11-5 Volumes of Pyramids and Cones

What You'll Learn

- To find the volume of a pyramid
- To find the volume of a cone

. . . And Why

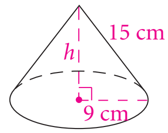
To find the volume of a structure in the shape of a pyramid, as in Example 1

Check Skills You'll Need

GO for Help Lesson 8-1

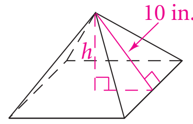
Use the Pythagorean Theorem to find the value of the variable.

1.



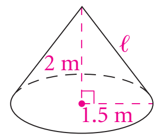
12 cm

2.



8 in.

3.



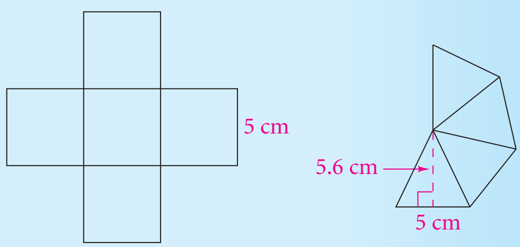
2.5 m

1 Finding Volume of a Pyramid

Hands-On Activity: Finding Volume

You know how to find the volume of a prism. Use the following to explore finding the volume of a pyramid.

- Draw the nets shown at the right on cardboard.
- Cut out the nets and tape them together to make a cube and a regular square pyramid. Each model will have one open face.



1. How do the areas of the bases of the cube and the pyramid compare? The areas are =.
2. How do the heights of the cube and pyramid compare? The heights are about =.
3. Fill the pyramid with rice or other material. Then pour the rice from the pyramid into the cube. How many pyramids full of rice does the cube hold? about 3 pyramids
4. The volume of the pyramid is what fractional part of the volume of the cube? $\frac{1}{3}$

The volume of a pyramid is a particular fraction of the volume of a prism that has the same base and height as the pyramid. The fraction is shown and this fact is stated as Theorem 11-8 at the top of the next page.

11-5

1. Plan

Objectives

1 To find the volume of a pyramid
2 To find the volume of a cone

Examples

1 Real-World Connection
2 Finding Volume of a Pyramid
3 Finding Volume of an Oblique Cone
4 Real-World Connection

Professional Development

Math Background

A hands-on activity will relate the volume of a prism to the volume of three pyramids with the same bases and heights. Students will generalize this relationship and relate the volume of a cylinder to the volume of three cones with the same bases and heights. Euclid included the proof of the latter relationship in Book XII of *The Elements.* Later, Archimedes attributed the proof to Eudoxus and his "method of exhaustion." This method is the forerunner of the modern method of limits.

More Math Background: p. 596D

Lesson Planning and Resources

See p. 596E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need
For intervention, direct students to:

The Pythagorean Theorem
Lesson 8-1: Example 1
Extra Skills, Word Problems, Proof Practice, Ch. 8

Differentiated Instruction Solutions for All Learners

Special Needs L1	**Below Level** L2
Have students repeat "the area of a triangle is $\frac{1}{2}$ the area of a rectangle in two-dimensions, and the volume of a pyramid is $\frac{1}{3}$ the volume of a prism in three-dimensions."	Write Theorems 11-8 and 11-9 on the board. Have students explain how they are alike and how they are different.
learning style: verbal	learning style: visual

2. Teach

Guided Instruction

Hands-on Activity

After students tape their nets, have them estimate the ratio of the pyramid's volume to the cube's volume. They may be surprised to find the ratio is $\frac{1}{3}$.

2 EXAMPLE Teaching Tip

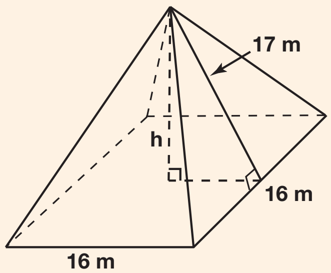
Have students explain why one leg of the right triangle is 20 ft.

PowerPoint

Additional Examples

1 Find the volume of a square pyramid with base edges 15 cm and height 22 cm. **1650 cm^3**

2 Find the volume of the square pyramid with base edges 16 m and slant height 17 m.



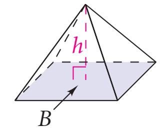
1280 m^3

Key Concepts

Theorem 11-8 Volume of a Pyramid

The volume of a pyramid is one third the product of the area of the base and the height of the pyramid.

