Warm-Up Oct. 1

- Decide whether the point (1, -3) is a solution to the equation $-4x + y = -5$.

- Daily Agenda:
  - Grade extra credit worksheet
  - Go over test
  - 3.1 notes / assignment
  - 3.2 notes / assignment??

Chapter 3

Systems of Linear Equations and Inequalities

Chapter 3.1

Solving Linear Systems by Graphing

Systems

- System of Two Linear Equations
  - Two equations with the same two variables
- Solution
  - An ordered pair (x, y) that satisfies BOTH equations

Example

- Check whether the ordered pair on solution of the following system.
  - (1, 4) NO
  - (-5, 0) yes

$$x - 3y = -5$$
$$-2x + 3y = 10$$

$1 - 3(4) = -5$
$1 - 12 = -5$
$-11 \neq -5$

$-5 - 3(0) = -5$
$-5 = -5$ ✓

$-2(-5) + 3(0) = 10$
$10 = 10$ ✓

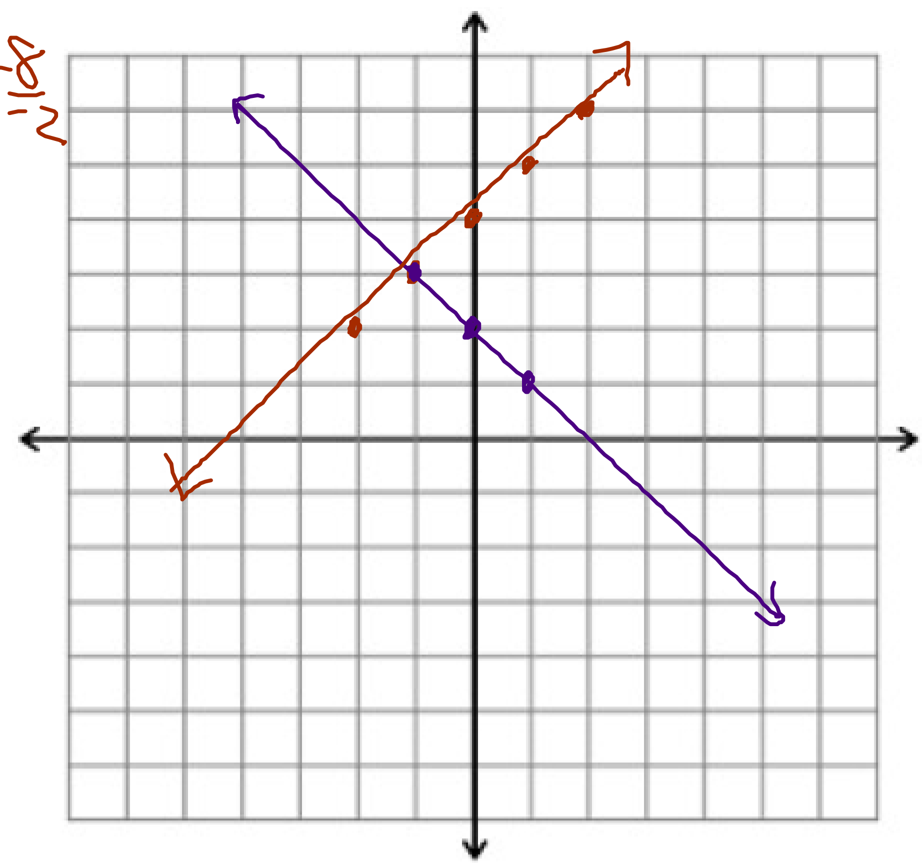
Solving a System Graphically

- Graph both equations on the same coordinate plane
- Number of solutions
  - One: the lines cross exactly one
  - No Solution: the lines are parallel
  - Infinitely Many Solutions: the lines are the same line

Example

- Solve the system by graphing:

$$2x - 2y = -8$$
$$2x + 2y = 4$$



Example

infinitely many!

- Solve the system by graphing:

$$2x + 4y = 12$$

$-2x \qquad -2x$

$$4y = -2x + 12$$

$$\frac{4y}{4} = \frac{-2x}{4} + \frac{12}{4}$$

$$y = -\frac{1}{2}x + 3$$

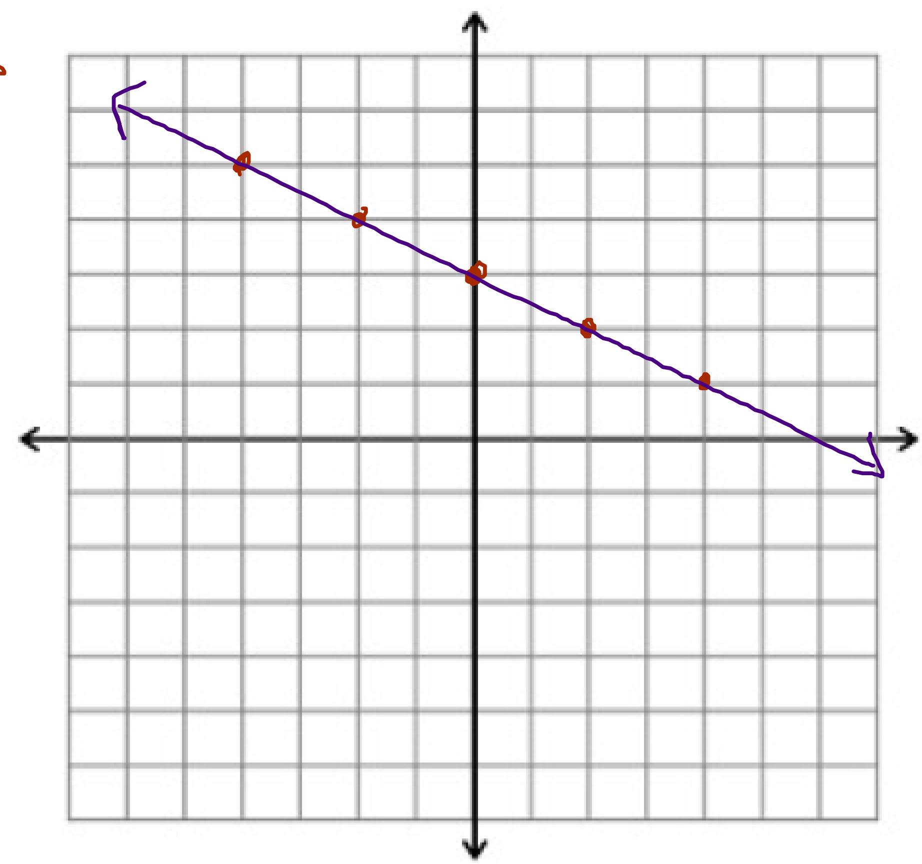
$$x + 2y = 6$$

$-x \qquad -x$

$$2y = -x + 6$$

$$\frac{2y}{2} = \frac{-x}{2} + \frac{6}{2}$$

$$y = -\frac{1}{2}x + 3$$



Example

- Solve the system by graphing:

$$x - y = 5$$

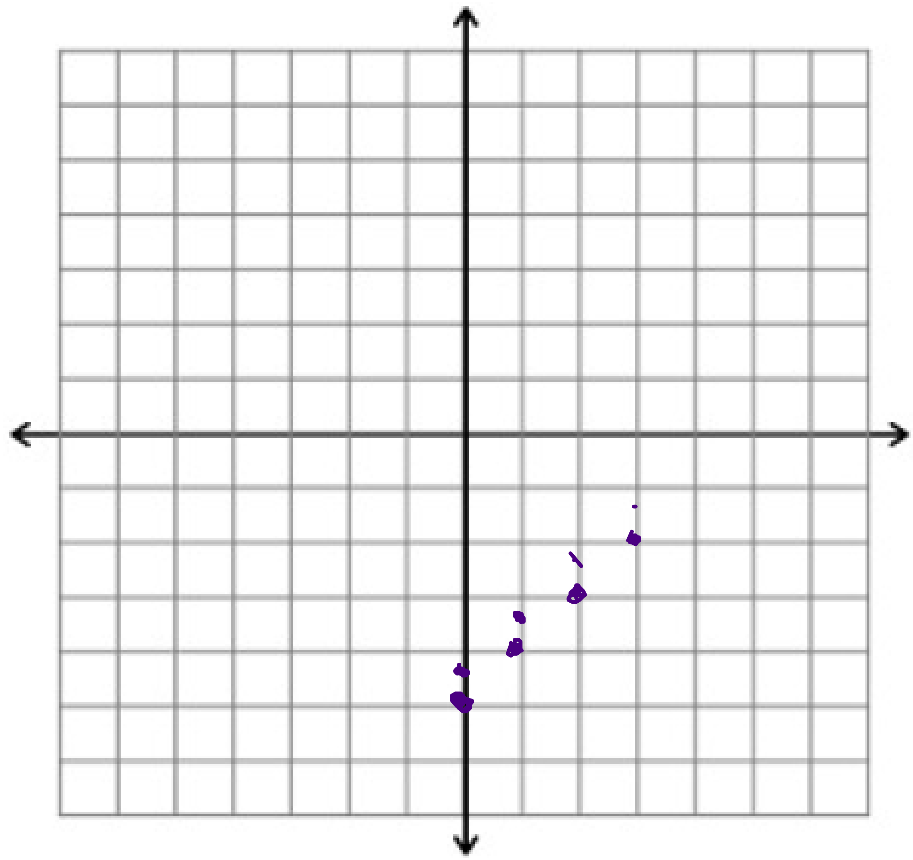
$$2x - 2y = 9$$

$-y = -x + 5$

$y = x - 5$

$-2y = -2x + 9$

$y = x - 4.5$



Example

- Solve the system by graphing:

