Monochromatic plane waves

Plane waves have straight wave fronts

– As opposed to spherical waves, etc.

– Suppose

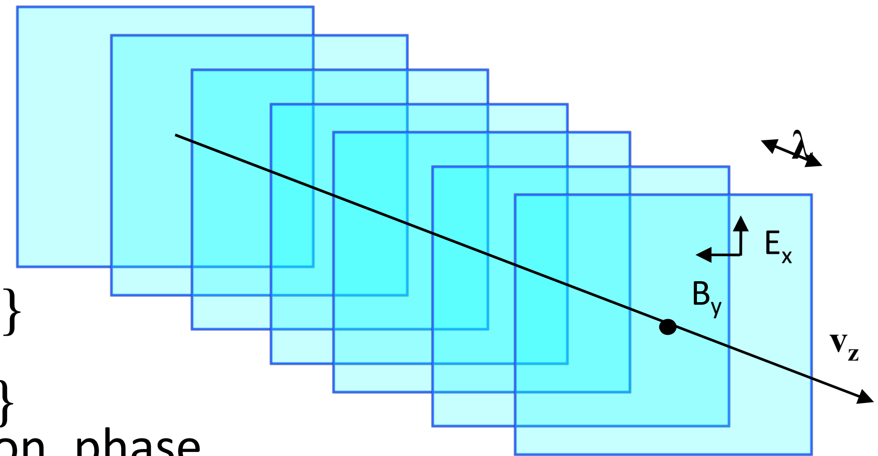
$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}}$$

➔

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\{\mathbf{E}(\mathbf{r})e^{-i\omega t}\}$$

$$= \mathrm{Re}\{\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}} e^{-i\omega t}\}$$

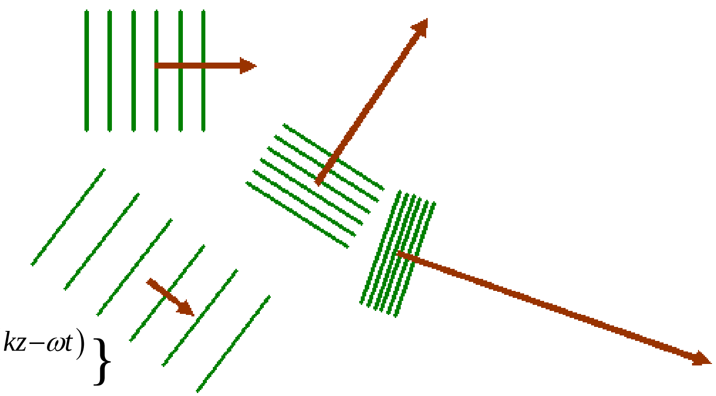
$$= \mathrm{Re}\{\mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\}$$



– $\mathbf{E}_0$ still contains: amplitude, polarization, phase

– Direction of propagation given by wavevector:

$$\mathbf{k} = (k_x, k_y, k_z) \text{ where } |\mathbf{k}| = 2\pi/\lambda = \omega/c$$



