Vibrational Response of Structures Exposed to Human-induced Loads

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Abstract

Structures like pedestrian bridges, staircases and floors become lighter and more slender. As a consequence human-induced vibrations in resonance with the structure become increasingly important when considering the serviceability of a structure. The present paper describes the vibrational response using models for pedestrian loads and the accompanying assessment of the serviceability of a structure exposed to human-induced vibrations. The presented approach uses natural frequencies, modal masses and structural damping to determine the structural vibrations. This allows for more flexible and elegant structures when considering human comfort, which often is dimensioning for light structures with large spans. A criteria based on frequency limits is not sufficient to ensure satisfactory vibration serviceability, especially for light structures. The approach is compared with vibration measurements on a structure before and after the installation of tuned mass dampers.

Keywords: human-induced vibrations; vibration measurements; tuned mass dampers; vibration criteria; structural dynamics.

1 Introduction

Human-induced vibrations become increasingly important as structures are getting lighter and longer. A spectral load model to predict human-induced vibrations was presented in [1]. The load model can be reduced to a simple analytical expression, which has been applied for practical applications for more than 15 years [2]. The simplified method may be applied to model rhythmic loading from people jumping, which is relevant for the design of grand stands and fitness centers [1]. The focus of this paper is to assess vertical human-induced vibrations of floors, pedestrian bridges and staircases with the spectral model and the criteria of human comfort on general office floors.

2 Serviceability Criteria for Structural Vibrations

The level of comfortable structural vibrations is influenced by several factors such as: individual perception, environment, serviceability, situation, duration and body position. Some of the more strict criteria are found for laboratories, while structures like grand stands and fitness centers have much more moderate criteria for satisfactory vibration levels. Continuous and intermittent vibrations are considered to be less tolerable than sudden impact vibrations in the same order of magnitude. Human-induced vibrations caused by walking in a general office are considered to be continuous or intermittent, as the events are expected to occur several times per day.

It is recommended in the Eurocode EN 1990 [3] to apply the International Standard ISO 10137 [4] for satisfactory vibrations in buildings. The ISO 10137 criteria for continuous vertical vibrations in general offices is applied in this

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paper, see Figure 1, which shows the standard deviation of the accelerations criteria as a function of the response frequency. The vibration limit of 0.02 m/s^2 between 4-8 Hz for general offices is established by the ISO base curve [4] with a multiplying factor of 4. The level of acceptable vibrations in general offices is similar to the recommended limit incorporated in the Danish National Annex [5], where the levels are based on measurements and tests on general office floors with employees working at their desks. The tests indicated that the amount of complains from employees sitting at their desks increase significantly if the level of accelerations become higher than the recommended limit. This recommended level of vibrations for general offices has been applied for more than 10 years in Danish building designs, and the specifications have turned out to be in line with the user expectations.

