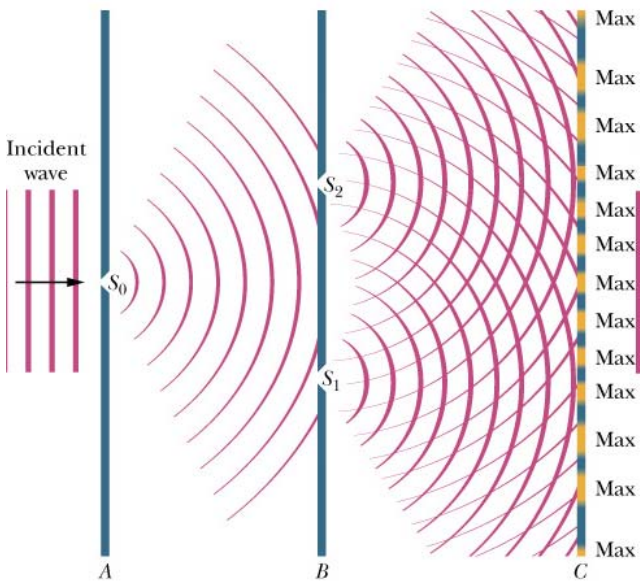
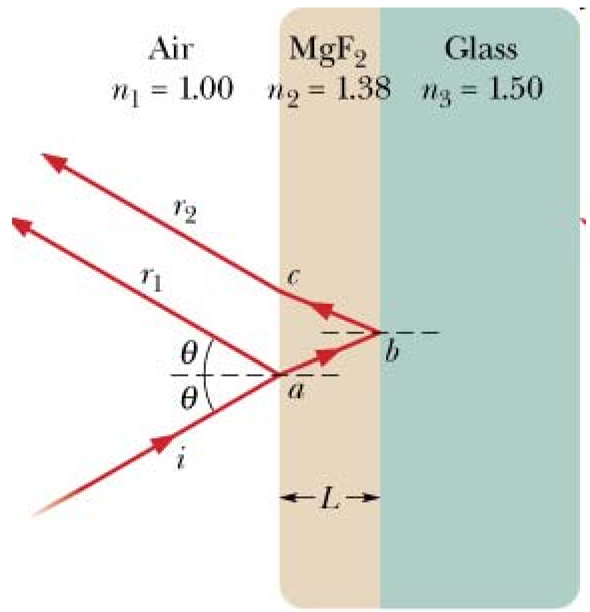
Wave Interference and Diffraction

Part 1: Introduction, Double Slt





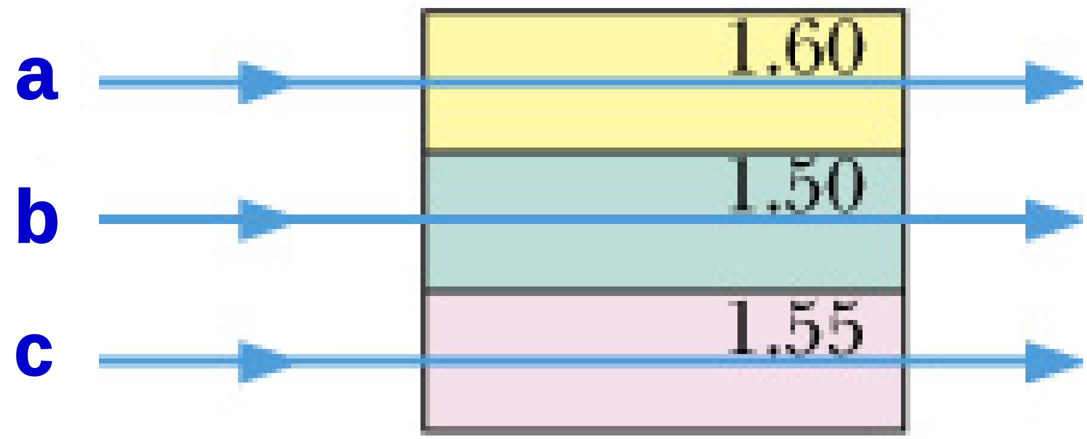
PHY 2049
Physics 2 with Calculus

Quiz

➔Three beams of light, a, b and c, of the same wavelength are sent through 3 layers of plastic with the indices of refraction as shown. Which material has the *most* number of wavelengths inside the material?

- 1. a
- 2. b
- 3. c
- 4. Same for all

Shortest wavelength in material, so fits most # of waves



Need to Understand Light as Wave!

