## VLSI Design Issues

Scaling/Moore's Law has limits due to the physics of material.

- Now L (L=2Onm??) affects $\dagger x$ delays (speed), noise, heat (power consumption)
- Scaling increases density of txs and requires "more" interconnect (highways \& buses)-more delays (lowering speed) and heat.

Possible Solutions:

- New fabrication solutions/material. E.g., Interconnect layers, new material (copper \& low k-material)
- Improve physical Designs at the transistor level. Create better cell libraries (min Power, min-delay, max speed)
- Exploit transister analog/physics characteristics
- Invent new transistors

Invent new architectures

## CMOS Inverter: DC Analysis

- Analyze DC Characteristics of CMOS Gates by studying an Inverter
pFET: $\quad V_{T p}<0$

$$
\beta_{p}=k_{p}^{\prime}\left(\frac{W}{L}\right)_{p}
$$

- DC Analysis
- DC value of a signal in static conditions
- DC Analysis of CMOS Inverter
- Vin, input voltage
- Vout, output voltage
- single power supply, VDD
- Ground reference
- find Vout = f(Vin)
- Voltage Transfer Characteristic (VTC)
- plot of Vout as a function of Vin
- vary Vin from 0 to VDD (and in reverse!)

nFET: $\quad V_{T n}>0$
$\beta_{n}=k_{n}^{\prime}\left(\frac{W}{L}\right)_{n}$
- find Vout at each value of Vin


## Inverter Voltage Transfer Characteristics

- Output High Voltage, $\mathrm{V}_{\mathrm{OH}}$
- maximum output voltage
- occurs when input is low (Vin = OV)
- pMOS is ON, nMOS is OFF
- pMOS pulls Vout to VDD
- $\mathrm{V}_{\mathrm{OH}}=\mathrm{VDD}$
- Output Low Voltage, $\mathrm{V}_{\text {OL }}$
- minimum output voltage
- occurs when input is high (Vin = VDD)
- pMOS is OFF, nMOS is ON
- nMOS pulls Vout to Ground
- $V_{O L}=0 \mathrm{~V}$
- Logic Swing
- Max swing of output signal

$$
\begin{aligned}
& \text { - } V_{L}=V_{O H}-V_{O L} \\
& -V_{L}=V D D
\end{aligned}
$$



## Inverter Voltage Transfer Characteristics

- Gate Voltage, $\mathrm{f}(\mathrm{Vin}) \quad$ - Drain Voltage, f (Vout)
- $V_{G S n}=V i n, V_{S G p}=V D D-\operatorname{Vin}$
$-V_{\text {DSn }}=$ Vout, $V_{\text {SDP }}=$ VDD-Vout
- Transition Region (between $\mathrm{V}_{\mathrm{OH}}$ and $\mathrm{V}_{\mathrm{OL}}$ )
- Vin low
- Vin < Vtn
- Mn in Cutoff, OFF
- Mp in Triode, Vout pulled to VDD
- Vin > Vtn < ~Vout
- Mn in Saturation, strong current
- Mp in Triode, $V_{S G}$ \& current reducing
- Vout decreases via current through Mn
- $\operatorname{Vin}=\operatorname{Vout}($ mid point $) \approx \frac{1}{2}$ VDD
- Mn and Mp both in Saturation
- maximum current at Vin = Vout $\quad$ Vin $<V_{\text {IL }}$
- Vin high
- Vin > ~Vout, Vin < VDD - |Vtp|
- Mn in Triode, Mp in Saturation
- Vin >VDD - |Vtp|
- Mn in Triode, Mp in Cutoff

$V_{I L} \quad V_{I H}$


## Noise Margin

- Input Low Voltage, $\mathrm{V}_{\text {IL }}$
- Vin such that Vin < $V_{I L}=$ logic 0
- point ' $a$ ' on the plot
- where slope, $\frac{\partial V i n}{\partial V o u t}=-1$
- Input High Voltage, $\mathrm{V}_{I H}$
- Vin such that Vin > $V_{I H}=$ logic 1
- point 'b' on the plot

