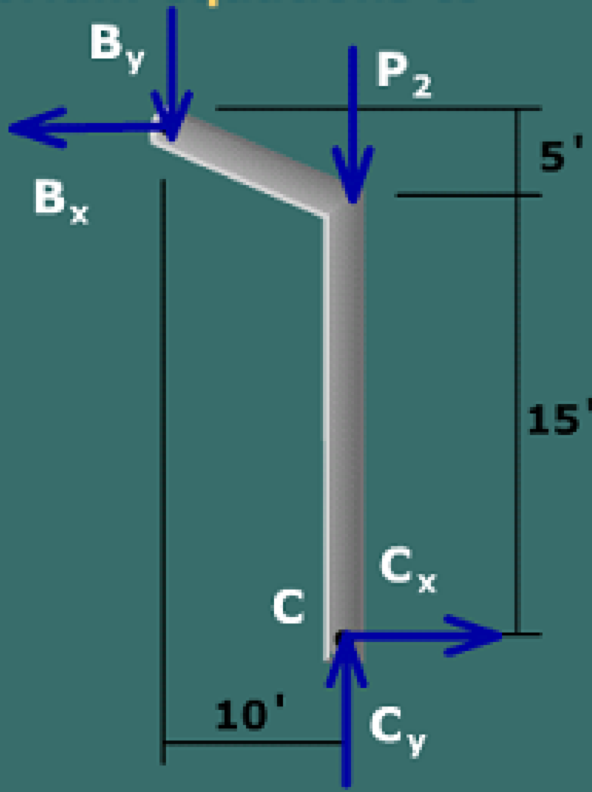
$B_y = 1375\ \text{lb}$; **and**

$C_x = -687.5\ \text{lb}$; **from overall FBD equilibrium.**



Frames - Table frame

An alternate approach is to apply the equilibrium equations to member BC.



$$\sum M_B = 0: \quad 20\,C_x + 10\,C_y - 10\,P_2 = 0$$

$$\sum F_x = 0: \quad C_x - B_x = 0$$

$$\sum F_y = 0: \quad C_y - B_y - P_2 = 0$$

Solving:

$C_x = -687.5\ \text{lb}$;

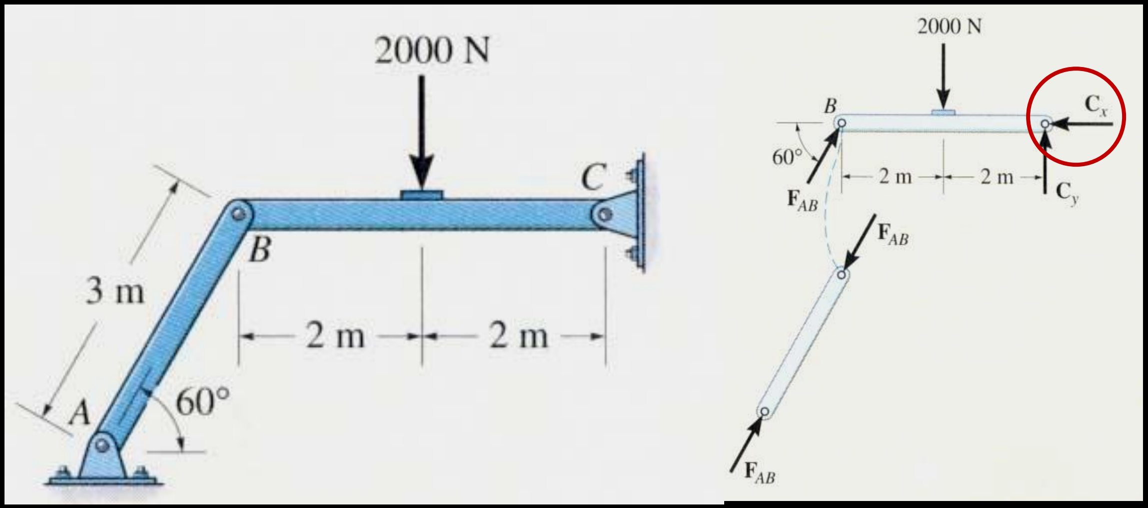
$B_x = -678.5\ \text{lb}$;