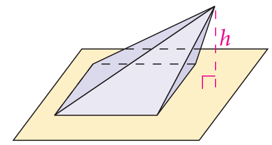
$$V = \frac{1}{3}Bh$$



GO for Help

You can find Cavalieri's Principle on page 625.

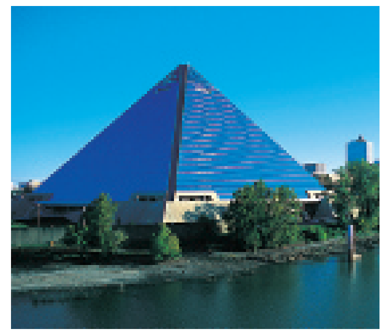
Because of Cavalieri's Principle, the volume formula is true for all pyramids, including oblique pyramids. The height h of an oblique pyramid is the length of the perpendicular segment from the vertex to the plane of the base.



Oblique Pyramid

1 EXAMPLE Real-World Connection

Architecture The Pyramid is an arena in Memphis, Tennessee. The area of the base of The Pyramid is about 300,000 ft^2. Its height is 321 ft. What is the volume of The Pyramid?



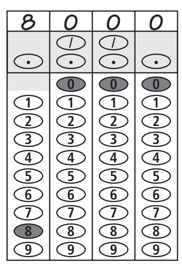
$V = \frac{1}{3}Bh$		**Use the formula for volume of a pyramid.**
	$= \frac{1}{3}(300{,}000)(321)$	**Substitute.**
	$= 32{,}100{,}000$	**Simplify.**

The volume is about 32,100,000 ft^3.

Quick Check 1 Find the volume of a square pyramid with base edges 12 in. and height 8 in. **384 in.^{3}**

To find the volume of a pyramid you may first need to find its height.

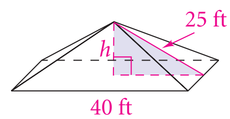
2 EXAMPLE Finding Volume of a Pyramid

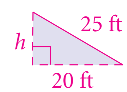


Gridded Response Find the volume in cubic feet of a square pyramid with base edges 40 ft and slant height 25 ft.

Step 1: Find the height of the pyramid.

$25^2 = h^2 + 20^2$	**Use the Pythagorean Theorem.**
$625 = h^2 + 400$	**Simplify.**
$h^2 = 225$	**Solve for h^2.**
$h = 15$	**Take square roots.**





Step 2: Find the volume of the pyramid.

$V = \frac{1}{3}Bh$	**Use the formula for volume of a pyramid.**
$= \frac{1}{3}(40 \cdot 40)15$	**Substitute.**
$= 8000$	**Simplify.**

The volume of the pyramid is 8000 ft^3.

Quick Check 2 Find the volume of a square pyramid with base edges 24 m and slant height 13 m. **960 m^3**

Differentiated Instruction Solutions for All Learners

Advanced Learners L4
Ask students to write a formula for the volume of a square pyramid whose base edge measures s and slant height measures h. $V = \frac{s^2}{6}\sqrt{4h^2 - s^2}$

learning style: verbal

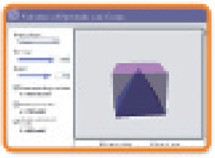
English Language Learners ELL
Watch for students who confuse *slant height* with *height* in the formula for the volume of a pyramid. Reinforce the relationship between the volume of a pyramid and the volume of a prism, which also uses height.

learning style: verbal

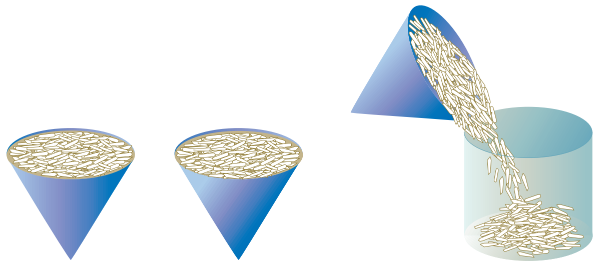
2 Finding Volume of a Cone

You have seen that the volume of a pyramid is one third the volume of a prism with the same base and height. Similarly, the volume of a cone is one third the volume of a cylinder with the same base and height.

