Venus Synodic Period and Ancient Mayan Calendars

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1 Introduction

The planet Venus is one of the most fascinating astronomical bodies. After the Moon, it is the brightest object in the nocturnal sky. Sometimes Venus appears as the morning star and can be seen before sunrise for several months. It then disappears from view for several weeks and reappears as the evening star, visible after sunset for several months. Venus disappears again for a few days and reappears as the morning star, and the cycle starts again after 584 days. One such cycle is called a synodic period or synodic revolution of Venus.

Since ancient times, astronomers in many cultures realized that the evening star and the morning star were the same object. The Maya astronomers carefully recorded the times for different stages of Venus. Mayan astronomers were aware that the average for the synodic period of Venus is very close to 584 days, and that 5 synodic revolutions of Venus correspond to 8 Earth years, because $5 \times 584 = 8 \times 365 = 2{,}920$. In addition to coordinating the synodic period of Venus with the solar calendar of 365 days, Maya astronomers coordinated also these two with the sacred almanac, an adivinatory and ritual calendar of 260 days. Due to this correspondence, Maya astronomers elaborated Venus tables in multiples of 260 and 2,920 days, for example, in the Dresden Codex (Thompson, 1972).

In this article we first present activities to explore the synodic period of Venus from a modern perspective. We use our understanding that Venus and the Earth orbit around the Sun and an interactive Geometer's Sketchpad file. Next we present Mayan numeration systems, including a vigesimal positional system to count everyday objects and a modified positional system to keep track of the number of days. Then we discuss different ways in which the Maya kept track of time, including the vague solar calendar of 365 days, the sacred almanac of 260 days, the Long Count, and the astronomical solar year. Then we will discuss the synodic period of Venus from the perspective of the ancient Maya, where the rise of the morning star after the disappearance of Venus at its lower conjunction was an ominous sign. The final section is about the coordination of the synodic period of Venus with the vague solar year, the adivinatory almanac, and the long count.

2 Venus synodic period in modern terms

If viewed from a point above the Sun's north pole, all the planets are orbiting around the Sun in a counter-clockwise direction. The Earth, like most planets, also rotates counter-clockwise around its own axis. Nowadays we know that the orbit of Venus is smaller than the orbit of the Earth, and also that Venus travels faster than the Earth. The semi major axis for Venus' orbit is about 108.21 million km, compared to the Earth's semi major axis of 149.60 million km. The mean orbital velocity of Venus is about 35.02 km/s whereas for the Earth the orbital velocity is about 29.78 km/s (Williams, 2014). The ratios $149.60 \div 29.78 = 5.0235$ and $108.21 \div 35.02 = 3.0899$ are proportional to the time each planet takes to complete an orbit around the Sun (explain in your own words why). Notice that the ratio of these two numbers $\frac{5.0235}{3.0899} \approx 1.6258$ is the ratio of the number of times Venus orbits around the Sun for each time the Earth completes one orbit. This ratio is very close to the ratio $\frac{13}{8} = 1.625$. The significance of the numbers 13 and 8 in terms of the motions of Venus and the Earth will become clear in the following paragraphs.

In order to let the reader have a more active role we have introduced some exercises. For the first you can use the interactive webpage Venus Earth (Flores, 2008) which simulates the movement of the Earth and Venus around the sun in a dynamical scale model. You can drag one of the planets by hand or you can press the animate button to see both planets move with velocities on the computer screen proportional to the real velocities of the planets in space.

Exercise. In a diagram showing both the orbits of Venus and the Earth around the sun, for a given position of the Earth, identify when Venus would be a morning star and when an evening star. Explain in your own words why Figure 1 corresponds to Venus as an evening star and why Figure 2 corresponds to Venus as a morning star.