$B_y = 1375\ \text{lb}$; **and**

$A_x = 187.5\ \text{lb}$; **from overall FBD equilibrium.**

Finished

CONCEPT QUIZ

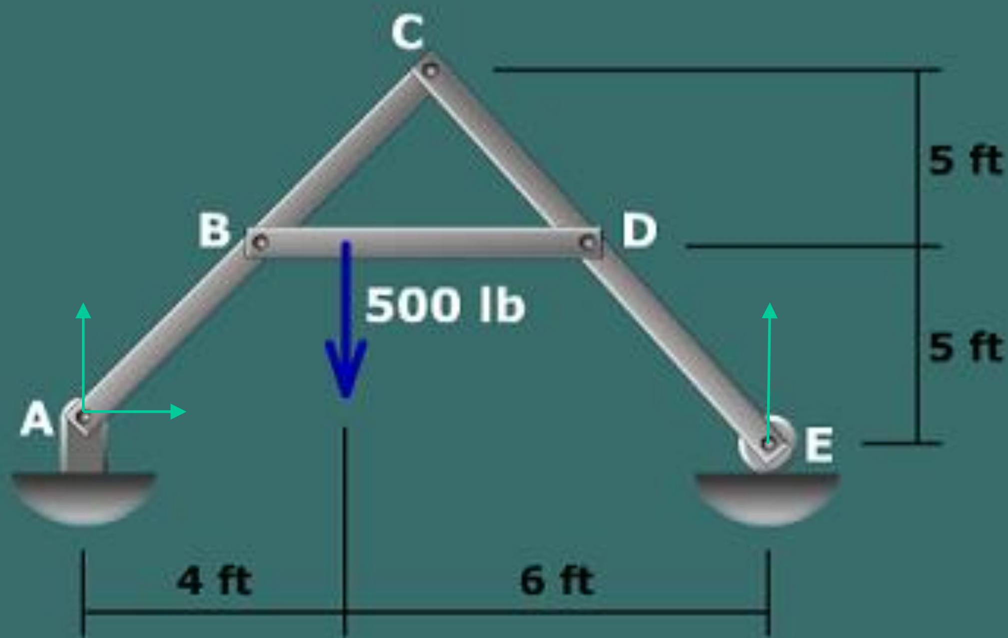


1. The figures show a frame and its FBDs. If an additional couple moment is applied at C, how will you change the FBD of member BC at B?

   A) No change, still just one force (F_{AB}) at B.
   B) Will have two forces, B_X and B_Y, at B.
   C) Will have two forces and a moment at B.
   D) Will add one moment at B.

Frames - A frame

Determine the horizontal and vertical components of force at all pins of the A-frame loaded with 500 lb force as shown.



$A_x = 0$

$A_y = 300$ lb

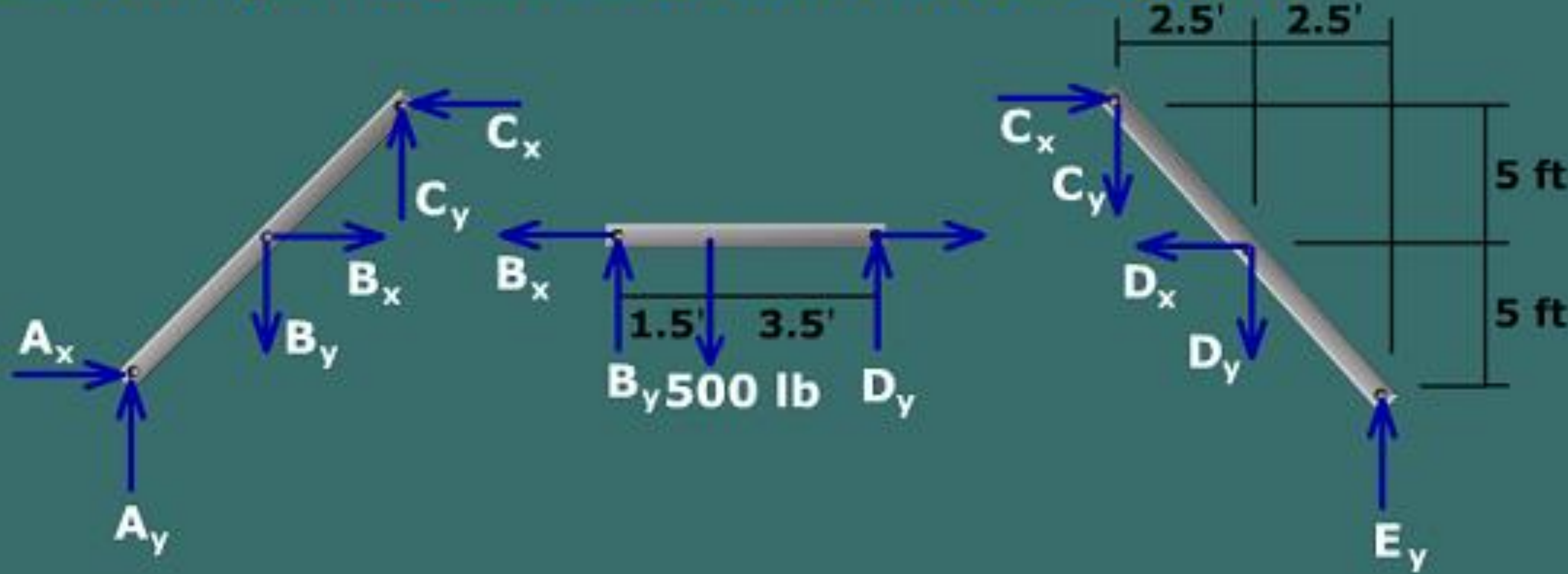
Solving for all pin forces will require that the frame be dismembered to show the forces that exist at the pins connecting the members. In addition a free-body of the overall frame can be used to advantage.