Online active math



For: Pyramid, Cone Activity
Use: Interactive Textbook, 11-5



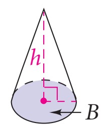
The cones and the cylinder have the same base and height. It takes three cones full of rice to fill the cylinder.

Key Concepts

Theorem 11-9 **Volume of a Cone**

The volume of a cone is one third the product of the area of the base and the height of the cone.

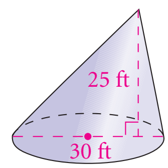
$$V = \frac{1}{3}Bh, \text{ or } V = \frac{1}{3}\pi r^2 h$$



This volume formula applies to all cones, including oblique cones.

3 EXAMPLE Finding Volume of an Oblique Cone

Find the volume of an oblique cone with diameter 30 ft and height 25 ft. Give your answer in terms of π and also rounded to the nearest cubic foot.



$V = \frac{1}{3}\pi r^2 h$ **Use the formula for volume of a cone.**

$= \frac{1}{3}\pi(15)^2 25$ **Substitute 15 for r and 25 for h.**

$= 1875\pi$ **Simplify.**

≈ 5890.4862 **Use a calculator.**

The volume of the cone is 1875π ft^3, or about 5890 ft^3.

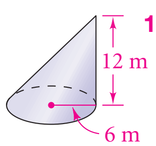
Test-Taking Tip

The formula $V = \frac{1}{3}Bh$ can be used to find the volume of a pyramid or a cone. The only difference is how you calculate B, the area of the base.

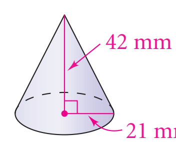
Quick Check

3 Find the volume of each cone in terms of π and also rounded as indicated.

a. to the nearest cubic meter **144π m^3; 452 m^3**



b. to the nearest cubic millimeter **6174π mm^3; 19,396 mm^3**



A cone-shaped structure can be particularly strong, as downward forces at the vertex are distributed to all points in its circular base.

Guided Instruction

3 EXAMPLE

The altitude of a right cone intersects the center of the base. Remind students that although the base of an oblique cone is still circular, the altitude of an oblique cone does not intersect the center of the base, and may not even intersect the base.

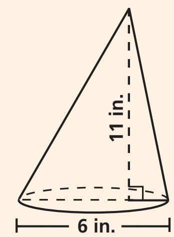
4 EXAMPLE **Diversity**

Point out that not all Native Americans lived in teepees. Encourage students to investigate other dwellings used by different tribes.

PowerPoint

Additional Examples

3 Find the volume of the cone in terms of π.



33π in.^{3}

4 An ice cream cone is 7 cm tall and 4 cm in diameter. About how much ice cream can fit entirely inside the cone? Find the volume to the nearest whole number. **29 cm^3**

Resources

- Daily Notetaking Guide 11-5 L3
- Daily Notetaking Guide 11-5—Adapted Instruction L1

Closure

Explain how to find the volume of a cone if you know its height and the circumference of its base. **Use $C = 2\pi r$ to find r, $B = \pi r^2$ to find B, and $V = \frac{1}{3}Bh$ to find V, or use $C = 2\pi r$ to find r and then $V = \frac{1}{3}\pi r^2 h$ to find V.**

3. Practice

Assignment Guide

1 A B 1-10, 16-18, 20, 21, 24

2 A B 11-15, 19, 22, 23, 25-30

C Challenge 31-36

Test Prep 37-42
Mixed Review 43-48

Homework Quick Check

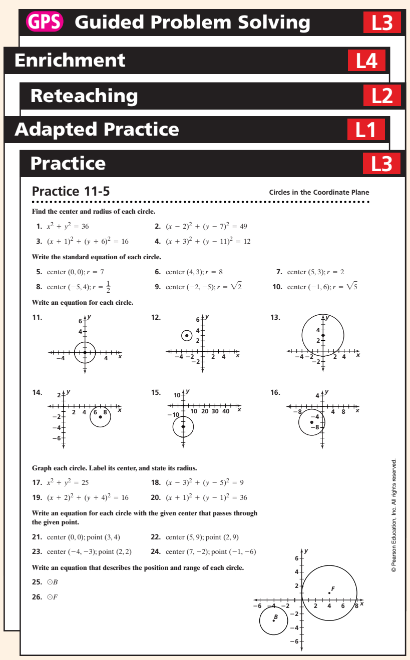
To check students' understanding of key skills and concepts, go over Exercises 6, 14, 19, 20, 24.