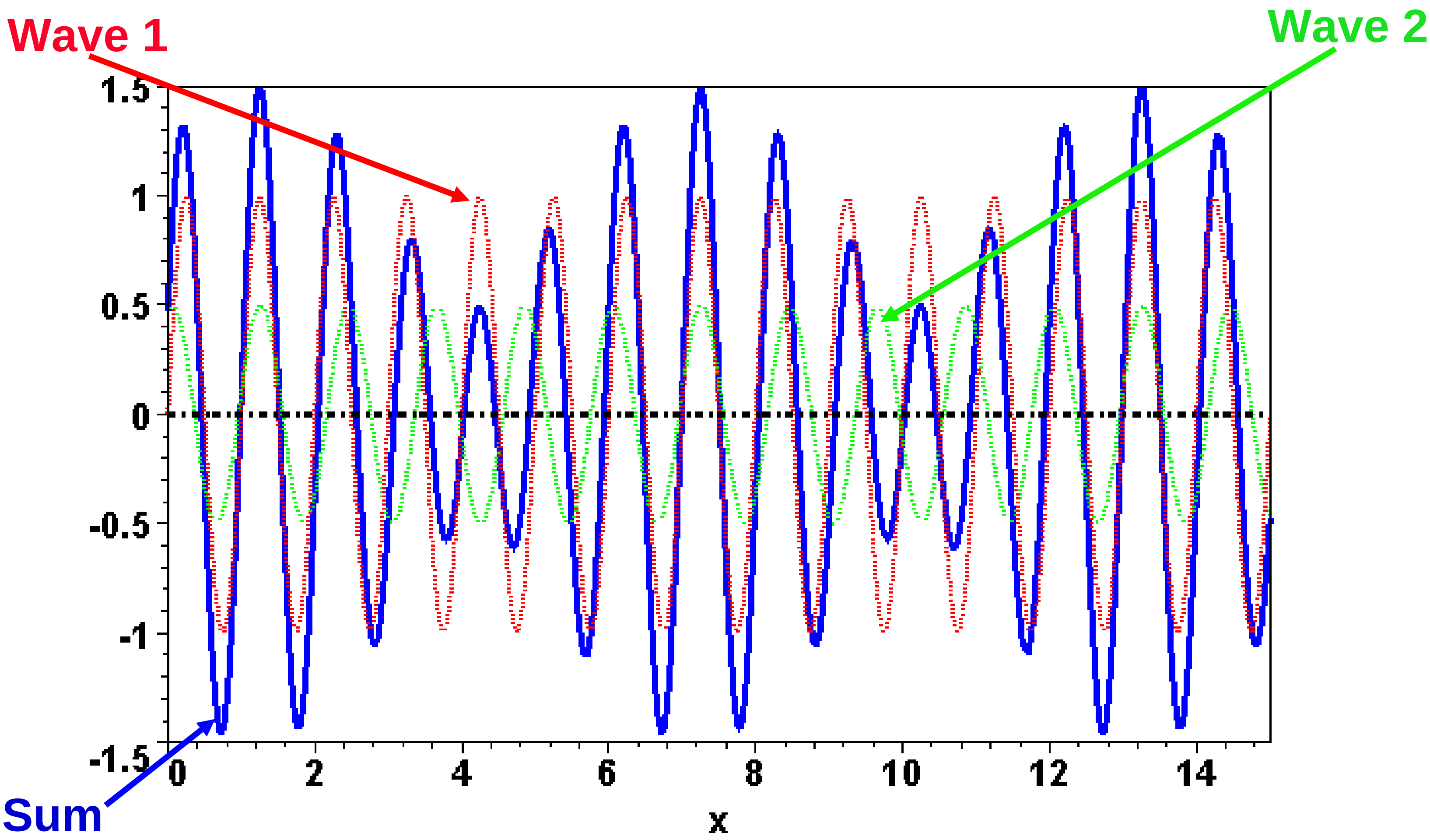
- (You already have read this material)
- Index of refraction
  - Speed of EM wave in medium: $c_n = c / n$
  - Wavelength of light: $\lambda_n = \lambda / n$
- Propagation of light: Huygens principle (36-2)
  - Explains reflection and refraction
  - Explains interference (from superposition)
  - Explains diffraction (spreading of light around barrier)

Interference as a Wave Phenomenon

- ➔ Interference of light waves
  - ◆ Caused by superposition of waves
  - ◆ Intensity can increase or decrease!
  - ◆ Contrast with particle model of light
- ➔ Effects and applications
  - ◆ Double slit
  - ◆ Single slit
  - ◆ Diffraction gratings
  - ◆ Anti-reflective coatings on lenses
  - ◆ Highly reflective coatings for mirrors
  - ◆ Iridescent coatings on insects
  - ◆ Colors on thin bubbles
  - ◆ Interferometry with multiple telescopes