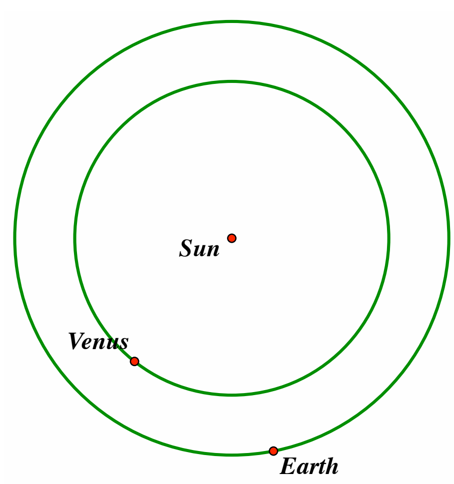


Figure 1. Venus as evening star

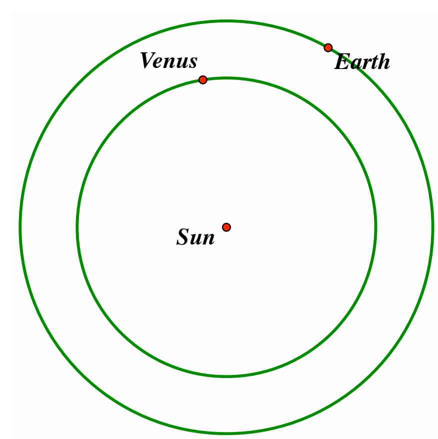


Figure 2. Venus as morning star

When Venus is between the Earth and the Sun, the position is known as "inferior conjunction" (Figure 3). When Venus is behind the Sun with respect to the Earth, the position is known as “upper conjunction” (Figure 4). With our modern understanding of planetary motion, we would say that Venus "overtakes" the Earth at its inferior conjunction every 584 days as they both orbit around the Sun. Readers may use the interactive web page or any other means to determine which disappearance of Venus from sight is longer, at its inferior conjunction or at its upper conjunction. In terms of conjunctions, when does Venus go from evening star to morning star? Readers may also use the interactive web page to estimate how many Earth years it takes for Venus to be again between the Earth and the Sun in a straight line. Figure 3 shows two such positions. Starting with positions as in Figure 3a, the next inferior conjunction would look like Figure 3b. Count how many Earth revolutions, how many times does Venus overtake the Earth, and how many revolutions of Venus are needed for both planets to be back exactly at the same initial positions with respect to the background. For example, if you start with the planets in positions like Figure 3a they both need to be back to look exactly like that and not like Figure 3b or any other inferior conjunction. In the following exercise readers can figure out the length of the tropical year of Venus by knowing the length of the year for the Earth and counting conjunctions of the two planets.