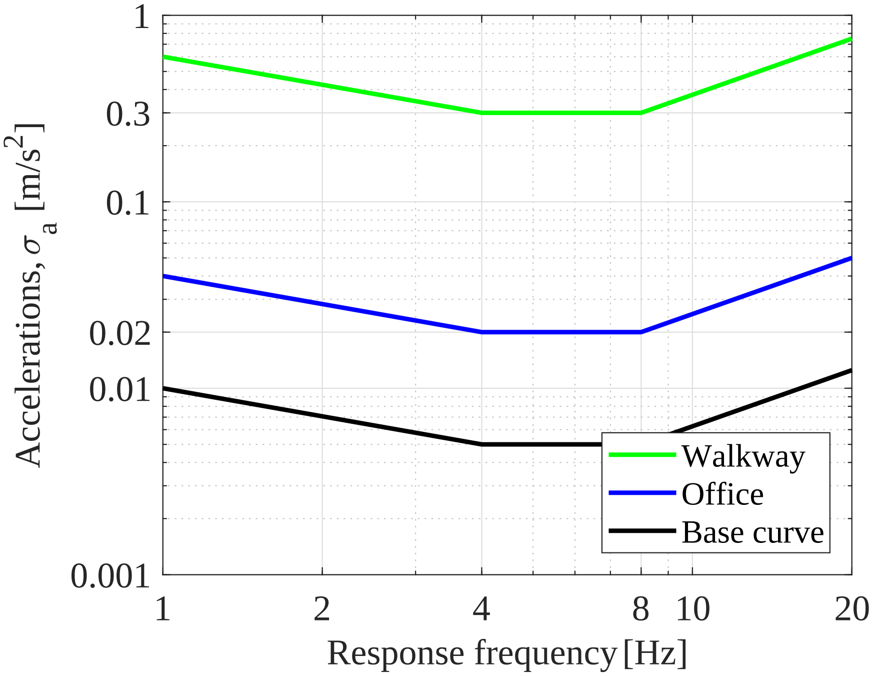


Fig. 1 ISO criteria [4] for the standard deviation of the vertical accelerations for continuous floor vibrations in general offices and walkways as a function of the structural response frequency. The criteria are based on the ISO base curve with a multiplying factor of 4 and 60. The response frequencies are divided into 1/3 octave band [4].

The curves in Figure 1 show the limits for the standard deviation of the accelerations for different frequencies, implying that people are mostly sensitive to vertical vibrations with a frequency of 4-8 Hz. This may be due to natural frequencies of the human body being in this frequency range. For a human-induced load in resonance with the structure, the response frequency in Figure 1 is equal to the relevant natural frequency of the structure.

3 Mathematical Model of Human-induced Vibrations

This section is divided into a spectral load model [1] and a response model presented in Section 3.1 and 3.2, respectively. The response model is based on a typical damped one degree-of-freedom system.

3.1 Human-induced Loads

A spectral approach may be used to describe the load and associated load effects of humans walking or jumping. Neglecting the static load, the human-induced vertical load from a group of people may be determined as a sum of k harmonic components [1]

$$F(t) = N m_{\mathrm{p}} g \sum_{j=1}^{k} \alpha_j C_j \cos(2\pi j n_{\mathrm{p}} t + \phi_j), \tag{1}$$

where t is time, N is the number of persons, n_{p} is the movement frequency, m_{p} is the human mass, g is the gravitational acceleration and α_j, C_j and ϕ_j are the amplitude factor, crowd reduction factor and phase lag for the

Table 1 Vertical load coefficients for human-induced vibrations.

	Walking on horizontal surface [2]	Walking on staircase [6]	Rhythmic jumping [2], free possibility of movement	Rhythmic jumping [2], reduced possibility of movement
n_{p}	1.6-2.4 Hz	1.6-2.4 Hz	0-3 Hz	0-3 Hz
α_1	0.4	1.00	1.6	0.4
α_2	0.1	0.21	1.0	0.25
α_3	0.06	0.08	0.2	0.05
ρ_1	0	0	1.0	1.0
ρ_2	0	0	0.3	0.1
ρ_3	0	0	0.03	0.01

j'th harmonic load component, respectively. Assuming no phase lag and that the resonant term is dominant, the magnitude of the single harmonic load component may be estimated as

$$\max|F(t)| \approx N m_{\mathrm{p}} g \alpha_j C_j, \tag{2}$$

where C_j is the crowd reduction factor describing the correlation between loads from various persons at different locations on the structure. The crowd reduction factor is defined as [1]

$$C_j = \sqrt{\rho_j + \frac{1-\rho_j}{N}\lambda}, \tag{3}$$

where ρ is the correlation coefficient and λ is a scaling of the equivalent number of persons given as

$$\lambda = \frac{1/N \sum_{i=1}^{N} \gamma_i^2}{(1/N \sum_{i=1}^{N} \gamma_i)^2}, \tag{4}$$

where the mode shape value γ_i is assumed to have the same sign for all N people. Constant mode shapes with $\lambda = 1$, are conservative when assessing the comfort of a vibrating structure. For a simply supported beam with uniform load distribution, λ is equal to 4/3. For a load from a single person, the crowd reduction factor is $C_j = 1$ and for a group of people walking, the crowd reduction factor is $C_j = \sqrt{\lambda/N}$. The relevant amplitude factors and correlation coefficients for people walking and jumping may be found in Table 1.