Moment about A: 4 ft * 500 lb – 10 ft * E = 0 gives E = 200 lb

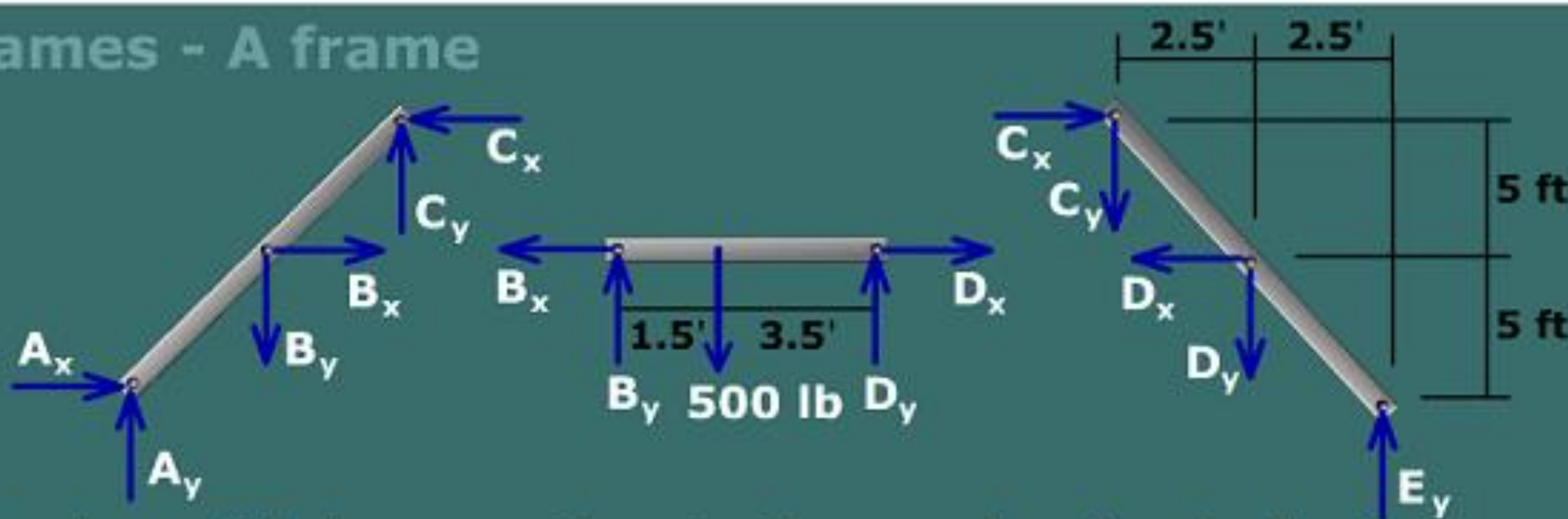
FBD for each member separately

Frames - A frame

To determine the pin forces at B, C, and D requires separate free-body diagrams to be drawn for each member. When members are separated, forces common to contacting members must have equal magnitude but opposite directions (Newton's Third Law). The force direction may be chosen arbitrarily on the first member but not the second.



Frames - A frame



Apply equilibrium equations to the member free body diagrams.

Member BD:

$\Sigma M_B = 0: \quad 5D_y - 1.5(500) = 0; \quad D_y = 150 \text{ lb}$

$\Sigma F_y = 0: \quad B_y + D_y - 500 = 0 \quad B_y = 350 \text{ lb}$

$\Sigma F_x = 0: \quad -B_x + D_x = 0$

Member CDE

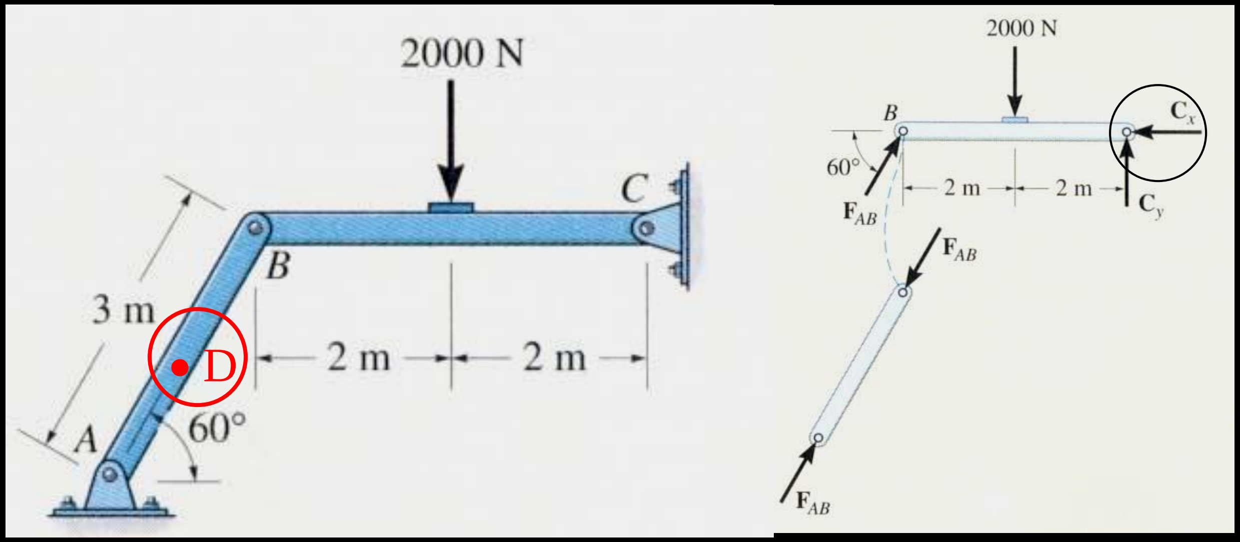
$\Sigma M_C = 0: \quad 200(5) - D_x(5) - D_y(2.5) = 0; \quad D_x = 125 \text{ lb} = B_x$

