4-1 Congruent Figures

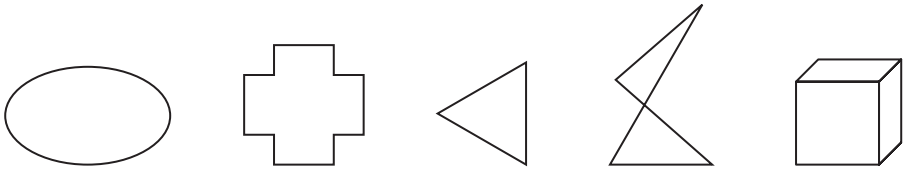
Vocabulary

Review

1. Underline the correct word to complete the sentence.

   A *polygon* is a two-dimensional figure with two / three or more segments that meet exactly at their endpoints.

2. Cross out the figure(s) that are NOT *polygons*.



Vocabulary Builder

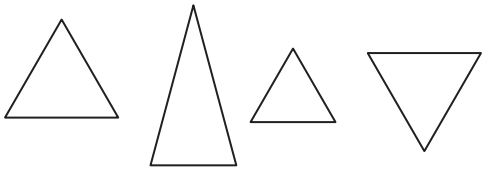
congruent (adjective) **kahng GROO unt**

Main Idea: **Congruent** figures have the same size and shape.

Related Word: congruence (noun)

Use Your Vocabulary

3. Circle the triangles that appear to be *congruent*.



Write T for *true* or F for *false*.

_____ 4. *Congruent* angles have different measures.

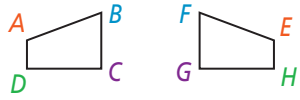
_____ 5. A prism and its net are *congruent* figures.

_____ 6. The corresponding sides of *congruent* figures have the same measure.

take note

Key Concept Congruent Figures

Congruent polygons have congruent corresponding parts—their matching sides and angles. When you name congruent polygons, you must list corresponding vertices in the same order.



$ABCD \cong EFGH$

7. Use the figures at the right to complete each congruence statement.

$\overline{AB} \cong$ ____ $\overline{BC} \cong$ ____ $\overline{CD} \cong$ ____ $\overline{DA} \cong$ ____

$\angle A \cong$ ____ $\angle B \cong$ ____ $\angle C \cong$ ____ $\angle D \cong$ ____

Problem 1 Using Congruent Parts

Got It? **If $\triangle WYS \cong \triangle MKV$, what are the congruent corresponding parts?**

8. Use the diagram at the right. Draw an arrow from each vertex of the first triangle to the corresponding vertex of the second triangle.

$\triangle W Y S \cong \triangle M K V$

9. Use the diagram from Exercise 8 to complete each congruence statement.

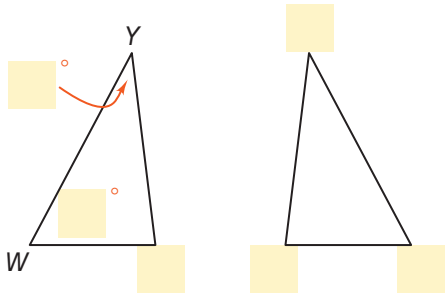
Sides $\overline{WY} \cong$ ____ $\overline{YS} \cong$ ____ $\overline{WS} \cong$ ____

Angles $\angle W \cong$ ____ $\angle Y \cong$ ____ $\angle S \cong$ ____

Problem 2 Finding Congruent Parts

Got It? **Suppose that $\triangle WYS \cong \triangle MKV$. If $m\angle W = 62$ and $m\angle Y = 35$, what is $m\angle V$? Explain.**

Use the congruent triangles at the right.



10. Use the given information to label the triangles. Remember to write corresponding vertices in order.

11. Complete each congruence statement.

$\angle W \cong$ ____

$\angle Y \cong$ ____

$\angle S \cong$ ____

12. Use the Triangle Angle-Sum theorem.

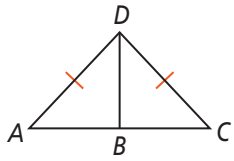
$m\angle S + m$ ____ $+ m$ ____ $= 180$, so $m\angle S = 180 - ($ ____ $+$ ____ $)$, or ____.

13. Complete.

Since $\angle S \cong$ ____ and $m\angle S =$ ____, $m\angle V =$ ____.

Problem 3 Finding Congruent Triangles

Got It? Is $\triangle ABD \cong \triangle CBD$? Justify your answer.



14. Underline the correct word to complete the sentence.

To prove two triangles congruent, show that all adjacent / corresponding parts are congruent.

15. Circle the name(s) for $\triangle ACD$.

acute | isosceles | right | scalene

16. Cross out the congruence statements that are NOT supported by the information in the figure.

$\overline{AD} \cong \overline{CD}$	$\overline{BD} \cong \overline{BD}$	$\overline{AB} \cong \overline{CB}$
$\angle A \cong \angle C$	$\angle ABD \cong \angle CBD$	$\angle ADB \cong \angle CDB$

17. You need _____ congruence statements to prove two triangles congruent, so you can / cannot prove that $\triangle ABD \cong \triangle CBD$.

take note

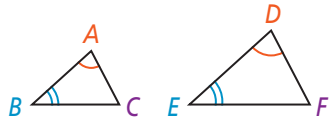
Theorem 4–1 Third Angles Theorem

Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.

If . . .

$\angle A \cong \angle D$ and $\angle B \cong \angle E$



Then . . .

$\angle C \cong \angle F$

Use $\triangle ABC$ and $\triangle DEF$ above.

18. If $m\angle A = 74$, then $m\angle D =$ _____.

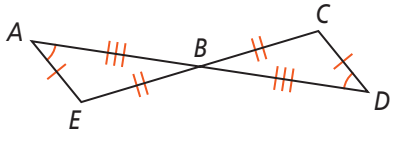
19. If $m\angle B = 44$, then $m\angle E =$ _____.

20. If $m\angle C = 62$, then $m\angle F =$ _____.

Problem 4 Proving Triangles Congruent

Got It? Given: $\angle A \cong \angle D$, $\overline{AE} \cong \overline{DC}$, $\overline{EB} \cong \overline{CB}$, $\overline{BA} \cong \overline{BD}$

Prove: $\triangle AEB \cong \triangle DCB$



21. You are given four pairs of congruent parts. Circle the additional information you need to prove the triangles congruent.