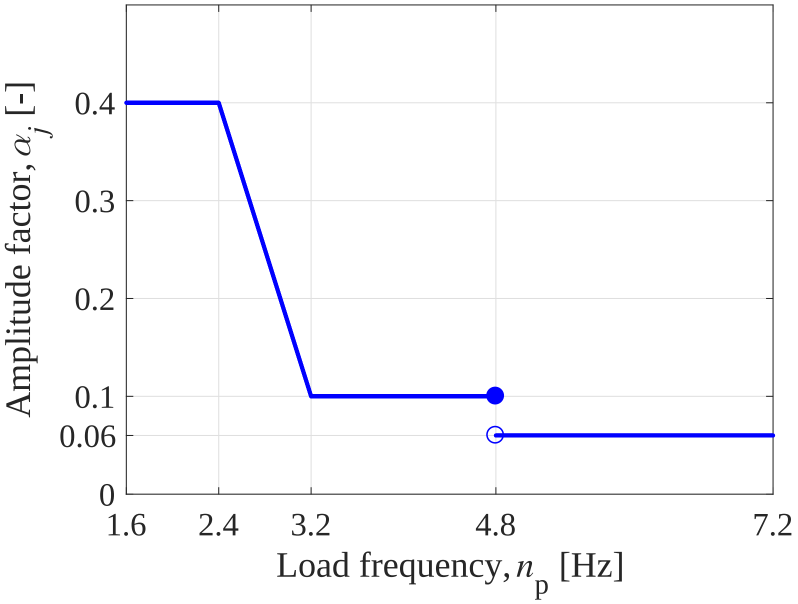


Fig. 2 Load amplitude factor α_j for people walking on a horizontal surface as a function of the load frequency. The gab of the amplitude factor near 4.8 Hz is caused by model uncertainties. A linear dependency is assumed for 2.4-3.2 Hz.

The amplitude factor for people walking on a horizontal surface is shown as a function of the load frequency in Figure 2. Load frequencies up to 7.2 Hz, corresponding to $k = 3$, will often result in accurate estimates of the human-induced vibrations. However, higher harmonics may be relevant for light structures with natural frequencies above 7.2 Hz. Values for jumping people should be applied for grand stands and fitness centers [1].

A correlation coefficient of $\rho = 0$, as shown in Table 1 for people walking, may be applied when the load from different persons is uncorrelated. As a consequence, the correlation coefficients in Table 1 are not applicable for people marching. The correlation coefficient is especially important for rhythmic loads such as people jumping, which is relevant for the design of grand stands and fitness centers.

3.2 Structural Response from Human-induced Loads

For a harmonic forced response of a damped one degree-of-freedom system, the standard deviation of the response acceleration may be calculated as a function of the load amplitude given by Eq. (1) and the modal mass m_{g} of structure

$$\sigma_{\mathrm{a}} = \frac{N m_{\mathrm{p}} g}{m_{\mathrm{g}}} \sqrt{\frac{1}{2} \sum_{j=1}^{k} (\alpha_j C_j H_j)^2}, \tag{5}$$

where H_j is the dynamic amplification factor defined as

$$H_j = \frac{1}{\sqrt{(1 - (\frac{j n_{\mathrm{p}}}{n_1})^2)^2 + (\frac{\delta}{\pi} \frac{j n_{\mathrm{p}}}{n_1})^2}}, \quad \delta << 2\pi, \tag{6}$$

where n_1 is the natural frequency and δ is the level of damping from people and structure given as the logarithmic decrement (LD). The modal mass m_{g} is often determined using the finite element method, but may be calculated for a simple plate as

$$m_{\mathrm{g}} = \int_0^a \int_0^b m(x) \gamma(x)^2 dx, \tag{7}$$

where x is the independent length coordinate, a and b are the plate dimensions, $m(x)$ is the mass per area and $\gamma(x)$ is the mode shape normalized with a maximum deflection equal to 1. The modal mass for the first mode of a simple supported plate with uniformly distributed mass maybe estimated by $m_{\mathrm{g}} = \frac{1}{4} m_{\mathrm{total}}$.