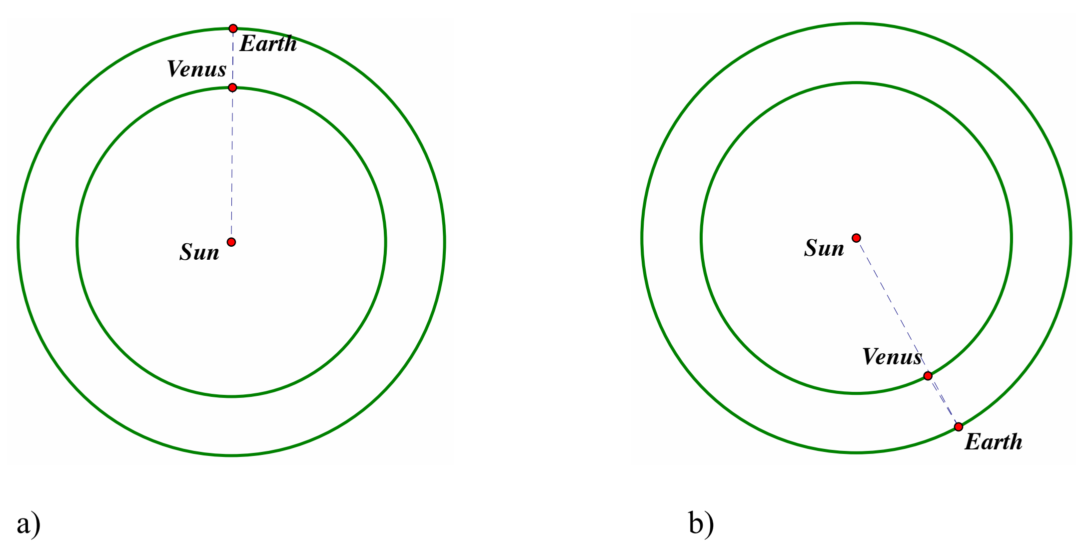


Figure 3. Two inferior conjunctions (Venus not visible from the Earth)

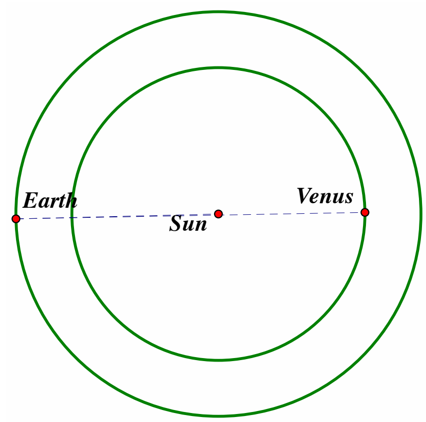


Figure 4. Upper conjunction (Venus not visible from the Earth)

Exercise. Using the fact that Venus overtakes the Earth five times in eight years, find how many times does Venus orbit around the sun in eight (Earth) years. Use this information to determine the duration of the tropical year for Venus, that is, the time Venus takes to complete one orbit around the Sun.

3 Number systems in Mesoamerica

3.1 Vigesimal systems

In Mesoamerica a base 20 numerical system was widely used. Twenty of one unit would give the next bigger unit. Thus numbers that are powers of 20, such as 1, 20, 400 and 8000 were especially important in Mesoamerica and had special symbols. Landa (1975) described how the Maya counted: "They count by fives up until twenty, by twenties until one hundred, and by hundreds until four hundred, and by four hundreds until eight thousand. They use this method of counting a great deal when trading cacao. They have other very long counts which may be extended ad infinitum, counting the number eight thousand twenty times, which makes 160,000, and multiplying the 160,000 by twenty, and thereafter multiplying by twenty until they reach an uncountable number.” (p. 72) Written numbers were represented by glyphs or by using dots and dashes.

Positional systems were also used by the Maya, where the value of a numerical symbol depends where it is placed in relation to the rest of the symbols. The Maya used a dot to represent one, and a dash to represent five. Numbers between one and nineteen were represented with a combination of dots and dashes. They also had a special symbol for zero (Figure 5).

one three five seven thirteen zero

Figure 5. Numbers in Mayan system

For numbers 20 and bigger, the value of a dot or dash depended on their position. Numbers were often written vertically. Figure 6a represents one unit of twenty and zero units. Figure 6b represents one unit of twenty and eight more units. Figure 6c represents one unit of four-hundred, one unit of twenty, and thirteen more units, 433 in our system.

a) twenty + zero b) twenty + eight c) four-hundred + twenty + thirteen

Figure 6. Positional values in Mayan system.

3.2 The Maya system for counting days

For counting days the Maya used a slightly different system. The smallest unit was 1 day which in Maya is 1 *kin*. 20 kin would give the next unit 1 *uinal*. But then 18 *uinals* would give the next unit the *tun*, so that the value of the tun was 360 days rather than 400. After that each next unit was reached in a regular way by multiplying by 20. To avoid confusion with our way to parcel days we will use Maya names in the following conversion list.