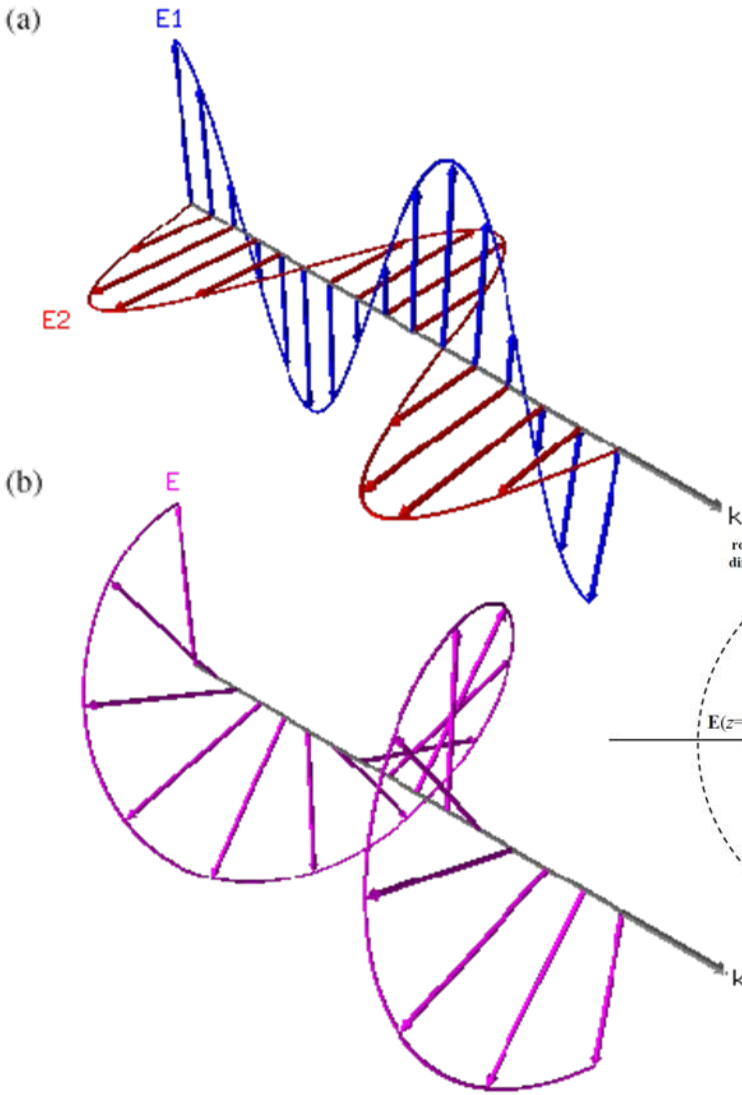
– Can also define

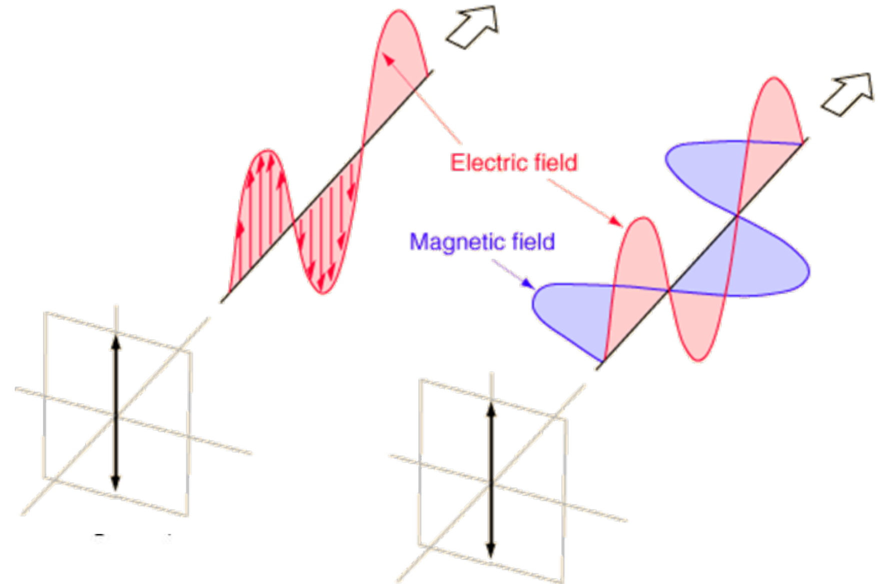
$$\mathbf{E} = (E_x, E_y, E_z)$$

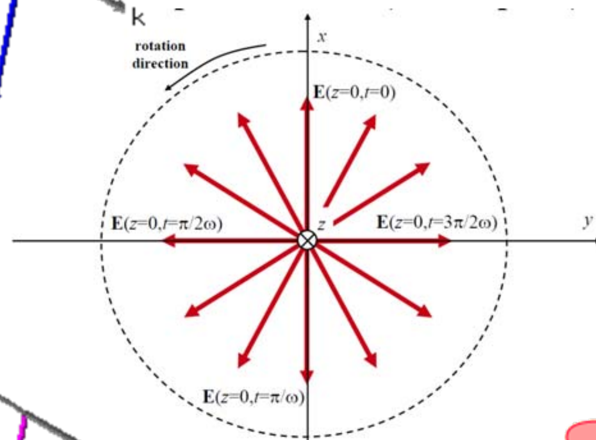
– Plane wave propagating in z-direction

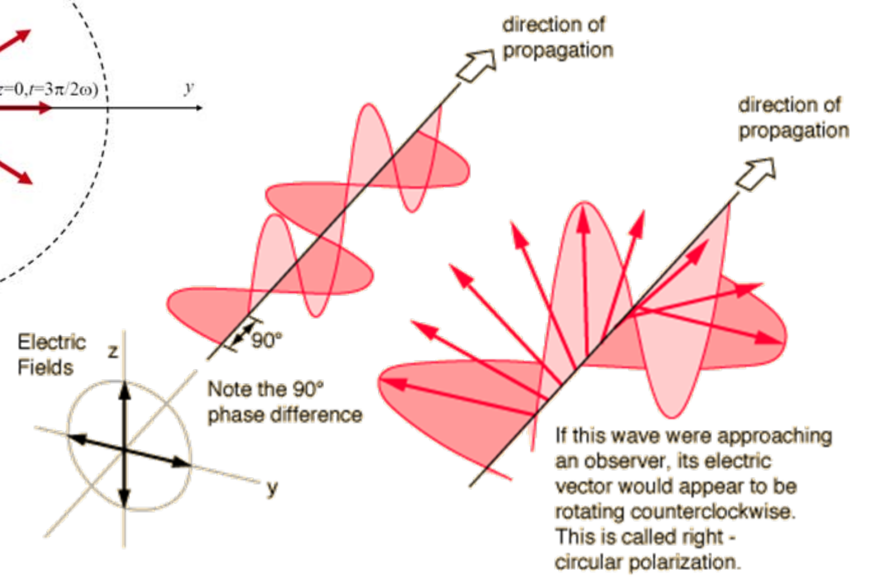
$$\mathbf{E}(z,t) = \mathrm{Re}\{\mathbf{E}_0 e^{i(kz-\omega t)}\} = \tfrac{1}{2}\{\mathbf{E}_0 