$$4x - 3y = -15$$
$$x + 2y = -1$$

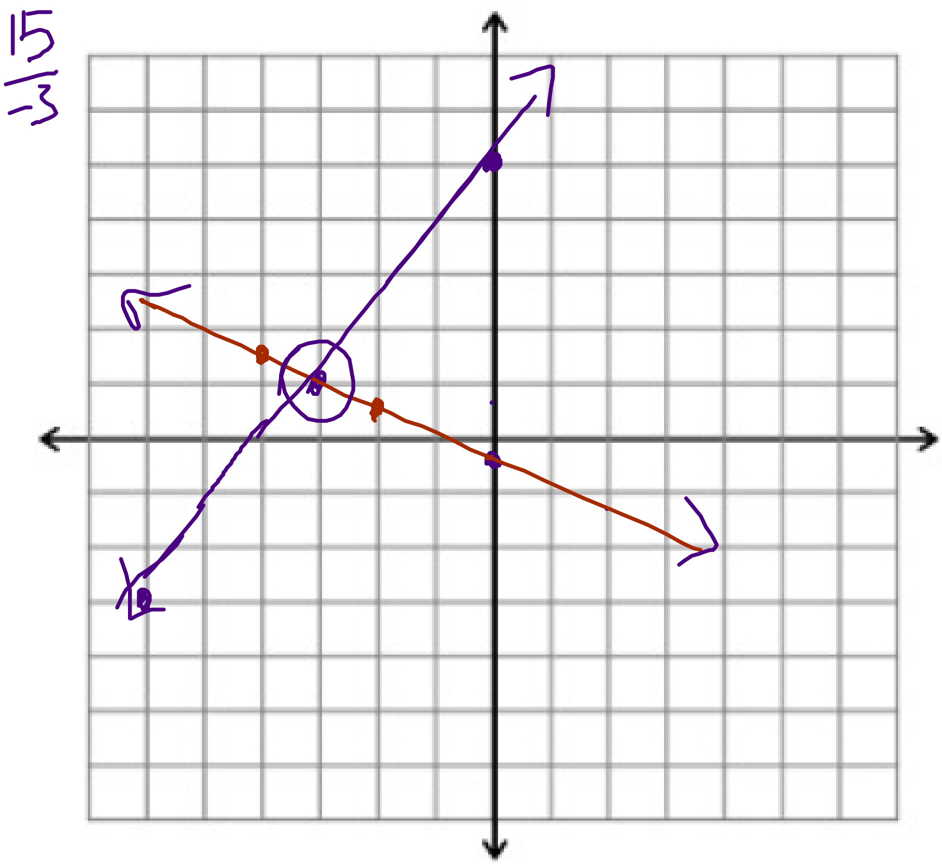
$\frac{-3y}{-3} = \frac{-4x}{-3} - \frac{15}{-3}$

$y = \frac{4}{3}x + 5$

$y = -\frac{1}{2}x - \frac{1}{2}$

$(-2\frac{1}{2}, 1\frac{1}{2})$

$(-3, 1)$



Example

x = hrs Babysitting

y = hrs mowing

150 hrs Babysitting

50 hrs mowing

- You plan to work 200 hours this summer mowing lawns and babysitting. You need to make a total of $1300. Babysitting pays $6 per hour and lawn mowing pays $8 per hour. How many hours should you work at each job?

$6x + 8y = 1300$

$-6x \qquad -6x$

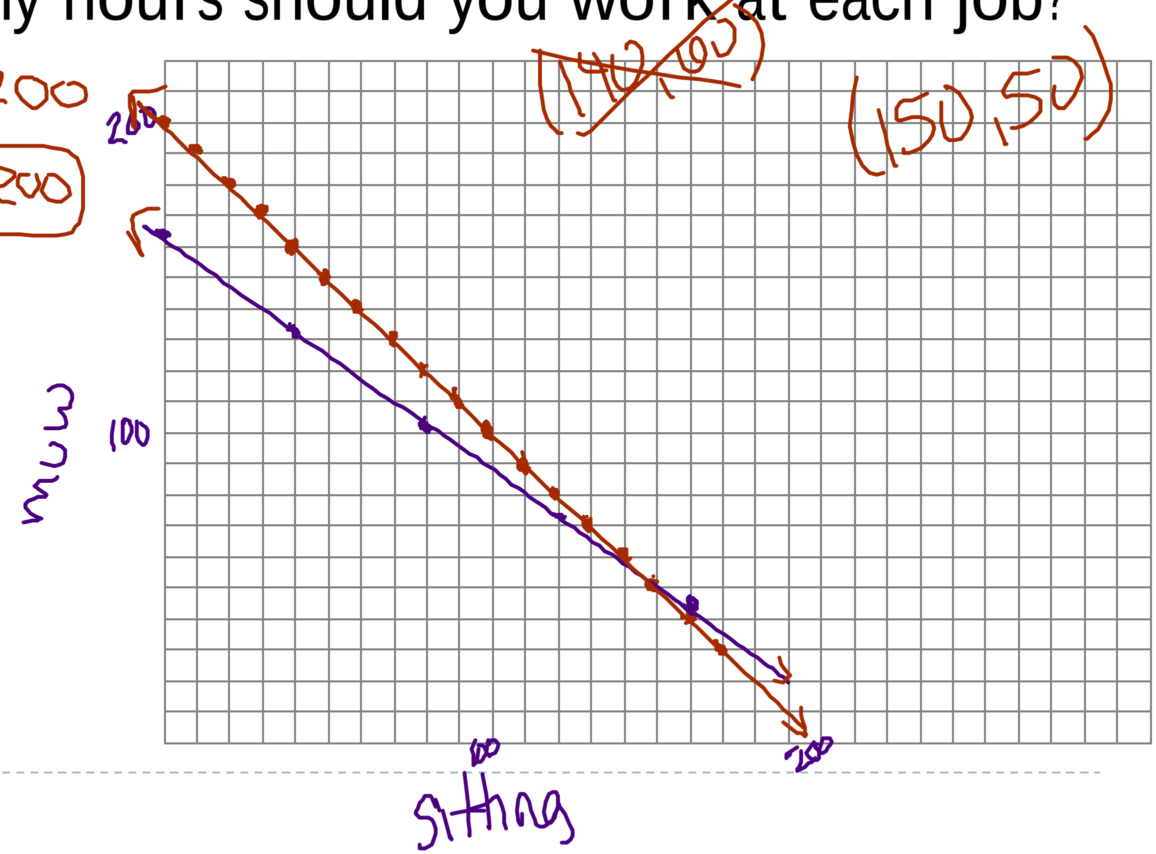
$8y = -6x + 1300$

$\frac{8y}{8} = \frac{-6x}{8} + \frac{1300}{8}$

$y = \frac{-3}{4}x + 162\frac{1}{2}$

$x + y = 200$

$y = -x + 200$



Assignment

- P142: 12, 18, 20, 22, 27, 30, 32 – 34, 42 – 52 even, 55, 67, 70, 71, 78

Warm-Up Oct. 3

- Solve each equation for the indicated variable.
  - $2x - y = 5$, y
  - $3x - 4y = 12$, y

- Daily Agenda:
  - Grade assignment
  - 3.2 notes / assignment
  - 3.3 notes / assignment
  - Quiz next time…

Chapter 3.2

Solving Linear Systems Algebraically

Substitution Method

1. Solve one equation for one of the variables
2. Substitute that expression into the other equation and solve for the remaining variable.
3. Substitute the known value back into one of the original equations to solve for the other variable.

