Wall-Modeled Large-Eddy Simulation of High-Speed Turbulent Flows with an Immersed Boundary Method

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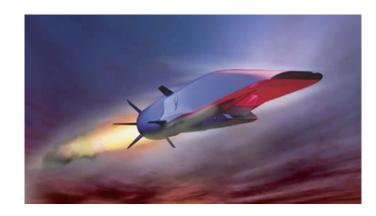
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Background





- Increasing interest in high-speed flows in the last few years, especially due to
 DoD relevance and space exploration
 - Rapid response and deployment capabilities
 - Space access and re-entry
- Combination of many relevant physical phenomena
 - Real-gas effects, shocks, SWBLI, large transition regions
- Challenges for hypersonic turbulence modeling (in addition to those from the low-speed regime):
 - Accurate prediction of regions with mixed laminar, transitional and turbulent flows
 - High temperature and compressibility effects, e.g., real gas effects
 - Aerodynamic heating
 - + all remaining challenges from the low-speed regime carry over, e.g., separation prediction, etc.
 - Limited development and testing of hybrid RANS-LES and WMLES approaches for high-speed flow problems





Artist's concepts of hypersonic cruise hardware.

High-Fidelity Simulations





- Coming years will see a substantial increase in the use of hybrid RANS-LES (HRLM) and wall-modeled LES (WMLES) methods for industrial applications due to:
 - Increased availability of HPC resources leading to faster turn-around times (\sim 24-48 hours)
 - Complex hypersonic flow phenomena, e.g., transition, turbulent separation, etc., cannot be predicted with current state-of-the-art RANS methods
- High-fidelity simulations for practical turbulent flow scenarios (high-Re) are computationally expensive¹
 - $(N_x N_t)_{DNS} \sim Re_{L_x}^{2.91}, (N_x N_t)_{WRLES} \sim Re_{L_x}^{2.72}$
- WMLES provides a viable alternative for high-fidelity simulations at reasonable cost
 - $(N_{\chi}N_t)_{WMLES} \sim Re_{L_{\chi}}^{1.14}$
 - Wall modeling allows for more fully-automated grid generation due to coarse grid resolution near the solid boundary
- Numerous challenges in LES/WMLES of high-speed flows, grid sensitivity

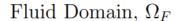
Immersed Boundary Methods

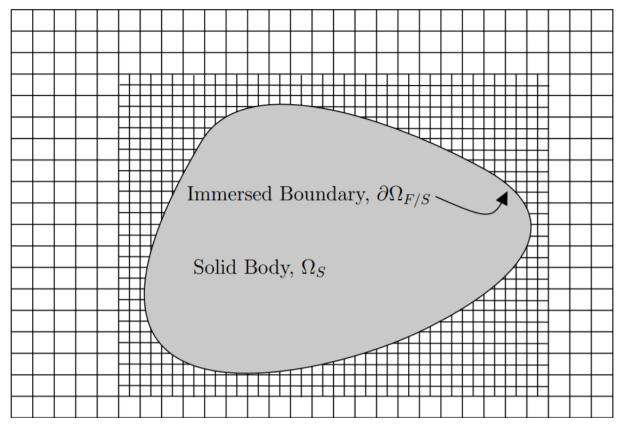






- Allow practitioners to side-step costly mesh generation process
 - Reduce mesh generation process to a small number of parameters, otherwise fully automated
- Arbitrary geometry is superimposed over a uniformly Cartesian grid
 - Apply a boundary closure rule to governing equations
 - Block-structure allows for simple AMR to resolve structures near boundary
- Simple, structured data layout with fixed spacing has favorable characteristics for HPC compared with e.g. unstructured approach
- Challenges
 - Limited to small aspect ratios
 - Numerical instability at boundaries
 - Reduction of order-of-accuracy at boundaries





Example of a solid object immersed in a Cartesian Grid

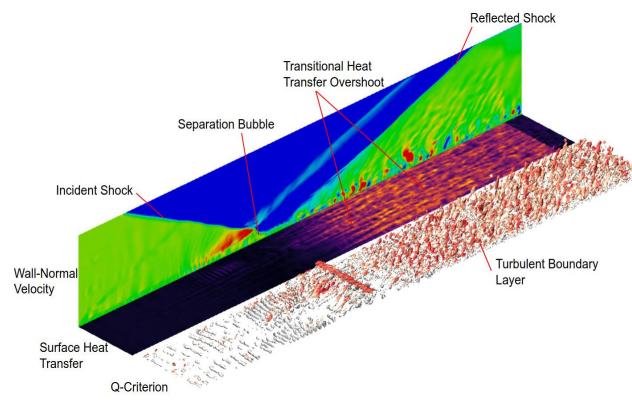
Numerical Methods for High-Fidelity Turbulence Modeling







- Conflicting requirements on numerical methods for WMLES of hypersonic flows:
 - Non-dissipative
 - Robust at solution discontinuities
- IBM-WMLES has additional conflicting requirements at boundaries:
 - IB treatment typically suffers numerical instabilities
 - Under-resolution of WMLES grid can give rise to discontinuities at IB
 - Strong stability requirements on inherently unstable numerical method
- Numerical methods must be solution-adaptive while maintaining efficient implementation
- Importance of secondary properties
 - Discrete kinetic energy conservation
 - Discrete entropy conservation



Hypersonic shock-wave boundary-layer interaction (SWBLI) highlights a number of challenging phenomena

Numerical Scheme for WMLES







Numerical scheme must be non-dissipative and provide sufficient dissipation to handle flow discontinuities:

$$\hat{f}_{i+1/2} = (1 - \alpha)\hat{f}_{i+1/2}^{Cent} + \alpha\hat{f}_{i+1/2}^{Diss}$$

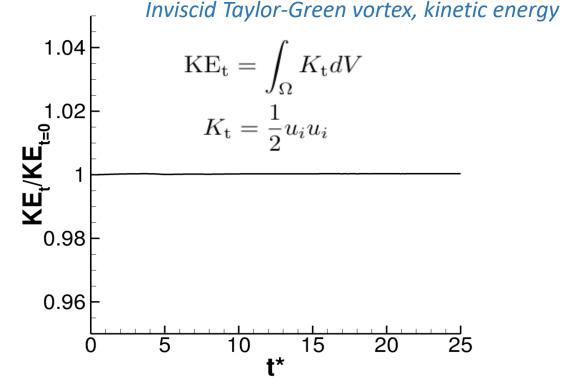
Interior

o Cent: Fourth-order, centered, kinetic energy and entropy preserving (KEEP) scheme^{1,2}

o Diss: Third-order WENO scheme

Boundary Closure

- Irregular boundary closure has a strong effect on solution accuracy
- o *Cent*: Reduce to 2nd-order at the wall
 - For one irregular point treatment higher-order (>2) is challenging
- Diss: Developed third-order boundary closure to be consistent with interior scheme
 - Irregular reconstruction and smoothness operators follow approach in Brehm 2017³



¹Kuya, Y., Totani, K., & Kawai, S. (2018). Kinetic energy and entropy preserving schemes for compressible flows by split convective forms. *Journal of Computational Physics*, 375, 823–853.