- where slope $=-1$
- Voltage Noise Margins
- measure of how stable inputs are with respect to signal interference
- $\mathrm{VNM}_{\mathrm{H}}=\mathrm{V}_{\mathrm{OH}}-\mathrm{V}_{\mathrm{IH}}=\mathrm{VDD}-\mathrm{V}_{\mathrm{IH}}$
$-V N M_{L}=V_{I L}-V_{O L}=V_{I L}$
- desire large $V N M_{H}$ and $V N M_{L}$ for best noise immunity


## Switching Threshold

- Switching threshold = point on VTC where Vout = Vin
- also called midpoint voltage, $\mathrm{V}_{\mathrm{M}}$
- here, Vin = Vout $=V_{M}$
- Calculating $V_{M}$
- at $V_{M}$, both $n M O S$ and $p M O S$ in Saturation
- in an inverter, $I_{D n}=I_{D p}$, always!
- solve equation for $V_{M}$

$I_{D n}=\frac{\mu_{n} C_{O X}}{2} \frac{W}{L}\left(V_{G S n}-V_{t n}\right)^{2}=\frac{\beta_{n}}{2}\left(V_{G S n}-V_{t n}\right)^{2}=\frac{\beta_{p}}{2}\left(V_{S G p}-\left|V_{t p}\right|\right)^{2}=I_{D p}$
$V_{I L} \quad V_{I H}$
- express in terms of $\mathrm{V}_{M}$

$$
\frac{\beta_{n}}{2}\left(V_{M}-V_{m}\right)^{2}=\frac{\beta_{p}}{2}\left(V_{D D}-V_{M}-\mid V_{p}\right)^{2} \Rightarrow \sqrt{\frac{\beta_{n}}{\beta_{p}}}\left(V_{M}-V_{M M}\right)=V_{D D}-V_{M}-\left|V_{p}\right|
$$

- solve for $V_{M} V_{M}=\frac{V D D-\left|V_{p_{p}}\right|+V_{m "} \sqrt{\frac{\beta_{n}}{\beta_{p}}}}{1+\sqrt{\frac{\beta_{n}}{\beta_{p}}}}$


## Effect of Transistor Size on VTC

- Recall

$$
\begin{array}{ll}
\text { Recall } & \beta_{n}=k_{n}^{\prime} \frac{W}{L}
\end{array} \quad \frac{\left.k_{n}^{\prime}, \frac{W}{\beta_{p}}\right)_{n}}{\beta_{p}}=\frac{k_{p}^{\prime}\left(\frac{W}{L}\right)_{p}}{}
$$

$$
V_{M}=\frac{V D D-\left|V_{t p}\right|+V_{t n} \sqrt{\frac{\beta_{n}}{\beta_{p}}}}{1+\sqrt{\frac{\beta_{n}}{\beta_{p}}}}
$$

- If nMOS and pMOS are same size
- $(W / L) n=(W / L) p$
- Coxn $=\operatorname{Coxp}$ (always) $\quad \frac{\beta_{n}}{\beta_{p}}=\frac{\mu_{n} C_{\operatorname{oxn}}\left(\frac{W}{L}\right)_{n}}{\mu_{p} C_{\text {orp }}\left(\frac{W}{L}\right)_{p}}=\frac{\mu_{n}}{\mu_{p}} \cong 20 r 3$
- If $\frac{\left(\frac{W}{L}\right)_{p}}{\left(\frac{W}{L}\right)_{n}}=\frac{\mu_{n}}{\mu_{p}}$, then $\frac{\beta_{n}}{\beta_{p}}=1$
since $L$ normally min. size for all $\dagger x$,
can get betas equal by making Wp larger than Wn
- Effect on switching threshold
- if $\beta_{n} \approx \beta_{p}$ and $V t n=|V+p|, V_{M}=V D D / 2$, exactly in the middle
- Effect on noise margin
- if $\beta_{n} \approx \beta_{p}, V_{I H}$ and $V_{I L}$ both close to $V_{M}$ and noise margin is good


## Example

- Given
$-k^{\prime} n=140 u A / V^{2}, V+n=0.7 \mathrm{~V}, \mathrm{VDD}=3 \mathrm{~V}$
$-k^{\prime} p=60 u A / V^{2}, V+p=-0.7 \mathrm{~V}$
- Find
- a) $t x$ size ratio so that $V_{M}=1.5 \mathrm{~V}$
- b) $V_{M}$ if $t \times$ are same size