20 kin = 1 uinal

18 uinal = 1 tun (360 days)

20 tun = 1 katun (7,200 days)

20 katun = 1 baktun (144,000 days)

20 baktun = 1 pictun

20 pictun = 1 calabtun

20 calabtun = 1 kinchiltun

Special glyphs or the position of a numerical symbol would indicate what time unit was being used. Thus the number represented by 5 katun, 5 tun, 8 uinal, 0 kin represents a total of $5 \times 7{,}200 + 5 \times 360 + 8 \times 20 + 0 = 36{,}000 + 1{,}800 + 160 = 37{,}960$ days. Mayanists write this date using modern notation as 5. 5. 8. 0 where the value of each digit depends on its position.

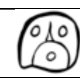
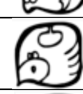
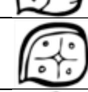
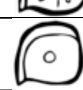
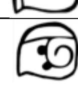
4 Mayan calendars

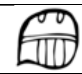
4.1 The sacred almanac

This adivinatory calendar of 260 days was widely used in Mesoamerica for almost 3000 years. It was used by cultures such as the Olmec in the Gulf coast, the Mixtec and Zapotec in Oaxaca, the Maya in Yucatan, Chiapas, Guatemala and Honduras, and the Toltec and Nahua in the central Mexican highlands. It is still used for adivinatory purposes in some Mayan regions (Stuart & Stuart, 1993, pp. 178-179). Among contemporary Maya the sacred almanac is still used by some women in conjunction with the time of gestation for human babies which has an average of 266 days (Foster, 2002, p. 251). Others use 13 months of 20 days to describe the time of gestation.

The sacred almanac was built by combining 13 numbers with 20 glyphs or names for days. The names of the days were chosen from deities, sacred animals, or natural objects. Table 1 gives the names of the days in Maya (Yucatec) and their meaning in English. The numbers would be used as a prefix to the names of the days in cyclical order. Thus, the name for a day in the sacred almanac would consist of a number together with a glyph. The series could start for example as 1 Ahau, 2 Imix, 3 Ik, …, 13 Eb, 1 Ben, 2 Ix, and so on. Because 13 and 20 do not have any common factors, there are $13 \times 20 = 260$ possible combinations. Thus each number and name combination appears only once for 260 days and the cycle repeats itself after 260 days. The glyphs in Table 1 are of the simplified type used in written documents (Morley, 1915). Different variations for some of the glyphs were also used.

Table 1. Names for Maya Days in the sacred calendar

Name of day (Yucatec Maya)	Glyph	English equivalent
Ahau		Lord, ruler
Imix		Alligator, Water
Ik		Air, life
Akbal		Night
Kan		Corn
Chicchan		Serpent
Cimi		Death
Manik		Deer
Lamat		Rabbit
Muluc		Rain
Oc		Dog

Chuen		Monkey
Eb		Broom
Ben		Reed
Ix		Jaguar
Men		Eagle
Cib		Owl
Caban		Quake
Eznab		Flint knife
Cauac		Storm

Each particular combination of number and glyph had special significance, in a similar way that in our culture some people attribute special significance to some combinations of numbers and names of the weekdays, for example Friday 13, or believe that certain activities need to be avoided on special weekdays ("martes, ni te cases ni te embarques"). The adivinatory almanac was used in conjunction with the occurrence of astronomical phenomena to predict who would be affected. For example, as we will discuss in section 5, the heliacal rise of Venus after its transit to the underworld was an ominous sign that bad things could happen. What group of people would be affected depended on the day of the sacred almanac when this happened. For example, if the heliacal rise was on a certain day of the sacred almanac, warriors would be affected, if it was on another day, old people would be affected instead.