Example

- Solve the system by substitution.

$$3x - y = 13$$
$$2x + 2y = -10$$

$3x = 13 + y$

$-13 \quad -13$

$y = 3x - 13$

$2x + 2(3x - 13) = -10$

$2x + 6x - 26 = -10$

$+26 \quad +26$

$8x = 16$

$\frac{8x}{8} = \frac{16}{8}$

$x = 2$

$3(2) - y = 13$

$6 - y = 13$

$-6 \quad -6$

$-y = 7$

$\frac{-y}{-1} = \frac{7}{-1}$

$y = -7$

$(2, -7)$

Example

- Solve the system by substitution.

$$-x+3y=1$$
$$4x+6y=8$$

$-x = -3y + 1$

$x = 3y - 1$

$4(3y-1)+6y=8$

$12y-4+6y=8$

$18y=12$

$y=\frac{2}{3}$

$-x+3\left(\frac{2}{3}\right)=1$

$-x+2=1$

$-x=-1$

$x=1$

$\left(1,\frac{2}{3}\right)$

Linear Combination Method

- Also called Elimination

1. Multiply one or both equations by a constant to obtain opposite coefficients.
2. Add the two equations together and solve the resulting equation.
3. Substitute the known value back into one of the original equations to solve for the other variable.

Example

- Solve the system by using linear combination.

$$2x - 6y = 19$$

$$3(-3x + 2y = 10)$$

$$\begin{aligned} 2x - 6y &= 19 \\ + \quad -9x + 6y &= 30 \\ \hline -7x &= 49 \end{aligned}$$

$$\frac{-7x}{-7} = \frac{49}{-7}$$

$$x = -7$$

$$2(-7) - 6y = 19$$

$$-14 - 6y = 19$$

$$+14 \qquad +14$$

$$\frac{-6y}{-6} = \frac{33}{-6}$$

$$y = \frac{-11}{2}$$

$$\left(-7, \frac{-11}{2}\right)$$

Example

- Solve the system by using linear combination.

$$4\,(9x - 5y = -7)$$
$$5\,(-6x + 4y = 2)$$

$$36x - 20y = -28$$
$$+\ -30x + 20y = 10$$

$$\frac{6x}{6} = \frac{-18}{6}$$

$$x = -3$$

$$9(-3) - 5y = -7$$
$$-27 - 5y = -7$$
$$+27 \qquad +27$$
$$\frac{-5y}{-5} = \frac{20}{-5}$$
$$y = -4$$

$$(-3, -4)$$

Example

- Solve the linear system using any method.

$$9x - 3y = 15$$
$$-3x + y = -5$$

$+3x \quad +3x$

$y = 3x - 5$

$9x - 3(3x - 5) = 15$

$9x - 9x + 15 = 15$

$15 = 15$

Infinitely Many

$$6x - 4y = 14$$
$$2(-3x + 2y = 7)$$

$6x - 4y = 14$

$+ \; -6x + 4y = 14$

$0 = 28$

No Solution

$\varnothing$

Example

- Solve the linear system using any method.

$$\begin{aligned} -9x + 14y &= -2 \\ 11x - 6y &= 8 \end{aligned}$$

$$\begin{aligned} 3x - 5y &= 4 \\ -6x + 10y &= -8 \end{aligned}$$

Example

$x = \#$ of oranges

$y = \#$ of grapefruit

13 oranges
9 grapefruit

- A citrus fruit company plans to make 13.25 lb gift boxes of oranges and grapefruits. Each box is to have a retail value of $21.00. Each orange weighs 0.50 lb and has a retail value of $.75, while each grapefruit weighs 0.75 lb and has a retail value of $1.25. How many oranges and grapefruits should be included in the box?

$3(.5x + .75y = 13.25)$

$-2(.75x + 1.25y = 21)$

$1.5x + 2.25y = 39.75$

$+ -1.5x - 2.5y = -42$

$-.25y = -2.25$

$\frac{-.25y}{-.25} = \frac{-2.25}{-.25}$

$y = 9$

$.75x + 1.25(9) = 21$

$.75x + 11.25 = 21$

$.75x = 9.75$

$x = 13$

Example

x = # of shrubs
y = # of plants

- You are planting a 160 ft^2 garden with shrubs and perennial plants. Each shrub costs \$42 and requires 16 ft^2 of space. Each perennial plant costs \$6 and requires 8 ft^2 of space. You plan to spend a total of \$270. How many of each type of plant should you buy to fill the garden?

$42x + 6y = 270$

$16x + 8y = 160$

Assignment

- P152: 12, 14, 18, 24, 28, 32, 35, 38, 41, 44, 47, 56, 59 – 61, 70, 72, 76, 78
- #59 – 61 require a graphing calculator

Chapter 3.3

Graphing and Solving Systems of Linear Inequalities

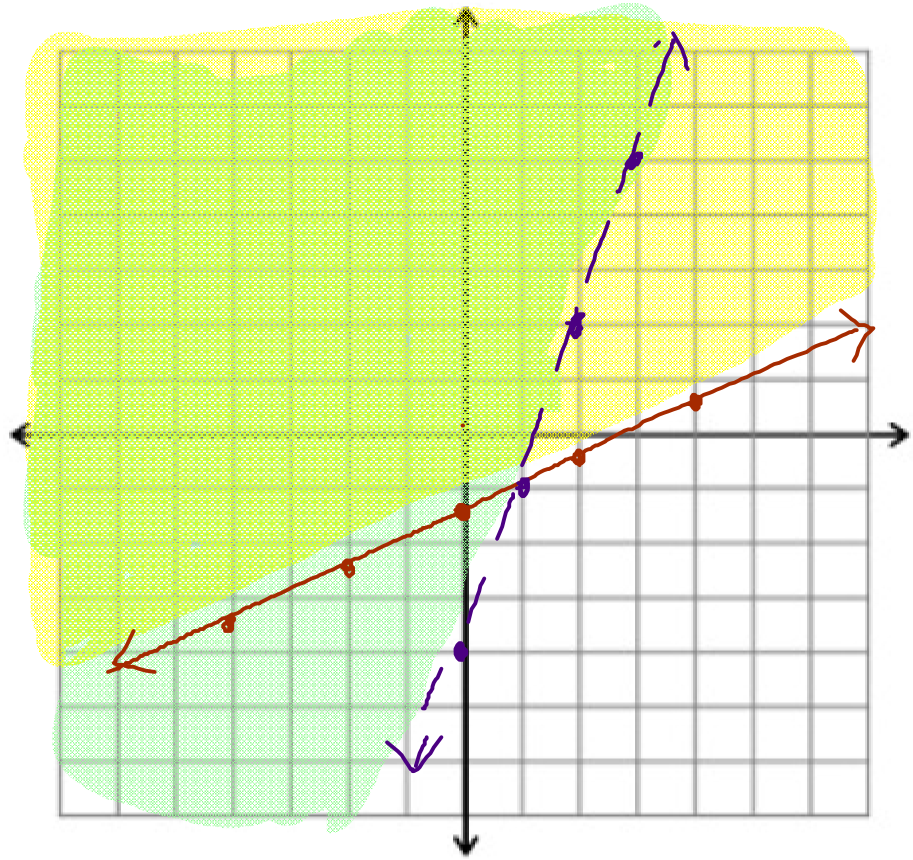
System of Inequalities

- System of Linear Inequalities
  - Two or more inequalities in two variables
- Solution
  - Any ordered pair that works on both inequalities
- Graph
  - The overlapping regions of both inequalities is the graph of all the ordered pairs that work in both
- To graph:
  - Completely graph each inequality, one at a time
    - When shading, shade lightly and in one direction

Example

- Graph the system:

$$x - 2y \le 3$$
$$y > 3x - 4$$



Example

- Graph the system:

$$x \le 0$$

$$y \ge 0$$

$0 \ge -2$ T

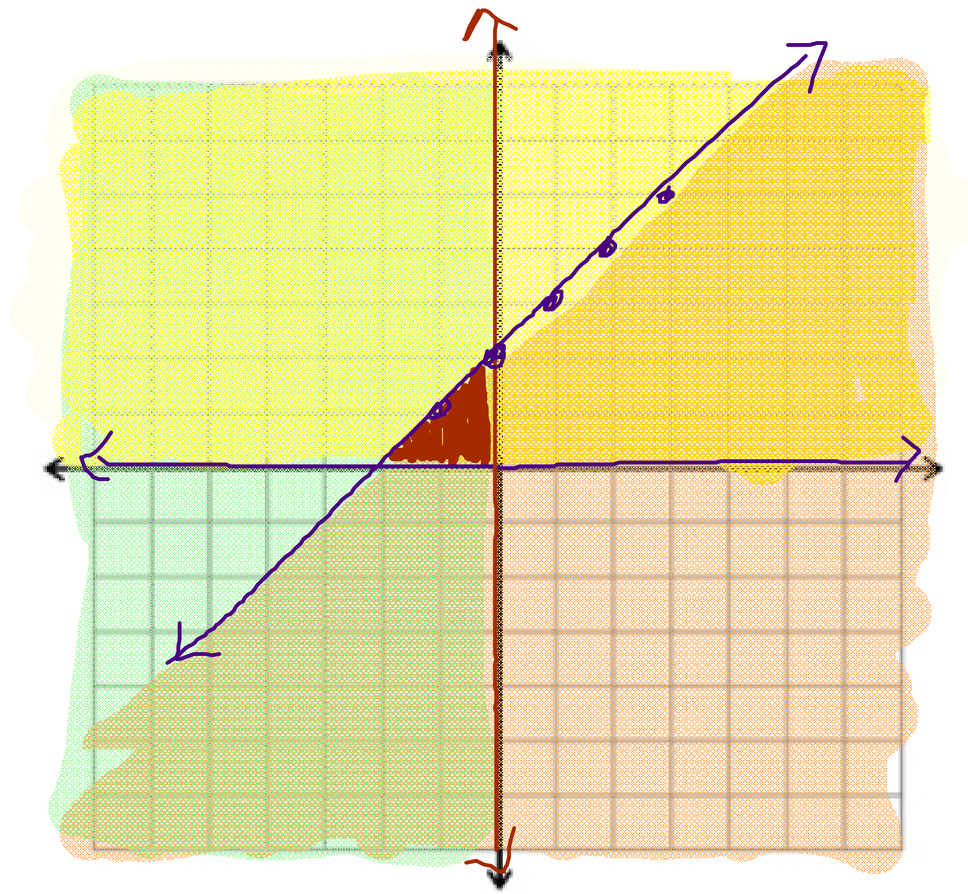
$$x - y \ge -2$$

$-x \qquad -x$

$-y = -x - 2$

$\overline{-1} \quad \overline{-1} \quad \overline{-1}$

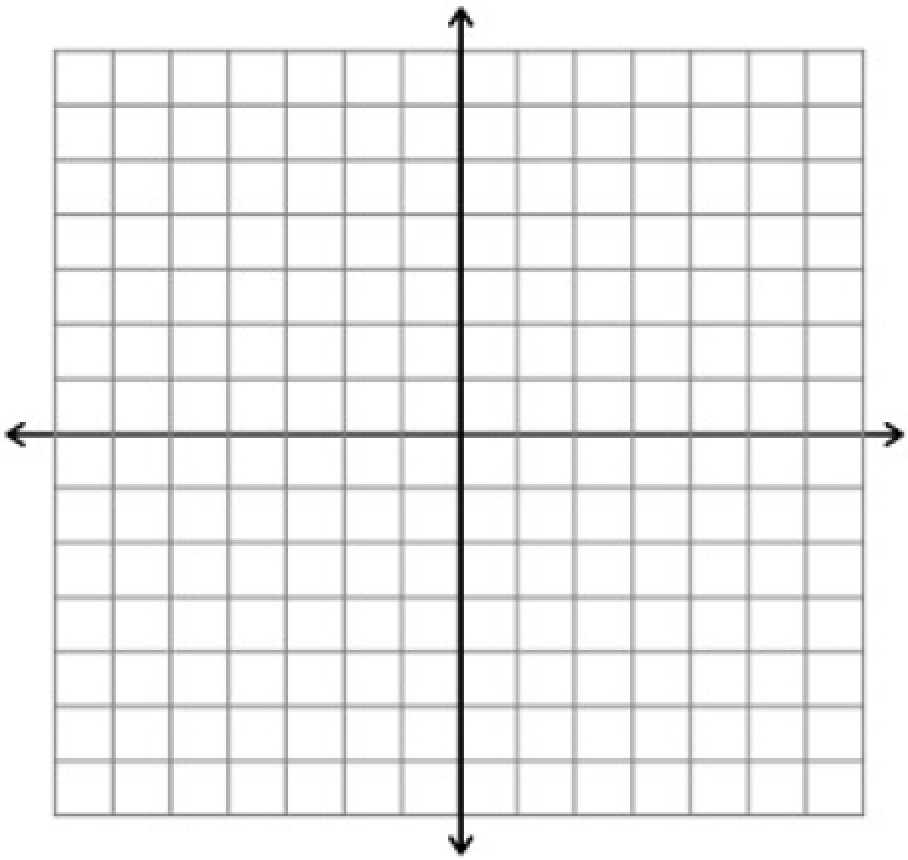
$y = x + 2$



Example

- Graph the system:

$$x \geq 0$$
$$y > 2x - 1$$
$$y \leq 2x + 3$$



Example

- At one college each class has between 20 and 140 students. From past data on attendance, it is expected that anywhere from 75% to 95% of students attend class any one day.
  - Write and graph a system of linear inequalities that describes the information.

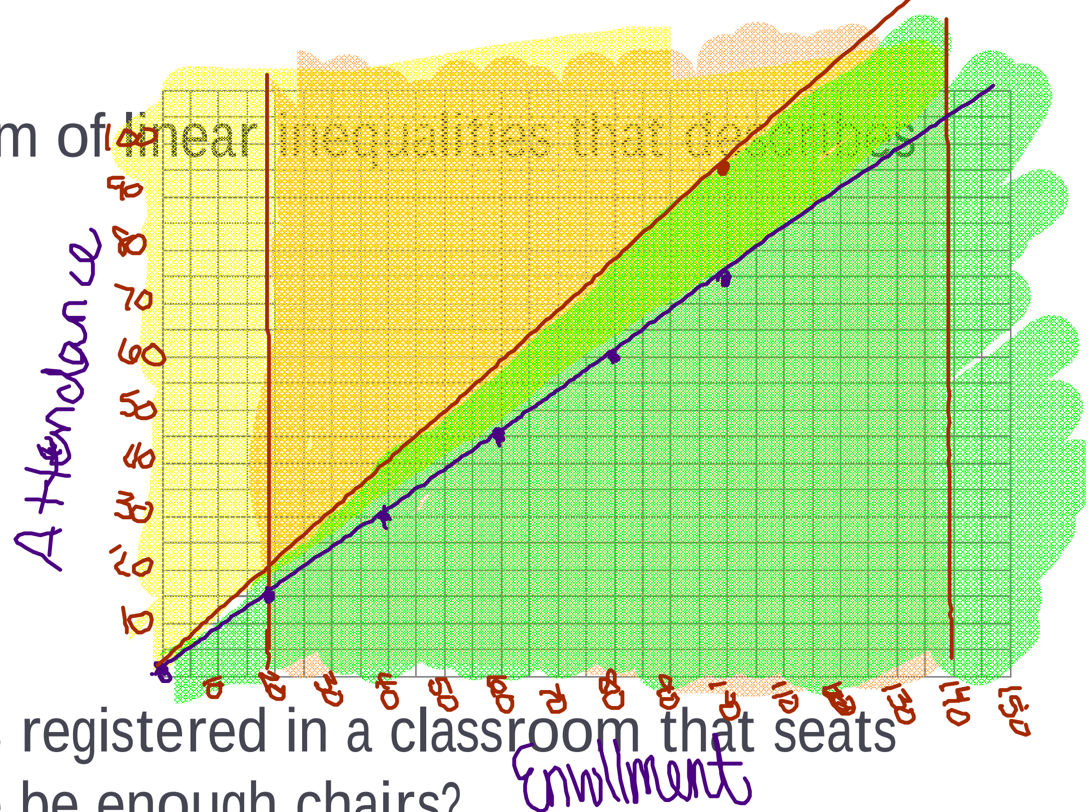
$20 \le x \le 140$

$x \ge 20$

$x \le 140$

$y \ge .75x$

$y \le .95x$



  - A class has 120 students registered in a classroom that seats 110 students. Will there be enough chairs?

yes

Assignment

- P159: 12 – 14, 21 – 26, 27 – 48 x 3s, 52 – 54, 62, 74, 77
- #52 requires graphing calculator

Warm-Up Oct. 5

- Solve the linear system:
  - $x - 2y = 5$
  - $-x + y = -1$

- Daily Agenda:
  - Grade assignments
  - Quiz
  - 3.4 notes / assignment

Chapter 3.4

Linear Programming

Linear Programming

- Optimization: means finding the maximum or minimum value of some quantity
  - Linear Programming is one type of optimization
- Linear Programming: the process of optimizing a linear objective function subject to a system of linear inequalities call constraints
  - The graph of the constraints is the feasible region
  - If an objective function has a maximum or minimum, it must occur at a vertex of the feasible region
    - It will have both a maximum and minimum if the region is bounded (P163 for picture)