Exercise 1 Point out that it is unnecessary to know the shape of the pyramid's base because the area of the base is given.

Error Prevention!

Exercises 8–10 Students may substitute the slant height for h in the volume formula. Remind them to use the Pythagorean Theorem to find h.

Differentiated Instruction Resources



4 EXAMPLE Real-World Connection

Dwelling The covering on a teepee rests on poles that come together like concurrent lines. The resulting structure approximates a cone. If the teepee pictured is 12 ft. high with a base diameter 14 ft, estimate its volume.

$V = \frac{1}{3}\pi r^2 h$ **Use the formula for the volume of a cone.**

$V = \frac{1}{3}\pi(7)^2(12)$ **Substitute; $r = \frac{1}{2}(14) = 7$.**

$= 615.75216$ **Use a calculator.**

The volume of the teepee is approximately 616 ft^3.

Quick Check 4 A small child's teepee is 6 ft tall and 7 ft in diameter. Find the volume of the teepee to the nearest cubic foot. **77 ft^3**

EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 632)

GO for Help

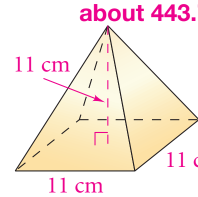
1. The entrance to the Louvre Museum in Paris, France is a pyramid. The height of the pyramid is about 70 ft and the area of its base is about 10,000 ft^2. What is the volume of the pyramid? **about 233,333 ft^3**

Find the volume of a square pyramid with the following dimensions.

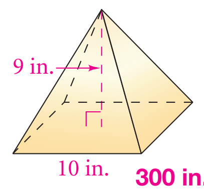
2. base edges 10 cm, height 6 cm **200 cm^3**
3. base edges 18 in., height 12 in. **1296 $in.^3$**
4. base edges 5 m, height 6 m **50 m^3**

Find the volume of each square pyramid.

5. **about 443.7 cm^3**

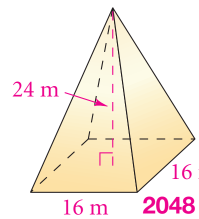


6.



300 $in.^3$

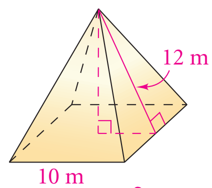
7.



2048 m^3

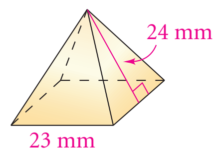
Example 2 (page 632)

8.



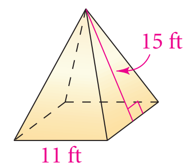
about 363.6 m^3

9.



about 3714.5 mm^3

10.

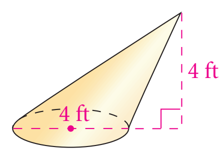


about 562.9 ft^3

Example 3 (page 633)

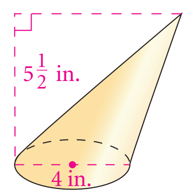
Find the volume of each cone in terms of π and also rounded as indicated.

11. nearest cubic foot



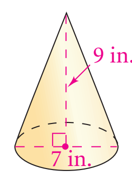
$\frac{16}{3}\pi$ ft^3; 17 ft^3

12. nearest cubic inch



$\frac{22}{3}\pi$ $in.^3$; 23 $in.^3$

13. nearest cubic inch

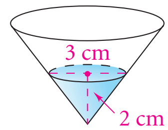


36.75π $in.^3$; 115 $in.^3$

Example 4 (page 634)

14. Chemistry In a chemistry lab you use a filter paper cone to filter a liquid. The diameter of the cone is 6.5 cm and its height is 6 cm. How much liquid will the cone hold when it is full? **about 66.4 cm^3**