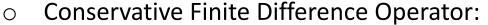
²Totani, K., Kuya, Y., & Kawai, S. (2019). High-order-accurate kinetic energy and entropy preserving schemes on curvilinear meshes. In AIAA Scitech 2019 Forum.

³Brehm, (2017), On consistent boundary closures for compact finite-difference WENO schemes, Journal of Computational Physics, 334, 573-581.

Numerical Methods – WENO IB Closure







$$\frac{\partial f}{\partial x} = \frac{1}{\Delta x} \left(h_{i+1/2} - h_{i-1/2} \right) = \frac{1}{\Delta x} \left(\hat{f}_{i+1/2} - \hat{f}_{i-1/2} \right) + \mathcal{O}(\Delta x^{2n-1})$$

- Scheme relies on error cancellation
 - Numerical flux derivative at x_i : $\frac{\partial f}{\partial x}\Big|_{x=x_i} = \frac{h_{i+1/2} h_{i-1/2}}{\Delta x} \approx \frac{\hat{f}_{i+1/2} \hat{f}_{i-1/2}}{\Delta x}$ (1)
 - Truncation error obtaining flux at $x_{i+1/2}$: $\hat{f}_{i+1/2} = h_{i+1/2} A \frac{\partial^3 f}{\partial x^3} \Big|_{x=x_i} \Delta x^3$ (scheme A) (2)
 - Truncation error obtaining flux at $x_{i-1/2}$: $\hat{f}_{i-1/2} = h_{i-1/2} B \frac{\partial^3 f}{\partial x^3} \Big|_{x=x_i} \Delta x^3$ (scheme B) (3)
 - Substituting (2) and (3) in (1) leads to: $\frac{\partial f}{\partial x}\Big|_{x=x_i} = \frac{h_{i+1/2} h_{i-1/2}}{\Delta x} (B A) \frac{\partial^3 f}{\partial x^3}\Big|_{x=x_i} \Delta x^2 + O(\Delta x^3)$
 - To recover formal order-of-accuracy match not only order but also leading term of truncation error

Numerical Methods – WENO IB Closure





Introduce weights on candidate FD stencils:

$$\frac{\partial f}{\partial x} = \sum_{l} \omega_{l} \sum_{k \in S_{1}} c_{k} \hat{f}_{k+1/2}$$

Weights computed according to the smoothness of the solution on stencil:

$$\beta_{l} = \sum_{j=1}^{n-1} \Delta x^{2j-1} \int_{x_{l,min}}^{x_{l,max}} \left(\frac{d\hat{f}_{k}}{dx^{j}}\right)^{2} dx, \qquad \omega_{l} = \frac{c_{l}}{(\beta_{l} + \varepsilon)^{p}}$$

- \circ Irregular smoothness indicator avoids the use of ghost cell information \to avoid erroneous stencil switching
- WENO procedure necessary for flows with shocks
- \circ Smoothness can be defined for a number of different variables (primitive/conservative/characteristic/...)

Immersed Boundary Method and Wall Model Coupling



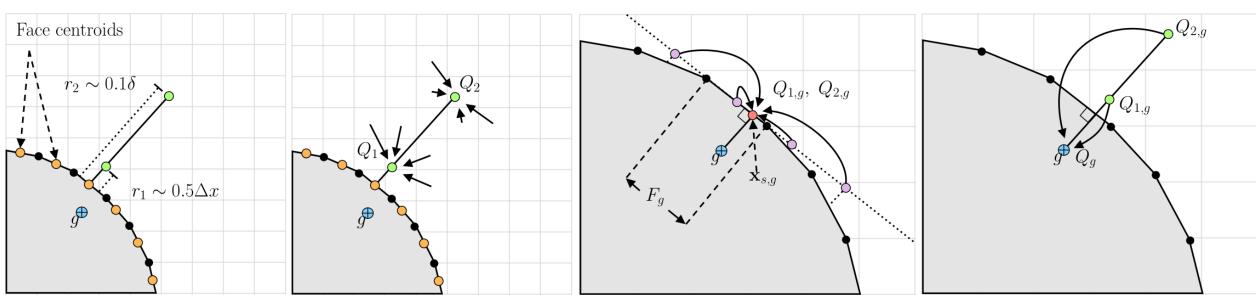




Flow Field Interpolation

On-Surface Face-Neighbor Interpolation

Computation of Boundary Conditions via Extrapolation



- No-slip wall boundary condition is not imposed
 - Tangential velocity is extrapolated to minimize effects of dissipation near the wall
 - Zero-penetration enforced through wall-normal velocity
 - Neumann boundary condition for pressure/temperature
- On-surface interpolation preserves spatial locality through ghost cells

WENO IB Closure — Stability Analysis



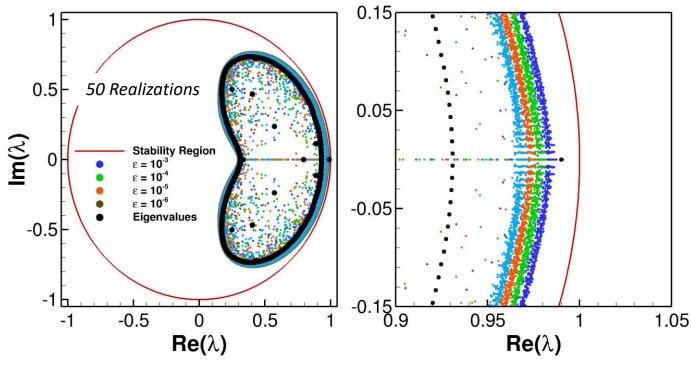




 Linear stability analysis performed on current IB closure (optimal stencils)

Upwind portion of flux shows finite-time stability for large CFL conditions; modulus of the largest pseudo-eigenvalues reside within stability region for $\varepsilon=10^{-3}$

Pseudo-Eigenvalue Spectrum of Upwind IB Closure.