Assuming resonance between the human-induced loads and the structural frequency implies

$$j n_{\mathrm{p}} = n_1, \quad j = 1, 2, ..., k, \tag{8}$$

where n_{p} is the moving frequency of the people. For a structure with a natural frequency of 4 Hz, the moving frequency of the people is assumed to be 2 Hz resulting in the 2nd harmonic load component to be in resonance with the structure.

Assuming that the resonance term of the loading is dominant, similar to Eq. (2), results in a simple analytical expression to estimate the standard deviation of the structural accelerations

$$\sigma_{\mathrm{a},j} \approx \frac{1}{\sqrt{2}} \frac{N m_{\mathrm{p}} g \alpha_j C_j}{m_{\mathrm{g}}} \frac{\pi}{\delta}, \tag{9}$$

A typical design load for general office floors is a single person walking in resonance [4] with the natural frequency of the floor.

4 Model Results

A comparison between the full model Eq. (5) and the single harmonic approach Eq. (9) is illustrated in Figure 3 based on the ISO criteria described in Section 2 for general offices. The simple expression in Eq. (9) is un-conservative as it underestimates the response. It is fairly accurate when considering heavy-weighing and lightly damped structures. For light-weighing and highly damped structures, the accuracy decreases, but will often give good indications to whether the comfort of a structure is satisfactory. The deviations become higher than 10 % for natural frequencies of 6 Hz with a damping of 0.12 LD and the deviations decrease for natural frequencies of 4 or 2 Hz, see Figure 3.

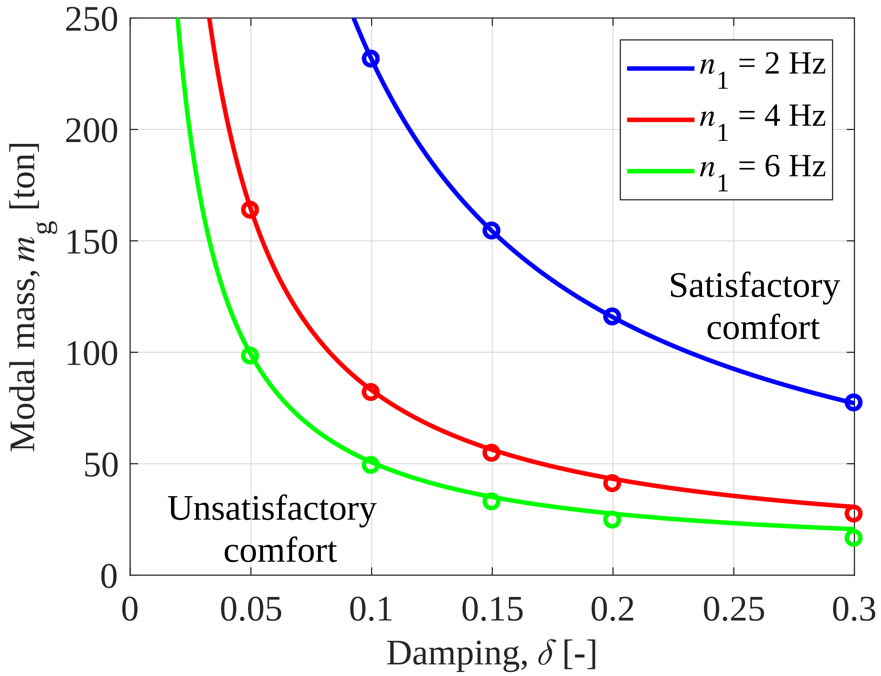


Fig. 3 Comfort of general office floors exposed to a load from a single person walking. Comparison of full model Eq. (5) marked by lines and the simple model Eq. (9) marked by circles. Points above the relevant curve imply satisfactory comfort.

