

7.2

Volume of a Prism

Focus on...

After this lesson, you will be able to...

- use a formula to determine the volume of a right rectangular prism
- use a formula to determine the volume of a right triangular prism

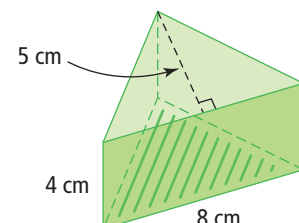
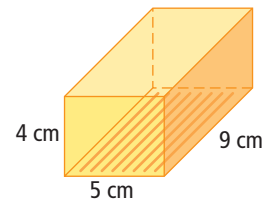


Recycling bins are located in most schools. Students and staff members fill them with paper, aluminum cans, plastic and glass bottles, and other recyclables. Schools probably recycle more paper than anything else. What would you need to know about a bin for recycled paper in order to determine how much paper was recycled?

Explore the Math

How can you use the dimensions to calculate the volumes of right prisms?

1. How is volume related to the area of the base and the height of a prism?
2. Show how you could determine the volume of the right rectangular prism shown?
3. Show how you could determine the volume of the right triangular prism shown?

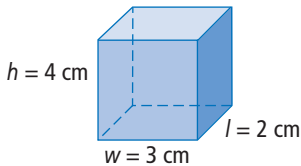


Reflect on Your Findings

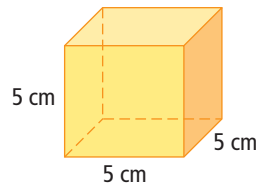
- If you know only the dimensions of a right rectangular prism, how can you determine the volume? Test your method using an example.
- If you know only the dimensions of the triangular base and the height of the prism, how can you determine the volume of a right triangular prism? Test your method using an example.
- How are the formulas for the volume of a right rectangular prism and a right triangular prism different? How are they the same?

Example 1: Use a Formula to Determine the Volume of a Right Rectangular Prism

- a) Determine the volume of the right rectangular prism.



- b) Determine the volume of the cube.



How do you calculate the area of a rectangle?

Solution

- a) Volume of a right rectangular prism = area of rectangular base \times height of prism

Volume of a right rectangular prism = (length \times width) \times height

$$V = l \times w \times h$$

$$V = 2 \times 3 \times 4$$

$$V = 24$$

The volume of the right rectangular prism is 24 cm^3 .

Strategies

Use a Variable

- b) A cube is also a right rectangular prism.

Volume of a cube = area of square base \times height of prism

Volume of a cube = (length \times width) \times height

$$V = s \times s \times s$$

$$V = 5 \times 5 \times 5$$

$$V = 125$$

The volume of the cube is 125 cm^3 .

Since $s \times s \times s = s^3$, you could write the formula as:

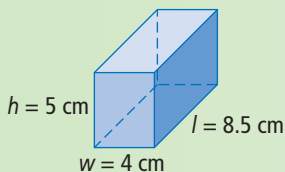
$$V = s^3$$

$$V = 5^3$$

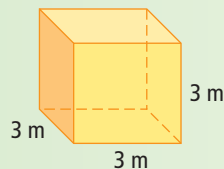
$$V = 125$$

Show You Know

- a) What is the volume of the right rectangular prism?



- b) What is the volume of a cube with edge length 3 m?



Example 2: Use a Formula to Determine the Volume of a Right Triangular Prism

Determine the volume of the right triangular prism.

Solution

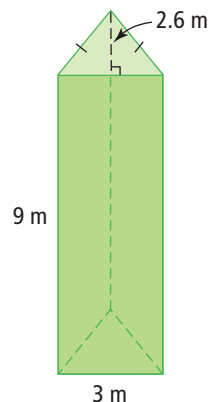
Volume of a triangular prism = area of triangular base
× height of prism

Volume of a triangular prism = $(\text{base} \times \text{height} \div 2)$
× height of prism

$$V = (3 \times 2.6 \div 2) \times 9$$

$$V = 35.1$$

The volume of the right triangular prism is 35.1 m^3 .



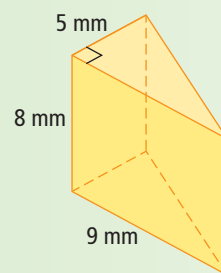
Strategies

Use a Variable

How do you calculate the area of a triangle?