Model Equation:
$$\frac{\partial u}{\partial t} + a \frac{\partial u}{\partial x} = 0 \rightarrow \frac{\partial \mathbf{u}}{\partial t} + a \frac{1}{\Delta x} D \mathbf{u} \rightarrow \mathbf{u}^{n+1} = A \mathbf{u}^n$$

Update Matrix:
$$A = I - \gamma D + \frac{1}{2!} \gamma^2 D^2 - \frac{1}{3!} \gamma^3 D^3 + \frac{1}{4!} \gamma^4 D^4$$

Pseudo-spectrum¹:
$$\frac{dist(\Lambda_{\varepsilon}\{A\},S)}{\Lambda_{\varepsilon}\{A\}} = \underbrace{O(\varepsilon)}_{\text{Im}(I)=\varepsilon} S = \{\omega \in \mathbb{C}: |\omega| \leq 1\}$$

$$\{\lambda \in \mathbb{C}: \det(\lambda I - A + M) = 0\}$$

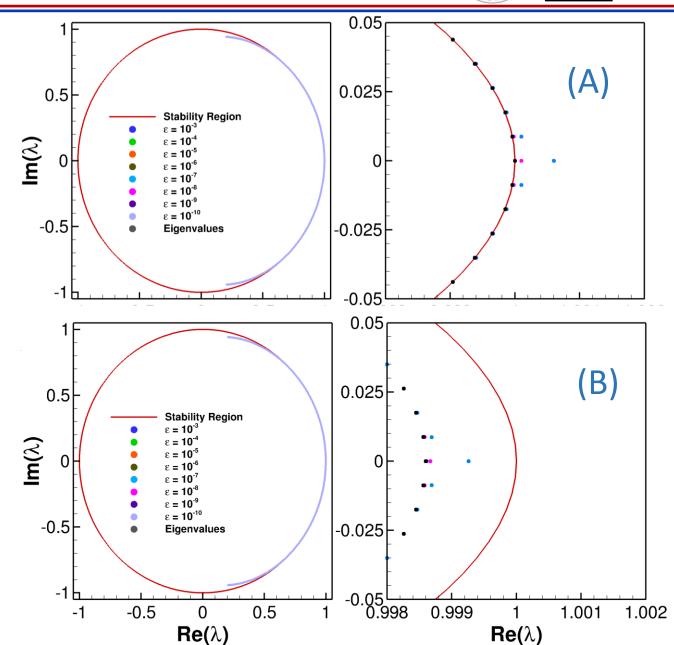
WENO IB Closure — Stability Analysis





- Centered operator spectrally stable, but boundary closure introduces pseudospectral instability – add minimal boundary dissipation to stabilize
- Upwind operator introduced at the boundary via blending parameter
 $\alpha_i = e^{-(j+1)}$

Zoomed view of the critical stability region for (A) centered IB operator and (B) modified IB operator



Numerical Methods – Viscous Discretization







Conservative discretization of viscous terms is essential for WMLES

$$\frac{\partial \boldsymbol{g}}{\partial x} \approx \frac{\widehat{\boldsymbol{g}}_{i+1/2} - \widehat{\boldsymbol{g}}_{i-1/2}}{\Delta x}$$

- If one of the faces is irregular then truncation error of viscous flux at the half point needs to match
 - Interpolation to face:

$$\phi_{i+1/2,j} = \frac{1}{2} (\phi_{i,j} + \phi_{i+1,j}) + \frac{1}{8} \Delta x^2 \phi_{xx} + \mathcal{O}(\Delta x^3)$$

Derivatives:

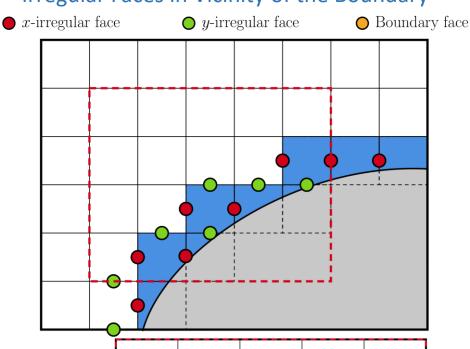
$$\begin{aligned} \frac{\partial \phi}{\partial y} \bigg|_{i+1/2j} &= \frac{1}{2} \left(\frac{\phi_{i+1,j+1} - \phi_{i+1,j-1}}{2\Delta y} + \frac{\phi_{i,j+1} - \phi_{i,j-1}}{2\Delta y} \right) + \\ & \frac{1}{6} \Delta y^2 \left. \phi_{yyy} \right|_{i+1/2,j} + \frac{1}{8} \Delta x^2 \left. \phi_{yxx} \right|_{i+1/2,j} + \mathcal{O}(\Delta y^3, \Delta x^3, \Delta x \Delta y^2) \\ \frac{\partial \phi}{\partial x} \bigg|_{i+1/2,j} &= \frac{\phi_{i+1,j} - \phi_{i,j}}{\Delta x} - \frac{1}{24} \Delta x^2 \phi_{xxx} + \mathcal{O}(\Delta x^3) \end{aligned}$$

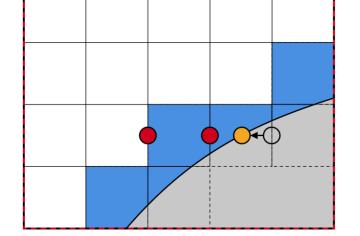
Matching truncation error at irregular face

$$\mathcal{R}\phi = \sum_{i,j\in J} c_{i,j}\phi_{i,j} + k_1\Delta x^2 + k_2\Delta y^2 + \mathcal{O}(\Delta x^3, \Delta x^2\Delta y, \Delta x\Delta y^2, \Delta y^3), \quad \mathcal{R} \in \left\{1, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right\}$$

Viscous flux on immersed boundary provided by wall model

Irregular Faces in Vicinity of the Boundary





Numerical Methods – Wall Model







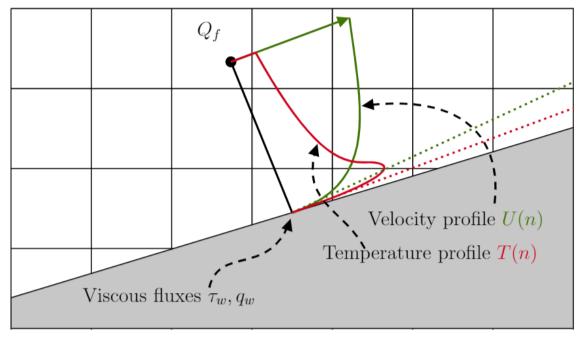
o Present method employs simple equilibrium wall model: viscous diffusive equilibrium

$$\frac{d}{dn}\left((\mu+\mu_t)\frac{du}{dn}\right)=0, \qquad \mu_t=\kappa\mu y^+\left(1-e^{\frac{y^+}{A^+}}\right)^2$$

O Contribution from aerodynamic heating significant in wall-model layer when $M^2C_f^{\frac{1}{2}}=O(1)^1$ (significant around $M\cong 3$):

$$\frac{d}{dn}\left((\mu + \mu_t)u\frac{du}{dn} + c_p\left(\frac{\mu}{Pr} + \frac{\mu_t}{Pr_t}\right)\frac{dT}{dn}\right) = 0$$

 Coupling of flow-field data performed via 3rd-order interpolation operation with matching truncation error