A third pair of congruent sides	A second pair of congruent angles	A third pair of congruent angles

22. Complete the steps of the proof.

1) $\overline{AE} \cong$ ____, $\overline{EB} \cong$ ____, $\overline{BA} \cong$ ____	1) Given
2) $\angle A \cong$ ____	2) Given
3) $\angle ABE \cong$ ____	3) Vertical angles are congruent.
4) $\angle E \cong$ ____	4) Third Angles Theorem
5) $\triangle AEB \cong$ ____	5) Definition of $\cong$ triangles

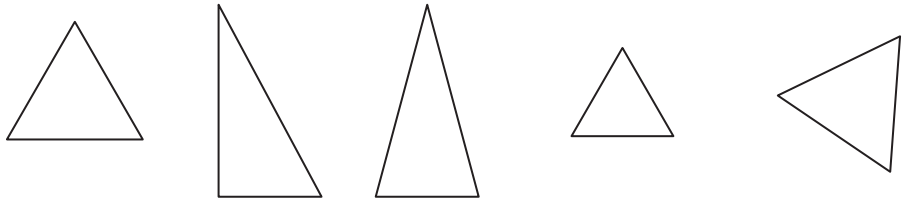
Lesson Check • Do you UNDERSTAND?

If each angle in one triangle is congruent to its corresponding angle in another triangle, are the two triangles congruent? Explain.

23. Underline the correct word to complete the sentence.

To disprove a conjecture, you need one / two / many counterexample(s).

24. An equilateral triangle has three congruent sides and three 60° angles. Circle the equilateral triangles below.



25. Use your answers to Exercise 24 to answer the question.

_____

_____

_____

Math Success

Check off the vocabulary words that you understand.

☐ congruent ☐ polygons

Rate how well you can ***identify congruent polygons.***

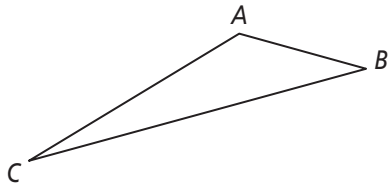
Need to review 0 2 4 6 8 10 Now I get it!

4-2 Triangle Congruence by SSS and SAS

Vocabulary

Review

1. Use the diagram at the right. Find each.



included angle between $\overline{AB}$ and $\overline{CA}$

included side between $\angle A$ and $\angle C$

included angle between $\overline{BC}$ and $\overline{CA}$

included side between $\angle B$ and $\angle C$

included angle between $\overline{BC}$ and $\overline{AB}$

included side between $\angle B$ and $\angle A$

Vocabulary Builder

postulate (noun) PAHS chuh lit

Definition: A **postulate** is a statement that is accepted as true without being proven true.

Main Idea: In geometry, you use what you know to be true to prove new things true. The statements that you accept as true without proof are called **postulates** or axioms.

Use Your Vocabulary

2. Underline the correct word to complete the sentence.

You can use properties, *postulates*, and previously proven theorems as reasons / statements in a proof.

3. **Multiple Choice** What is a *postulate*?

(A) a convincing argument using deductive reasoning

(B) a conjecture or statement that you can prove true

(C) a statement accepted as true without being proven true

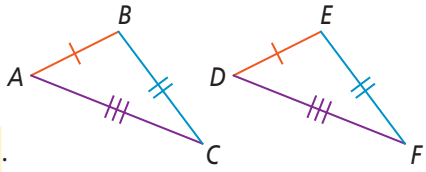
(D) a conclusion reached by using inductive reasoning

take note

Postulate 4–1 Side-Side-Side (SSS) Postulate

Postulate 4-1 Side-Side-Side (SSS) Postulate

If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.

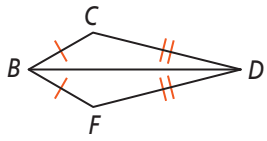


4. Use the figures at the right to complete the sentence.

If $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong$ ____, then $\triangle ABC \cong \triangle$ ____.

Problem 1 Using SSS

Got It? Given: $\overline{BC} \cong \overline{BF}$, $\overline{CD} \cong \overline{FD}$
Prove: $\triangle BCD \cong \triangle BFD$



5. You know two pairs of sides that are congruent. What else do you need to prove the triangles congruent by SSS?

_____

6. The triangles share side ____.

7. Complete the steps of the proof.

Statement	Reason
1) $\overline{BC} \cong$ ____	1) Given
2) $\overline{CD} \cong$ ____	2) Given
3) $\overline{BD} \cong$ ____	3) Reflexive Property of $\cong$
4) $\triangle BCD \cong$ ____	4) SSS

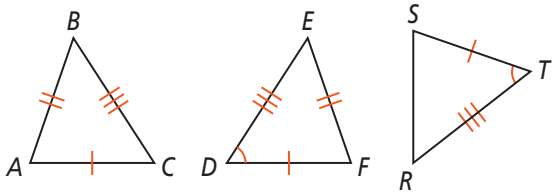
take note

Postulate 4–2 Side-Angle-Side (SAS) Postulate

Postulate 4–2 Side-Angle-Side (SAS) Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

Use the figures below to complete each statement.

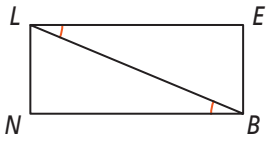


8. $\triangle DEF \cong$ ____ by SAS.

9. $\triangle ABC \cong$ ____ by SSS.

Problem 2 Using SAS

Got It? **What other information do you need to prove $\triangle LEB \cong \triangle BNL$ by SAS?**



10. Circle the angles that are marked congruent in the diagram.

$\angle EBL$ $\angle ELB$ $\angle NBL$ $\angle NLB$

11. Circle the sides that form the angles that are marked congruent in the diagram.

$\overline{BE}$ $\overline{BL}$ $\overline{BN}$ $\overline{LB}$ $\overline{LE}$ $\overline{LN}$

12. Complete each congruence statements.

$\overline{LB} \cong$ ____ $\angle BLE \cong$ ____

Underline the correct word(s) to complete each sentence.

13. Proving $\triangle LEB \cong \triangle BNL$ by SAS requires one / two pair(s) of congruent sides and one / two pair(s) of congruent angles.

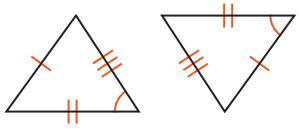
14. The diagram shows congruency of zero / one / two pair(s) of congruent sides and zero / one / two pair(s) of congruent angles.

15. To prove the triangles congruent by SAS, you still need zero / one / two pair(s) of congruent sides and zero / one / two pair(s) of congruent angles .

16. To prove the triangles congruent, you need to prove ____ and ____ congruent.

Problem 3 Identifying Congruent Triangles

Got It? **Would you use SSS or SAS to prove the triangles below congruent? Explain.**



Complete each statement with *SSS* or *SAS*.

17. Use ___?___ if you have three pairs of sides congruent. ____

18. Use ___?___ if you have two pairs of sides and the included angle congruent. ____

Write T for *true* or F for *false*.