Show You Know

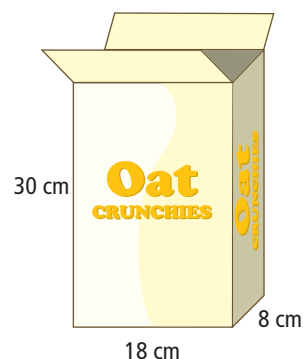
What is the volume of the right triangular prism?



Example 3: Use Volume to Solve a Problem

When Katie opened a new box of Oat Crunchies, she noticed that the box was only $\frac{5}{6}$ full.

- How much space was empty?
- Why do you think packages often seem to have empty space when first opened?



Solution

- a) The cereal box is a right rectangular prism.

$$V = l \times w \times h$$


$$V = 18 \times 8 \times 30$$

$$V = 4320$$

The volume of the cereal box is 4320 cm^3 .

The package is $\frac{5}{6}$ full. Therefore, $\frac{1}{6}$ of the package is empty.

$$\begin{aligned}\text{Amount of empty space} &= \frac{1}{6} \times 4320 \\ &= \frac{4320}{6} \\ &= 720\end{aligned}$$

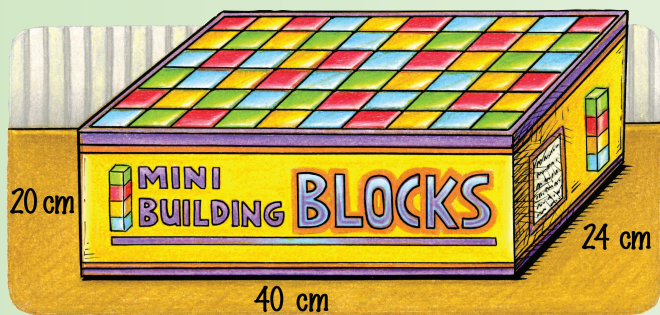

$$\frac{6}{6} - \frac{5}{6} = \frac{1}{6}$$

There was 720 cm^3 of empty space.

- b) Packages often seem to have empty space because the contents settle when being shipped.

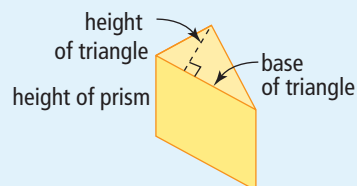
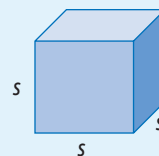
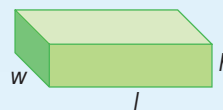
Show You Know

Mr. Chin bought a box of small building blocks for his four children. He will give an equal number to each of them. What volume of blocks will each child get?



Key Ideas

- The volume of a right rectangular prism can be determined using the formula:
 $V = l \times w \times h$
- The volume of a cube can be determined using the formula:
 $V = s \times s \times s$
 $V = s^3$
- The volume of a right triangular prism can be determined using the formula:
 $V = (\text{base of triangle} \times \text{height of triangle} \div 2) \times \text{height of prism}$



Communicate the Ideas

- Grace tells Dakota that the volume of a cube can be found using the formula: $V = l \times w \times h$. Do you agree? Explain.
- Kwan wants to build a concrete ramp to his back door. He wants to determine the volume of concrete needed for the ramp. What measurements does he need to know? Justify your response.
- Jack's family opened a full carton of frozen yogurt for dessert. After they ate, there was $\frac{3}{4}$ left. Jack wants to know what volume of frozen yogurt they ate. He does the following calculation.

Volume of carton:

$$V = 12 \times 9 \times 18$$

$$V = 1944$$

The volume of the carton is 1944 cm^3 .

Volume of frozen yogurt eaten:

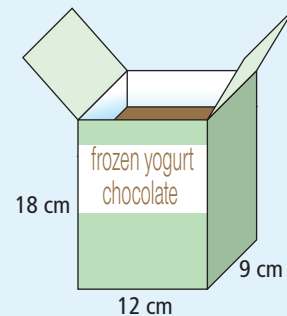
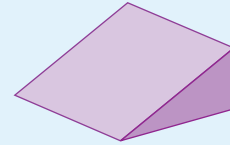
$$V = 1944 \times \frac{3}{4}$$

$$V = \frac{1944 \times 3}{4}$$

$$V = \frac{5832}{4}$$

$$V = 1458$$

They ate 1458 cm^3 of frozen yogurt.