Cartesian Grid WM Coupling

WMLES Simulations – Overview

1.25 (1.25 1 0.75

0.25

1.5

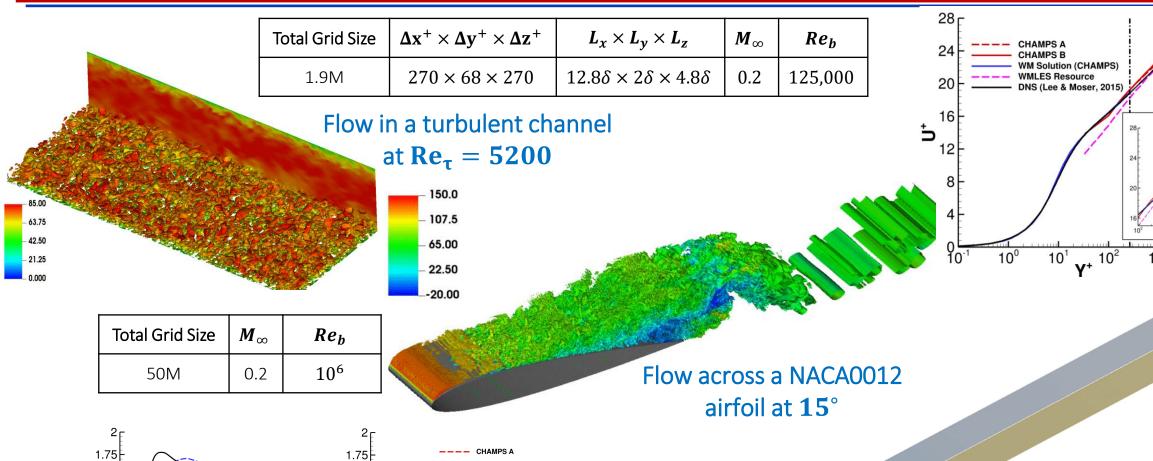
(x 1.25 1 0.75

0.25

Re_x (x10⁻⁶)







DNS (Subareddy 8

Re_x (x10⁻⁶)

Hypersonic transitional boundary layer flow at M = 6

Total Grid Size	$\Delta x \times \Delta y \times \Delta z$	M_{∞}	Re_b
32M	$1.14\delta_0 \times 0.2\delta_0 \times 0.55\delta_0$	6	2.2×10^7

WMLES Simulations – Overview

1.5

 $\mathbf{y}/\delta_{\mathbf{0}}^{\star}$

0.5

8.9

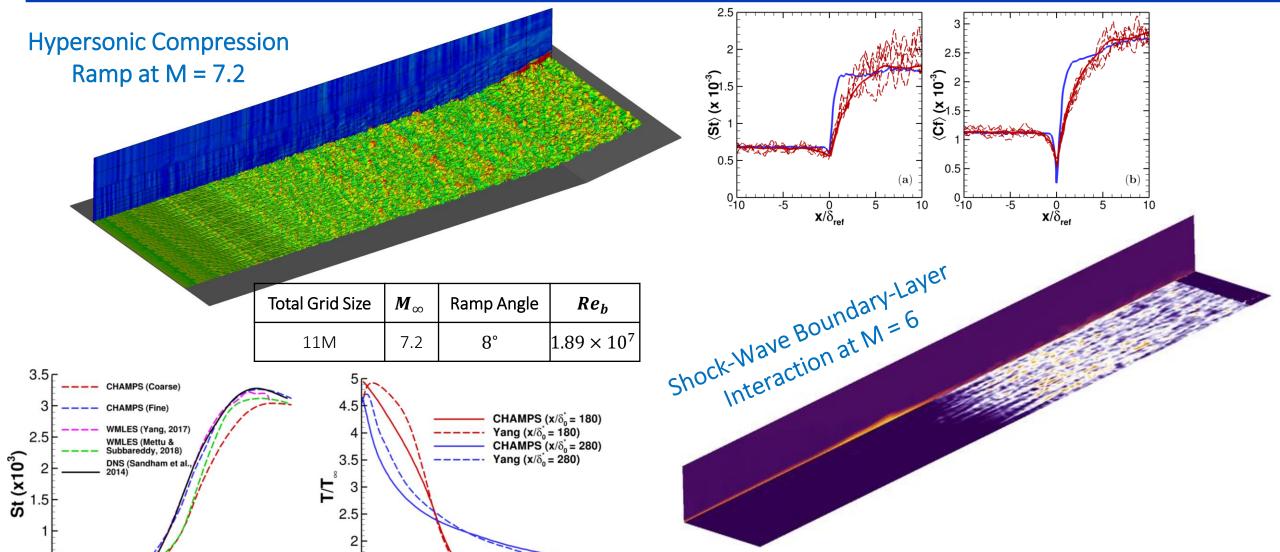
1.4

 $Re_{x}(x10^{-6})$

1.8







Total Grid Size

4.6M

 $\Delta x^+ \times \Delta y^+ \times \Delta z^+$

 $148 \times 50 \times 85$

 M_{∞}

6

 Re_{b}

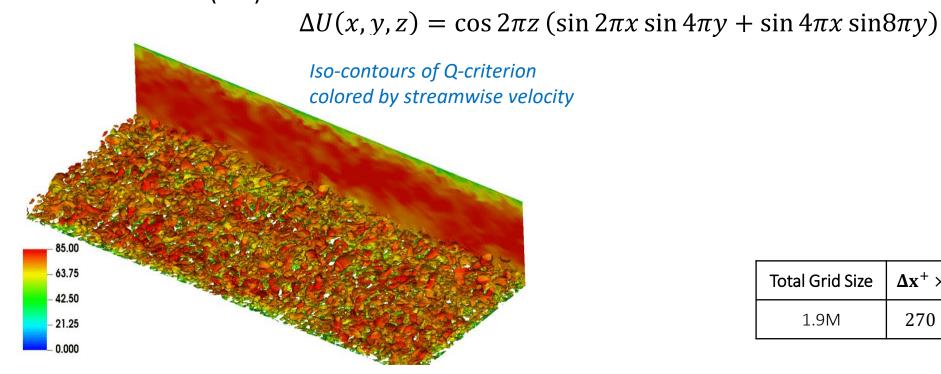
 6.0×10^{6}

WMLES Simulations – Turbulent Channel Flow

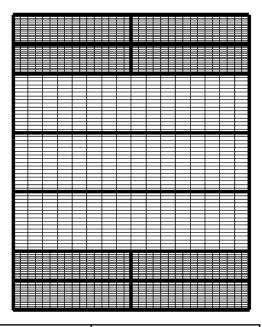




- Well-established test case in CFD community
- \circ Skin friction Reynolds number $Re_{ au}=5200$
- Serves as basic validation case and development platform for interior schemes
- Initial condition is a perturbation at most amplified wavenumber (LST):