(2,8) C = -2 + 3(8) = 22   (5,0) C = -5

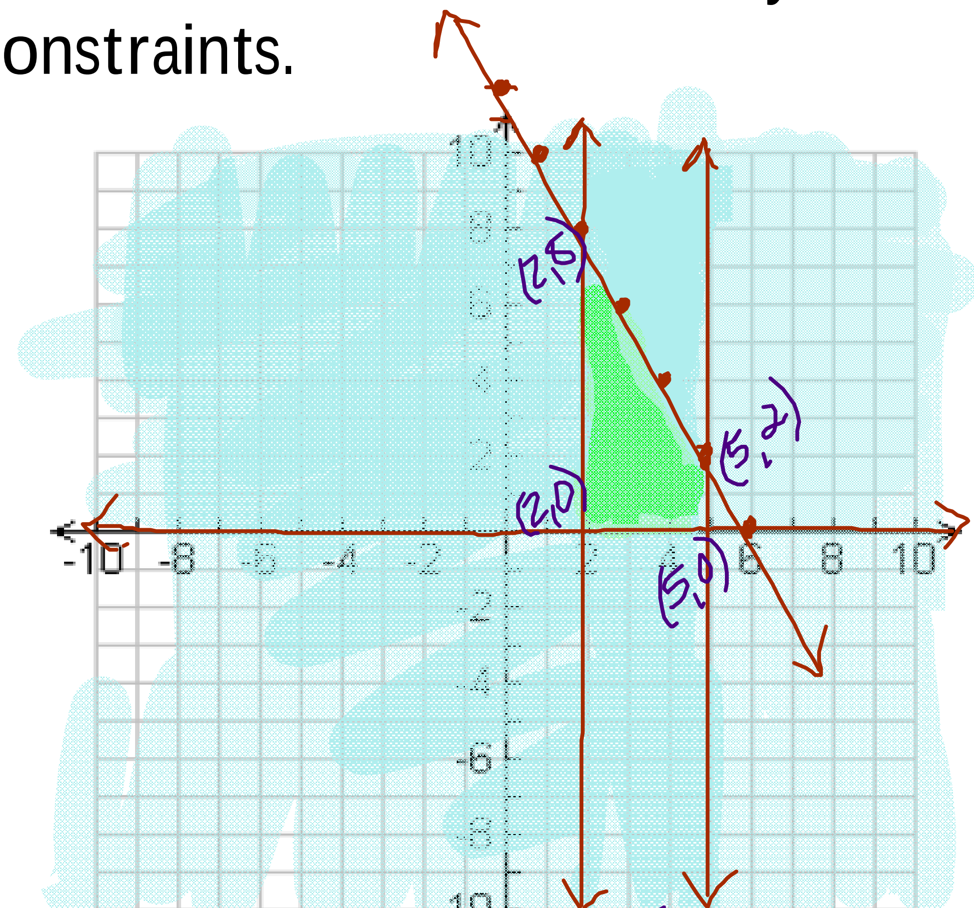
Example

(2,0) C = -2 + 3(0) = -2   (5,2) C = 1

- Find the minimum and maximum value of C = -x + 3y subject to the following constraints.

1. Graph the constraints

$$x \geq 2$$
$$x \leq 5$$
$$y \geq 0$$
$$y \leq -2x + 12$$



2. Find the vertices
3. Plug each vertex into objective function

Min = -5
Max = 22

4. Results are the minimum and maximum C.

Example

- Find the minimum and maximum value of C = x + 5y subject to the following constraints.

$$x \ge 0$$

$$y \le 2x + 2$$

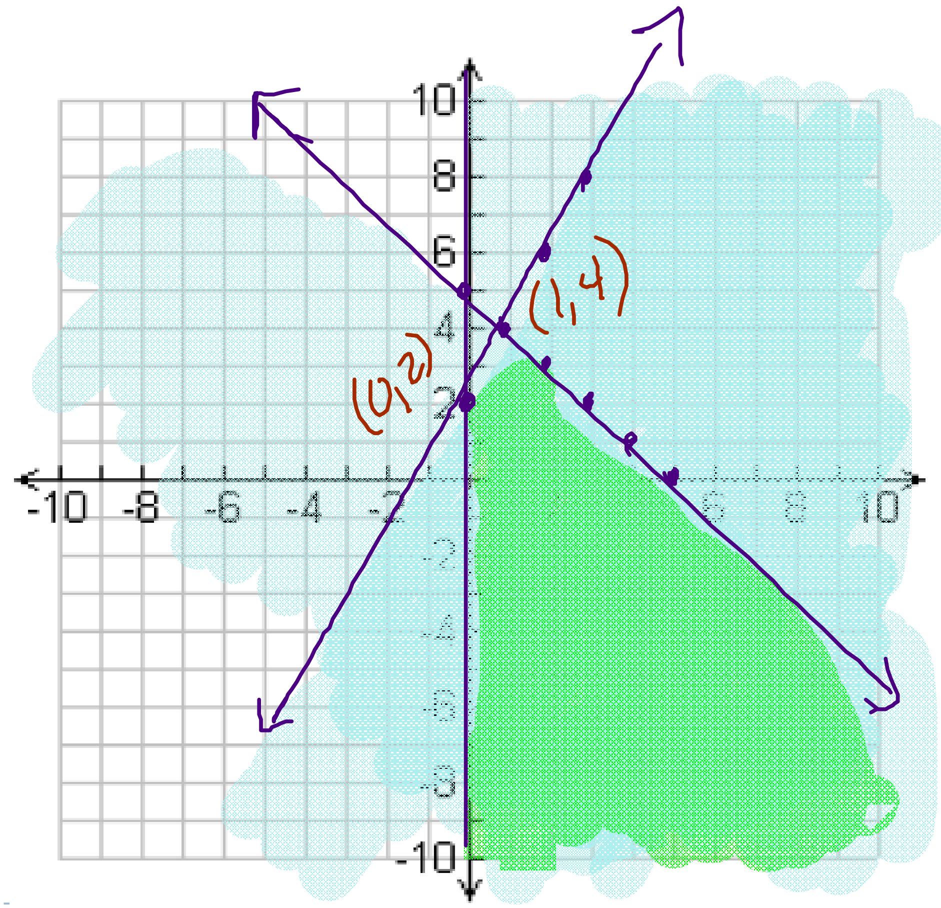
$$5 \ge x + y$$

y = -x+5

(0,2) C= 10

(1,4) C=21

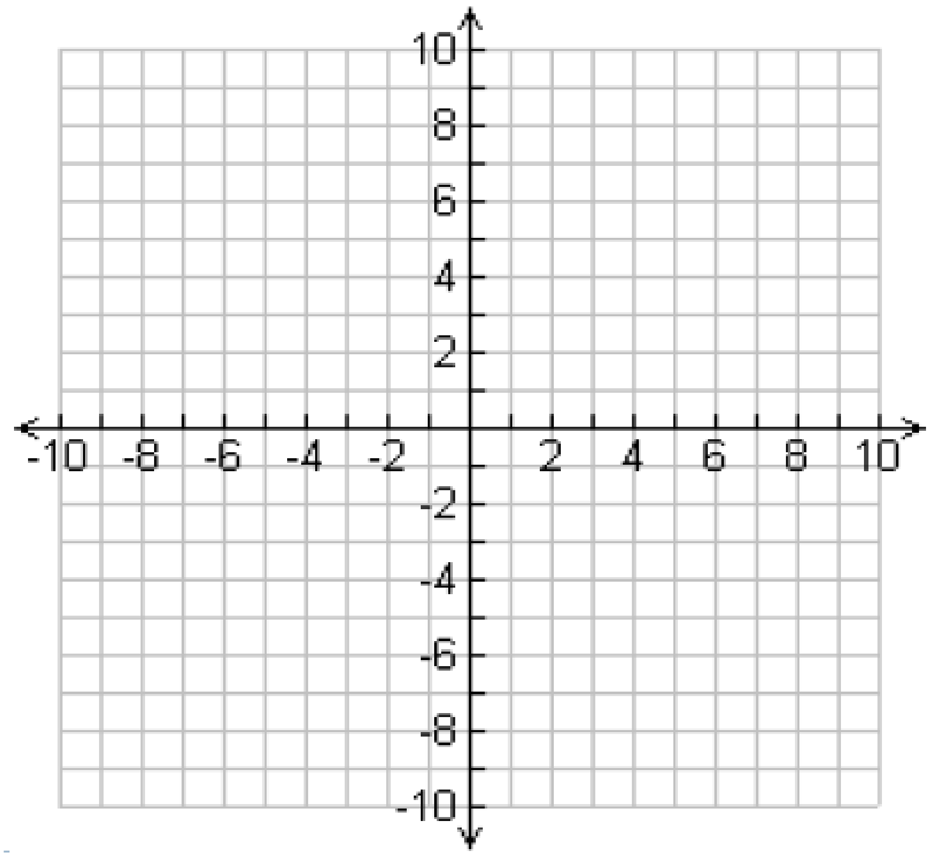
max=21
min=none



Example

- Find the minimum and maximum value of C = 2x – y subject to the following constraints.

$$x \geq 0$$
$$y \geq x + 2$$
$$y \leq -x + 6$$



Example

- A furniture manufacturer makes chairs and sofas from prepackaged parts. The table gives the number of packages of wood parts, stuffing, and material required for each chair or sofa. The packages are delivered weekly and the manufacturer has room to store 1300 packages of wood parts, 2000 packages of stuffing, and 800 packages of fabric. The manufacturer earns $200 per chair and $350 per sofa. How many chairs and sofas should they make each week to maximize profit?