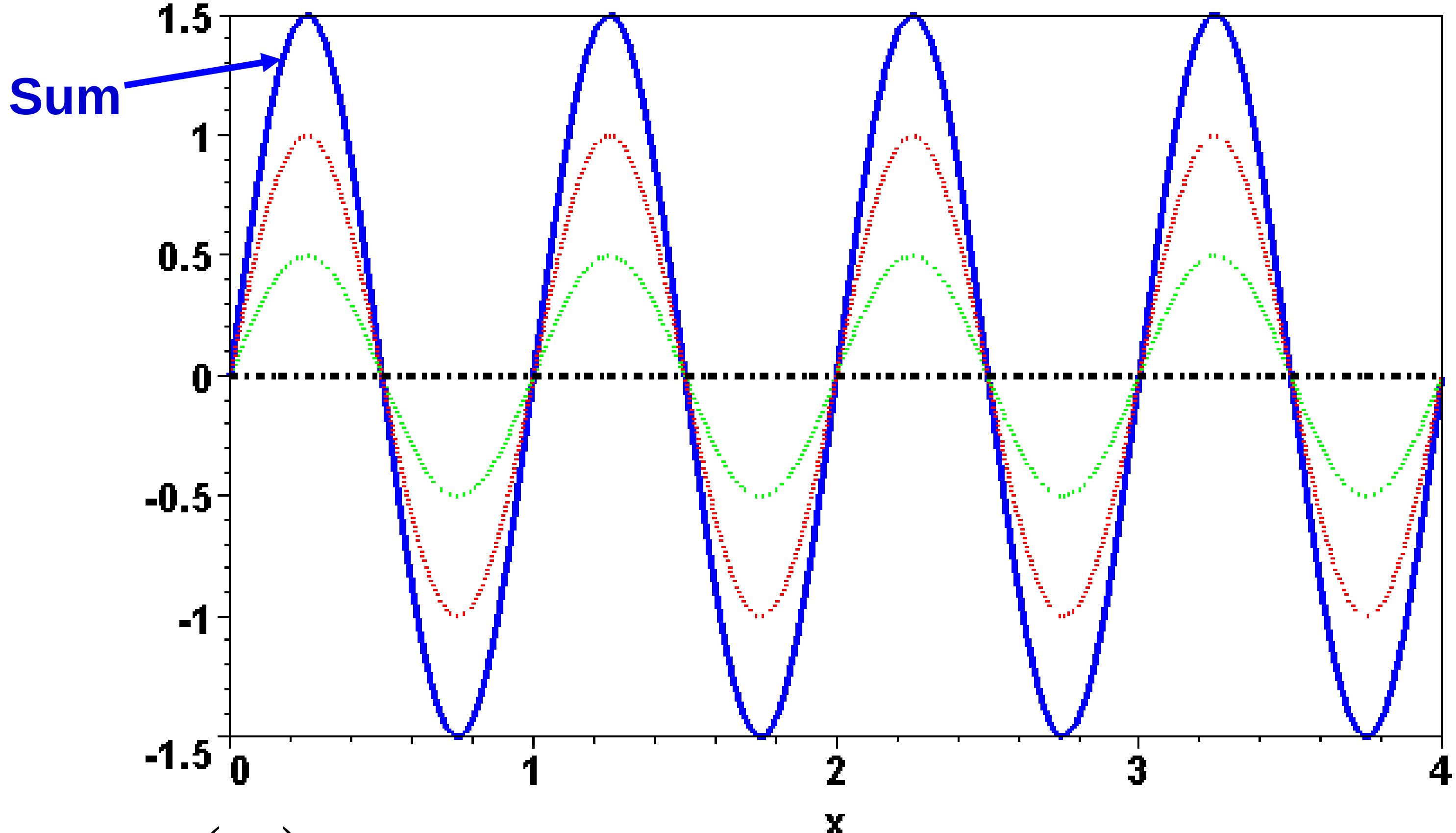
Interference from Wave Superposition

Basic rule: Add displacement at every point



Constructive Interference

- ➔Same wavelength, phase difference = 0°
- ➔Amplitude larger: Higher intensity

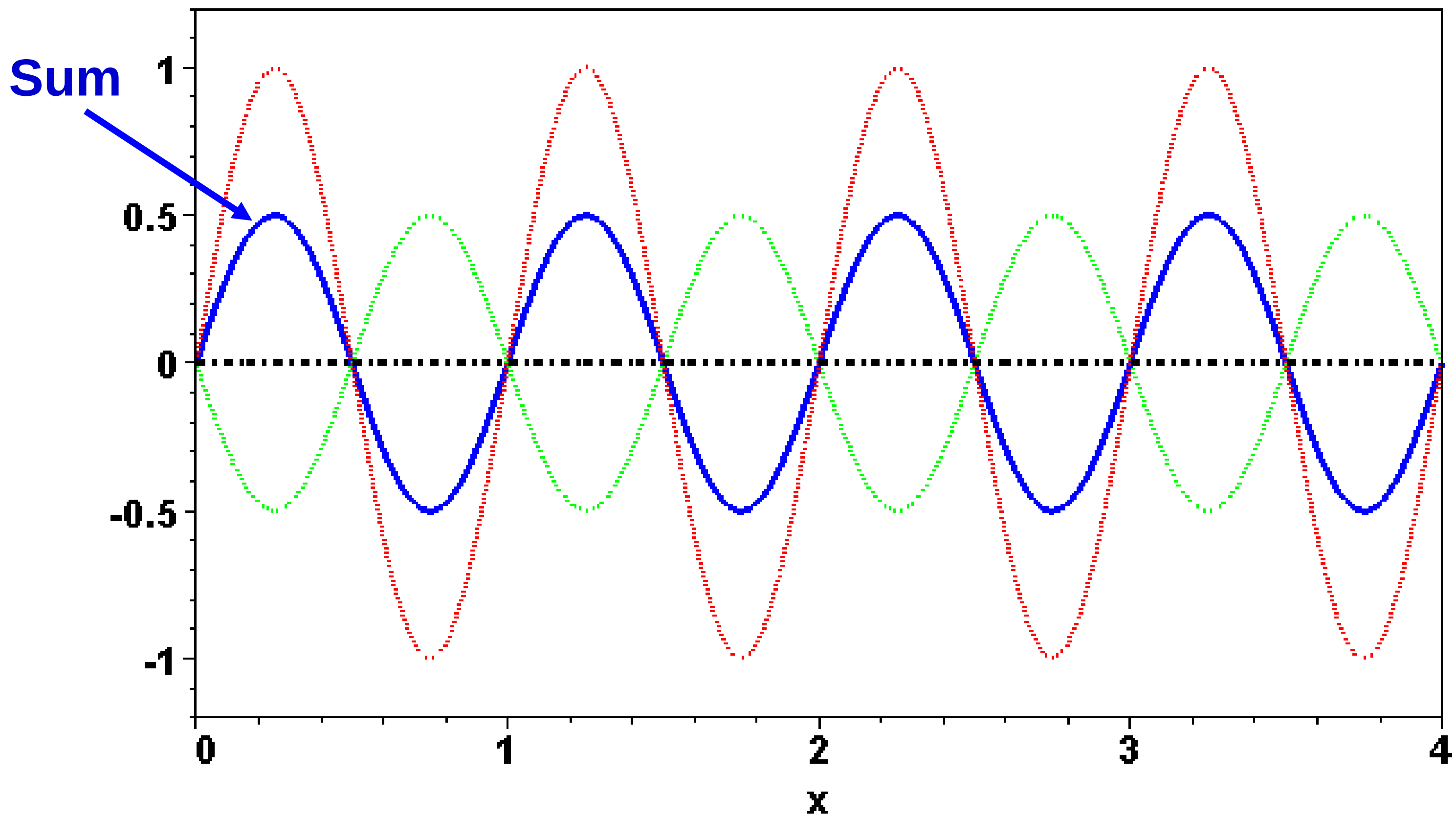


$$E(x) = \sin(kx) + 0.5\sin(kx) = 1.5\sin(kx)$$

Destructive Interference

➔Same wavelength, phase difference = 180° (1/2 λ)

➔Amplitude smaller: Lower intensity



$$E\left(x\right)=\sin(kx)+0.5\sin(kx+\pi)=0.5\sin(kx)$$

Examples

- ➔Two waves, same λ, with amplitudes 2A and A
  - ◆ Initial intensities 4I and I, respectively ($I \propto A^2$)
- ➔No interference
  - ◆ Combined intensity: $I_{new} = 4I + I = 5I$
- ➔Maximum constructive interference ($\phi = 0$)
  - ◆ New amplitude: $A_{new} = 2A + A = 3A$
  - ◆ New intensity: $I_{new} = 9I$
- ➔Maximum destructive interference ($\phi = \pi$)
  - ◆ New amplitude: $A_{new} = 2A - A = A$
  - ◆ New intensity: $I_{new} = I$

General Treatment of Interference

- ➔Most interference is partial
  - ◆Amplitudes for 2 waves are generally different
  - ◆Phase difference : $0 < \phi < 180°$