e^{i(kz-\omega t)} + \mathbf{E}_0^* e^{-i(kz-\omega t)}\}$$

Linear versus Circular polarization



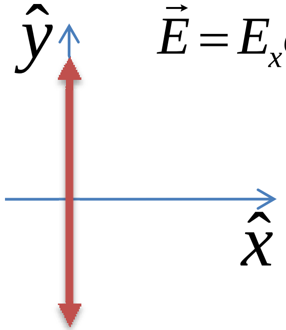




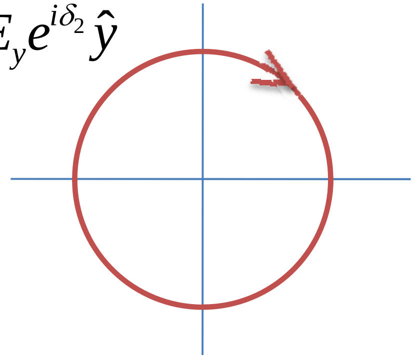


Polarization: Summary

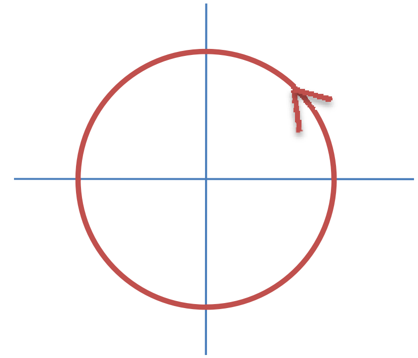
$$\vec{E} = E_x e^{i\delta_1}\hat{x} + E_y e^{i\delta_2}\hat{y}$$