Grid topology for turbulent channel mesh

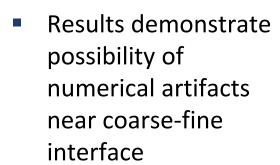


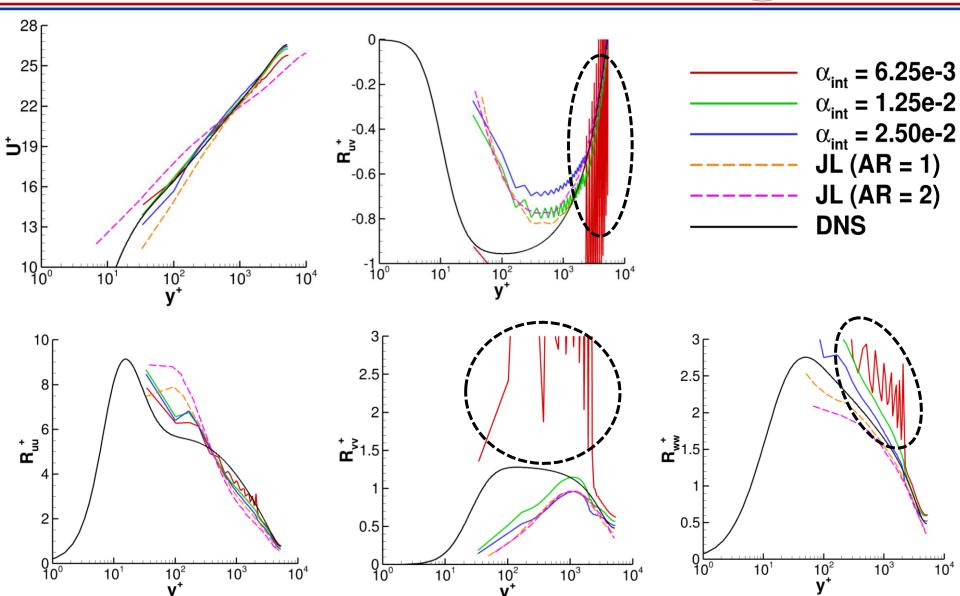
Total Grid Size	$\Delta x^+ \times \Delta y^+ \times \Delta z^+$	$L_x \times L_y \times L_z$
1.9M	$270 \times 68 \times 270$	$12.8\delta \times 2\delta \times 4.8\delta$

WMLES Simulations – Turbulent Channel Flow









DNS: Lee, M., & Moser, R. D. (2015). Direct numerical simulation of turbulent channel flow up to $Re_{\tau} \approx 5200$. Journal of Fluid Mechanics, 774, 395–415. https://doi.org/10.1017/jfm.2015.268

JL: Wall-Modeled Large Eddy Simulation Resource: https://wmles.umd.edu

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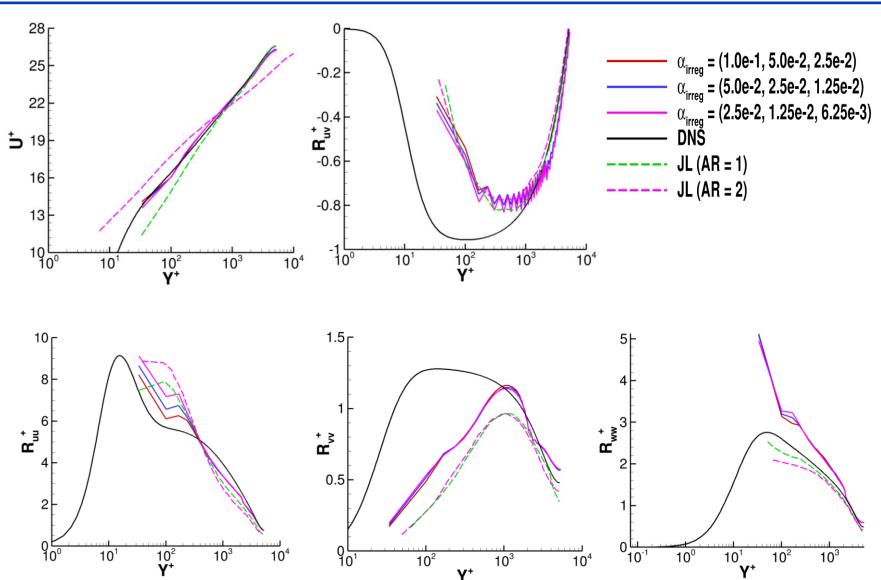
WMLES Simulations – Turbulent Channel Flow







- For low enough boundary dissipation coefficient, mean profile is invariant
- Reynolds stresses show dependence on the boundary dissipation term



WMLES Simulations – M6 Transitional Boundary Layer



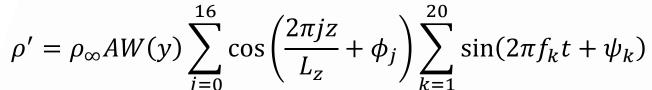


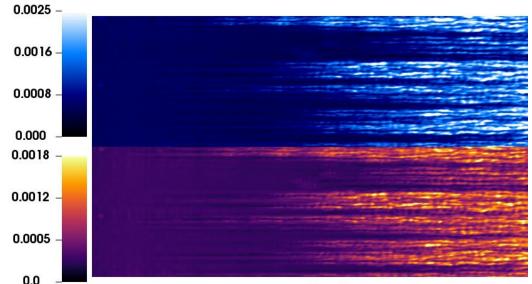


- Assess capability of IBM-WMLES to handle transitional flows
- Transition handled in the wall-model via the transition sensor, turbulent eddy viscosity suppressed in the wall model when below a prescribed threshold

$$T_r = 0.15 \frac{\hat{\rho}\hat{k}}{\hat{\mu}|\hat{S}|}, \qquad k = u_i \hat{u}_i - \hat{u}_i \hat{u}_i$$

Promotion of transition via time-varying density perturbations





 C_f (top) and St (bottom) on the flat-plate surface

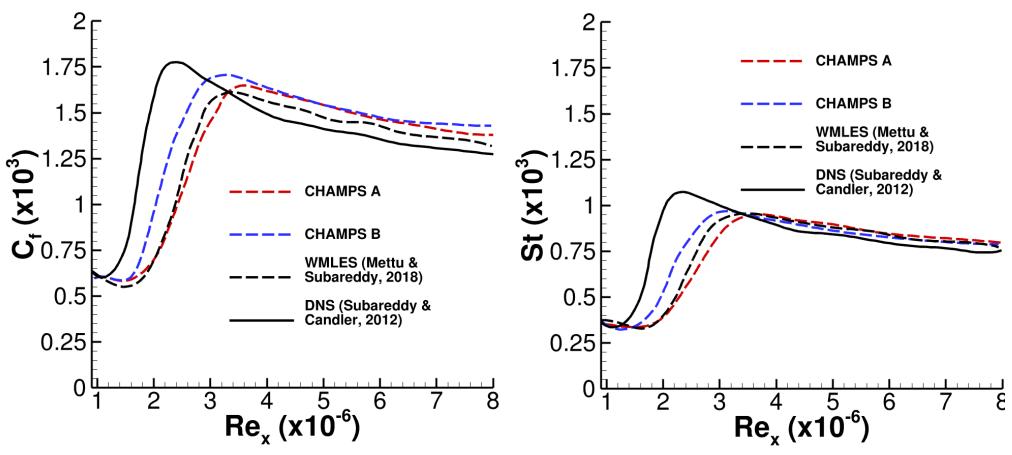
WMLES Simulations – M6 Transitional Boundary Layer







- Surface quantities are in good agreement with the reference DNS and WMLES simulations
- Effect of increasing transition sensor threshold is to delay transition