____ **19.** The diagram shows congruence of three sides.

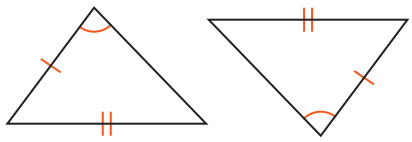
____ **20.** In the triangle on the left, the marked angle is the included angle of the side with two marks and the side with three marks.

____ **21.** In the triangle on the right, the marked angle is the included angle of the side with two marks and the side with three marks.

22. Would you use SSS or SAS to prove the triangles congruent? Explain.

Lesson Check • Do you UNDERSTAND?

Error Analysis **Your friend thinks that the triangles below are congruent by SAS. Is your friend correct? Explain.**



23. Are two pairs of corresponding sides congruent? Yes / No

24. Is there a pair of congruent angles? Yes / No

25. Are the congruent angles the included angles between the corresponding congruent sides? Yes / No

26. Are the triangles congruent by SAS? Explain.

Math Success

Check off the vocabulary words that you understand.

☐ congruent ☐ corresponding

Rate how well you can *use SSS and SAS to prove triangles congruent*.

Need to review 0 2 4 6 8 10 Now I get it!

4-3 Triangle Congruence by ASA and AAS

Vocabulary

Review

1. Cross out the figure(s) that are NOT *triangle*(s).

2. A *triangle* is a polygon with _____ sides.

3. A *triangle* with a right angle is called a(n) obtuse / right / scalene *triangle*.

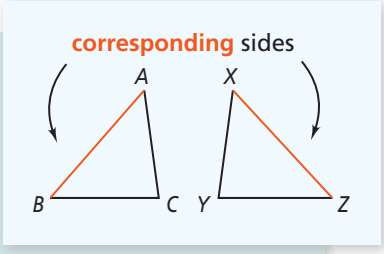
Vocabulary Builder

corresponding (adjective) **kawr uh SPAHN ding**

Other Word Forms: correspond (verb); correspondence (noun)

Definition: **Corresponding** means similar in position, purpose, or form.

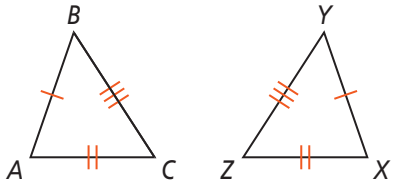
Math Usage: Congruent figures have congruent **corresponding** parts.



Use Your Vocabulary

Draw a line from each part of $\triangle ABC$ in Column A to the *corresponding* part of $\triangle XYZ$ in Column B.

Column A	Column B
4. $\overline{BC}$	$\angle Z$
5. $\angle A$	$\angle Y$
6. $\overline{AB}$	$\overline{YZ}$
7. $\angle C$	$\angle X$
8. $\overline{AC}$	$\overline{XY}$
9. $\angle B$	$\overline{XZ}$



take note

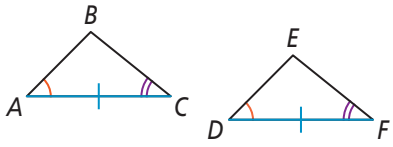
Postulate 4–3 Angle-Side-Angle (ASA) Postulate

Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

If . . .

$\angle A \cong \angle D, \overline{AC} \cong \overline{DF}, \angle C \cong \angle F$



Then . . .

$\triangle ABC \cong \triangle DEF$

10. Explain how the ASA Postulate is different from the SAS Postulate.

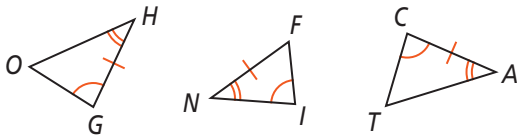
_____

_____

_____

Problem 1 Using ASA

Got It? **Which two triangles are congruent by ASA? Explain.**



11. Name the triangles. List the vertices in corresponding order: list the vertex with the one arc first, the vertex with the two arcs second, and the third vertex last.

12. $\angle G \cong \angle$ ____ $\cong \angle$ ____

13. $\angle H \cong \angle$ ____ $\cong \angle$ ____

14. $\overline{HG} \cong$ ____ $\cong$ ____

15. The congruent sides that are included between congruent angles are ____ and ____.

16. Write a congruence statement. Justify your reasoning.

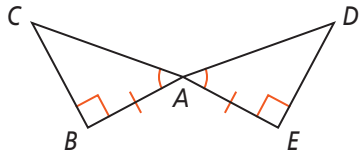
$\triangle$ ____ $\cong \triangle$ ____

_____

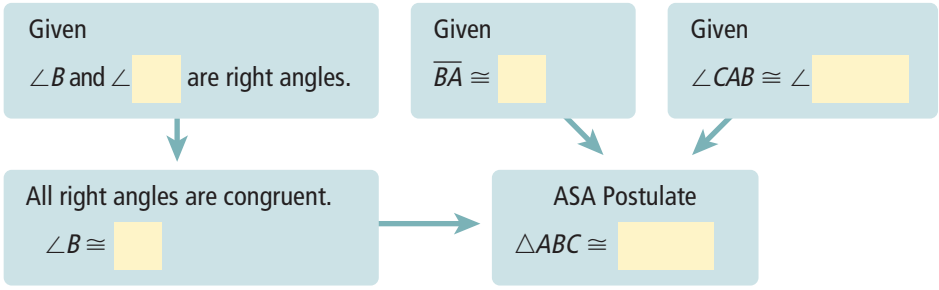
_____

Problem 2 Writing a Proof Using ASA

Got It? Given: $\angle CAB \cong \angle DAE$, $\overline{BA} \cong \overline{EA}$, $\angle B$ and $\angle E$ are right angles
Prove: $\triangle ABC \cong \triangle AED$



17. Complete the flow chart to prove $\triangle ABC \cong \triangle AED$.



take note

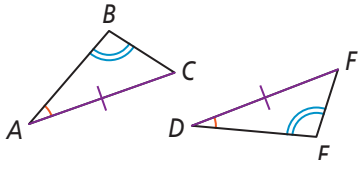
Theorem 4–2 Angle-Angle-Side (AAS) Theorem

Theorem

If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the two triangles are congruent.

If . . .

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\overline{AC} \cong \overline{DF}$



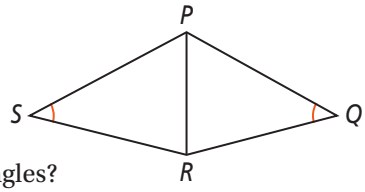
Then . . .