The results in Figure 3 are found using the load from a single person with mass $m_{\mathrm{p}} = 75$ kg. Points above the relevant curve imply that the comfort of the structure is satisfactory. For example, a floor with a natural frequency of 4 Hz, modal mass of 100 ton and a damping of 0.2 LD have satisfactory comfort. However, if the natural frequency is 2 Hz and the other parameters remain unchanged, the comfort will be unsatisfactory.

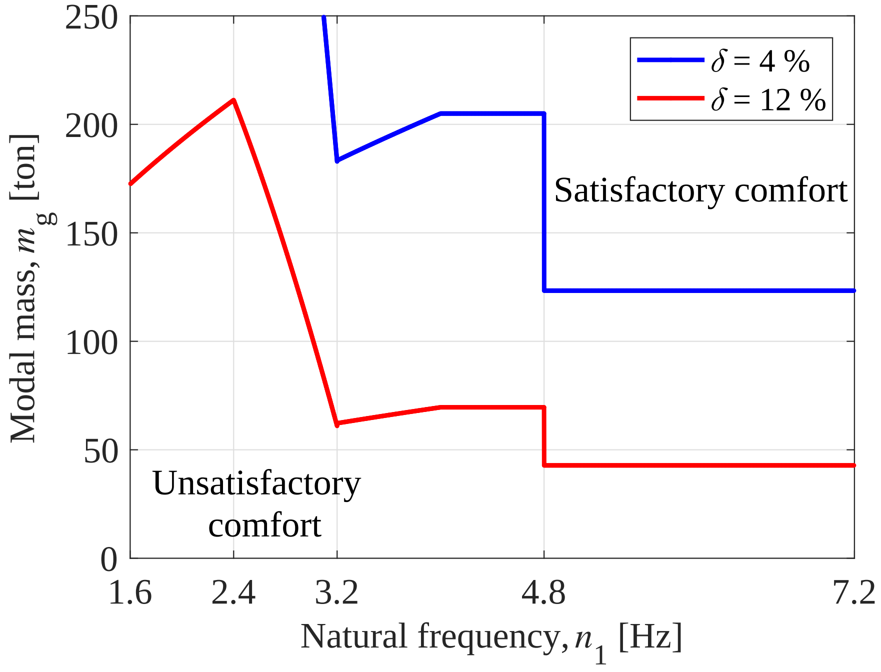


Fig. 4 Comfort of general office floors exposed to a load from a single person walking. The comfort is calculated by the full model Eq. (5). Points above the relevant curve imply that the comfort of the structure is satisfactory.