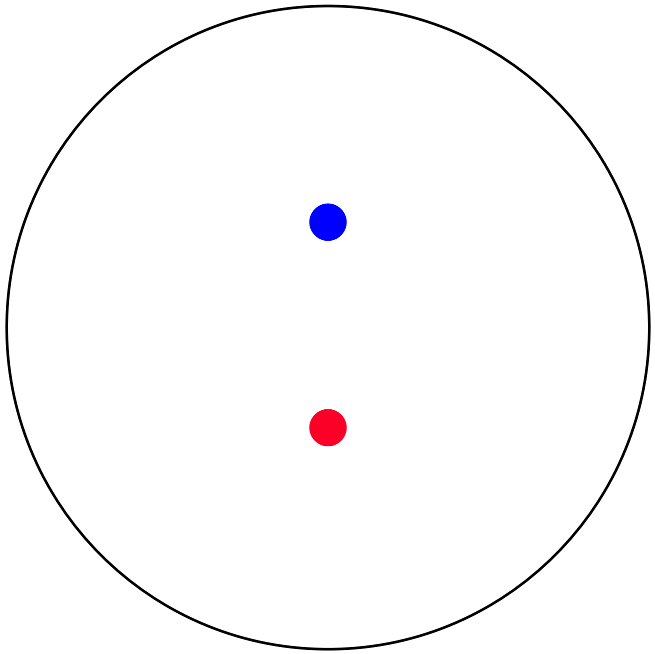
$$E(x,t) = E_1 \cos(kx - \omega t) + E_2 \cos(kx - \omega t + \phi)$$

- ➔Additional considerations
  - ◆Wavelengths can be different
  - ◆Multiple waves may interfere (e.g., diffraction grating)
  - ◆But easy to accommodate: just sum over all waves

$$E(x,t) = \sum_i E_i \cos(k_i x - \omega_i t + \phi_i)$$

Interference and Path Length

Two sources, spaced 3 wavelengths apart, emit waves with the same wavelength and phase. In how many places on the circle will the net intensity be a relative maximum?



Answer = 12

Can you see why?

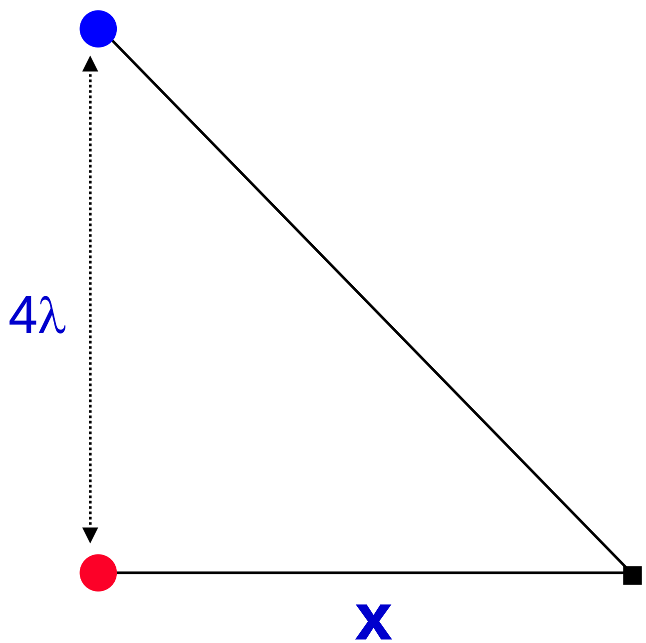
Hint: Start at far right and move counterclockwise towards top, noting path length changes.

Key idea: Path difference leads to phase difference

Interference and Path Length

➔ Two sources, separated by 4λ, emit waves at same wavelength and phase. Find relative minima on +x axis.

- ◆ Solution: path *difference* must be a half-multiple of λ



$$\Delta L = \sqrt{x^2 + (4\lambda)^2} - x = \left(n + \tfrac{1}{2}\right)\lambda$$

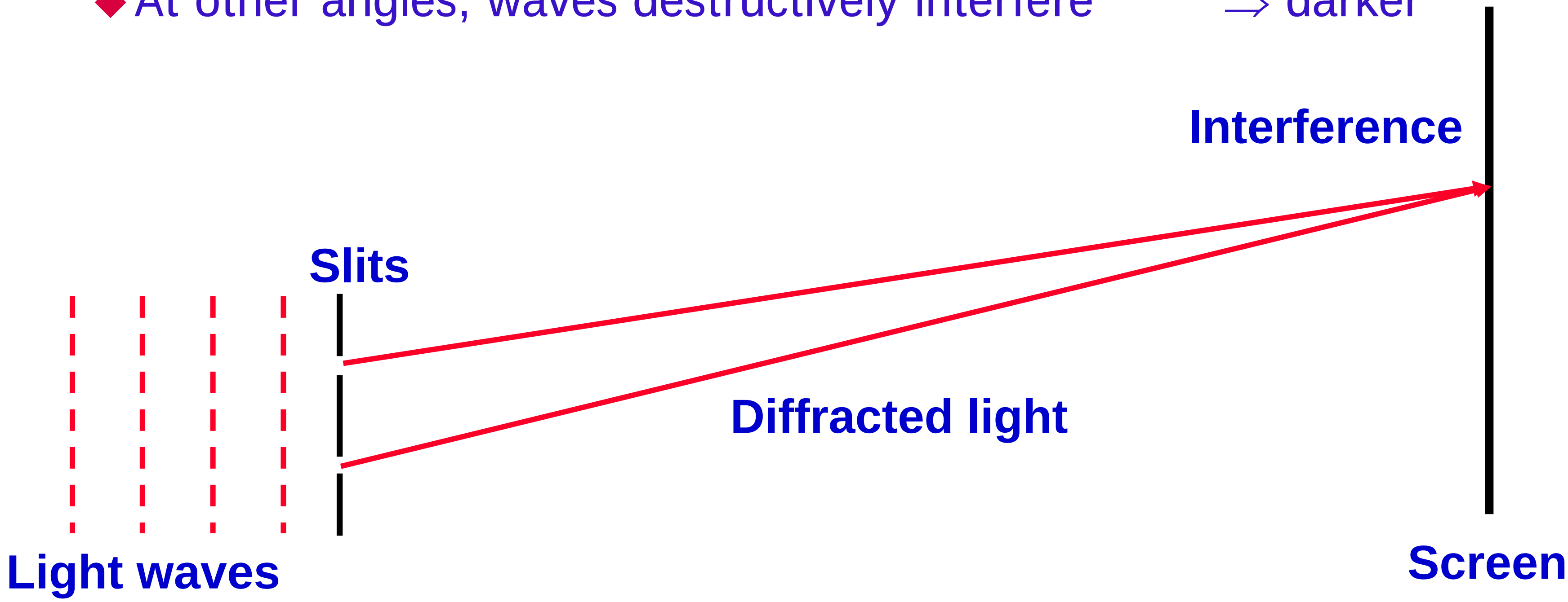
$$x = \frac{16 - \left(n + \tfrac{1}{2}\right)^2}{2n+1}\lambda$$