$\triangle ABC \cong \triangle DEF$

18. The nonincluded congruent sides of $\triangle ABC$ and $\triangle DEF$ are ____ and ____.

Problem 3 Writing a Proof Using AAS

Got It? Given: $\angle S \cong \angle Q$, $\overline{RP}$ bisects $\angle SRQ$
Prove: $\triangle SRP \cong \triangle QRP$



19. How do you know which angles in the diagram are corresponding angles?

_____

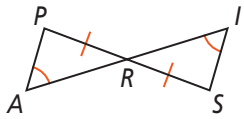
_____

20. Complete the statements to prove $\triangle SRP \cong \triangle QRP$.

Statements	Reasons
1) $\angle S \cong$ ____	1) Given
2) $\overline{RP}$ bisects ____	2) Given
3) $\angle SRP \cong$ ____	3) Definition of an angle bisector
4) $\overline{RP} \cong$ ____	4) Reflexive Property of Congruence
5) $\triangle SRP \cong$ ____	5) AAS

Problem 4 Determining Whether Triangles Are Congruent

Got It? Are $\triangle PAR$ and $\triangle SIR$ congruent? Explain.



21. The congruence marks show that $\angle A \cong$ ____ and $\overline{PR} \cong$ ____.

22. What other corresponding congruent parts exist? Explain.

_____

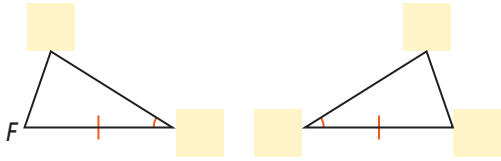
23. Are $\triangle PAR$ and $\triangle SIR$ congruent? If so, what theorem proves them congruent?

_____

Lesson Check • Do you UNDERSTAND?

Reasoning **Suppose $\angle E \cong \angle I$ and $\overline{FE} \cong \overline{GI}$. What else must you know in order to prove $\triangle FDE$ and $\triangle GHI$ are congruent by ASA? By AAS?**

24. Label the diagram at the right.



25. To prove the triangles congruent by ASA, what do you need?

_____

_____

26. To prove the triangles congruent by AAS, what do you need?

_____

_____

27. If you want to use ASA, $\angle$ ____ and $\angle$ ____ must also be congruent.

28. If you want to use AAS, $\angle$ ____ and $\angle$ ____ must also be congruent.

Math Success

Check off the vocabulary words that you understand.

☐ included ☐ nonincluded ☐ corresponding

Rate how well you can *use ASA and AAS*.

Need to review 0 2 4 6 8 10 Now I get it!

4-4 Using Corresponding Parts of Congruent Triangles

Vocabulary

Review

Underline the correct word(s) to complete each sentence.

1. The *Reflexive* Property of Congruence states that any geometric figure is congruent / similar to itself.
2. The *Reflexive* Property of Equality states that any quantity is equal to / greater than / less than itself.
3. Circle the expressions that illustrate the *Reflexive* Property of Equality.

$a = a$ | If $AB = 2$, then $2 = AB$.

$3(x + y) = 3x + 3y$ | $5 + c = 5 + c$

4. Circle the expressions that illustrate the *Reflexive* Property of Congruence.

If $\angle A \cong \angle B$, then $\angle B \cong \angle A$. | If $\overline{CD} \cong \overline{LM}$ and $\overline{LM} \cong \overline{XY}$, then $\overline{CD} \cong \overline{XY}$.

$\angle ABC \cong \angle ABC$ | $\overline{CD} \cong \overline{CD}$

Vocabulary Builder

proof (noun) **proof**

Related Word: prove (verb)

Definition: A **proof** is convincing evidence that a statement or theory is true.

Math Usage: A **proof** is a convincing argument that uses deductive reasoning.

Use Your Vocabulary

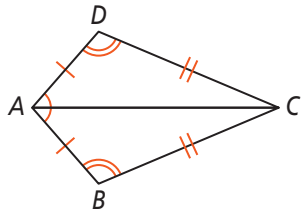
Complete each statement with *proof* or *prove*.

5. In geometry, a ? uses definitions, postulates, and theorems to prove theorems. _____
6. No one can ? how our universe started. _____
7. He can ? when he bought the computer because he has a receipt. _____

8. Complete the steps in the *proof*.

Given: $\overline{AB} \cong \overline{AD}$, $\overline{BC} \cong \overline{DC}$, $\angle D \cong \angle B$, $\angle DAC \cong \angle BAC$

Prove: $\triangle ABC \cong \triangle ADC$

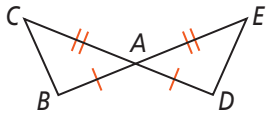


Statements	Reasons
1) $\overline{AB} \cong$ ____ $\overline{BC} \cong$ ____	1) Given
2) $\overline{AC} \cong$ ____	2) Reflexive Property of $\cong$
3) $\angle D \cong$ ____ $\angle DAC \cong$ ____	3) Given
4) $\angle DCA \cong$ ____	4) Third Angles Theorem
5) $\triangle ABC \cong$ ____	5) Definition of $\cong$ triangles

Problem 1 Proving Parts of Triangles Congruent

Got It? **Given:** $\overline{BA} \cong \overline{DA}$, $\overline{CA} \cong \overline{EA}$

Prove: $\angle C \cong \angle E$



9. Name four ways you can use congruent parts of two triangles to prove that the triangles are congruent.

____ ____

____ ____

10. To prove triangles are congruent when you know two pairs of congruent corresponding sides, you can use ____ or ____.

Underline the correct word to complete the sentence.

11. The *Given* states and the diagram shows that there are one / two / three pairs of congruent sides.

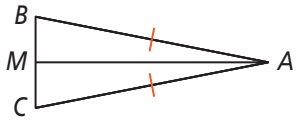
12. Give a reason for each statement of the proof.

Statements	Reasons
1) $\overline{BA} \cong \overline{DA}$	1) _____
2) $\overline{CA} \cong \overline{EA}$	2) _____
3) $\angle CAB \cong \angle EAD$	3) _____
4) $\triangle CAB \cong \triangle EAD$	4) _____
5) $\angle C \cong \angle E$	5) _____