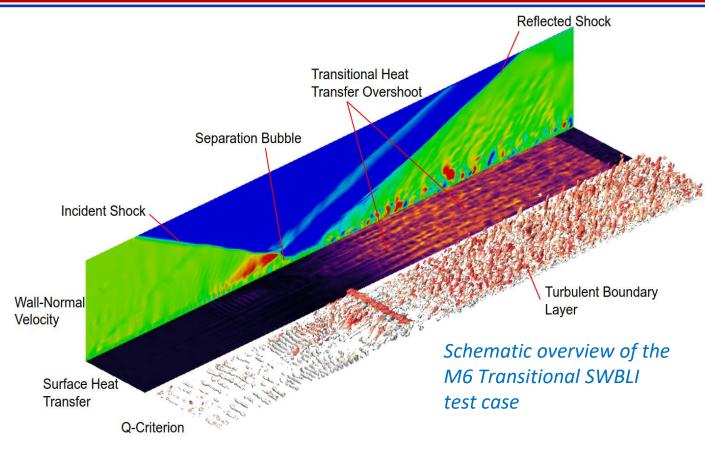
Skin friction (left) and Stanton number (right) for the M6 transitional boundary layer test case

WMLES Simulations – M6 Transitional SWBLI





- Test case has been the subject of a number of studies^{1,2,3,...}
- Increase complexity over a simple boundary layer
- Presence of shock wave poses a challenge to the interior scheme
- Induced separation bubble introduces challenge of predicting point of separation
- Non-equilibrium effects present throughout the wall-model layer

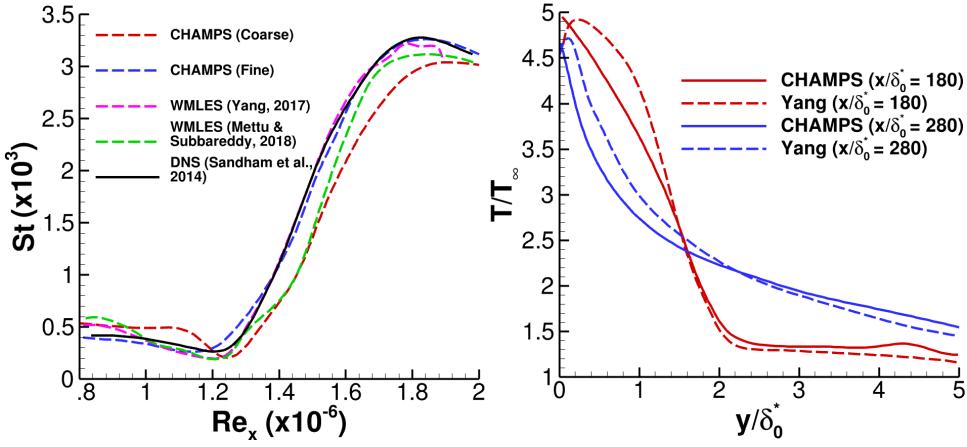


WMLES Simulations – M6 Transitional SWBLI





- Overall, good agreement in surface quantities with the DNS and WMLES references
- Temperature profile mismatch possibly due to difference in the energy equation coupling
- Coarser grid resolution under-predicted separation length, peak heat transfer



Stanton number (left) and temperature profile (right) for the M6 SWBLI test case

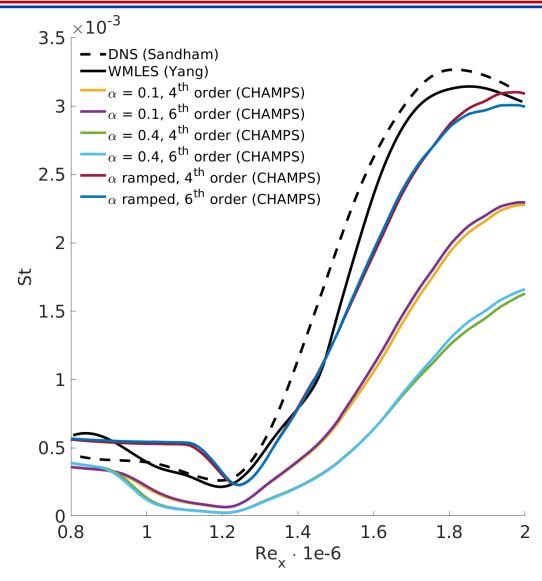
WMLES Simulations – M6 Transitional SWBLI







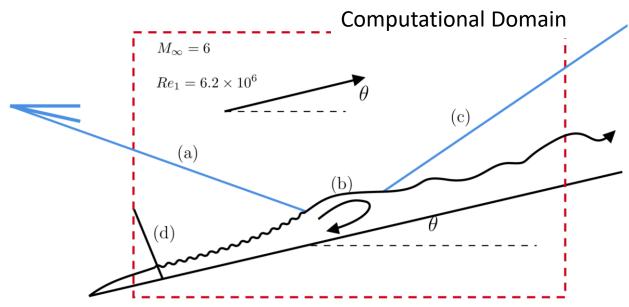
- A test was run to investigate the effect of boundary dissipation for this test case
- Coarse mesh
- Significant effect of boundary dissipation term
- Dissipation coefficient manipulated only in the first three grid cells
- Effect of dissipation apparently more significant than order-of-accuracy beyond 4th order



Stanton number predictions showing variations of dissipation coefficient at the immersed boundary

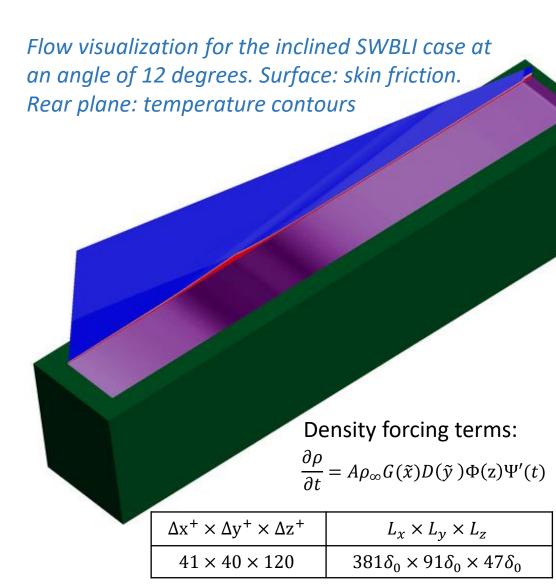






Schematic of the SWBLI ramp test-case: (a) incoming shock wave, (b) induced separation bubble, (c) reflected shock wave, and (d) density disturbances.