(a) Larger pFET design

transition pushed lower as beta ratio increases


## CMOS Inverter: Transient Analysis

- Analyze Transient Characteristics of CMOS Gates by studying an Inverter
- Transient Analysis
- signal value as a function of time
- Transient Analysis of CMOS Inverter

- $\operatorname{Vin}(t)$, input voltage, function of time
- Vout(t), output voltage, function of time
- VDD and Ground, DC (not function of time) ${ }_{V_{D D}} \notin$
- find $\operatorname{Vout}(\dagger)=f(\operatorname{Vin}(t))$
- Transient Parameters
- output signal rise and fall time
- propagation delay


## Transient Response

- Recall: the RC nMOS Transistor Model



## Transient Response

- Response to step change in input
- delays in output due to parasitic R \& C
- Inverter RC Model
- Resistances (linear model)

$$
\begin{aligned}
& -R n=1 /\left[\beta_{n}\left(V_{D D}-V+n\right)\right] \\
& -R p=1 /\left[\beta_{p}\left(V_{D D^{-}}|V+p|\right)\right]
\end{aligned}
$$

- Output Cap. (only output is important)
- $C_{D n}$ (nMOS drain capacitance)

$$
-C_{D n}=\frac{1}{2} C o x W_{n} L+C_{j} A_{\text {Dnbot }}+C_{j s w} P_{\text {Dnsw }}
$$

- $C_{D p}$ (pMOS drain capacitance)

$$
-C_{D p}=\frac{1}{2} C o x W_{p} L+C_{j} A_{D p b o t}+C_{j s w} P_{D p s w}
$$

- Load capacitance, due to gates attached at the output

$$
-C_{L}=3 \text { Cin }=3\left(C_{G n}+C_{G p}\right), 3 \text { is a "typical" load }
$$

- Total Output Capacitance

$$
\text { Cout }=C_{D n}+C_{D P}+C_{L}
$$

term "fan-out" describes \# gates attached at output

## Fall Time

- Fall Time, $t_{f}$
- time for output to fall from '1' to '0'
- derivation: $i=-C_{\text {out }} \frac{\partial V_{\text {out }}}{\partial t}=\frac{V_{\text {out }}}{R_{n}}$
- initial condition, $\operatorname{Vout(0)}=\operatorname{VDD}$

$$
\begin{aligned}
& \text { - solution } \\
& \operatorname{Vout}(t)=V_{D D} e^{-t / \tau_{n}} \quad \begin{array}{l}
\text { time constant } \\
\tau_{\mathrm{n}}=\mathrm{R}_{\mathrm{n}} \mathrm{C}_{\text {out }} \\
t=\tau_{n} \ln \left(\frac{V_{D D}}{\text { Vout }}\right)
\end{array}
\end{aligned}
$$



- definition
- $t_{f}$ is time to fall from
$90 \%$ value $\left[\mathrm{V}_{1}, \mathrm{t}_{x}\right]$ to $10 \%$ value $\left[\mathrm{V}_{0}, \mathrm{t}_{y}\right]$
$t=\tau_{n}\left[\ln \left(\frac{V_{D D}}{0.1 V_{D D}}\right)-\ln \left(\frac{V_{D D}}{0.9 V_{D D}}\right)\right]$
- $\mathrm{t}_{\mathrm{f}}=2.2 \tau_{\mathrm{n}}$

(a) Discharge circuit

(b) Output waveform


## Rise Time

- Rise Time, $t_{r}$
- time for output to rise from '0' to '1'
- derivation: ${ }_{i=C_{\text {out }}} \frac{\partial V_{\text {out }}}{\partial t}=\frac{V_{D D}-V_{\text {out }}}{R_{p}}$
- initial condition, $\operatorname{Vout}(0)=0 \mathrm{~V}$

$$
\begin{aligned}
& \text { solution } \\
& \operatorname{Vout}(t)=V_{D D}\left[1-e^{-t / \tau_{p}}\right] \tau_{\mathrm{p}}=\mathrm{R}_{\mathrm{p}} \mathrm{C}_{\text {out }}
\end{aligned}
$$