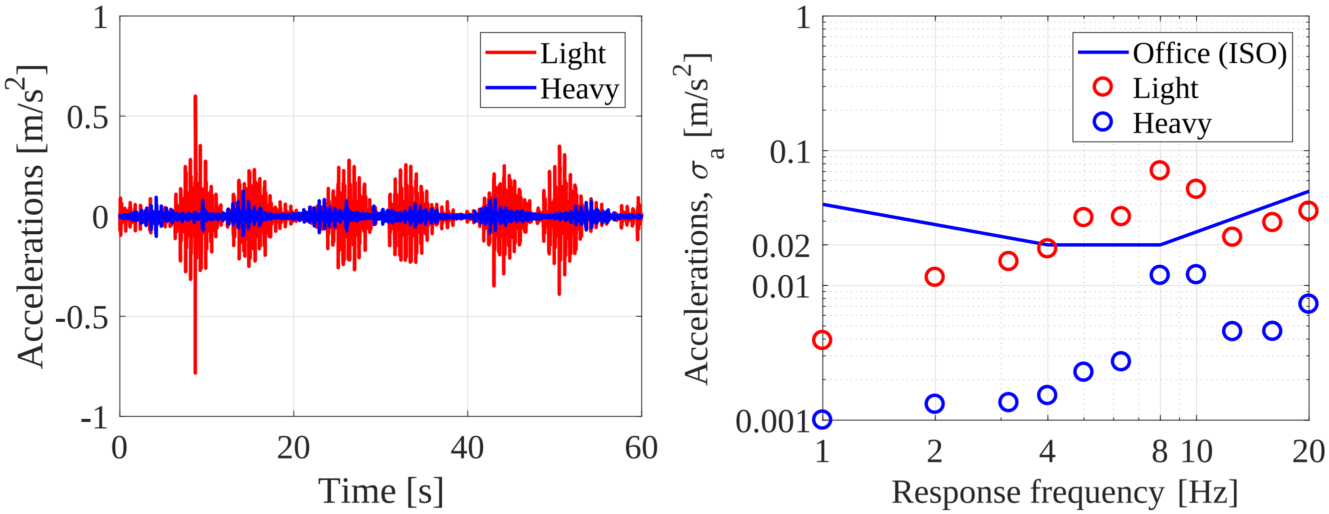
The model shows that the important parameters to determine the magnitude of structural vibrations are the natural frequency, modal mass and damping. Applying the model and the ISO criteria shown in Figure 1 result in specific parameters to determine the vibration comfort in general offices, see Figures 3 and 4. Figure 4 shows that relatively large modal mass and damping is criteria for natural frequencies up to 4.8 Hz where the relevant load harmonic shifts from $j = 2$ to $j = 3$. The necessary modal masses are approximately 125 and 45 ton for damping levels of 0.04 and 0.12 LD respectively, as the natural frequencies become higher than 4.8 Hz.

5 Measurements

Calculated structural vibrations are associated with uncertainties from constrain-types, connections, masses, structural damping levels etc. Some examples of the measured response on structures such as a pedestrian bridge, a staircase and general office floors are presented in this section.

5.1 General Office Floor

The measured accelerations on a light and heavy general office floor are presented in Figure 5. The accelerations are measured during a single person walking continuously with a walking frequency of approximately 2 Hz. The light-weighing floor has a modal mass of 5 ton, whereas the heavy-weighing floor has a modal mass of 50 ton. The two general office floors have similar damping levels and natural frequencies of 7 Hz. The employees were complaining about the vibrations on the light general office floor, whereas no employees complained about the vibrations on the heavier floor. The vibrations of the light-weighing floor exceed the vibration criteria for general offices [4], see Figure 5. The level of vibrations on the heavy floor is below the vibration criteria.



(a) (b)

Fig. 5 Measured accelerations during a single person walking continuously with a walking frequency of approximately 2 Hz. The accelerations are measured on a light and heavy general office floor. The response frequencies are divided into 1/3 octave band [4].

5.2 Tuned Mass Dampers

Structural damping is influenced by the type of material, connections, amplitude etc. Typically levels of structural damping are 0.02 and 0.12 LD for welded steel and reinforced concrete structures, respectively. In some design situations it might be advantageously to add damping to reduce vibrations. Tuned mass dampers (TMD) are very usable to increase damping levels as they are cost effective and the mechanical components are relatively simple. Tuned mass dampers have been used during the last 70 years on structures such as: bridges, chimneys, tall buildings, floors, cables etc. A tuned mass damper weighs typically 1-10 % of the structural modal mass and contains simple elements such as: a mass, springs, dashpots. The design of a typical TMD is shown in Figure 8.

The theory about tuned mass dampers is well known. It was determined and improved by Den Hartog [7] during the 1950s, and the theory was based on harmonic excitation loading. Tuned mass dampers are often installed on structures exposed to stochastic loads such as wind gust. The load type is not without consequence as the effectiveness of TMDs decreases for structures exposed to stochastic loads compared to harmonic loads. The behavior of TMDs exposed to stochastic loads is described by Krenk [8]. The load caused by several persons walking on a horizontal surface is similar to stochastic loads. The influence of reduced effectiveness should be considered when designing TMDs for human-induced vibrations.