$\Sigma F_x = 0: \quad C_x - D_x = 0; \quad C_x = 125 \text{ lb}$

$\Sigma F_y = 0: \quad -C_y - D_y + E_y = 0; \quad C_y = 50 \text{ lb}$

Note: The free body diagram of member ABC could have been used instead of the free body diagram of member CDE.

CONCEPT QUIZ (continued)

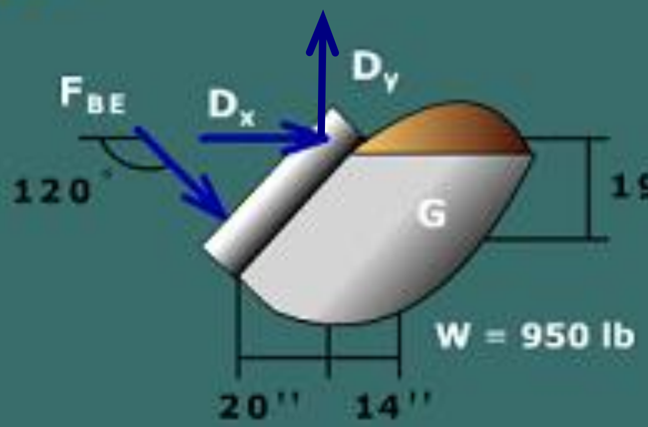


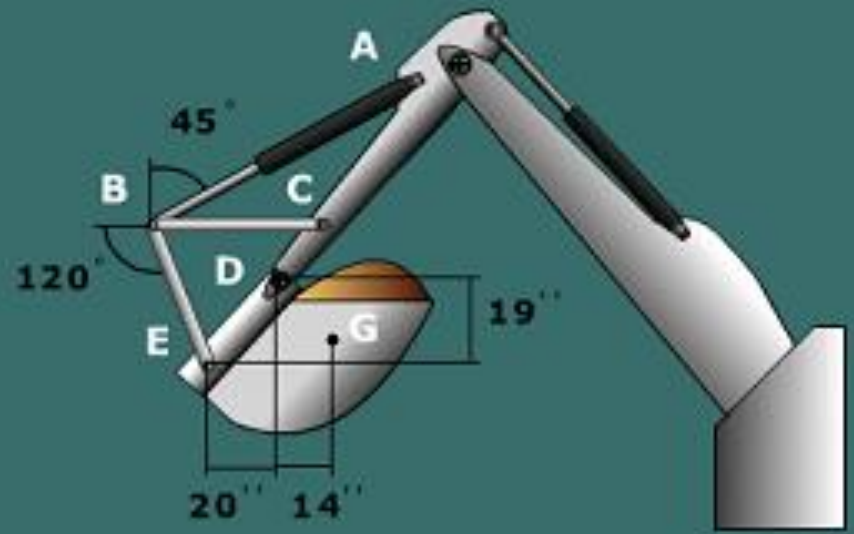
2. The figures show a frame and its FBDs. If an additional force is applied at D, then how will you change the FBD of member BC at B?