← **4 values**

$n = 0$	$x = 15.8\lambda$	$\Delta L = \lambda/2$
$n = 1$	$x = 4.58\lambda$	$\Delta L = 3\lambda/2$
$n = 2$	$x = 1.95\lambda$	$\Delta L = 5\lambda/2$
$n = 3$	$x = 0.54\lambda$	$\Delta L = 7\lambda/2$

Double Slit Interference

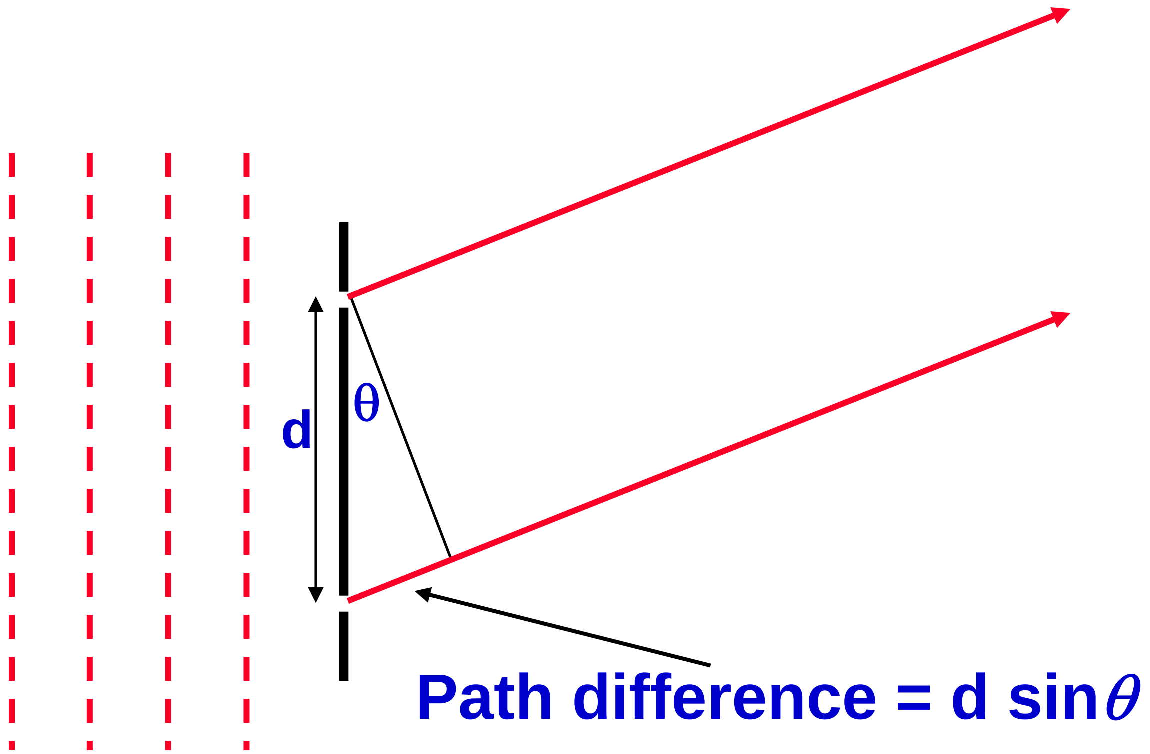
- Incident light
  - Light waves strike 2 narrow slits close together
  - Light goes through both slits, diffracts in all directions
- Interference
  - At certain angles, waves constructively interfere $\Rightarrow$ brighter
  - At other angles, waves destructively interfere $\Rightarrow$ darker



Basic Requirements for Two Slit Setup

- ➔ Light beam strikes normal to slits
- ➔ Light beam illuminates both slits equally
- ➔ Light beam is in phase at both slits: coherent
  - ◆ Young used small slit in front of 2 slits to get coherence
  - ◆ Modern versions use laser for coherence (much brighter)