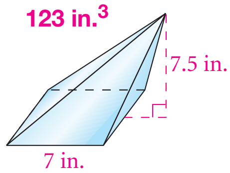
15. Chemistry This funnel has a filter that was being used to remove impurities from a solution but became clogged and stopped draining. The remaining solution is represented by the shaded region. How many cubic centimeters of the solution remain in the funnel? **about 4.7 cm^3**



B Apply Your Skills

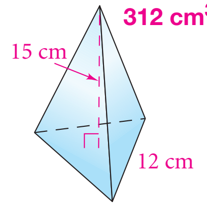
Find the volume to the nearest whole number.

16. **123 in.^{3}**



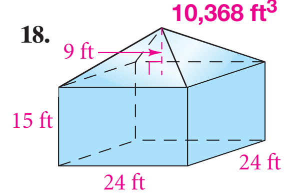
Square base

17. **312 cm^3**



Equilateral base

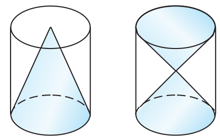
18. **10,368 ft^3**



Square base

19. They are equal; both volumes are $\frac{1}{3}\pi r^2 h$.

19. Writing The two cylinders pictured at the right are congruent. How does the volume of the larger cone compare to the total volume of the two smaller cones? Explain. **See left.**



Exercise 20

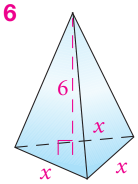
20. Architecture The Transamerica Pyramid in San Francisco (see photo at left) is 853 ft tall with a square base that is 149 ft on each side.

a. What is its volume to the nearest thousand cubic feet? **6,312,000 ft^3**

b. How tall would a prism-shaped building with the same square base as the Pyramid have to be to have the same volume as the Pyramid? **about 284 ft**

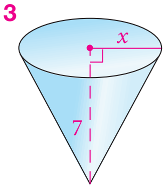
Algebra **Find the value of the variable in each figure. Leave answers in simplest radical form. The diagrams are not to scale.**

21. **6**



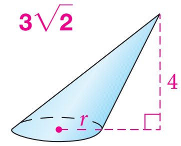
Volume = $18\sqrt{3}$

22. **3**



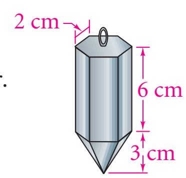
Volume = 21π

23. **$3\sqrt{2}$**



Volume = 24π

24. Hardware Builders use a plumb bob to find a vertical line. GPS The plumb bob shown combines a regular hexagonal prism with a pyramid. Find its volume to the nearest cubic centimeter. **73 cm^3**



25. Multiple Choice A cone has a volume of 600π in.^{3} and a height of 50 in. What is the radius of the cone? **B**

(A) 3.5 in. (B) 6.0 in. (C) 10.6 in. (D) 36.0 in.

26. cube: 8 units^3, cone: $\frac{2}{3}\pi$ units^3, pyramid: $\frac{8}{3}$ units^3

26. A cone with radius 1 fits snugly inside a square pyramid which fits snugly inside a cube. What are the volumes of the three figures? **See left.**

27. A cone with radius 3 ft and height 10 ft has a volume of 30π ft^3. What is the volume of the cone formed when the following happens to the original cone?

a. The radius is doubled. **120π ft^3** **b.** The height is doubled. **60π ft^3**

c. The radius and the height are both doubled. **240π ft^3**

GO Online Homework Help
Visit: PHSchool.com
Web Code: aue-1105

Exercise 18 Students must find the volumes of two space figures. Ask: *What figure is on the bottom?* **rectangular prism** *What figure is on the top?* **pyramid**

Connection to Mental Math

Exercise 20 Do part b as a class exercise. Write the formulas for the volume of a prism and the volume of a pyramid side by side. Then challenge students to solve the problem using mental math and explain their methods.

Careers

Exercise 24 Builders must know how to construct level floors and vertical walls. Have students investigate how carpentry tools such as plumb bobs and levels use principles of geometry to do this.

Exercise 27 The answer to part c will be generalized to all similar cones in Lesson 11-7.