Material	Chair	Sofa
Wood	2 boxes	3 boxes
Stuffing	4 boxes	3 boxes
Fabric	1 box	2 boxes

$P = 200x + 350y$

Ex. Cont'd

Material	Chair, x	Sofa, y
Wood	2 boxes	3 boxes
Stuffing	4 boxes	3 boxes
Fabric	1 box	2 boxes

- Room to store 1300 packages of wood parts, 2000 packages of stuffing, and 800 packages of fabric. The manufacturer earns \$200 per chair and \$350 per sofa. How many chairs and sofas should they make each week to maximize profit?

$2x + 3y \le 1300$

$y = -\frac{2}{3}x + 433\frac{1}{3}$

$4x + 3y \le 2000$

$y = -\frac{4}{3}x + 666\frac{2}{3}$

$x + 2y \le 800$

$y = -\frac{1}{2}x + 400$

$x \ge 0$

$y \ge 0$

Assignment

- P166: 12, 16, 18, 20, 24, 34 – 41

Warm-Up Oct. 10

- Find the x- and y-intercepts of graph of the equation: $2x + 4y = 20$

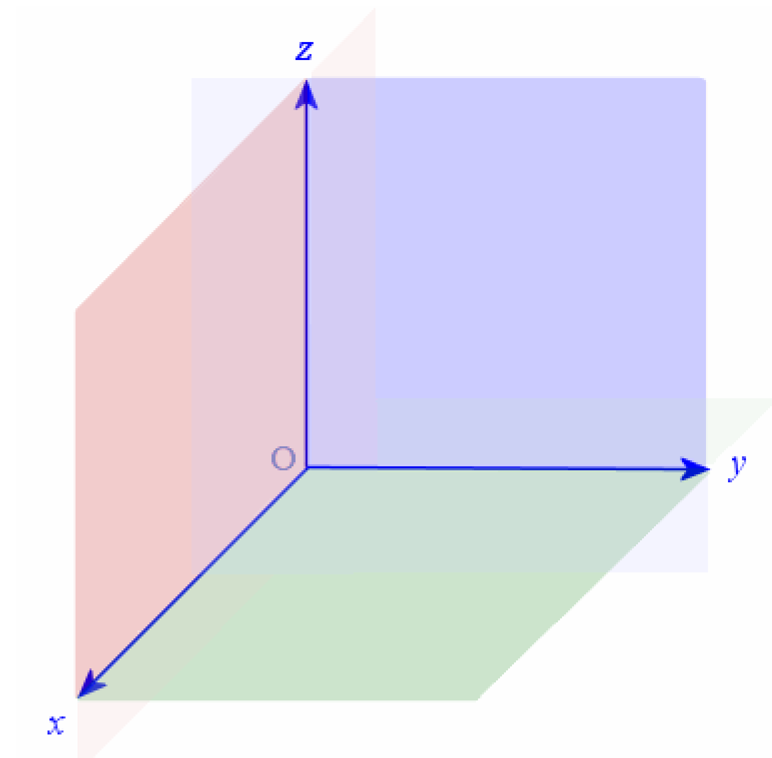
- Daily Agenda:
  - Grade assignment
  - Get quizzes back
  - 3.5 notes / assignment
  - 3.6 notes / assignment

Chapter 3.5

Graphing Linear Equations in Three Variables

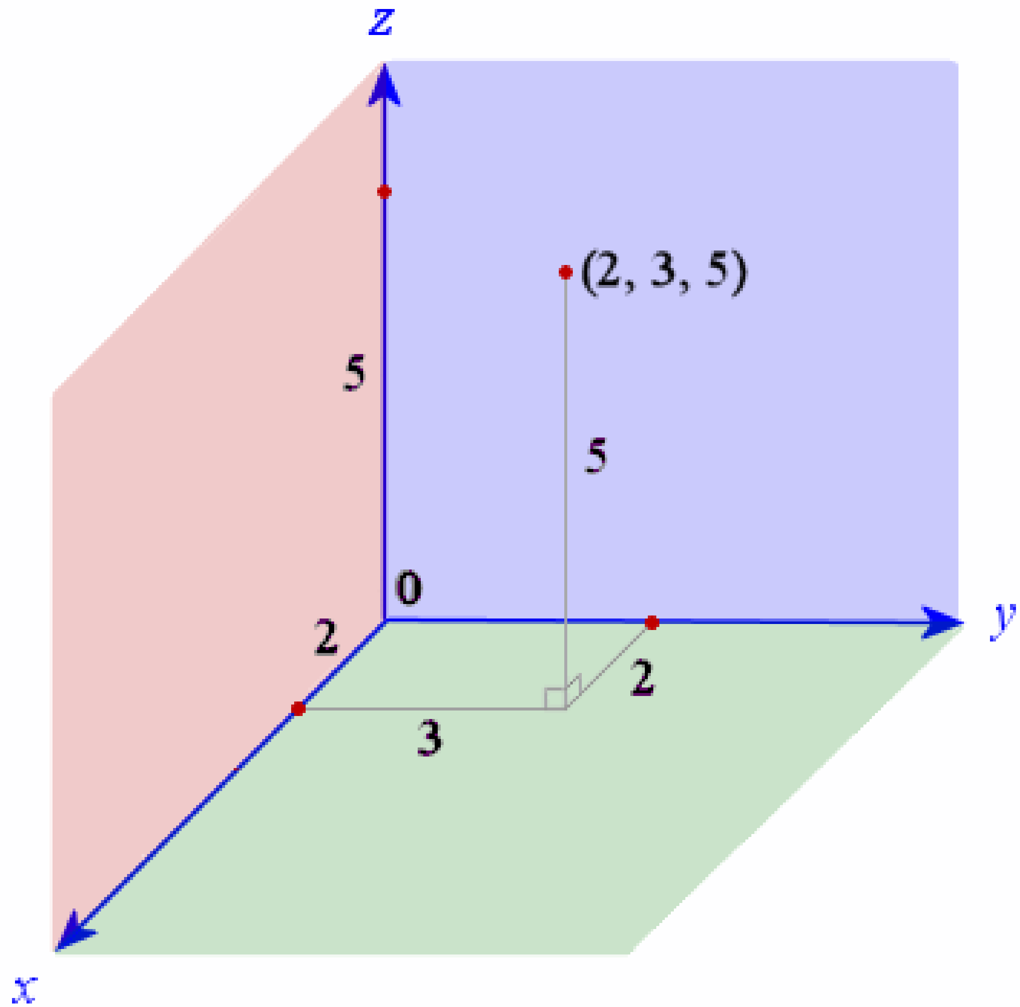
Three-Dimensional Coordinate System

- xy-coordinate plane is in the horizontal plane
- z-axis is vertical
- Ordered triple: (x, y, z)
- Octants: the eight spaces the three axes make
  - First octant is where all are positive



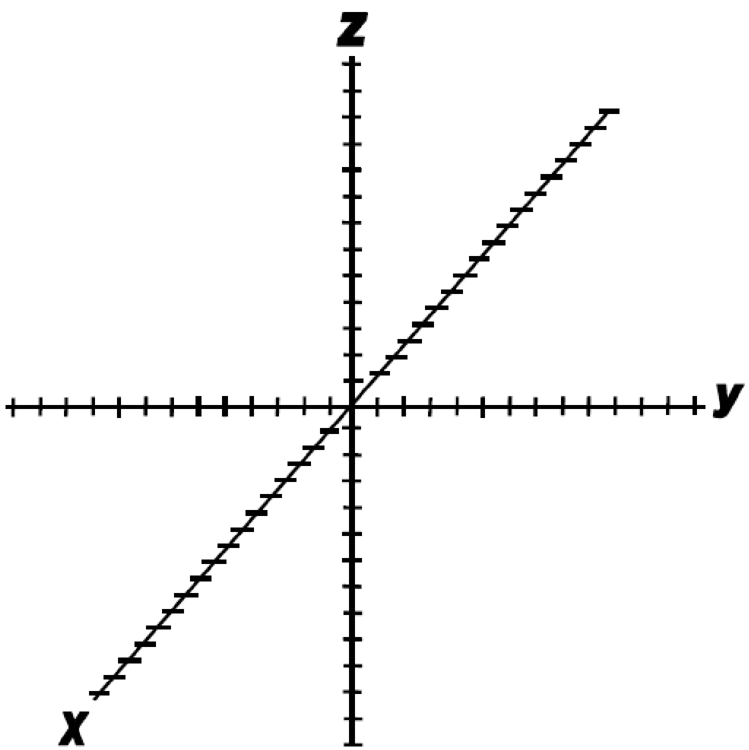
Example of Plotting in 3D

- Plot (2, 3, 5)