- definition
- $t_{f}$ is time to rise from
$10 \%$ value $\left[\mathrm{V}_{0}, \mathrm{t}_{\mathrm{u}}\right]$ to $90 \%$ value $\left[\mathrm{V}_{1}, \mathrm{t}_{\mathrm{v}}\right]$
- $t_{r}=2.2 \tau_{p}$
- Maximum Signal Frequency
$-f_{\max }=1 /\left(\dagger_{r}+\dagger_{f}\right)$

$V_{\text {out }}(0)=0 \mathrm{~V}$
(a) Charse circuit
- taster than this and the output can't settle


## Propagation Delay

- Propagation Delay, $\dagger_{p}$
- measures speed of output reaction to input change
- $t_{p}=\frac{1}{2}\left(\dagger_{p f}+t_{p r}\right)$
- Fall propagation delay, $t_{p f}$
- time for output to fall by $50 \%$
- reference to input switch
- Rise propagation delay, $t_{p r}$
- time for output to rise by $50 \%$
- reference to input switch

- Ideal expression (if input is step change)
- $\dagger_{p f}=\ln (2) \tau_{n}$
- $\mathrm{t}_{\mathrm{pr}}=\ln (2) \tau_{\mathrm{p}}$
- Total Propagation Delay

Propagation delay measurement:

- from time input reaches $50 \%$ value
- to time output reaches $50 \%$ value
$-\dagger_{p}=0.35\left(\tau_{n}+\tau_{p}\right)$
Add rise and fall propagation delays for total value

Switching Speed -Resistance

- Rise \& Fall Time

$$
-t_{f}=2.2 \tau_{n}, t_{r}=2.2 \tau_{p},
$$

- Propagation Delay

$$
-t_{p}=0.35\left(\tau_{n}+\tau_{p}\right)
$$

- In General

$$
\begin{aligned}
& \text { - delay } \propto \tau_{\mathrm{n}}+\tau_{\mathrm{p}} \\
& -\tau_{\mathrm{n}}+\tau_{\mathrm{p}}=\operatorname{Cout}(R n+R p)
\end{aligned}
$$

- Define delay in terms of design parameters

$$
\begin{aligned}
& -R n+R p=\frac{\left(V_{D D}-V t\right)\left(\beta_{n}+\beta_{p}\right)}{\beta_{n} \beta_{p}\left(V_{D D}-V t\right)^{2}} \\
& -R n+R p=\frac{\beta_{n}+\beta_{p}}{\beta_{n} \beta_{p}\left(V_{D D}-V t\right)}
\end{aligned}
$$

$$
\tau_{\mathrm{n}}=\mathrm{R}_{\mathrm{n}} \mathrm{C}_{\text {out }} \quad \tau_{\mathrm{p}}=\mathrm{R}_{\mathrm{p}} \mathrm{C}_{\text {out }}
$$

$R n=1 /\left[\beta_{n}\left(V_{D D}-V+n\right)\right]$
$\beta=\mu \operatorname{Cox}(W / L)$
$R p=1 /\left[\beta_{p}\left(V_{D D}-|V+p|\right)\right]$

$$
\text { Gout }=C_{D n}+C_{D p}+C_{L}
$$

Beta Matched if $\beta_{n}=\beta_{p}=\beta$,

$$
R n+R p=\frac{2}{\beta\left(V_{D D}-V t\right)}=\frac{2 L}{\mu C o \times W\left(V_{D D}-V t\right)}
$$

Width Matched if $W_{n}=W_{p}=W$, and $L=L_{n}=L_{p}$

$$
R n+R p=\frac{L\left(\mu_{n}+\mu_{p}\right)}{\left(\mu_{n} \mu_{p}\right) \operatorname{Cox} W\left(V_{D D}-V t\right)}
$$

- if $\mathrm{V}+=\mathrm{V}+n=|\mathrm{V}+\mathrm{p}| \quad$ To decrease $\mathrm{R}^{\prime} \mathrm{s}, \Downarrow \mathrm{L}, \Uparrow \mathrm{W}, \Uparrow \mathrm{VDD},\left(\Uparrow \mu_{\mathrm{p}}, \Uparrow C_{0 x}\right)$