4.2 Calendar of 365 days.

The wandering or vague solar year consisted of 365 days. Because the tropical year is 365.2422 by using 365 days, the seasons would wander slowly through the year. Although Maya astronomers were well aware that the duration of the solar year was a fraction longer than 365 days, and kept careful records (see section 4.5), Maya society did not introduce compensation days into the calendar for use by the people in everyday life the way we introduce leap years. The year was divided into18 months of 20 days plus 5 days considered of bad luck, called Uayab days. Each day of the solar year was identified by the name of the month and the number of the day within the month. The glyphs and names for the months were different from the glyphs and names for the days used in the sacred almanac, so there was no confusion to what calendar did the name belong. The days within a month were numbered 0, 1, 2, …, 19, and the Uayab days 0, 1, …, 4. The names for the 18 months in Maya are Pop, Uo, Zip, Zotz, Tzec, Xul, Yaxkin, Mol, Chen, Yax, Sac, Ceh, Mac, Kankin, Muan, Pax, Kayab, Cumhu. Some of the glyphs used in codices are given in Figure 7.

Figure 7. Glyphs for 18 months plus the glyph for the Uayab days (Morley, 1915)

4.3 Coordination of the calendars of 260 and 365 days

In addition to the number and month used to identify a day in the calendar of 365 days, the same day had also a different pair (number and name) associated with the adivinatory almanac. Each day would thus be identified by a combination of a number and a glyph from the sacred almanac and a number and a glyph from the other calendar (see Figure 8). The combinations would be unique for 52 years. Because 52 × 365 = 73 × 260 the cycle of combinations would repeat after 52 years. This cycle is called the calendar round. The beginning of each cycle of 52 years, the New Fire, was a very important event in Mesoamerica.

Figure 8. Numbers and glyphs for 9 Kan 12 Kayab (Morley,1915)

Because 365 gives a remainder of 5 when divided by 20, only four names of the sacred almanac were possible to mark the beginning of a solar year. These were called the year bearers. Because 365 gives a remainder of 1 when divided by 13, the numbers of the sacred almanac corresponding to the beginning of the solar year would increase by 1 each year. Thus, each solar year in each cycle of 52 years could be identified by a number and one of the four possible names from the sacred almanac.

4.4 The Long Count

Because days were not uniquely determined by their identification in terms of the adivinatory almanac and the calendar of 365 days beyond cycles of 52 years, the Maya used a system to locate any date with respect to an "absolute" series of dates, called the Long Count. The beginning of this count corresponds to a date well before the actual flourishing of Mayan civilization. The Maya did not invent the Long Count; the use of the Long Count predates the Maya classic period. Preclassic examples of Long Count dates have been found between Veracruz and the Pacific slope of Guatemala (Stuart & Stuart, 1988, p. 38). There is a date found at Copán, Honduras, written using Mayan numbers and glyphs as nine baktun, fifteen katun, five tun, zero uinal, zero kin, which we write using modern notation as 9.15.5.0.0, represents 9 × 144,000 + 15 × 7,200 + 5 × 360 + 0 × 20 + 0 × 1 = 1,405,800 days elapsed from the Long Count starting date. Usually the Long Count is given with five places, and according to Stuart and Stuart (1993, p. 177) the beginning of the Long Count using five places corresponds to August 13, 3114 BC using the Gregorian calendar extended backwards. The Copán date described above would correspond to July 27, 736 AC, during the Classic Maya period.

A few years ago some fuss was created because the day corresponding to December 21, 2012 of our Gregorian calendar would have a representation in the Long Count involving many zeros, 13.0.0.0.0, marking the end of a baktun. Of course there is no reason to believe the world will come to an end or that something special will happen just because in one of the several calendars used by different people in the world the number for that year is a round number. December 21, 2012 corresponds to 1,872,000 days since the beginning of the Long Count.

In some places the Long Count was expanded from the usual five places to 24 places, as in Stella 1 in Copan (Stuart and Stuart, 1993, p. 177). By using 24 places of a vigesimal system, the Maya scribes represented such a huge number of years that it is hard to imagine. The number would be bigger than the number represented in our system by 40 followed by 27 zeros. Compared to this number of years, the inferred time span of 14,000,000,000 years passed since the Big Bang seems like a blink.