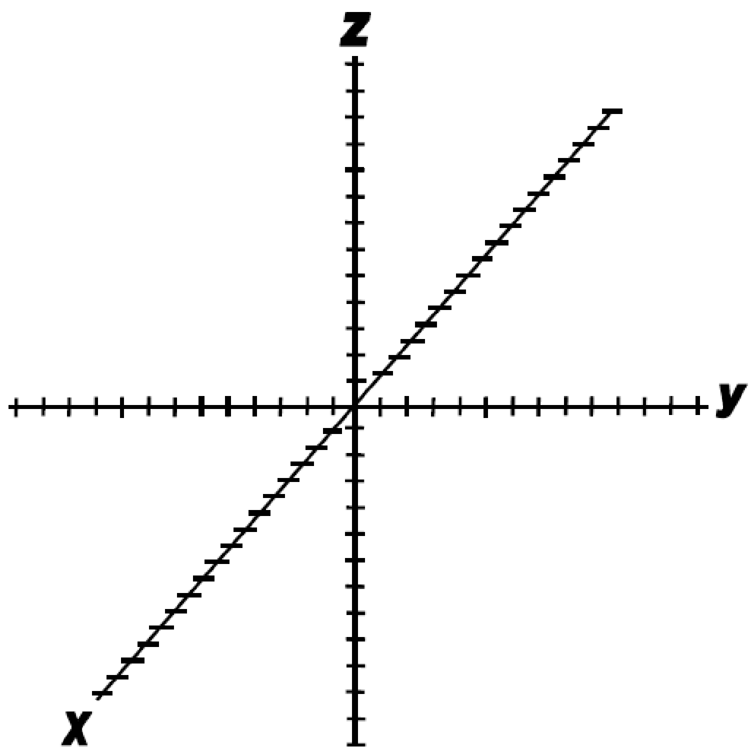


Example

- Plot (3, -1, -5)

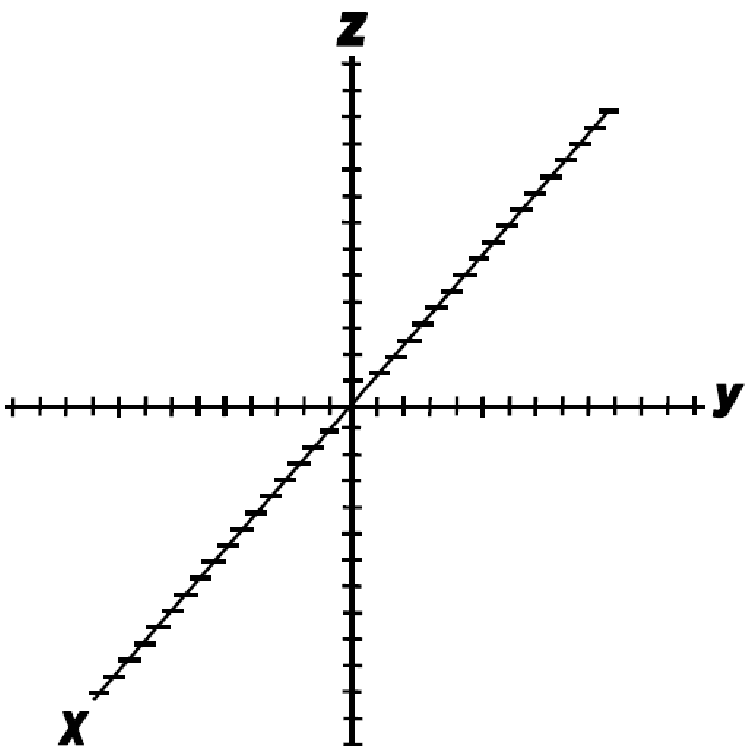


- Plot (-5, 3, 4)

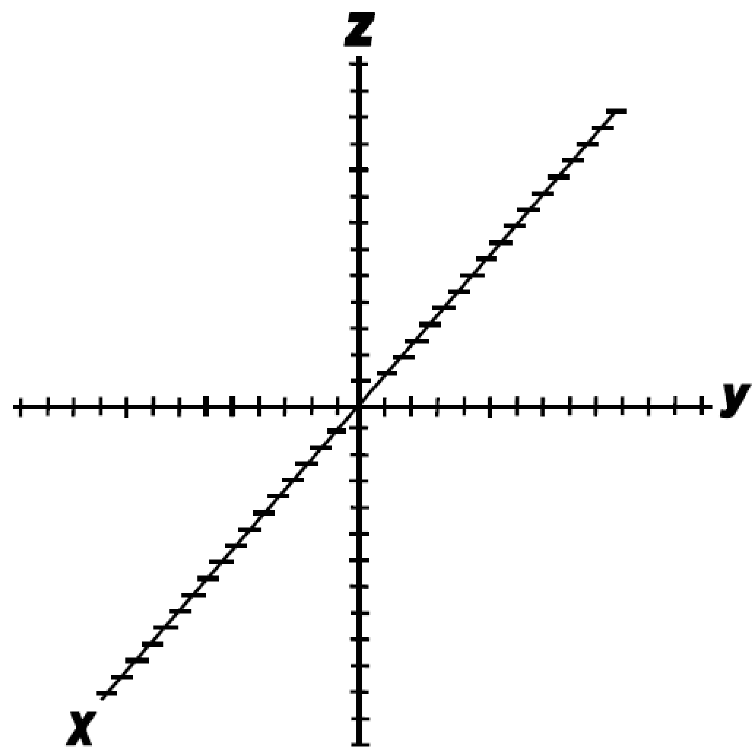


Example

- Plot (1, 3, -2)



- Plot (-2, -3, 4)

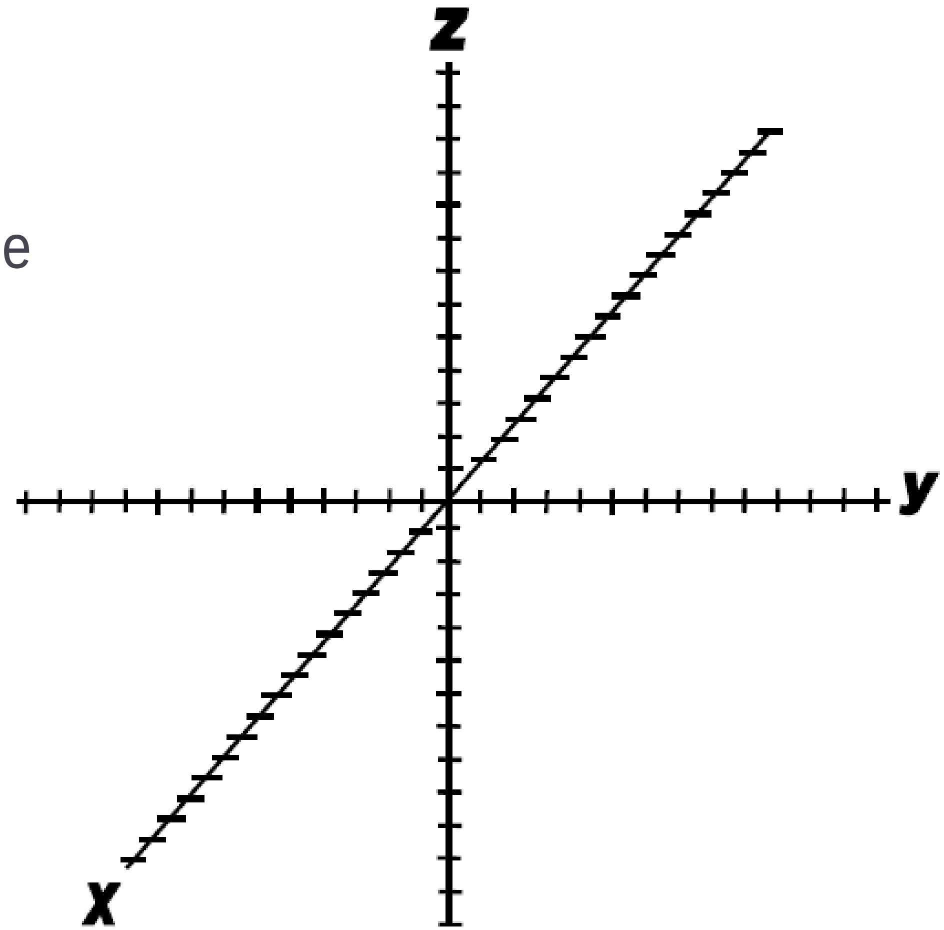


Linear Equation in Three Variables

- ax +by + cz = d
  - a, b, and c cannot ALL be zero
  - Ordered triple, (x, y, z), is a solution
  - The graph is the graph of all of the solutions
  - The graph of a linear equation in three variables is a plane