Visual Learners

Exercise 33 Have students with strong visual skills explain to the rest of the class what the solid looks like, then cut off the tip of a conical ice cream cone to illustrate it.

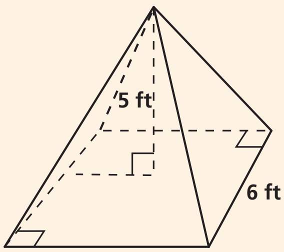
4. Assess & Reteach

PowerPoint

Lesson Quiz

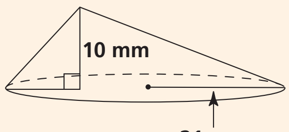
Find the volume of each figure. When appropriate, leave your answer in terms of π.

1.



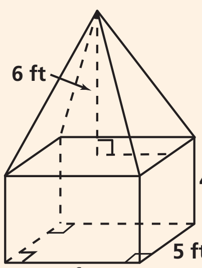
60 ft^3

2.



$1470\pi\text{ mm}^3$

3. square pyramid with base edges 24 in. long and slant height 15 in. 1728 in.^3
4. cone with diameter 3 m and height 4 m $3\pi\text{ m}^3$
5.



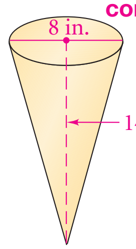
150 ft^3

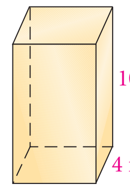
Alternative Assessment

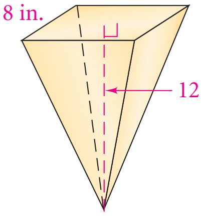
Have partners solve the following exercise and show their work in detail: A cone and a square pyramid each have height 12 cm. The perimeter of the base of the pyramid and the circumference of the base of the cone are each 24 cm. Which has the greater volume?

29. cone with $r = 4$ and $h = 3$; 16π
30. cone with $r = 3$ and $h = 4$; 12π
31. cylinder with $r = 4$, $h = 3$, with a cone of $r = 4$, $h = 3$ removed from it; 32π
32. cone with $r = 4$, $h = 5\frac{1}{3}$, with a cone of $r = 1$, $h = 1\frac{1}{3}$ cut off the top, and a cylinder of $r = 1$ cut out of its center; 24π

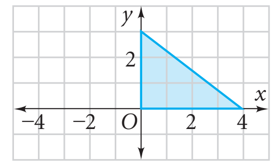
28. List the volumes of the cone, prism, and pyramid in order from least to greatest.
cone: 234.6 in.^3; prism: 240 in.^3; pyramid: 256 in.^3







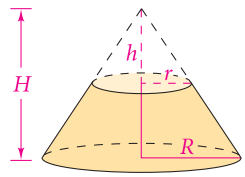
Visualization **The plane region is revolved completely about the given line to sweep out a solid of revolution. Describe the solid. Then find its volume in terms of π.**
29–32. See left.



29. the y-axis
30. the x-axis
31. the line $x = 4$
32. the line $y = -1$

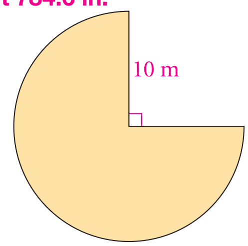
Challenge

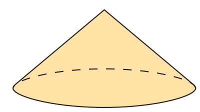
33. A *frustum* of a cone is the part that remains when the vertex is cut off by a plane parallel to the base.
33a. See margin.
a. Explain how to use the formula for the volume of a cone to find the volume of a frustum of a cone.
b. **Containers** A 9-in. tall popcorn container is the frustum of a cone. Its small radius is 4.5 in. and its large radius is 6 in. What is its volume? about 784.6 in.^3



Frustum of cone

34. A disk has radius 10 m. A 90° sector is cut away, and a cone is formed. 47.1 m
a. What is the circumference of the base of the cone?
b. What is the area of the base of the cone? 176.7 m^2
c. What is the volume of the cone? (*Hint:* Use the slant height and the radius of the base to find the height.) 389.6 m^3





Graphing Calculator **In Exercises 35 and 36, the volume of the solid is 1000 cm^3. Use the Activity Lab on page 616 to help you complete each exercise.**