4.5 Mayan astronomical solar year

Although as we mentioned no corrections were introduced for the vague solar calendar used in daily life, astronomers in Mesoamerica were aware that the Earth's tropical year is not exactly 365 days. They inserted an additional day every four years about 200 years before Julius Cesar's calendar reform, which introduced a leap year every four years. This gives 365.25 days per year. Later, Mayan astronomers refined the duration of the solar year even more by doing two things. They inserted an additional day every four years, and subtracted 4 days every 520 years. This gives 365.2423 days per year. In our present day Gregorian calendar (introduced in parts of Europe in 1582) we insert an additional day every four years except when it is a multiple of 100 but not a multiple of 400 (thus the year 1900 had 365 days, but the years 1600 and 2000 had 366 days). That is, in our system we subtract 3 days every 400 years from the leap year correction. This gives 365.2425 days per year. Modern astronomers compute the tropical year as 365.2422 days, so the approximation by Mayan astronomers is slightly better than the Gregorian calendar once the correction after 520 years is made.

5 Venus synodic period in ancient Mayan culture

In ancient Mesoamerica the rising morning star was identified with a dangerous and harmful deity. The disappearance of Venus from the sky at the inferior conjunction corresponded to the visit of the deity to the underworld. The heliacal rising of Venus after this conjunction was considered a dangerous time. In pages 25 – 29 of the Dresden Codex, in the second drawing, different manifestations of the god Venus are represented with spears and spear thrower in his hands, menacing the victim. According to Thompson (1972, p. 67) “spears symbolize the planet’s death-dealing shafts of light.” The third picture on each page represents the different victims according to the corresponding years determined by the sacred almanac. The possible victims include among others the jaguar (warriors), the god of corn, and the frog deity (rain) (see Figure 9 for two examples).

Figure 9. Manifestations of the morning star and their victims underneath them.

These concepts are very close to those found in codices from the Nahua cultures in the central highlands of Mexico, who also used a sacred calendar with 13 names and 20 numbers. For example, in the *Anales de Cuauhtitlán* (Codex Chimalpopoca, 1992) the disappearance for eight days of Venus is associated with the death of Quetzalcoatl and his visit to the dead land for four days, and four additional days he spent making darts for himself. Quetzalcoatl in his rise converted as the morning star "casts light on certain people, venting his anger against them, shooting them with darts." (p. 36). If the rise occurs "on 1 Alligator he shoots old men and old women…. If on 1 Jaguar or 1 Deer or 1 Flower he shoots little children. And if on 1 Reed he shoots nobles. The same with everybody, if on 1 Death. And if on 1 Rain, he shoots the rain. No rain will fall." (p. 36)

In the Dresden Codex the synodic period stages of Venus were divided as follows. Venus shines 236 days as the morning star, followed by 90 days of invisibility, then it shines 250 days as the evening star, followed by 8 days of invisibility, after which Venus appears again as the morning star (Thompson, 1972, p. 65).

Venus was very important in the lives of the Maya and their astronomers kept careful records of when the heliacal rise occurred, both for the past and for the future. They were well aware that the average for the time for consecutive heliacal rises, that is the synodic period of Venus, was very close to, but not exactly, 584 days. They were able to make corrections with remarkable precision as we will see in the next section.

6 Coordination of Venus synodic period with the calendars of 260 and 365 days

As we described in section 3, the wrath of Venus at the heliacal rising did affect other deities or different groups of people according to the corresponding date in the sacred almanac. Because 584 leaves a remainder of 4 when divided by 20, the names of the days were separated by 4 for each successive inferior conjunction. Thus only five of the 20 names in the sacred calendar are possible for the heliacal rising of Venus. Each of the pages 25 – 29 (46 - 50 in the former incorrect numeration) of the Dresden Codex is dedicated to one of these names. Because 584 leaves a remainder of 12 when divided by 13, the numbers of the days increase by 12 (and reduced by 13) in cycles 1, 13, 12, 11, 10,…, 2.