Switching Speed -Capacitance

- From Resistance we have
- $\downarrow l, \Uparrow W, \Uparrow V D D,\left(\Uparrow \mu_{\mathrm{p}}, \Uparrow C o x\right)$
- but 介VDD increases power $\Uparrow$ W increases Cout
- Cout
- Cout $=\frac{1}{2} \operatorname{CoxL}\left(W_{n}+W_{p}\right)+C_{j} 2 L$ $\left(W_{n}+W_{p}\right)+3 \operatorname{Cox} L\left(W_{n}+W_{p}\right)$
- assuming junction area $\sim W \cdot 2 L$
- neglecting sidewall capacitance
- Cout $\approx L\left(W_{n}+W_{p}\right)\left[3 \frac{1}{2} C o x+2 C_{j}\right]$
- Cout $\propto L\left(W_{n}+W_{p}\right)$

To decrease Cout, $\Downarrow \downarrow, \Downarrow W,(\Downarrow C j, \Downarrow C o x)$

- Delay $\propto \operatorname{Cout}(R n+R p) \propto L W \frac{L}{W V D D}=\frac{L^{2}}{V D D}$


## Switching Speed -Local Modification

- Previous analysis applies to the overall design
- shows that reducing feature size is critical for higher speed
- general result useful for creating cell libraries
- How do you improve speed within a specific gate?
- increasing $W$ in one gate will not increase $C_{G}$ of the load gates
- Cout $=C_{D n}+C_{D p}+C_{L}$
- increasing $W$ in one logic gate will increase $C_{D n / p}$ but not $C_{L}$
- $C_{L}$ depends on the size of the $t \times$ gates at the output
- as long as they keep minimum $W, C_{L}$ will be constant
- thus, increasing W is a good way to improve the speed within a local point
- But, increasing W increases chip area needed, which is bad
- fast circuits need more chip area (chip "real estate")
- Increasing VDD is not a good choice because it increases power consumption


## CMOS Power Consumption

- $P=P_{D C}+P_{d y n}$
- $P_{D C}: D C$ (static) term
- $P_{\text {dyn: }}$ dynamic (signal changing) term
- $P_{D C}$
$-P=I_{D D} V_{D D}$

- $I_{D D} D C$ current from power supply
- ideally, $I_{D D}=0$ in CMOS: ideally only current during switching action
- leakage currents cause $I_{D D}>0$, define quiescent leakage current, $I_{D D Q}$ (due largely to leakage at substrate junctions)
- $P_{D C}=I_{D D Q} V_{D D}$
- Pdyn, power required to switch the state of a gate
- charge transferred during transition, Qe = Cout VDD
- assume each gate must transfer this charge $1 \times /$ clock cycle
- Paverage $=V_{D D}$ Qe $f=$ Cout $V_{D D}{ }^{2} f, f=$ frequency of signal change

> Power increases with Cout and frequency, and strongly with VDD (second order).

## Multi-Input Gate Signal Transitions

- In multi-input gates multiple signal transitions produce output changes
- What signal transitions need to be analyzed?
- for a general $N$-input gate with $M_{0}$ low output states and $M_{1}$ high output states
- $\#$ high-to-low output transitions $=M_{0} \cdot M_{1}$
- \# low-to-high output transitions $=M_{1} \cdot M_{0}$
- total transitions to be characterized $=2 \cdot M_{0} \cdot M_{1}$

(ii) | $V_{A}$ | $V_{B}$ | $V_{\text {out }}$ |
| :---: | :---: | :---: |
| 0 | 0 | $V_{D D}$ |
| (ii) |  |  |
| 0 | $V_{D D}$ | $V_{D D}$ |
| (iii) |  |  |
| $V_{D D}$ | 0 | $V_{D D}$ |
| $V_{D D}$ | $V_{D D}$ | 0 |

- example: NAND has $M_{0}=1, M_{1}=3$
- don't test/characterize cases without output transifions
- Worst-case delay is the slowest of all possible cases
- worst-case high-to-low
- worst-case low-to-high
- often different input transitions for each of these cases


## Series/Parallel Equivalent Circuits

- Scale both W and L
- no effective change in W/L

- increases gate capacitance inputs must be at same value/voltage
- Series Transistors
- increases effective L