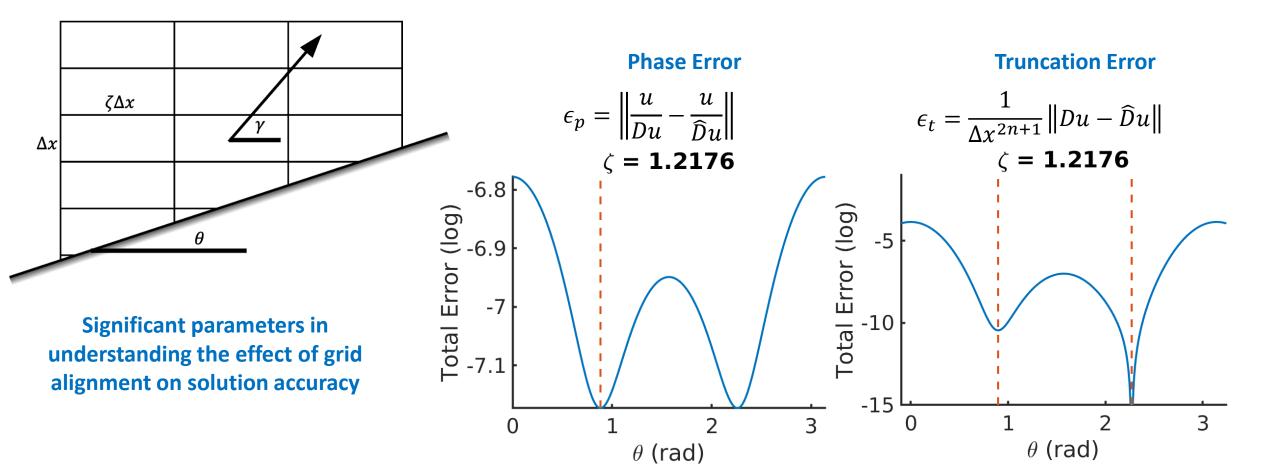
- To test IB treatment, an angle of inclination is introduced
- Shock wave at inflow imposed via Rankine-Hugoniot relations
- Impingement of shock wave induces a separation bubble
- Present method has been validated previously for grid aligned variation³





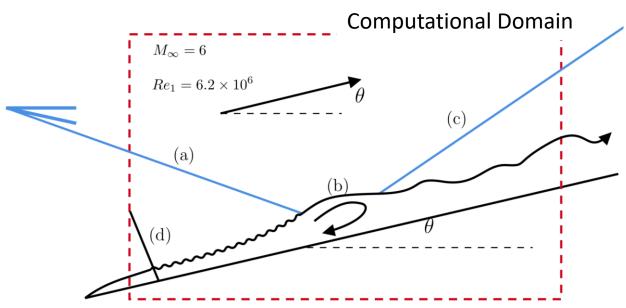


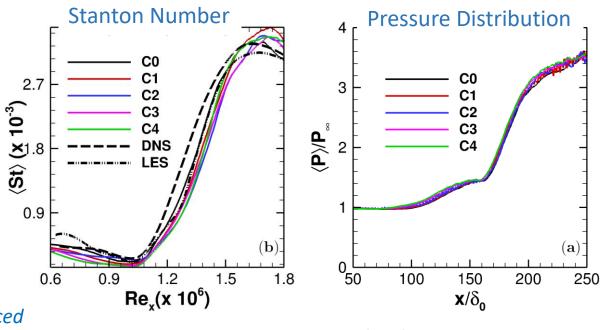
- Grid alignment was briefly analyzed to assess suitability of fixed grid spacing for varying angle of inclination
- Unit aspect ratio gives small variation in the dependence of numerical errors on angle of inclination
- Variation of numerical errors less significant when avoiding inclinations near various critical angles





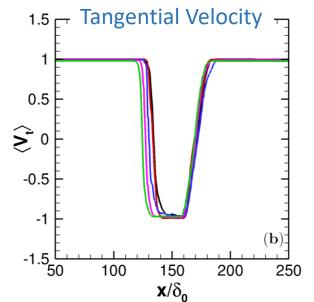






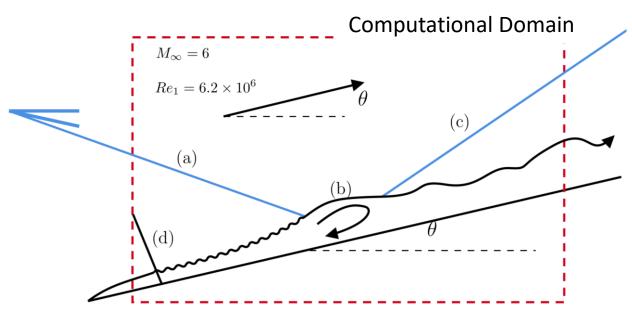
Schematic of the SWBLI ramp test-case: (a) incoming shock wave, (b) induced separation bubble, (c) reflected shock wave, and (d) density disturbances.

- Good agreement in peak turbulent/transitional heat transfer among all angles of inclination
- Consistency in surface pressure prediction
- Mean separation location indicated by the timeand space-averaged streamwise component of normalized velocity at wall
- Small disagreement ($\approx 8\delta_0$) in separation location



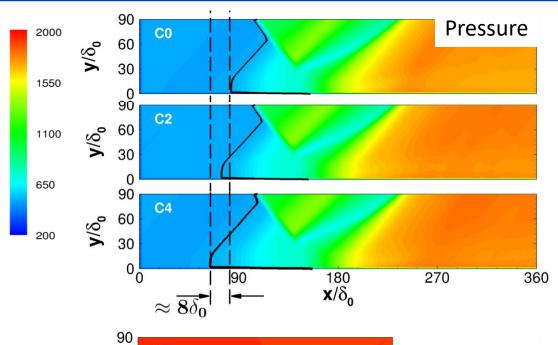


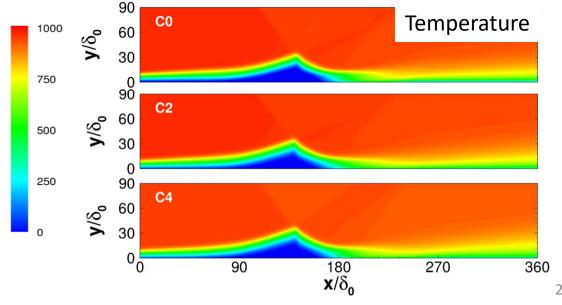




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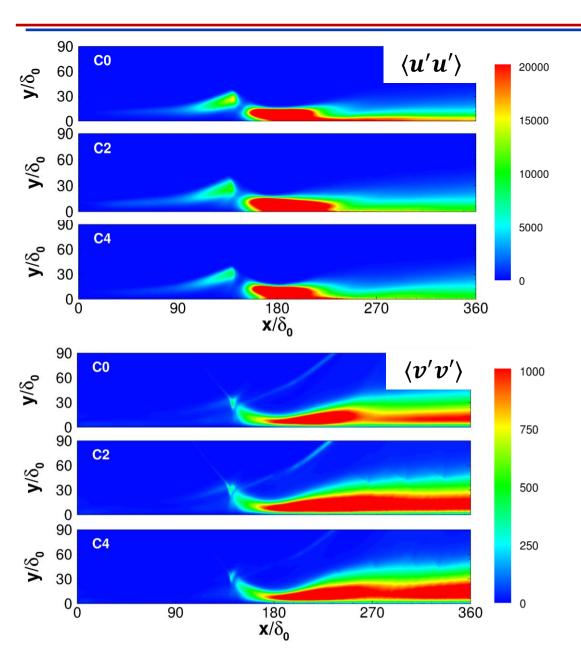