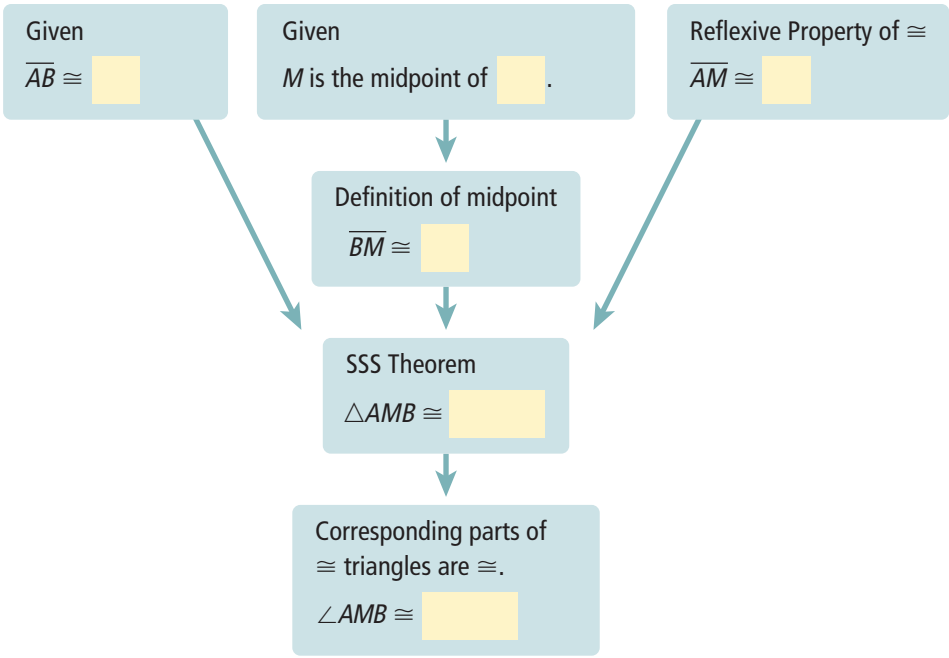
Problem 2 Proving Triangle Parts Congruent to Measure Distance

Got It? Given: $\overline{AB} \cong \overline{AC}$, M is the midpoint of $\overline{BC}$

Prove: $\angle AMB \cong \angle AMC$

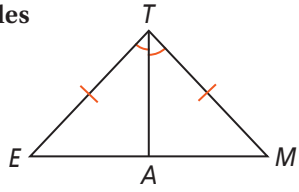


13. Use the flow chart to complete the proof.



Lesson Check • Do you know HOW?

Name the postulate or theorem that you can use to show the triangles are congruent. Then explain why $\overline{EA} \cong \overline{MA}$.



14. Circle the angles that are marked congruent.

$\angle E$ $\angle ETA$ $\angle M$ $\angle EAT$ $\angle MTA$

15. Circle the sides that are marked congruent.

$\overline{ET}$ $\overline{MT}$ $\overline{EA}$ $\overline{MA}$ $\overline{AT}$

16. Circle the sides that are congruent by the Reflexive Property of Congruence.

$\overline{ET}$ and $\overline{MT}$ $\overline{EA}$ and $\overline{MA}$ $\overline{AT}$ and $\overline{AT}$

17. Underline the correct postulate or theorem to complete the sentence.

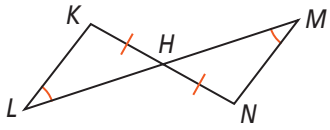
$\triangle EAT \cong \triangle MAT$ by SAS / AAS / ASA / SSS .

18. Now explain why $\overline{EA} \cong \overline{MA}$.

_____

Lesson Check • Do you UNDERSTAND?

Error Analysis **Find and correct the error(s) in the proof.**



Given: $\overline{KH} \cong \overline{NH}$, $\angle L \cong \angle M$ **Prove:** H is the midpoint of $\overline{LM}$.

Proof: $\overline{KH} \cong \overline{NH}$ because it is given. $\angle L \cong \angle M$ because it is given. $\angle KHL \cong \angle NHM$ because vertical angles are congruent. So, $\triangle KHL \cong \triangle MHN$ by ASA Postulate. Since corresponding parts of congruent triangles are congruent, $\overline{LH} \cong \overline{MH}$. By the definition of midpoint, H is the midpoint of $\overline{LM}$.

Place a ✓ in the box if the statement is correct. Place an ✗ if it is incorrect.

☐ **19.** $\angle KHL \cong \angle NHM$ because vertical angels are congruent.

☐ **20.** $\triangle KHL \cong \triangle MHN$ by ASA Postulate.

Underline the correct word to complete each sentence.

21. When you name congruent triangles, you must name corresponding vertices in a different / the same order.

22. To use the ASA Postulate, you need two pairs of congruent angles and a pair of included / nonincluded congruent sides.

23. To use the AAS Theorem, you need two pairs of congruent angles and a pair of included / nonincluded congruent sides.

24. Identify the error(s) in the proof.

_____

_____

_____

25. Correct the error(s) in the proof.

_____

Math Success

Check off the vocabulary words that you understand.

☐ congruent ☐ corresponding ☐ proof

Rate how well you can *use congruent triangles*.

Need to review 0 2 4 6 8 10 Now I get it!

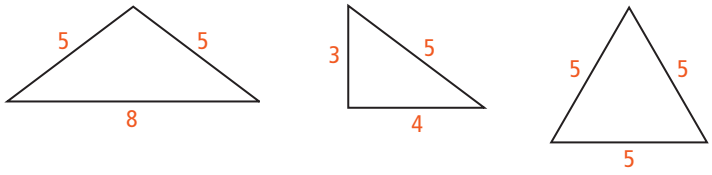
4-5 Isosceles and Equilateral Triangles

Vocabulary

Review

Underline the correct word to complete each sentence.

1. An *equilateral* triangle has two/ three congruent sides.
2. An *equilateral* triangle has acute / obtuse angles.
3. Circle the *equilateral* triangle.



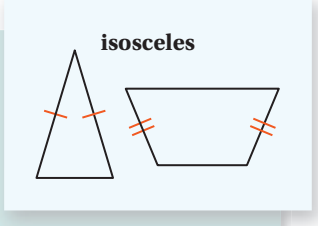
Vocabulary Builder

isosceles (adjective) **eye SAHS uh leez**

Related Words: equilateral, scalene

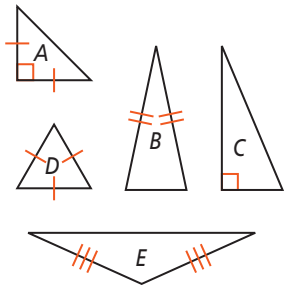
Definition A triangle is **isosceles** if it has two congruent sides.

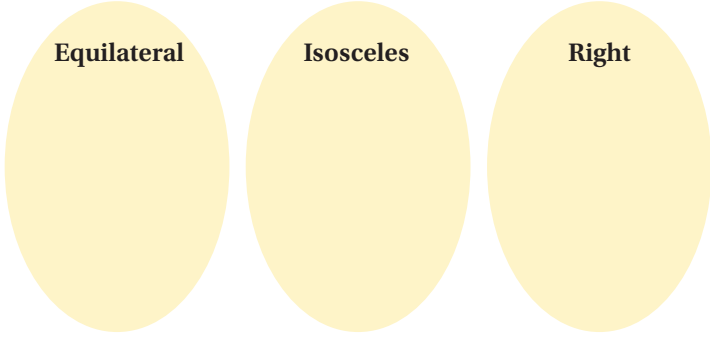
Main Idea: The angles and sides of **isosceles** triangles have special relationships.