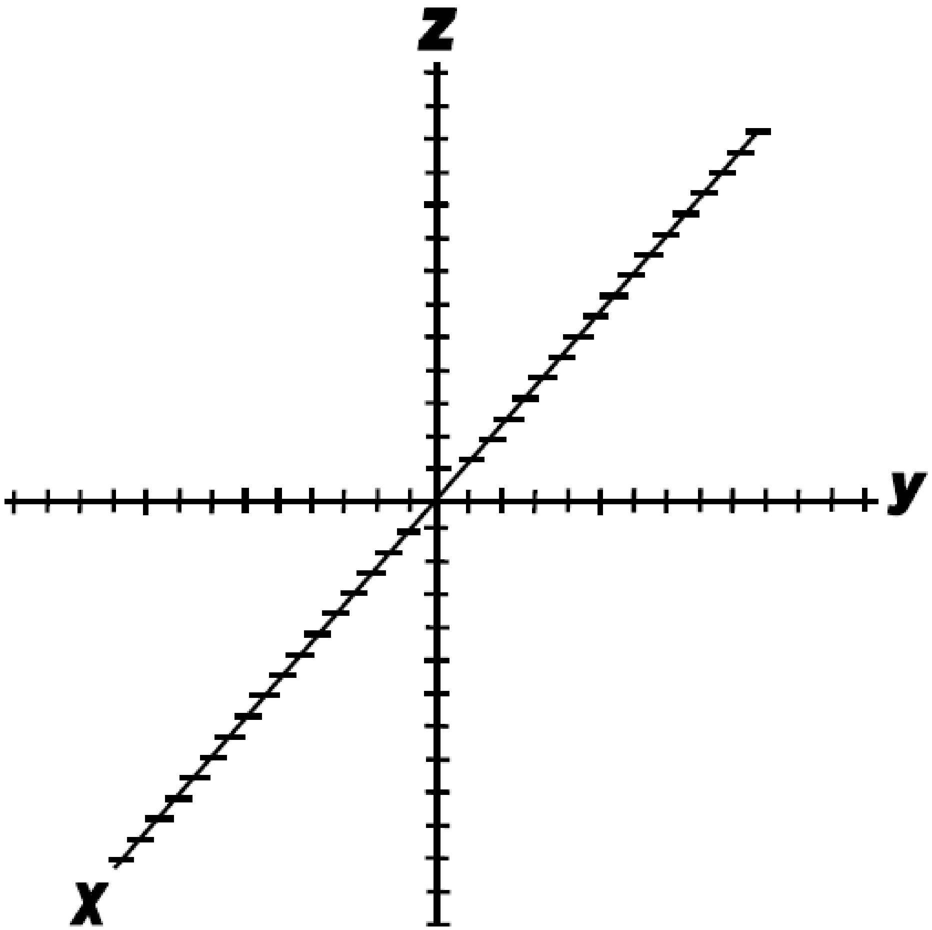
Example

- Graph 3x – 12y + 5z = 30
  - Graph using the intercepts
  - And only graph a triangular region of answers that will be in the plane of solutions



Example

- Graph $x - 2y + 2z = 6$



Function of two variables

- f(x,y)
- To write a linear equation in x, y, and z as a function of two variables, solve for z and replace z with f(x, y).

Example

- Write $3x - 12y + 5z = 30$ as a function of x and y.

- Evaluate the function when $x = -2$ and $y = 2$.

Example

- Write $x - 2y + 2z = 6$ as a function of x and y.

- Evaluate $f(0, -2)$.

Example

- You are packing a food supply crate for a canoe trip. The crate weighs 12 lb and it will be filled with boxes of granola bars, each weighing 1.5 lb, and boxes of macaroni each weighing 0.75 lb. Write a model for the total weight of the crate as a function of the number of boxes of granola bars and macaroni.

- How much will a crate with 15 boxes of granola bars and 25 boxes of macaroni weigh?

Assignment

- P173: 18 – 24 even, 27 – 36 x 3s, 38 – 44 even, 48, 50, 67

Warm-Up Oct. 12

- Solve the linear system $\begin{cases} x - y = -5 \\ x + 3y = 11 \end{cases}$

- Daily Agenda:
  - Grade assignment
  - 3.6 notes / assignment

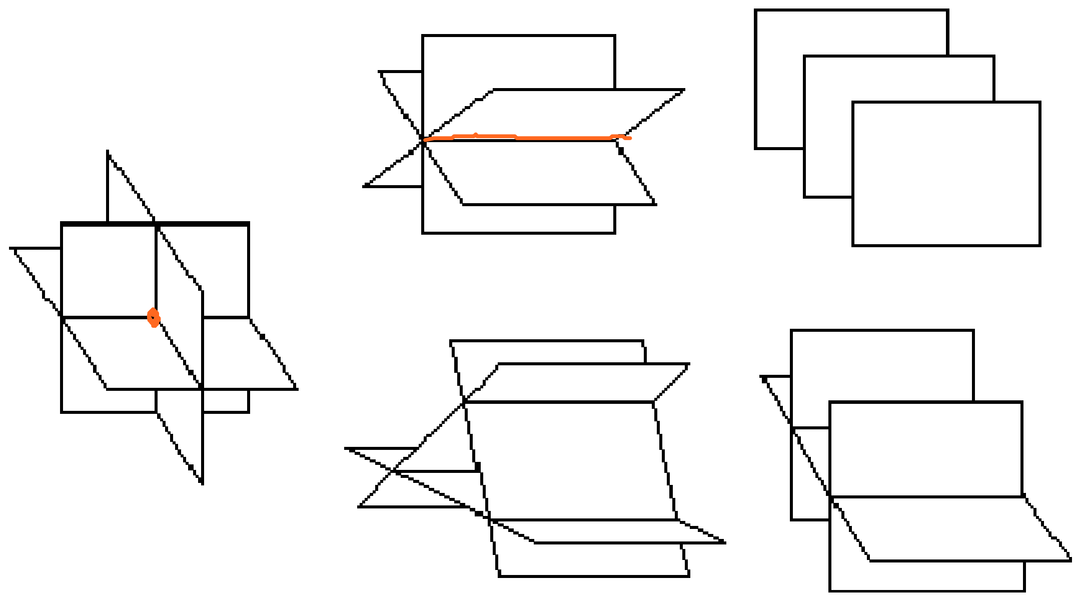
  - Review Tuesday / Test Thursday

Chapter 3.6

Solving Systems of Linear Equations in Three Variables

System of Three Linear Equations

- Solution: an ordered triple that is a solution to all three equations
- Three types of solutions:



Solving a System of Three Variables using Linear Combination

1. Choose a set of two original equations and eliminate one variable.
2. Choose a different set of two original equations and eliminate the SAME variable.
3. Solve the two new equations, that now each have two variables, for both variables.
4. Substitute the two known values back into one of the original equations and solve for the third value.

Example

- Solve the system:

$x + 3y - z = -11$

$2x + y + z = 1$

$5x - 2y + 3z = 21$

Example

- Solve the system:

  $2x + 3y + 7z = -3$

  $x - 6y + z = -4$

  $-x - 3y + 8z = 1$

Example

- Solve the system:

  $-x + 2y + z = 3$

  $2x + 2y + z = 5$

  $4x + 4y + 2z = 6$

Example

- Solve the system:

(1) $-2x + 4y + z = 1$

(2) $3x - 3y - z = 2$

(3) $5x - y - z = 8$

(1) $-2x + 4y + z = 1$
(2) $+\ 3x - 3y - z = 2$
(4) $x + y = 3$

(1) $-2x + 4y + z = 1$
(3) $+\ 5x - y - z = 8$
(5) $3x + 3y = 9$

(4) $-3(x + y = 3)$
(5) $3x + 3y = 9$

$-3x - 3y = -9$
$+\ 3x + 3y = 9$
$0 = 0$

$y = -x + 3$

$-2x + 4(-x + 3) + z = 1$
$-2x - 4x + 12 + z = 1$
$-6x + 12 + z = 1$
$+6x - 12 \quad -12 + 6x$
$z = 6x - 11$

$(x, -x + 3, 6x - 11)$

Example

- Solve the system:

  $x + y + 2z = 10$

  $-x + 2y + z = 5$

  $-x + 4y + 3z = 15$

Example

$x = 3(y+z)$

$x = 3y + 3z$

330 reg.
46 prem.
64 elite

- A theater group sold a total of 440 tickets for $3940. Each regular ticket costs $5, each premium ticket costs $15, and each elite ticket costs $25. The number of regular tickets was three times the number of premium and elite tickets combined. How many of each type of ticket were sold?

① $5x + 15y + 25z = 3940$

② $3(x + y + z = 440)$

③ $5(x - 3y - 3z = 0)$

① $5x + 15y + 25z = 3940$

③ $+\ 5x - 15y - 15z = 0$

④ $10x + 10z = 3940$

② $3x + 3y + 3z = 1320$

③ $x - 3y - 3z = 0$

⑤ $4x = 1320$

$x = 330$

$z = 64$

$y = 46$

Assignment

- P181: 12, 14, 16, 26, 28, 30, 36, 44, 58, 60

- #58, 60: number line