- Parallel Transistors
- increases effective W


(a) Separate transistors
(b) Single equivalent FET

(b) Single equivalent FET


## NAND: DC Analysis

- Multiple Inputs
- Multiple Transitions
- Multiple VTCs
- VTC varies with transition

| (i)$V_{A}$ $V_{B}$ $V_{\text {out }}$ <br> 0 0 $V_{D D}$ <br> (ii) $\rightarrow$  <br> 0 $V_{D D}$ $V_{D D}$ <br> $V_{D D}$ 0 $V_{D D}$ <br> $V_{D D}$ $V_{D D}$ 0 |
| :--- |

(a) Transition table

(b) VTC family

- transition from 0,0 to 1,1 pushed right of others
- why?
- $V_{M}$ varies with transition
- assume all $\dagger x$ have same $L$
- $\mathrm{V}_{\mathrm{M}}=\mathrm{V}_{\mathrm{A}}=\mathrm{V}_{\mathrm{B}}=$ Vout
- can merge transistors at this point
- if $W_{p A}=W_{p B}$ and $W_{n A}=W_{n B}$
- series nMOS, $\beta n \Rightarrow 1 / 2 \beta n$
- parallel pMOS, $\beta$ p $\Rightarrow 2 \beta$ p
- can now calculate the NAND $V_{M}$



## NAND Switching Point

- Calculate VM for NAND
- 0,0 to 1,1 transition
- all tx change states (on, off)
- in other transitions, only 2 change
- $V_{M}=V_{A}=V_{B}=$ Vout
- set $I_{D n}=I_{D p}$, solve for $V_{M}$

$$
V_{M}=\frac{V D D-\left|V_{t p}\right|+V_{t n} \frac{1}{2} \sqrt{\frac{\beta_{n}}{\beta_{p}}}}{1+\frac{1}{2} \sqrt{\frac{\beta_{n}}{\beta_{p}}}}
$$

- denominator reduced more
- VTC shifts right
- For NAND with N inputs
series nMOS means more resistance to output falling,
shifts VTC to right
to balance this effect and set $\mathrm{V}_{\mathrm{M}}$ to $\mathrm{V}_{\mathrm{DD}} / 2$, can increase $\beta$ by increasing Wn

(b) VTC family



## NOR: DC Analysis

- Similar Analysis to NAND
- Critical Transition
- 0,0 to 1,1
- when all transistors change
- $\mathrm{V}_{\mathrm{M}}$ for NOR2 critical transition
- if $W_{p A}=W_{p B}$ and $W_{n A}=W_{n B}$

- parallel $n M O S, \beta n \Rightarrow 2 \beta n$
- series pMOS, $\beta$ p $\Rightarrow 1 / 2 \beta$ p
$V_{M}=\frac{V D D-\left|V_{t p}\right|+2 V_{t r} \sqrt{\frac{\beta_{n}}{\beta_{p}}}}{1+2 \sqrt{\frac{\beta_{n}}{\beta_{p}}}} V_{M}=\frac{V D D-\left|V_{t p}\right|+N V_{t r} \sqrt{\frac{\beta_{n}}{\beta_{p}}}}{1+N \sqrt{\frac{\beta_{n}}{\beta_{p}}}}$
for NOR2
for NOR-N
- series pMOS resistance means slower rise
- VTC shifted to the left
- to set $\mathrm{V}_{\mathrm{M}}$ to $\mathrm{V}_{\mathrm{DD}} / 2$, increase Wp
- this will increase $\beta$ p


## NAND: Transient Analysis

- NAND RC Circuit
- R: standard channel resistance
- C: Cout $=C_{L}+C_{D n}+2 C_{D P}$
- Rise Time, $t_{r}$
- Worst case charge circuit - 1 pMOS ON
$-\dagger_{r}=2.2 \tau_{p}$
- $\tau_{p}=R_{p}$ Cout
- best case charge circuit
- $2 \mathrm{pMOS} \mathrm{ON}, \mathrm{Rp} \Rightarrow \mathrm{Rp} / 2$

(a) Charging circuit ${ }_{\text {dis. }}$


$$
\begin{aligned}
\mathrm{t}_{f} & =2.2 \tau_{n} \\
\cdot & \tau_{n}=\operatorname{Cout}\left(2 R_{n}\right)+C \times R_{n}
\end{aligned}
$$