35. For a square pyramid, find the length of a side of the base for which the lateral area is as small as possible. about 16.2 cm

36. For a cone, find the radius for which the lateral area is as small as possible. about 8.8 cm

Test Prep

Multiple Choice

37. What is the volume of a 6-ft high square pyramid with base edges 8 ft? A
A. 128 ft^3 B. 192 ft^3 C. 256 ft^3 D. 384 ft^3

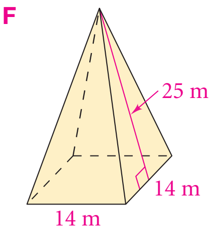
38. What is the volume of a cone with diameter 21 m and height 4 m? F
F. $147\pi\text{ m}^3$ G. $220.5\pi\text{ m}^3$ H. $294\pi\text{ m}^3$ J. $441\pi\text{ m}^3$

39. What is the volume of an oblique cone with radius 9 cm and height 12 cm?
A. $324\pi\text{ cm}^3$ B. $486\pi\text{ cm}^3$ C. $648\pi\text{ cm}^3$ D. $972\pi\text{ cm}^3$ A

33. a. The frustum has vol. $V = \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(R^2 H - r^2 h)$. Now if $h_1 = H - h$ is the frustum's height, $V = \frac{1}{3}\pi(R^2(h_1 + h) - r^2 h) = \frac{1}{3}\pi(R^2 h_1 + h(R^2 - r^2))$.
By similar $\triangle$s, $\frac{h}{r} = \frac{h_1 + h}{R}$, or $h = \frac{rh_1}{R - r}$.
Simplifying, $V = \frac{1}{3}\pi h_1(r^2 + rR + R^2)$.

40. What is the volume of the square pyramid at the right? **F**

F. 1568 m^3 **G.** 1633 m^3
H. 2352 m^3 **J.** 2450 m^3



41. What is the volume of an oblique square pyramid with base edges 25 in. and height 24 in.? **A**

A. 5000 $in.^3$ **B.** 7500 $in.^3$
C. 10,000 $in.^3$ **D.** 15,000 $in.^3$

Short Response

42. The volume of a cone is $82{,}418\pi$ cm^3. Its diameter is 203 cm. What is its height? Show all your work, including any formulas that you use. **See margin.**

Mixed Review

GO for Help

Lesson 11-4

43. **Sports** A cylindrical hockey puck is 1 in. high and 3 in. in diameter. What is its volume in cubic inches? Round your answer to the nearest tenth. **7.1 $in.^3$**

44. A triangular prism has height 30 cm. Its base is a right triangle with legs 10 cm and 24 cm. Find the volume of the prism. **3600 cm^3**

Lesson 10-5

45. Find the area of a regular pentagon with a radius 5 in. Give your answer to the nearest tenth of a square inch. **59.4 $in.^2$**

Lesson 8-2

Find the area of each equilateral triangle to the nearest tenth of a square unit.

46. The triangle has 12 cm sides. **62.4 cm^2**

47. The triangle has 10-in. altitudes. **57.7 $in.^2$**

48. Find the area of a 30°-60°-90° triangle with shorter leg of length 4 cm. **13.9 cm^2**

Geometry at Work

Package Designer

Each year, more than one trillion dollars in manufactured goods are packaged in containers. To create each new box, bag, or carton, package designers must balance such factors as safety, environmental impact, and attractiveness against cost of production.

Consider the three boxes of dishwasher detergent. All three boxes have standard volumes of 108 $in.^3$. The boxes have different shapes, however, and different surface areas. The box on the left has the greatest surface area and therefore costs the most to produce. Despite the higher cost, the box on the left has become standard. In this case, the least expensive package on the right is too difficult for a consumer to pick up and pour.

Go Online PHSchool.com **For:** Information about package design **Web Code:** aub-2031

Test Prep

Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 657
- Test-Taking Strategies, p. 652
- Test-Taking Strategies with Transparencies

42. [2] $\frac{1}{2}d = r$;
$r = \frac{1}{2}(203) = 101.5$;
$V = \frac{1}{3}\pi r^2 h$, so
$82{,}418\pi = \frac{1}{3}\pi(101.5)^2 h$, and
$h = 24$ cm.

[1] one computational error