For long periods of time, the coordination of Venus synodic period with the two calendars used by the Maya required the use of common multiples of 260, 365, and 584. In addition, because the cycle of Venus is not exactly 584 days but a little less (583.92), it required periodic adjustments. These adjustments were made in multiples of 4 days so that the names for days for the beginning of cycles be the same.

On the lower part of p. 24 of the Dresden Codex, second number on the left, there is the number 9.9.16.0.0. This number corresponds to 72 calendar rounds of 52 years, that is, it is a multiple of both 260 and 365. It is also equivalent to 2340 synodic revolutions of Venus.

On the right side of the page appear multiples of 2920 (5 × 584 = 8 × 365). In the codex they need to be read from the bottom, from right to left. There is a sequence in multiples of 5 (5 × 2920, 10 × 2960, …) until 60 × 2960 (see Figure 10). There is another short sequence in multiples of 65 (65, 130, 195, 260), now mostly damaged. In the following exercise readers can find important numbers used to coordinate the sacred almanac, the solar year, and the synodic cycle of Venus.

Figure 10. Numbers corresponding to 5, 10, and 15 revolutions of Venus (right to left)

Exercise. Find the least common multiple of 260, 365 and 584. Write the result using Maya positional system for time. Solution: 584 = 73 × 8, 260 = 13 × 5 × 4, 365 = 5 × 73. The least common multiple is 73 × 13 × 5 × 8 = 37,960. Because 360 = 9 × 8 × 5, a common multiple of this number and 360 will be 9 × 37,960 = 341,640 = 949 × 360. Using Mayan positional system for time this would be written as 2.7.9.0.0.

Some of the entries of the table on p. 24 of the Dresden Codex correspond to needed corrections for the fact that the average cycle of Venus is not exactly 584 days but 583.92. According to Thompson (p. 63), the table indicates that Maya astronomers subtracted four days after 61 cycles of Venus four times, in addition to subtraction 8 days after 57 cycles of Venus, for a total correction of 24 days in 301 Venus cycles. The exact correction is 24.08 days in 301 Venus cycles, which gives an error of one day in about six thousand years. Quite an accomplishment for Mayan astronomers!

The pages on Venus in the Dresden Codex also have numbers that allow readers to correlate the tables with a date in the Long Count. On p. 24 the Long Count date is given as 9.9.9.16.0 1 Ahau 18 Kayab (see Figure 11).

Figure 11. Long Count date 9.9.9.16.0 1 Ahau 18 Kayab

7 Final remarks

Both in our modern conception of Venus as a planet orbiting around the Sun, and in ancient Maya identification of Venus with a deity, mathematics plays a central role in our ability to predict when certain events would happen. Our modern astronomical theories of planetary motion are based on mathematics and allow us to predict with great accuracy when events such as the transit of Venus will occur. The transit of Venus is when the planet crosses the face of the sun. Because the orbit of Venus is slightly inclined with respect to the plane of the orbit of the Earth (the ecliptic), at its inferior conjunction Venus usually does not cross the face of the sun at is lower conjunction. The transit of Venus occurs in cycles of 243 years which can be predicted with accuracy and corroborated by direct observation.

According to the Maya systems of beliefs, the heliacal rise of Venus would have different consequences according to the date in the sacred calendar when such event occurred. It was thus very important to coordinate the synodic cycle of Venus of 584 with the calendars of 260 and 365 days to be able to predict the exact date when Venus would rise. The ability to coordinate the various systems for keeping track of time, and make precise corrections over long periods of time, was made possible for Mayan astronomers by their number knowledge and understanding of common multiples, their use of —what we would call using modern mathematical terminology— modular arithmetic and congruences, as well as careful observations and records kept over centuries. The needed computations were greatly facilitated by their efficient positional system to record numbers, which was far better suited for computation than the system used in Europe at that time.

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