- Fall Time, $\mathrm{t}_{\mathrm{f}}$
- Discharge Circuit
- 2 series $n M O S, R n \Rightarrow 2 R n$
- must account for internal cap, Cx



## NOR: Transient Analysis

- NAND RC Circuit
- R: standard channel resistance
- C: Cout $=C_{L}+2 C_{D n}+C_{D P}$
- Fall Time, $\dagger_{f}$
- Worst case discharge circuit - 1 nMOSON
$-t_{f}=2.2 \tau_{n}$ - $\tau_{n}=R_{n}$ Cout
- best case discharge circuit - $2 \mathrm{nMOS} \mathrm{ON}, \mathrm{Rn} \Rightarrow \mathrm{Rn} / 2$
- Rise Time, $t_{r}$
- Charge Circuit
- 2 series pMOS, $R p \Rightarrow 2 R p$
- must account for internal cap, Cy

$$
\begin{aligned}
& \dagger_{r}=2.2 \tau_{p} \\
& \cdot \tau_{p}=\operatorname{Cout}\left(2 R_{p}\right)+C y R_{p}
\end{aligned}
$$



## NAND/NOR Performance

- Inverter: symmetry $\left(\mathrm{V}_{M}=\mathrm{V}_{D D} / 2\right), \beta n=\beta p$
- $(W / L)_{p}=\mu_{n} / \mu_{p}(W / L)_{n}$
- Match INV performance with NAND
- pMOS, $\beta_{\mathrm{p}}=\beta \mathrm{p}$, same as inverter
- $n$ MOS, $\beta_{N}=2 \beta n$, to balance for 2 series nMOS
$\beta$ is adjusted by
changing transistor
size (width)
- Match INV performance with NOR
- pMOS, $\beta_{\mathrm{p}}=2 \beta$ p, to balance for 2 series pMOS
- nMOS, $\beta_{N}=\beta n$, same as inverter
- NAND and NOR will still be slower due to larger Cout
- This can be extended to 3, 4, ... N input NAND/NOR gates

(b) NAND2

(c) NOR2


## NAND/NOR Transient Summary

- Critical Delay Path
- paths through series transistors will be slower
- more series transistors means worse delays
- Tx Sizing Considerations
- increase $W$ in series transistors
- balance $\beta_{n} / \beta_{p}$ for each cell
- Worst Case Transition

- when all series transistor go from OFF to ON
- and all internal caps have to be
- charged (NOR)
- discharged (NAND)


## Performance Considerations

- Speed based on $\beta n, \beta p$ and parasitic caps
- DC performance ( $V_{M}$, noise) based on $\beta n / \beta p$
- Design for speed not necessarily provide good DC performance
- Generally set $\dagger x$ size to optimize speed and then test $D C$ characteristics to ensure adequate noise immunity
- Review Inverter: Our performance reference point
- for symmetry $\left(V_{M}=V_{D D} / 2\right), \beta n=\beta p$
- which requires $(W / L)_{p}=\mu_{n} / \mu_{p}(W / L)_{n}$
- Use inverter as reference point for more complex gates
- Apply slowest arriving inputs to series node closest to output
- let faster signals begin to charge/discharge nodes closer to VDD and Ground



## Timing in Complex Logic Gates

- Critical delay path is due to series-connected transistors
- Example: $f=\overline{x(y+z)}$
- assume all $+x$ are same size
- Fall time critical delay
- worst case, $x$ ON, and y or $z O N$
- $t_{f}=2.2 \tau_{n}$

$$
\begin{aligned}
-\tau_{n} & =R n C n+2 R n C_{\text {out }} \\
& -C_{\text {out }}=2 C_{D P}+C_{D n}+C_{L} \\
& -C_{n}=2 C_{D n}+C_{S n}
\end{aligned}
$$