Use Your Vocabulary

4. Use the triangles below. Write the letter of each triangle in the correct circle(s) at the right.



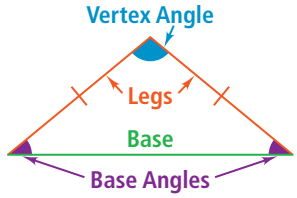


take note

Theorems 4–3, 4–4, 4–5

Theorem 4-3 Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.



Theorem 4-4 Converse of Isosceles Triangle Theorem

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Theorem 4-5

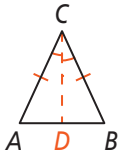
If a line bisects the vertex angle of an isosceles triangle, then the line is also the perpendicular bisector of the base.

5. If $\overline{PQ} \cong \overline{RQ}$ in $\triangle PQR$, then $\angle$ ____ $\cong \angle$ ____.

6. Underline the correct theorem number to complete the sentence.

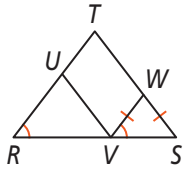
The theorem illustrated below is Theorem 4–3 / 4–4 / 4–5.

If . . .



Then . . .

Problem 1 Using the Isosceles Triangle Theorems



Got It? **Is $\angle WVS$ congruent to $\angle S$? Is $\overline{TR}$ congruent to $\overline{TS}$? Explain.**

7. The markings show that $\overline{WV} \cong$ ____.

8. Is $\angle WVS \cong \angle S$? Explain.

_____

_____

9. Is $\angle R \cong \angle S$? Explain.

_____

_____

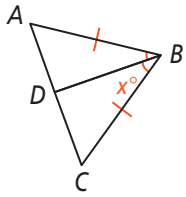
10. Is $\overline{TR} \cong \overline{TS}$? Explain.

_____

_____

Problem 2 Using Algebra

Got It? **Suppose $m\angle A = 27$. What is the value of x?**



11. Since $\overline{CB} \cong$ ____, $\triangle ABC$ is isosceles.

12. Since $\triangle ABC$ is isosceles, $m\angle A = m\angle$ ____ = ____.

13. Since $\overline{BD}$ bisects the vertex of an isosceles triangle, $\overline{BD} \perp$ ____ and $m\angle BDC =$ ____.

14. Use the justifications below to find the value of x.

$m\angle$ ____ $+ m\angle BDC + x = 180$	Triangle Angle-Sum Theorem
____ $+$ ____ $+ x = 180$	Substitute.
____ $+ x = 180$	Simplify.
$x =$ ____	Subtract 117 from each side.

take note

Corollaries to Theorems 4–3 and 4–4

Corollary to Theorem 4-3

If a triangle is equilateral, then the triangle is equiangular.

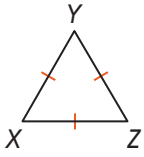
Corollary to Theorem 4-4

If a triangle is equiangular, then the triangle is equilateral.

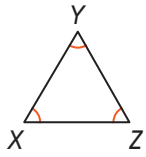
15. Underline the correct number to complete the sentence.

The corollary illustrated below is Corollary to Theorem 4-3 / 4-4.

If . . .

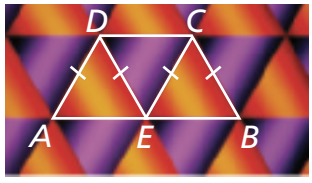


Then . . .



Problem 3 Finding Angle Measures

Got It? **Suppose the triangles at the right are isosceles triangles, where $\angle ADE$, $\angle DEC$, and $\angle ECB$ are vertex angles. If the vertex angles each have a measure of 58, what are $m\angle A$ and $m\angle BCD$?**



16. Which triangles are congruent by the Side-Angle-Side Theorem?

17. Which angles are congruent by the Isosceles Triangle Theorem?

18. By the Triangle Angle-Sum Theorem, $m\angle A + 58 + m\angle DEA =$ ____.

19. Solve for $m\angle A$.

20. Since ____ $\cong \angle ECD$, $m\angle ECD =$ ____.

21. Using the Angle Addition Postulate, $m\angle BCD = 58 + m\angle ECD =$ ____.

Lesson Check • Do you UNDERSTAND?

What is the relationship between sides and angles for each type of triangle?

isosceles **equilateral**

Complete.

22. An isosceles triangle has ____ congruent sides.

23. An equilateral triangle has ____ congruent sides.

Complete each statement with *congruent, isosceles,* or *equilateral*.

24. The Isosceles Triangle Theorem states that the angles opposite the congruent sides are ___?___.

25. Equilateral triangles are also ___?___ triangles.

26. The sides and angles of an ___?___ triangle are ___?___.

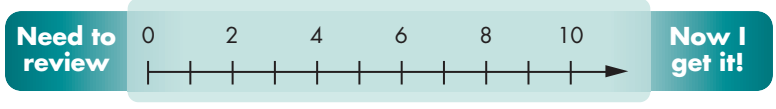
Math Success

Check off the vocabulary words that you understand.

☐ corollary ☐ legs of an isosceles triangle ☐ base of an isosceles triangle

☐ vertex angle of an isosceles triangle ☐ base angles of an isosceles triangle

Rate how well you understand *isosceles and equilateral triangles*.



4-6 Congruence in Right Triangles

Vocabulary

Review

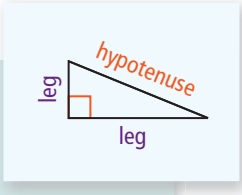
Write T for *true* or F for *false*.

_____ **1.** Segments that are *congruent* have the same length.

_____ **2.** Polygons that are *congruent* have the same shape but are not always the same size.

_____ **3.** In *congruent* figures, corresponding angles have the same measure.

Vocabulary Builder



hypotenuse (noun) **hy PAH tuh noos**

Related Word: leg

Definition: The **hypotenuse** is the side opposite the right angle in a right triangle.

Main Idea: The **hypotenuse** is the longest side in a right triangle.

Use Your Vocabulary

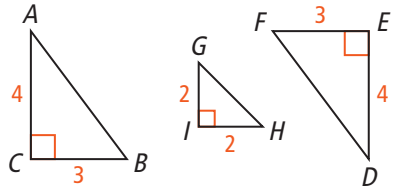
Underline the correct word(s) to complete each sentence.

4. One side of a right triangle is / is not a *hypotenuse*.

5. A right triangle has one / two / three *legs*.

6. The length of the *hypotenuse* is always equal to / greater than / less than the lengths of the *legs*.

Use the triangles at the right for Exercises 7 and 8.