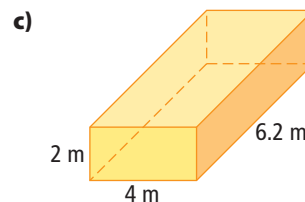
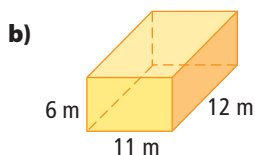
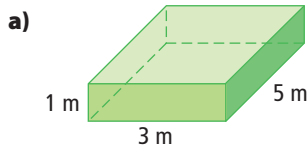
- What mistake did Jack make?
- Show the correct calculation.
- Show an alternative way to calculate the answer.

Check Your Understanding

Practise

For help with #4 to #6, refer to Example 1 on page 255.

- Determine the volume of each right rectangular prism.



- What is the volume of each right rectangular prism?

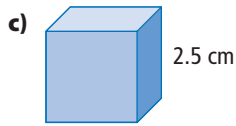
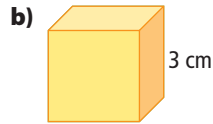
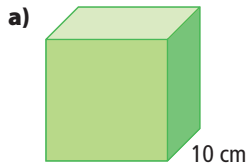
a) $l = 2 \text{ m}$, $w = 2 \text{ m}$, $h = 10 \text{ m}$

b) $l = 8 \text{ cm}$, $w = 7 \text{ cm}$, $h = 9 \text{ cm}$

c) $l = 11.7 \text{ mm}$, $w = 6.3 \text{ mm}$,
 $h = 2.9 \text{ mm}$

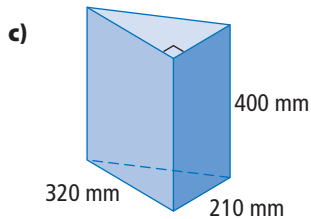
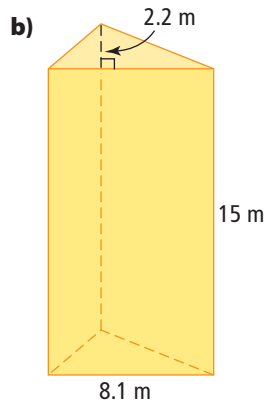
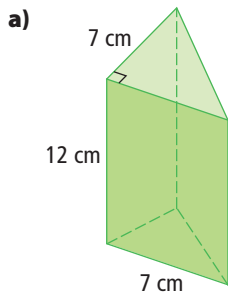
d) $l = 6.2 \text{ cm}$, $w = 6.4 \text{ cm}$, $h = 6.4 \text{ cm}$

6. Determine the volume of each cube.



For help with #7 and #8, refer to Example 2 on page 256.

7. Determine the volume of each right triangular prism.



8. What is the volume of each right triangular prism?

a) base of triangle = 3 m
height of triangle = 7 m
height of prism = 8 m

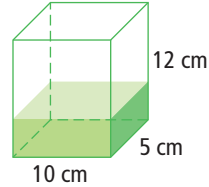
b) base of triangle = 15 cm
height of triangle = 8 cm
height of prism = 20 cm

c) base of triangle = 10 mm
height of triangle = 9.1 mm
height of prism = 11.3 mm

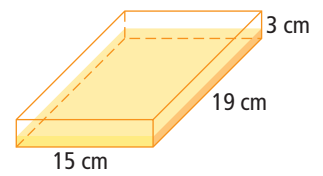
For help with #9 and #10, refer to Example 3 on pages 256–257.

9. Determine the volume of the contents of each right prism.

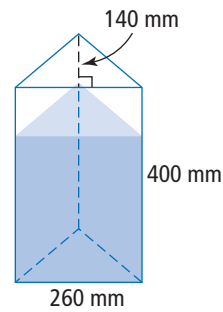
a) $\frac{1}{3}$ full



b) $\frac{3}{8}$ full

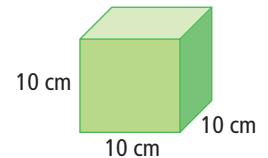


c) $\frac{3}{4}$ full

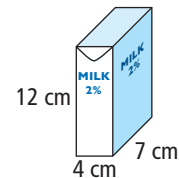


10. Determine the volume of the empty space in each object.

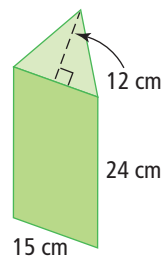
a) $\frac{4}{5}$ full of facial tissues



b) $\frac{3}{4}$ full of milk



c) $\frac{1}{6}$ full of water



Apply

11. Copy and complete the following table.

Right Rectangular Prism

	Length (cm)	Width (cm)	Height (cm)	Volume (cm ³)
a)	7	2		70
b)	12		10	1080
c)		15	5	1200