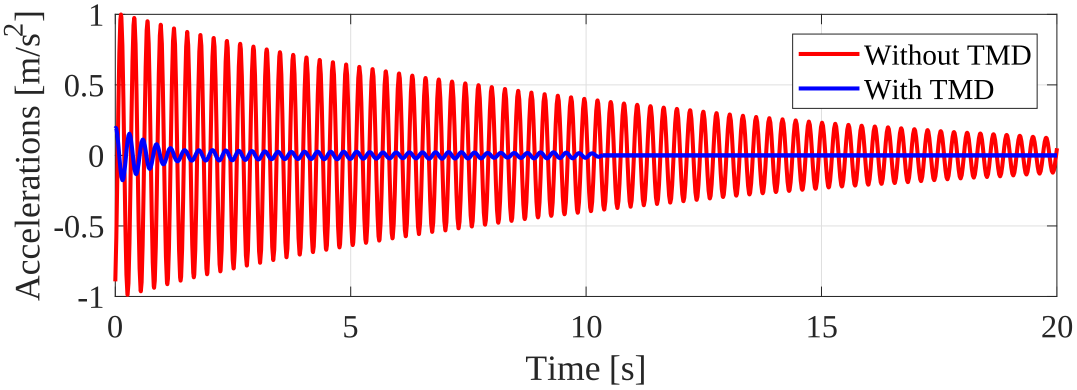
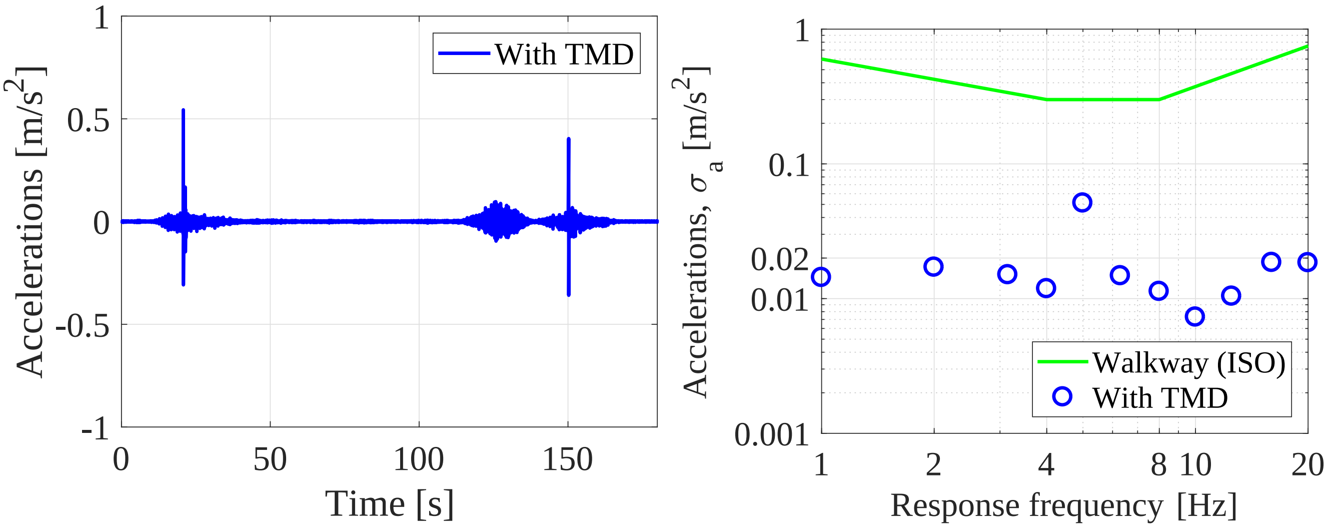


Fig. 6 Measured accelerations during decay tests on a pedestrian bridge with and without a TMD. Both decay tests are carried out with an identical excitation force.