- Rise time critical delay
- worst case, y and z ON, x OFF
$-t_{r}=2.2 \tau_{p}$

$$
\begin{aligned}
-\tau_{\mathrm{p}} & =R p C_{p}+2 R p C_{\text {out }} \\
& -C_{\text {out }}=2 C_{\text {Dp }}+C_{\text {Dn }}+C_{L} \\
& -C p=C_{D p}+C_{S p}
\end{aligned}
$$


size vs. tx speed considerations $\Uparrow W n x \Rightarrow \Downarrow$ Rn but $\Uparrow$ Cout and $\Uparrow$ Cn $\downarrow$ Wny $\Rightarrow \downarrow$ Cn but $\Uparrow$ Rn
$\Uparrow W p z \Rightarrow \|_{R p}$ but $\Uparrow$ Cout and $\Uparrow \subset p$
$\Downarrow \mathrm{Wpx} \Rightarrow$ no effect on critical path!

## Sizing in Complex Logic Gates

- Improving speed within a single logic gate
- An Example: $f=\overline{(a b+c d) x}$
- nMOS
- discharge through 3 series nMOS
- set $\beta_{N}=3 \beta n$
- pMOS
- charge through 2 series pMOS
- set $\beta_{p}=2 \beta p$
- but, $M \times p$ is alone so $\beta_{P 1}=\beta p$
- but setting $\beta_{p 1}=2 \beta$ p might make layout easier

- These large transistors will increase capacitance and layout area and may only give a small increase in speed
Advanced logic structures are best way to improve speed


## Timing in Multi-Gate Circuits

- What is the worst-case delay in multi-gate circuits?

- too many transitions to test manually
- Critical Path
- longest delay through a circuit block

- largest sum of delays, from input to output
- intuitive analysis: signal that passes through most gates
- not always true. can be slower path through fewer gates

path through most gates
critical path if delay at
D input is very slow


## Power in Multi-Input Logic Gates

- Inverter Power Consumption
$-P=P_{D C}+P_{\text {dyn }}=V_{D D} I_{D D Q}+C_{\text {out }} V^{2}{ }_{D D} f$
- assumes gates switch output state once per clock cycle, $f$
- Multi-Input Gates
- same DC component as inverter, $P_{D C}=V_{D D} I_{D D Q}$
- for dynamic power, need to estimate "activity" of the gate, how often will the output be switching
- $P_{\text {dyn }}=a C_{\text {out }} V^{2}{ }_{\text {DD }} f, a=$ activity coefficient NOR NAND
- estimate activity from truth table

$$
\text { - } \begin{aligned}
a & =p_{0} p_{1} \\
& -p_{0}=\text { prob. output is at } 0 \\
& -p_{1}=\text { prob. of transition to } 1
\end{aligned}
$$

| $\boldsymbol{A}$ | $\boldsymbol{B}$ | $\overline{A+B}$ | $\overline{\boldsymbol{A} \cdot \boldsymbol{B}}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ |  |  |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |  |  |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ |  |  |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ |  |  |
| $\mathrm{p} 0=0.75$ |  |  |  |  | $\mathrm{p} 0=0.25$ |
|  | $\mathrm{p} 1=0.25$ | $\mathrm{p} 1=0.75$ |  |  |  |
| $\mathrm{a}=3 / 16$ | $\mathrm{a}=3 / 16$ |  |  |  |  |

## Timing Analysis of Transmission Gates

- TG = parallel nMOS and pMOS
- RC Model
- in general, only one tx active at same time $\frac{\stackrel{\rightharpoonup}{E}}{}$

- nMOS pulls output low
- pMOS pushes output high
$-R_{T G}=\max (R n, R p)$
- Cin $=C_{S n}+C_{D p}$
- if output at higher voltage than input ${ }^{-}$

- larger W will decrease $R$ but increase Cin
- Note: no connections to VDD-Ground. Input signal, Vin, must drive TG output; TG just adds extra delay


## Pass Transistor

- Single nMOS or pMOS tx
- Often used in place of TGs
- less area and wiring
- can't pull to both VDD and Ground

- typically use nMOS for better speed
- Rise and Fall Times
- $\tau_{n}=\operatorname{Rn} C_{\text {out }}$

- $\dagger_{f}=2.94 \tau_{n}$
- $t_{r}=18 \tau_{n}$

- much slower than fall time $\quad x=1 \quad y=0 \Rightarrow 1$

- nMOS can't pull output to VDD
- rise time suffers from threshold loss in nMOS