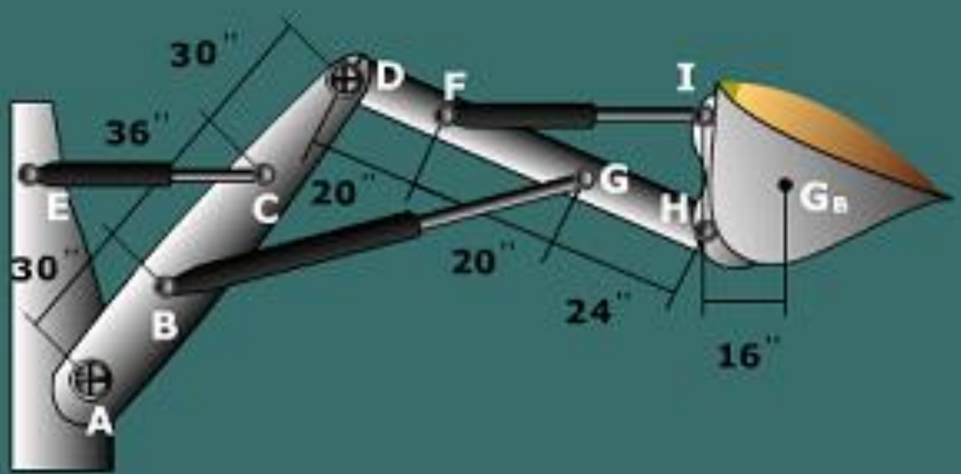
A) No change, still just one force (F_{AB}) at B.
B) Will have two forces, B_X and B_Y, at B.
C) Will have two forces and a moment at B.
D) Will add one moment at B.

ANALYSIS OF MACHNES

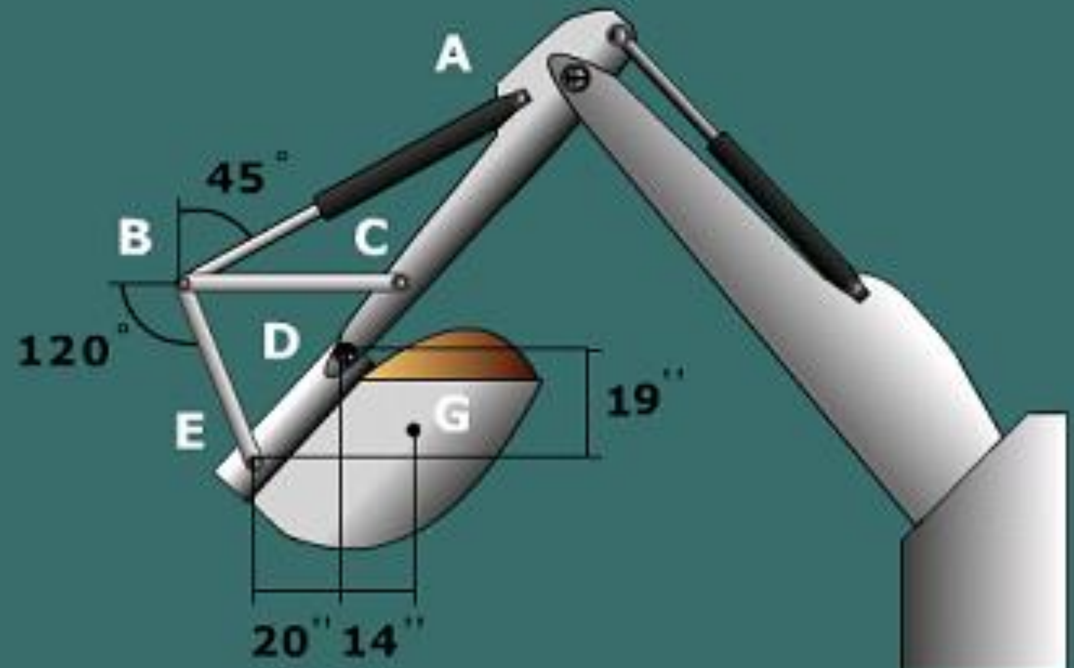
Machines are structures composed of pin-connected, multi-force members. Machines differ from frames in that they contain moving parts and are designed to transmit and modify forces.





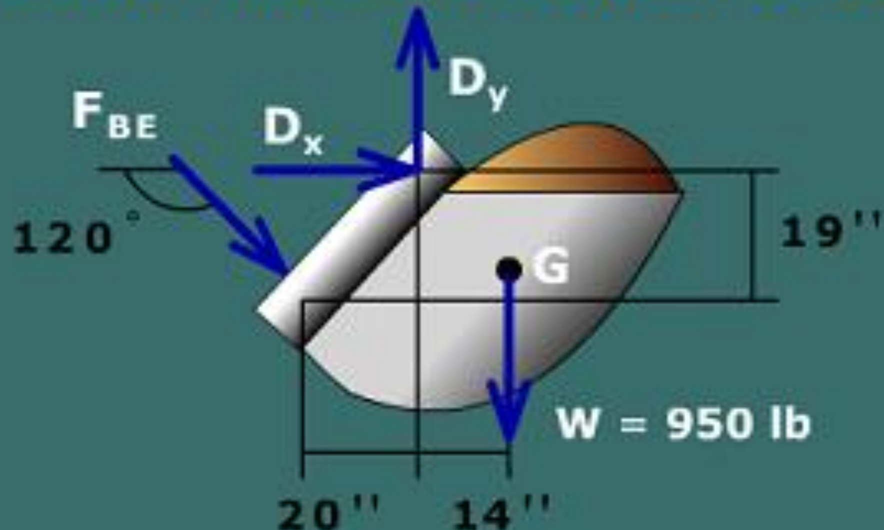


The backhoe bucket and contents weigh 950 lb with center of gravity at G. The link BC is horizontal for the position shown. Determine the force in the hydraulic cylinder AB and the horizontal and the vertical pin forces at pin D.