7. Cross out the side that is NOT a *hypotenuse*.

$\overline{BC}$ $\overline{AB}$ $\overline{GH}$ $\overline{FD}$

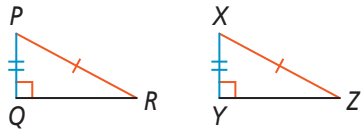
8. Circle the *leg(s)*.

$\overline{AC}$ $\overline{AB}$ $\overline{HI}$ $\overline{ED}$

You can prove that two triangles are congruent without having to show that *all* corresponding parts are congruent. In this lesson, you will prove right triangles congruent by using one pair of right angles, a pair of hypotenuses, and a pair of legs.

take note

Theorem 4-6 Hypotenuse-Leg (HL) Theorem and Conditions

Theorem	If . . .	Then . . .
If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent.	$\triangle PQR$ and $\triangle XYZ$ are right triangles, $\overline{PR} \cong \overline{XZ}$, and $\overline{PQ} \cong \overline{XY}$ 	$\triangle PQR \cong \triangle XYZ$

9. To use the HL Theorem, the triangles must meet three conditions. Complete each sentence with *right* or *congruent*.

There are two _?_ triangles. _____

The triangles have _?_ hypotenuses. _____

There is one pair of _?_ legs. _____

Use the information in the Take Note for Exercises 10–12.

10. How do the triangles in the Take Note meet the first condition in Exercise 9? Explain.

_____

_____

11. How do the triangles in the Take Note meet the second condition in Exercise 9? Explain.

_____

_____

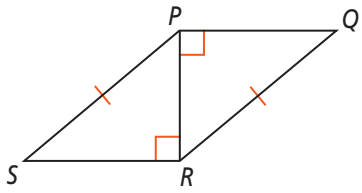
12. How do the triangles in the Take Note meet the third condition in Exercise 9? Explain.

_____

_____

Problem 1 Using the HL Theorem

Got It? Given: $\angle PRS$ and $\angle RPQ$ are right angles, $\overline{SP} \cong \overline{QR}$
Prove: $\triangle PRS \cong \triangle RPQ$



13. Complete each step of the proof.

Given
$\angle PRS$ and $\angle$ _____ are right angles.

Given
$\overline{SP} \cong$ _____

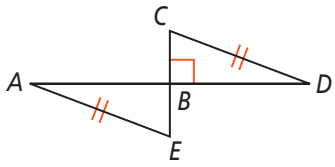
Reflexive Prop. of $\cong$
$\overline{PR} \cong$ _____

Definition of right triangle
$\triangle PRS$ and $\triangle$ _____ are right triangles.

HL Theorem
$\triangle PRS \cong \triangle$ _____

Problem 2 Writing a Proof Using the HL Theorem

Got It? Given: $\overline{CD} \cong \overline{EA}$, $\overline{AD}$ is the perpendicular bisector of $\overline{CE}$
Prove: $\triangle CBD \cong \triangle EBA$



14. Circle what you know because $\overline{AD}$ is the perpendicular bisector of $\overline{CE}$.

$\angle CBD$ and $\angle EBA$ are right angles. $\angle CBD$ and $\angle EBA$ are acute angles.

B is the midpoint of $\overline{AD}$. B is the midpoint of $\overline{CE}$.

15. Circle the congruent legs.

$\overline{AB}$ $\overline{CB}$ $\overline{DB}$ $\overline{EB}$

16. Write the hypotenuse of each triangle.

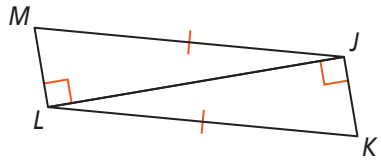
$\triangle CBD$ _____ $\triangle EBA$ _____

17. Complete the proof.

Statements	Reasons
1) $\overline{CD} \cong$ _____	1) Given
2) $\angle CBD$ and $\angle$ _____ are right $\angle$s.	2) Definition of $\perp$ bisector
3) $\triangle CBD$ and $\triangle$ _____ are right $\triangle$s.	3) Definition of right $\triangle$
4) $\overline{CB} \cong$ _____	4) Definition of $\perp$ bisector
5) $\triangle CBD \cong$ _____	5) HL Theorem

Lesson Check • Do you UNDERSTAND?

Error Analysis **Your classmate says that there is not enough information to determine whether the two triangles at the right are congruent. Is your classmate correct? Explain.**



Write T for *true* or F for *false*.

____ **18.** There are three right angles.

____ **19.** There are two right triangles.

____ **20.** There are two congruent hypotenuses.

____ **21.** There are no congruent legs.

____ **22.** You need to use the Reflexive Property of Congruence.

____ **23.** $\overline{LJ} \cong \overline{LJ}$ is given.

24. Do you always need three congruent corresponding parts to prove triangles congruent? Explain.

_____

_____

25. Is your classmate correct? Explain.

_____

_____

_____

_____

Math Success

Check off the vocabulary words that you understand.

☐ hypotenuse ☐ legs of a right triangle

Rate how well you can *use the Hypotenuse-Leg (HL) Theorem.*

Need to review 0 2 4 6 8 10 Now I get it!

4-7 Congruence in Overlapping Triangles

Vocabulary

Review

1. Circle the *common* side of $\triangle ABC$ and $\triangle ADC$.

$\overline{AB}$	$\overline{AC}$	$\overline{AD}$	$\overline{BC}$

2. Circle the *common* side of $\triangle XWZ$ and $\triangle YWZ$.

$\overline{WZ}$	$\overline{WX}$	$\overline{WY}$	$\overline{ZY}$

3. Circle the *common* side of $\triangle RST$ and $\triangle RPT$.

$\overline{RP}$	$\overline{RS}$	$\overline{RT}$	$\overline{ST}$

Vocabulary Builder

overlapping (adjective) **oh vur LAP ing**

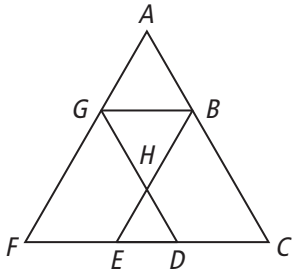
Other Word Form: overlap (noun)

Definition: **Overlapping** events or figures have parts in common.

Math Usage: Two or more figures with common regions are **overlapping** figures.

Use Your Vocabulary

Circle the common regions of the *overlapping* figures in the diagram at the right.



4. $\triangle FGD$ and $\triangle CBE$

$\triangle ABG$	$\triangle ACF$	$\triangle EHD$	$\triangle GHB$

5. $\triangle BEC$ and $\triangle HED$

$\triangle BEC$	$\triangle GBH$	$\triangle GDF$	$\triangle HED$

6. $\triangle ACF$ and $\triangle ABG$

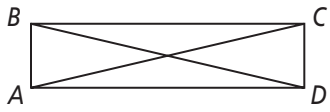
$\triangle ABG$	$\triangle ACF$	$\triangle GBH$	$\triangle EHD$

7. $\triangle ACF$ and $\triangle GBH$

$\triangle ABG$	$\triangle ACF$	$\triangle GBH$	$\triangle HED$

Problem 1 Identifying Common Parts

Got It? What is the common side in $\triangle ABD$ and $\triangle DCA$?