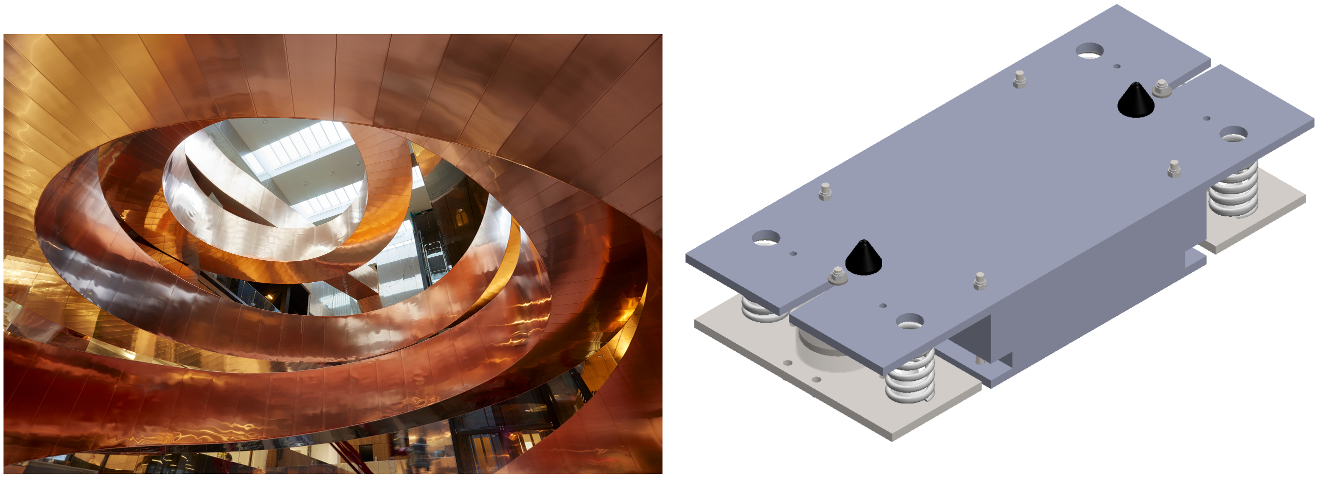
Measured accelerations are shown in Figure 6 during a decay test on a Danish pedestrian bridge with a modal mass of approximately 15 ton and a vertical natural frequency of 3 Hz. The bridge is exited by an identical harmonic load with and without a tuned mass damper. The consequence of the TMD is significant, as the damping level is increased from 0.025 to approximately 0.25 LD.



(a)

(b)

Fig. 7 Accelerations measured on a staircase with installed TMDs during a single person walking continuously with a walking frequency of approximately 2 Hz. The response frequencies are divided into 1/3 octave band [4].



(a) Photo by: CSK Stålindustri A/S, Tom Jersøe.

(b) Tuned mass damper.

Fig. 8 Left: Copper plated helical staircase located inside the public science center *Experimentarium* in Copenhagen, Denmark. Right: Typical design of a TMD.

The level of vibrations from a single person walking on a staircase with installed TMDs are shown in Figure 7. The staircase is located inside the public science center *Experimentarium* in Copenhagen, see Figure 8. The staircase has a modal mass of approximately 10 ton and a natural frequency of 5 Hz. The accelerations are measured during a single person walking continuously with a walking frequency of approximately 2 Hz, see Figure 7. The measured level of vibration is clearly lower than the criteria [4].

6 Conclusion

A model is presented to predict human-induced vibrations of floors, pedestrian bridges, grand stands and staircases. The model can be reduced to a simple analytical expression, which often provides accurate estimates to determine if the comfort of a structure exposed to human-induced vibrations is satisfactory. The model has been applied for more than 15 years [2] and the single harmonic approach may quickly yield results to determine if the comfort of a structure is satisfactory. The vibration level for general offices suggested in ISO 10137 is adequate as a design criteria.

The vibration comfort on structures such as general office floors is influenced by the structural mass, especially for structures with natural frequencies lower than 8 Hz. A criteria based on frequency limits is not sufficient to ensure satisfactory vibration serviceability, especially for light structures. Tuned mass dampers may be installed to increase the damping levels to obtain a better comfort. Figures 3 and 4 may provide a simple design approach to determine whether a general office floor will have satisfactory comfort when exposed to people walking.

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