12. Copy and complete the following table.

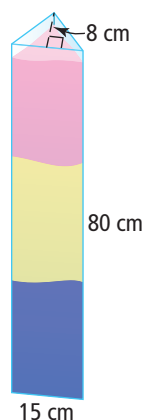
Right Triangular Prism

	Base (cm)	Height of Triangle (cm)	Height of Prism (cm)	Volume (cm ³)
a)	7	2		70
b)	18		10	1080
c)		14	5	700

13. A landscaper has 0.5 m³ of gravel to use as the base of a patio. If the gravel base must be 10 cm deep and the patio is 2.6 m wide and 2.8 m long, does she have enough gravel? How much extra gravel does she have, or how much more will she need?

The word *deep* is sometimes used to describe the height.

14. A glass vase in the shape of a right triangular prism is filled with coloured sand as a decoration in a living room. What is the volume of the vase?



15. Calculate the volume of concrete used to make a sidewalk 1.5 m wide and 120 m long. The concrete is poured 0.1 m thick.
16. Cindy's aquarium stands 75 cm tall and has a base that measures 1.2 m \times 80 cm. At one point during the initial filling, the aquarium has a 12-cm depth of water in it. Cindy needs to fill it to 15 cm from the top before she adds the fish. Draw a diagram and label the dimensions of the aquarium. Determine how much more water Cindy must add before she puts in the fish.



17. A contractor is excavating a rectangular hole 10 m \times 12 m \times 3 m to pour the foundation for a house. A dump truck with a capacity of 9 m³ is used to haul away the excavated soil. How many trips does the truck need to make?

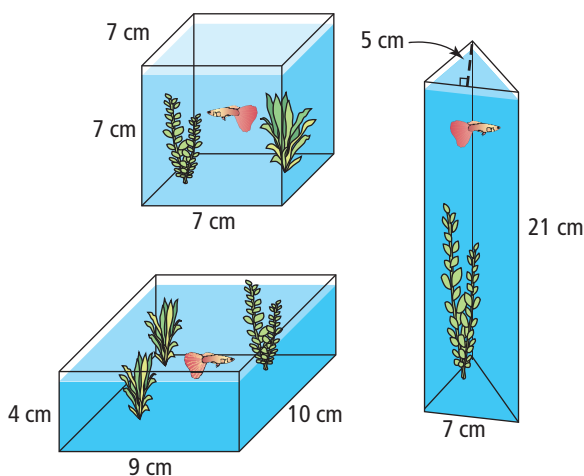
Literacy Link

Capacity refers to the greatest volume that a container such as a tank, a truck, or a measuring cup can contain.

18. Suki has 30 small linking cubes.

- She wants to use 18 of them to make a large cube. Is this possible? Why or why not?
- What number of linking cubes would she use to construct the largest cube she can possibly make?

19. Melissa has three glass vases. She wants to use one as a decorative fish tank for Harvey the guppy. Which will give Harvey the most water to swim in?



20. The ratio of length:width:height of a box is 6:3:2. What is its volume if the height is 5 cm?

21. Sketch and label all possible rectangular prisms with dimensions that are whole numbers of centimetres that have a volume of 120 cm^3 . Do not consider a change of orientation as a new figure; for example, $4 \text{ cm} \times 5 \text{ cm} \times 6 \text{ cm}$ is the same as $6 \text{ cm} \times 4 \text{ cm} \times 5 \text{ cm}$.

22. A tank made of concrete has outside dimensions measuring $6 \text{ m} \times 3 \text{ m} \times 1 \text{ m}$. It has no lid. The concrete is 8 cm thick. What is the maximum volume the concrete tank can hold? What assumptions did you make?

Extend

23. Rectangular Prism A and Rectangular Prism B have the same length. The width of A is half the width of B. The height of A is twice the height of B. What is the difference in volume?

24. A rectangular tank, 40 m long by 30 m wide, is filled with 960 m^3 of water.

- Determine the depth of water.
- If the water drains out at a rate of $60 \text{ m}^3/\text{h}$, how much water is left after 2.5 h? What is the new depth of water?
- Later, the depth of the water is 0.2 m. For how long has the tank been draining?

MATH LINK

The Parks Committee is considering putting 12 of these recycling bins throughout the park. If the bins are filled to the brim and emptied twice weekly, what volume of waste is recycled each week?