Solving for the pin forces at D and the force in the hydraulic cylinder will require that the machine be dismembered into component parts.

Begin with a free-body diagram of the backhoe bucket and its contents. The member BE is a short link (two force member) hence the unknown force acts between B and E.



Apply the equilibrium equations to solve.

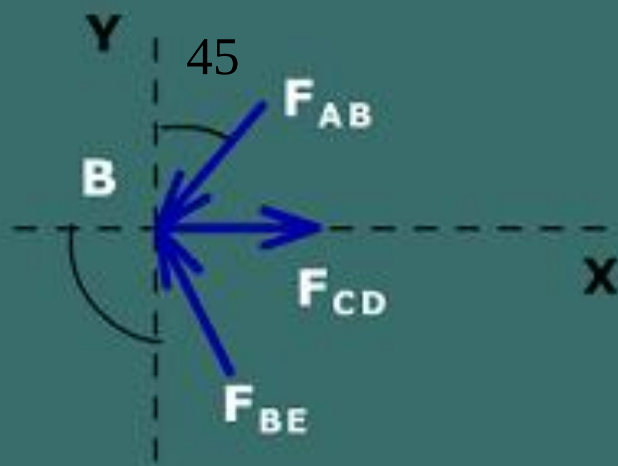
$$\Sigma M_D = 0: \quad F_{BE}\cos 30^\circ(20) + F_{BE}\sin 30^\circ(19) - 950(14) = 0$$

$$\Sigma F_x = 0: \quad F_{BE}\sin 30^\circ + D_X = 0$$

$$\Sigma F_y = 0: \quad D_Y - F_{BE}\cos 30^\circ - 950 = 0$$

Solving: F_{BE} = 495.9 lb; D_X = −247.9 lb; D_Y = 1379.5 lb.

The three members that are joined at B are short links BC and BE and a two force member AB. A free-body diagram of the pin has three forces of known direction.



Since F_{BE} is known, the force equilibrium in the y-direction will yield the force in the hydraulic cylinder.