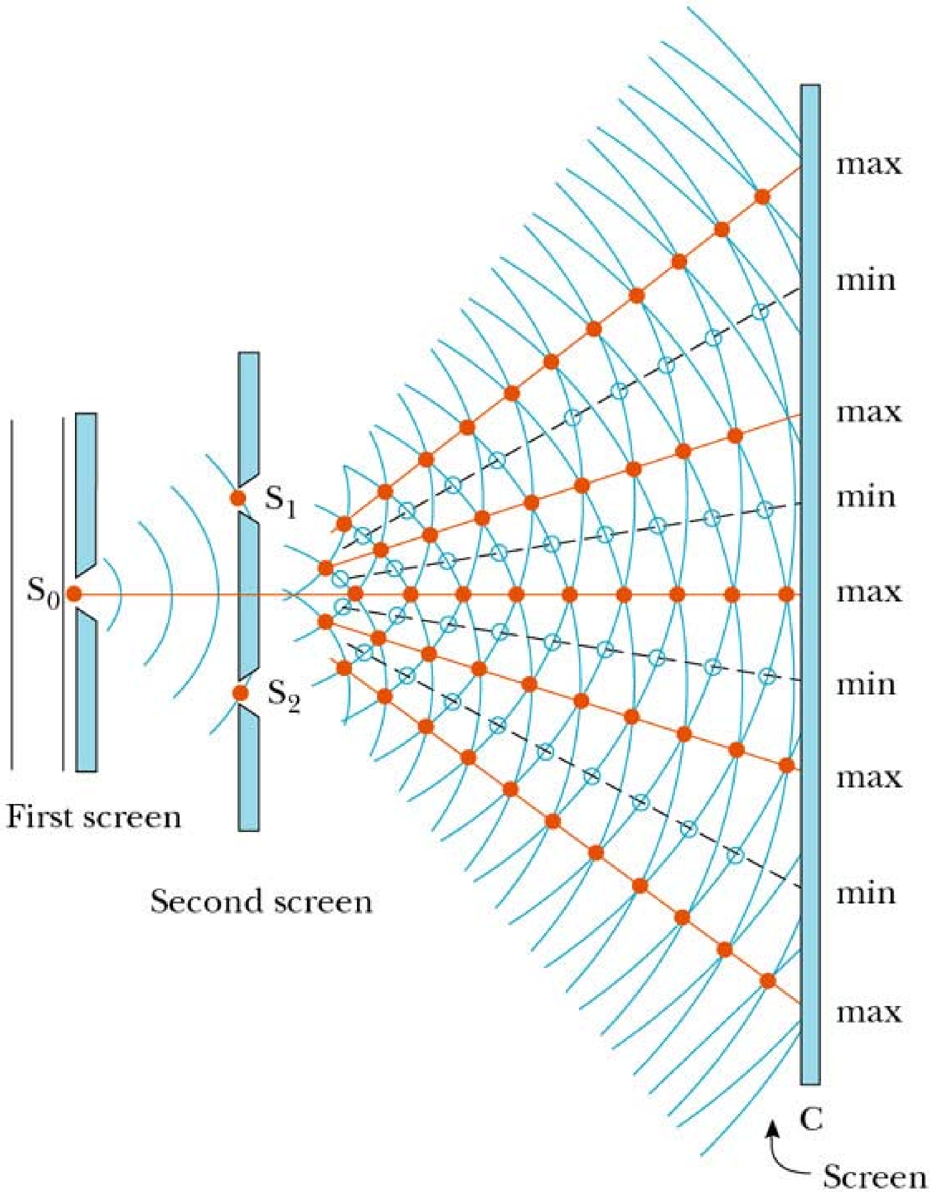
Two Slit Analysis



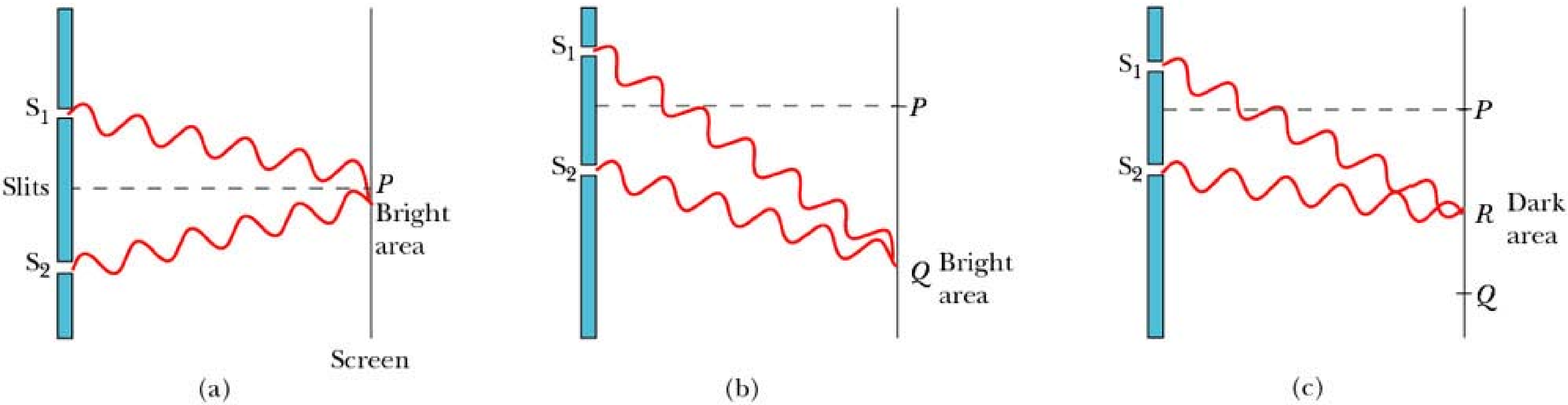
$$d \sin\theta = m\lambda \qquad \text{Maximum}$$

$$d \sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \qquad \text{Minimum}$$