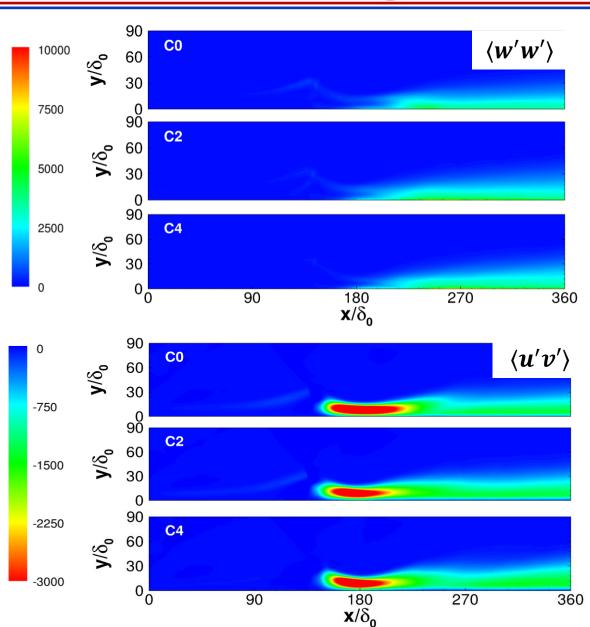












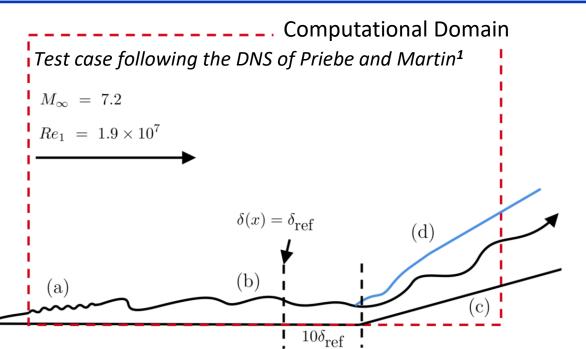
WMLES Simulations – Compression Corner





Grid Details





Flow visualization for the compression ramp test case.
Contours of heat flux and temperature in rear plane

Schematic of the compression ramp test-case. (a) Density disturbances, (b) turbulent boundary layer, (c) 8° compression ramp, and (d) induced shock wave

- Freestream-to-wall temperature ratio of 0.185
- Spatially, ~5% cost of the reference DNS (WMLES 11M, DNS 235M)

	$\Delta x^+ \times \Delta y^+ \times \Delta z^+$	$L_x \times L_y \times L_z$
Coarse	$160 \times 80 \times 190$	$68\delta_{ref} \times 9\delta_{ref} \times 10\delta_{ref}$
Fine	90 × 40 × 110	$68\delta_{ref} \times 9\delta_{ref} \times 10\delta_{ref}$

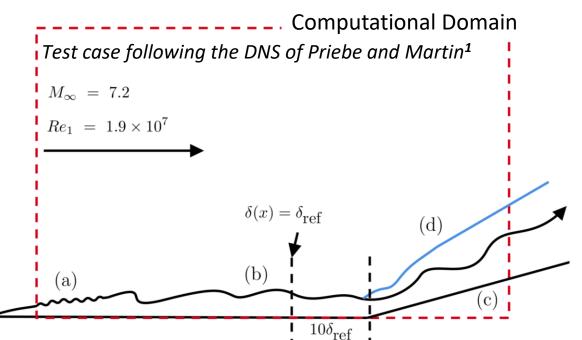
Mach 7.2 Compression Ramp

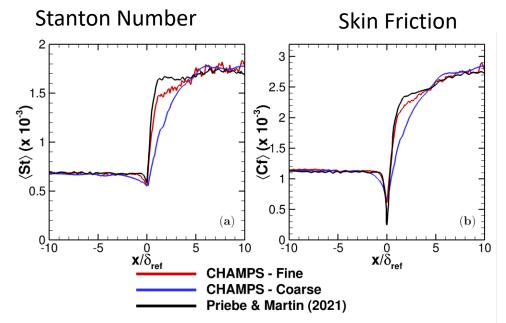
WMLES Simulations – Compression Corner





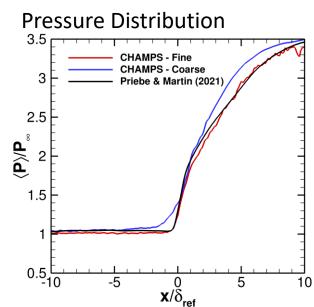






Schematic of the compression ramp test-case. (a) Density disturbances, (b) turbulent boundary layer, (c) 8° compression ramp, and (d) induced shock wave

- Flow remains attached in the mean, but significant instantaneous separation
- Present IB treatment remained stable through transition, as well as at the corner location
- Ramp geometry not aligned with the Cartesian grid
- Equilibrium assumptions appear to break down at the corner location



Future Work



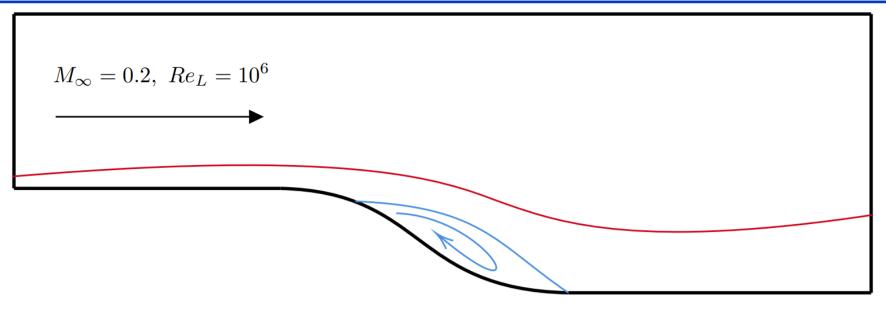


- Move towards complex, practical geometries
- O Non-equilibrium wall modeling and wall-modeling for laminar flows
- High-enthalpy and chemical non-equilibrium flows

AIAA Smooth-Body Separation Workshop (AIAA SciTech 2022)

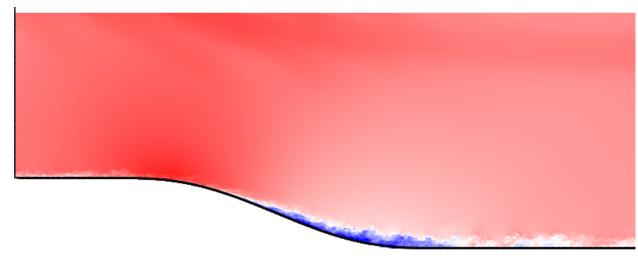






Schematic diagram for smooth-body flow separation test case

- To be held at SciTech 2022
- Understanding various effects on smooth-body separation
- Not too late to participate!
- wmles.umd.edu







Any Questions?