$$\Sigma F_y = 0: \quad F_{BE}\cos 30^\circ - F_{AB}\cos 45^\circ = 0$$

$$495.9\cos 30^\circ - F_{AB}\cos 45^\circ = 0$$

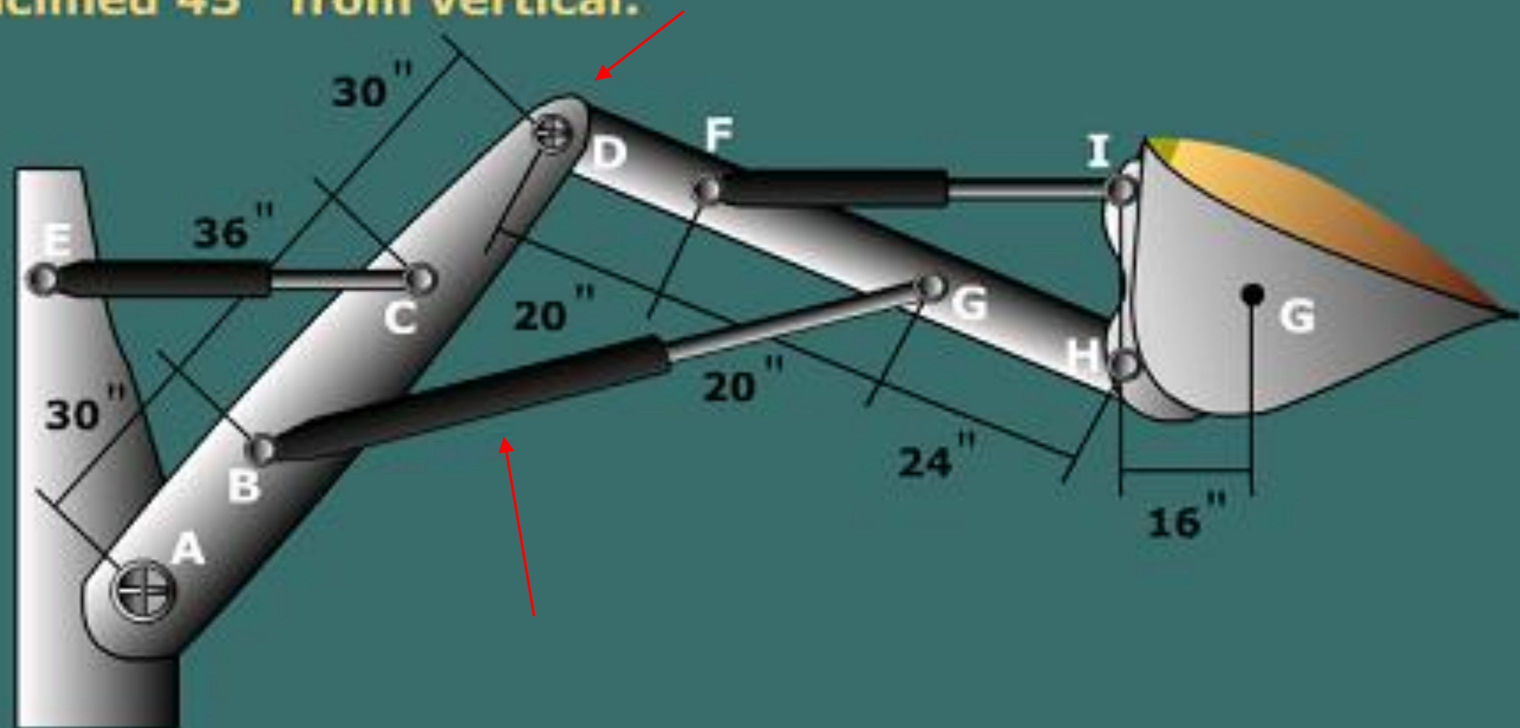
$$F_{AB} = 607.3 \text{ lb}$$

Finished

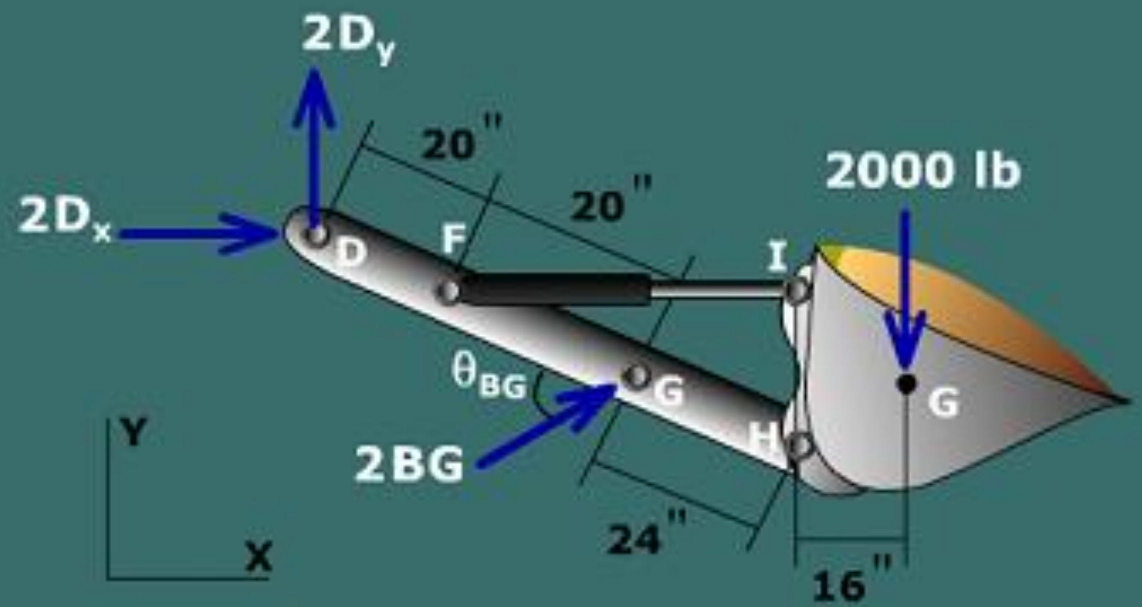
Machines - front end loader

Two symmetric mechanisms (one shown) support the bucket of the front-end loader. The bucket and the contents weigh 2000 lb. The hydraulic cylinders EC and FI are horizontal at the instant shown. Determine the horizontal and vertical pin reactions at D and the force in the hydraulic cylinder BG.

Member ABCD is inclined 20° from vertical and member DFGH is inclined 45° from vertical.