Double Slit Intensity Pattern on Screen



Example of Double Slit Max and Min



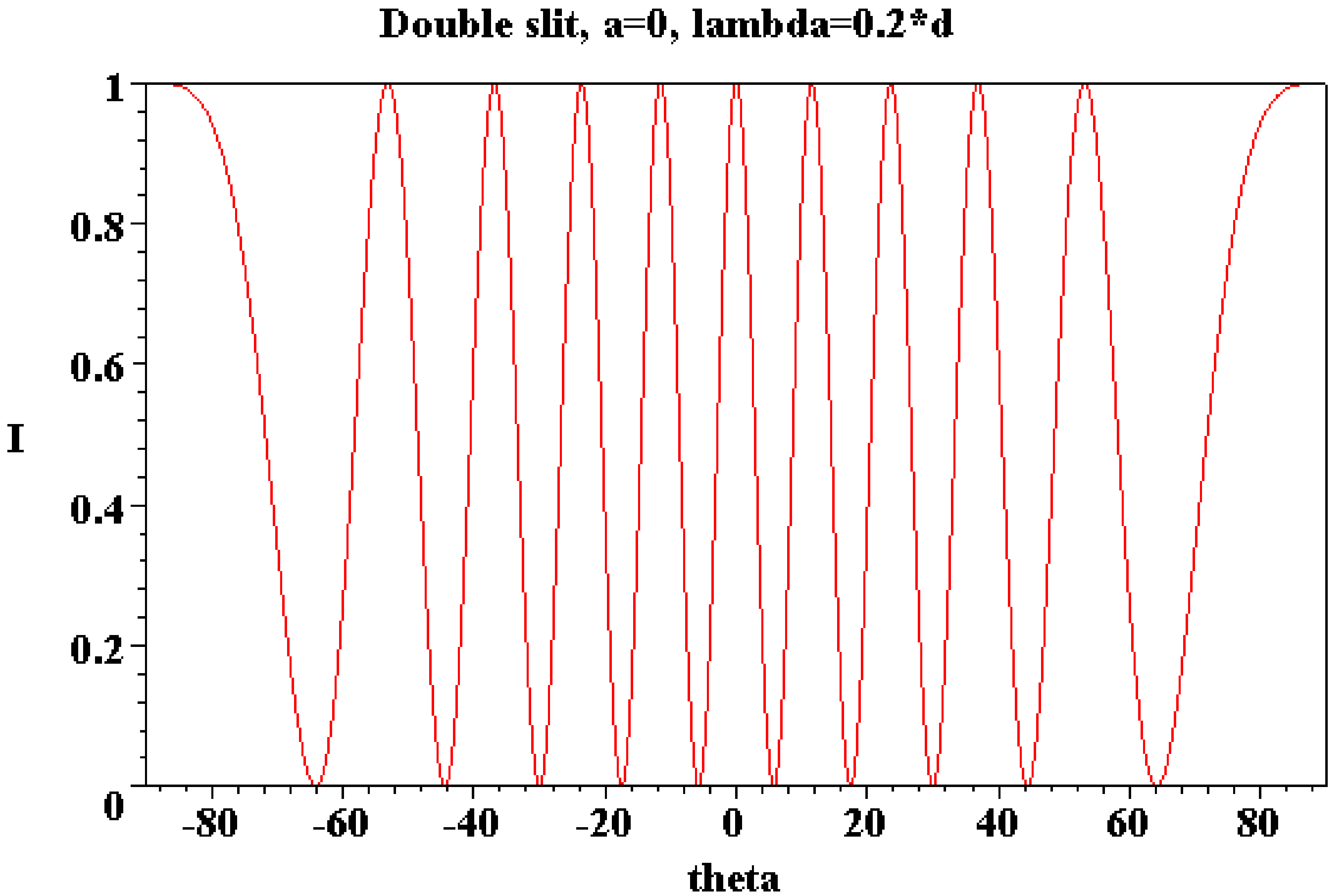
Example 1: $d = 5\lambda$

$$\text{Max} \quad \sin\theta = m(\lambda/d) = 0.2m$$

$$\text{Min} \quad \sin\theta = \left(m+\tfrac{1}{2}\right)(\lambda/d) = 0.2\left(m+\tfrac{1}{2}\right)$$

m	$\sin\theta_{max}$	θ_{max}	$\sin\theta_{min}$	θ_{min}
0	0	0	±0.1	±5.7
±1	±0.2	±11.5	±0.3	±17.5
±2	±0.4	±23.6	±0.5	±30
±3	±0.6	±36.9	±0.7	±44.4
±4	±0.8	±53.1	±0.9	±64.2
±5	±1.0	±90	±1.1	--

Intensity vs Angle for d = 5λ



Example 2: $d = 2.0\ \mu m$, $\lambda = 550$ nm

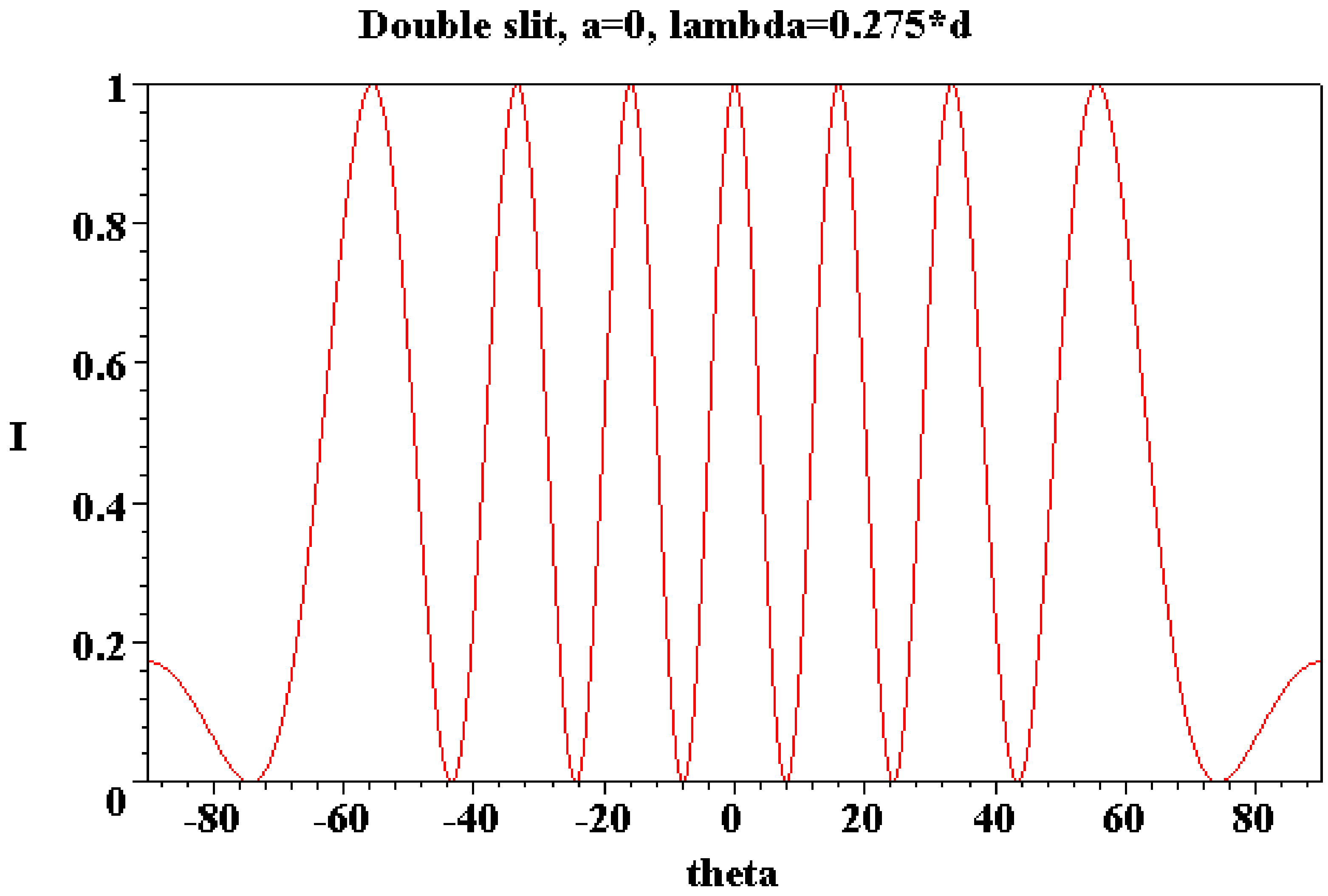
➔How many bright fringes? Where are they?