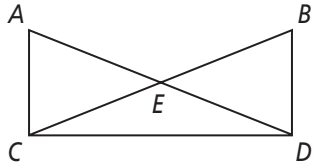
8. Separate and redraw $\triangle ABD$ and $\triangle DCA$.

9. You drew ____ twice, so the common side is ____.

Problem 2 Using Common Parts

Got It? Given: $\triangle ACD \cong \triangle BDC$

Prove: $\overline{CE} \cong \overline{DE}$



10. Use the information in the problem to complete the problem-solving model below.

Know	Need	Plan

11. Use the justifications below to complete each statement.

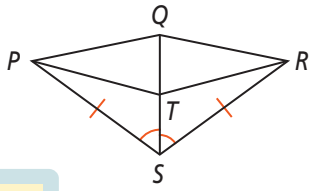
Statements	Reasons
1) $\triangle ACD \cong$ ____	1) Given
2) $\overline{AC} \cong$ ____	2) Corresponding parts of $\cong$ triangles are $\cong$.
3) $\angle A \cong$ ____	3) Corresponding parts of $\cong$ triangles are $\cong$.
4) ____ $\cong \angle BED$	4) Vertical angles are congruent.
5) ____ $\cong \triangle BED$	5) Angle-Angle-Side (AAS) Theorem
6) $\overline{CE} \cong$ ____	6) Corresponding parts of $\cong$ triangles are $\cong$.

12. How could you use the Converse of the Isosceles Triangle Theorem to prove $\overline{CE} \cong \overline{DE}$?

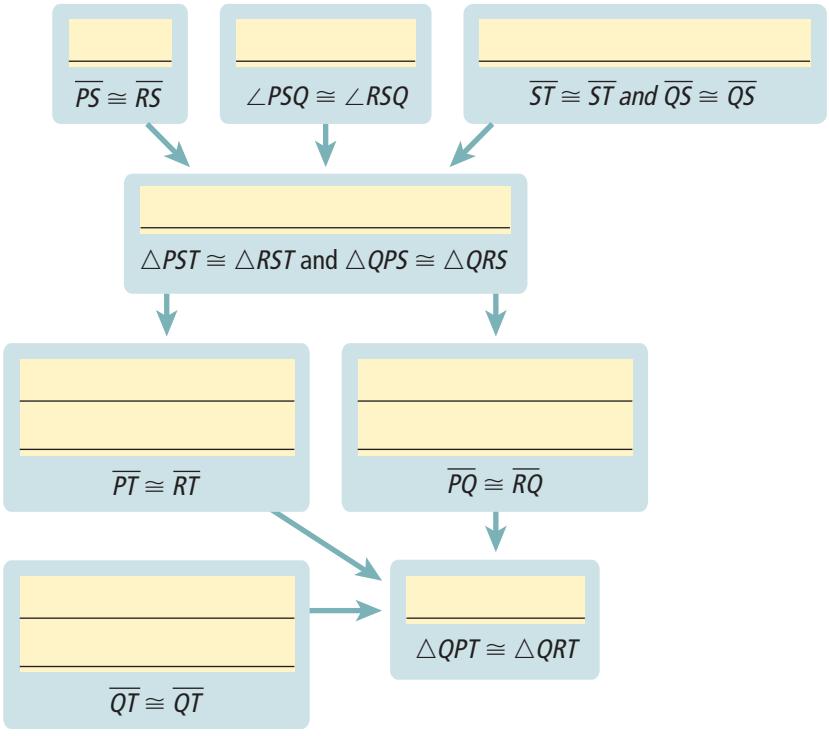
Problem 3 Using Two Pairs of Triangles

Got It? Given: $\overline{PS} \cong \overline{RS}$, $\angle PSQ \cong \angle RSQ$

Prove: $\triangle QPT \cong \triangle QRT$



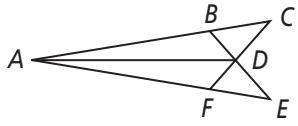
13. Give the reason for each statement in the proof.



Problem 4 Separating Overlapping Triangles

Got It? Given: $\angle CAD \cong \angle EAD$, $\angle C \cong \angle E$

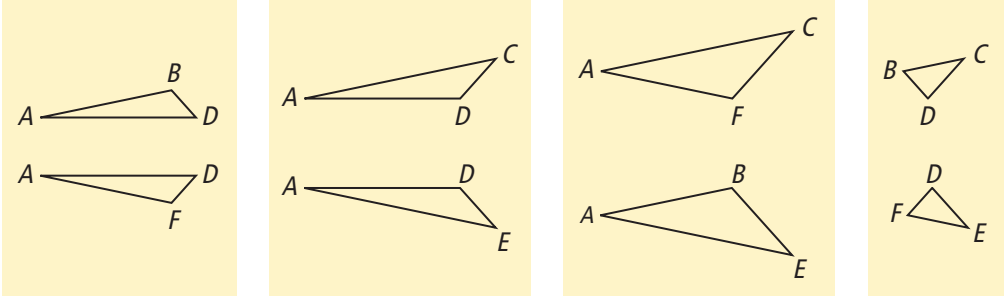
Prove: $\overline{BD} \cong \overline{FD}$



14. Circle the angles that are vertical angles.

$\angle ADB$ $\angle ADC$ $\angle ADE$ $\angle ADF$ $\angle BDC$ $\angle FDE$

15. Mark the angles that you know are congruent in each pair of separated triangles below.



16. Which triangles are congruent by AAS? Explain.

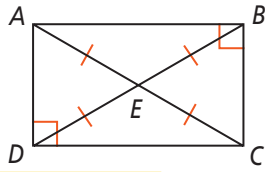
_____

17. Which triangles are congruent by ASA? Explain.

18. How can you prove $\overline{BD} \cong \overline{FD}$?

Lesson Check • Do you UNDERSTAND?

In the figure at the right, which pair of triangles could you prove congruent first in order to prove that $\triangle ACD \cong \triangle CAB$? Explain.



19. Is the hypotenuse of $\triangle ACD$ congruent to the hypotenuse of $\triangle CAB$? Explain.

20. What else do you need to prove right angles congruent using HL?

21. Which triangles can you prove congruent to find this? Explain.

Math Success

Check off the vocabulary words that you understand.

☐ congruent ☐ corresponding ☐ overlapping

Rate how well you can *identify congruent overlapping triangles*.