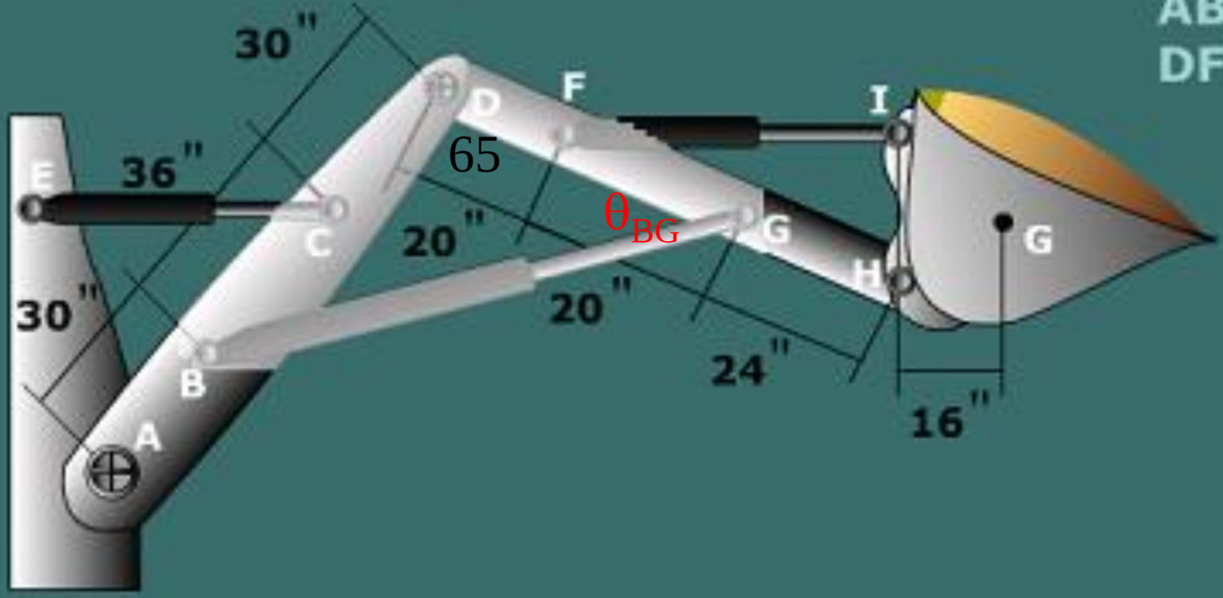
Begin the solution by drawing a free-body diagram of the arm DH, the loaded bucket, and the cylinder FI. This requires removing the pin at D and the hydraulic cylinder BG and showing the unknown forces at those locations.



The angle between the hydraulic cylinder BG and the arm DFGH must be determined from geometry.

The angle can be determined using the law of cosines and the law of sines on the original figure.

Note : The angle between ABCD and DFGH is 65°.



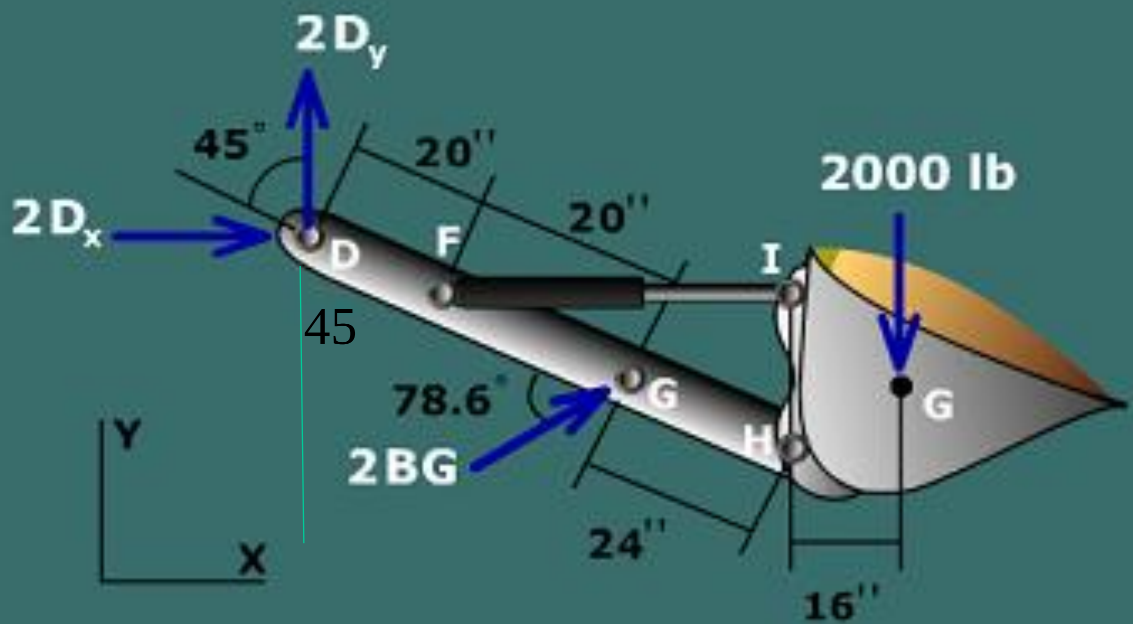
The law of cosines gives the length of BG:

$$L_{BG} = (40^2 + 66^2 - 2(40)(66)\cos 65^\circ)^{1/2} = 61.0''$$

The law of sines then gives the angle between BG and DFG:

$$\theta_{BG} = \sin^{-1}(66 \sin 65^\circ / 61.0) = 78.6^\circ$$

Machines - front end loader



$\Sigma M_D = 0$:

$2BG \sin 78.6^\circ (20+20) - 2000\,(16+(24+20+20)\sin 45^\circ) = 0$

$\Sigma F_x = 0$: $2D_x + 2BG \sin(180^\circ - (45^\circ + 78.6) = 0$

$\Sigma F_y = 0$: $2D_y - 2000 + 2BG \cos(180^\circ - (45^\circ + 78.6) = 0$

Solving : BG = 1562.4 lb ; D_x = −1302 lb ; D_y = 136.4 lb

Finished