$$\sin\theta = m(\lambda / d) = 0.275m$$

➔m can equal 0, ±1, ±2, ±3 ⇒ 7 maxima

m = 0	$\sin\theta = 0$	$\theta = 0$
m = ±1	$\sin\theta = 0.275$	$\theta = 16.0°$
m = ±2	$\sin\theta = 0.55$	$\theta = 33.4°$
m = ±3	$\sin\theta = 0.825$	$\theta = 55.6°$

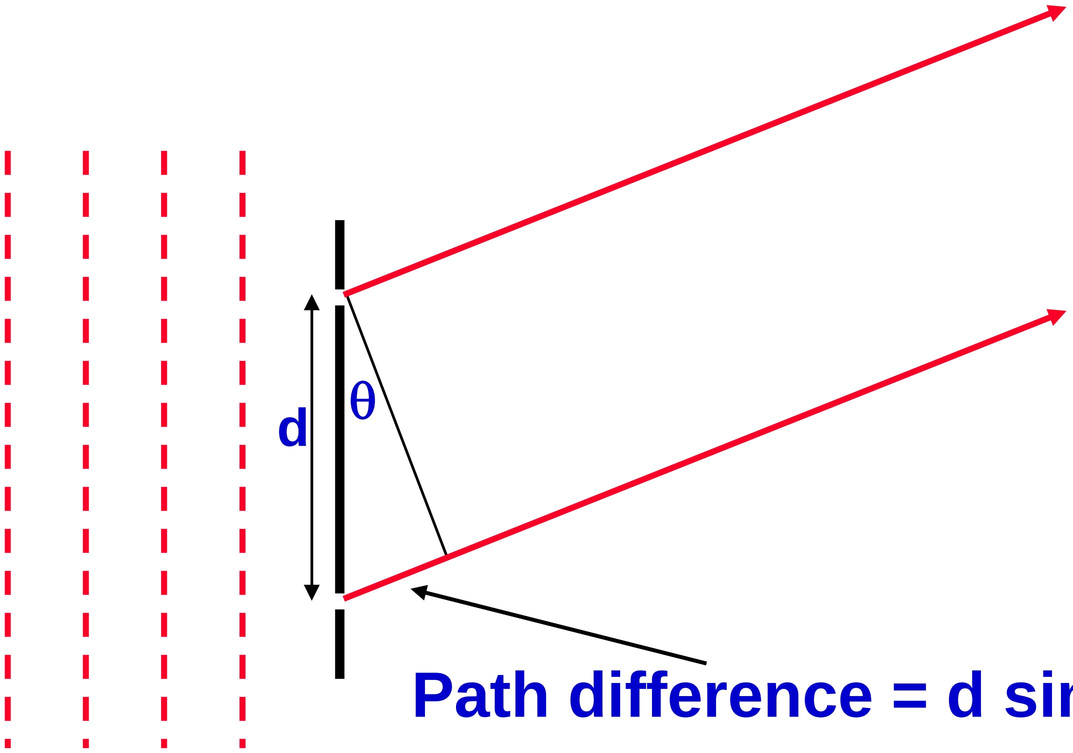
Intensity vs θ for d = 2.0 μm , λ = 550 nm



Calculating Double Slit Intensity

➔ Assumptions

- Each slit acts as a source of waves
- Waves radiate equally in all directions



Path difference = d sinθ

Double Slit Intensity (2)

➔ Add amplitudes, include phase difference
- Assume equal size slit widths
- Phase difference from path difference: $2\pi \times$ # wavelengths
- We ignore x dependence here (analysis does not depend on it)

$$E(t) = E_0 \cos(\omega t) + E_0 \cos(\omega t + \phi)$$

$$\phi = 2\pi\left(\frac{d\sin\theta}{\lambda}\right)$$

Double Slit Intensity (3)

➔ Intensity is time average of amplitude *squared*

- Consider single wave of amplitude $E = E_0 \cos\omega t$
- Intensity from time average of E^2: $I_0 = K^2 E_0^2 \left\langle \cos^2 \omega t \right\rangle = \frac{1}{2} K^2 E_0^2$
- <...> is time average over a period, K is a constant
- Use these to calculate total intensity

$$I_{\text{tot}} = K^2 \left\langle \left(E_0 \cos(\omega t) + E_0 \cos(\omega t + \phi) \right)^2 \right\rangle$$

$$= K^2 E_0^2 \left\langle \cos^2 \omega t \right\rangle + K^2 E_0^2 \left\langle \cos^2 \left(\omega t + \phi \right) \right\rangle$$

$$+ 2K^2 E_0^2 \left\langle \cos \omega t \cos\left(\omega t + \phi \right) \right\rangle$$

We work out these 3 terms on next page

Double Slit Intensity (4)

$$\left\langle \cos^2 \omega t \right\rangle = \frac{1}{2}$$

$$\phi = 2\pi \left(\frac{d \sin\theta}{\lambda} \right)$$

$$\left\langle \cos^2 (\omega t + \phi) \right\rangle = \frac{1}{2}$$

$$\left\langle \cos \omega t \cos (\omega t + \phi) \right\rangle = \frac{1}{2} \cos\phi$$

From expanding cos(ωt+φ) term

$$\begin{aligned} I_{\text{tot}} &= K^2 E_0^2 (1 + \cos\phi) = 2I_0 (1 + \cos\phi) \\ &= 4I_0 \cos^2 \tfrac{1}{2}\phi \\ &= 4I_0 \cos^2 (\pi d \sin\theta / \lambda) \end{aligned}$$

Double Slit Intensity (5)

➔So the intensity is $I = 4I_0 \cos^2(\pi d \sin\theta / \lambda)$

➔Maxima occur when argument inside cos() is nπ

$$d \sin\theta = n\lambda$$

➔Minima occur when argument inside cos() is (n+1/2)π

$$d \sin\theta = \left(n + \tfrac{1}{2}\right)\lambda$$