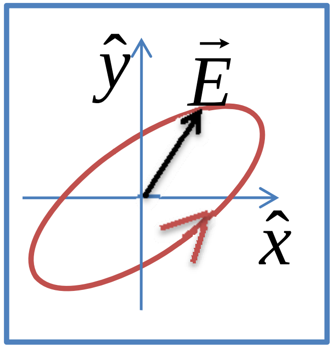
linear polarization
y-direction



right circular
polarization

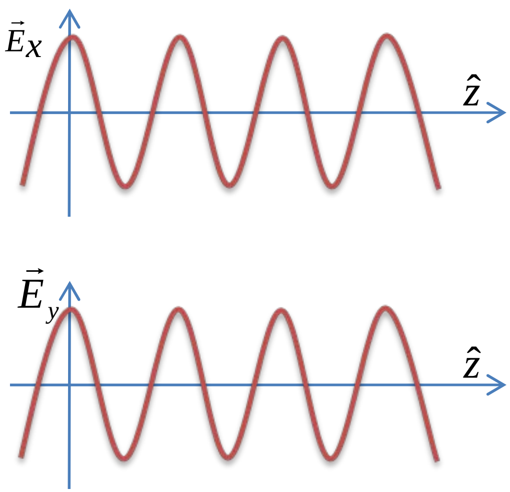


left circular
polarization

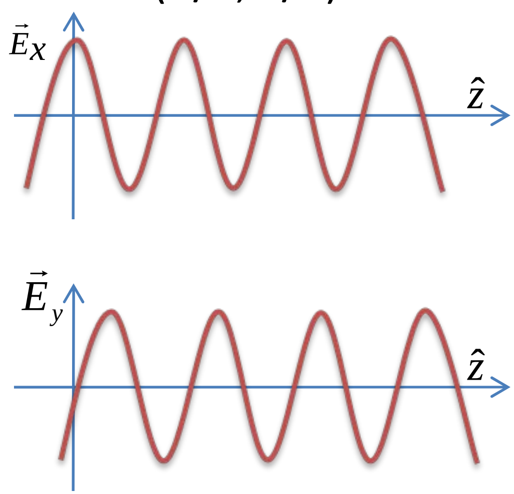


left elliptical
polarization

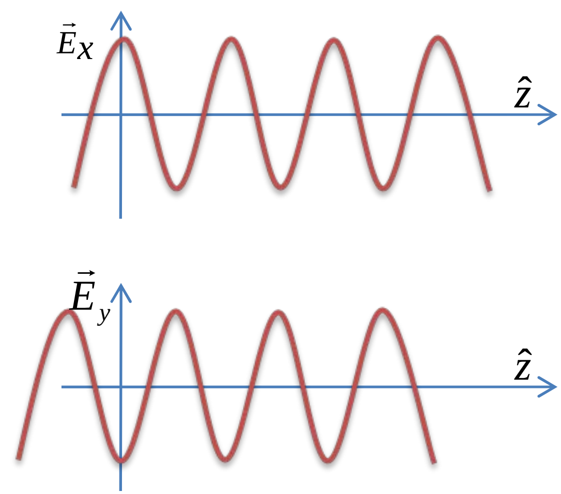
Phase difference = 0^0



Phase difference ➔
90^0 ($\pi/2$, $\lambda/4$)



Phase difference ➔
180^0 (π, $\lambda/2$)

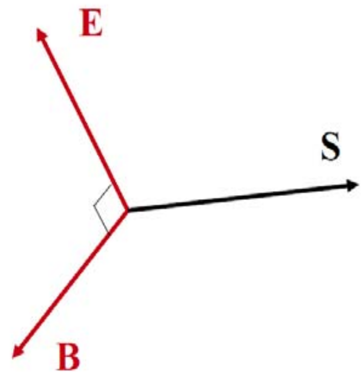


Poynting vector & Intensity of Light $\mathbf{S} = \mathbf{E} \times \mathbf{H}$

Summary (free space or isotropic media)

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}; \quad \|\mathbf{S}\| = c\varepsilon_0\|\mathbf{E}\|^2$$ Poynting vec

$$\langle\|\mathbf{S}\|\rangle = \frac{1}{T}\int_t^{t+T}\|\mathbf{S}\|\,\mathrm{d}t$$ Irradiance (or intensity)



$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} = c^2\varepsilon_0\mathbf{E}\times\mathbf{B}$$

so in free space

$$\mathbf{S} \,\|\, \mathbf{k}$$

S has units of W/m^2 so it represents energy flux (energy per unit time & unit area)

- Poynting vector describes flows of E-M power
- Power flow is directed along this vector
    - Usually parallel to **k**
- Intensity is equal to the magnitude of the time averaged Poyning vector: I=<**S**>

$$|\langle\mathbf{S}\rangle| = I \equiv |\langle\mathbf{E}(t)\times\mathbf{H}(t)\rangle| = \frac{c\varepsilon_0}{2}E^2 = \frac{c\varepsilon_0}{2}\left(E_x{}^2 + E_y{}^2\right)$$

$$c\varepsilon_0 \approx 2.654\times10^{-3}\,A/V$$

example

$$E = 1V/m$$

$$I = ?\,W/m^2$$

$$\hbar\omega[eV] = \frac{1239.85}{\lambda[nm]}$$

$$\hbar = 1.05457266\times10^{-34}\,Js$$

Maxwell's equations in a medium (source-free)

The induced polarization, P, contains the effect of the medium:

$$\vec{\nabla}\cdot\vec{E}=0 \qquad \vec{\nabla}\times\vec{E}=-\mu\frac{\partial\vec{H}}{\partial t} \qquad \vec{B}=\mu\vec{H}$$

$$\vec{\nabla}\cdot\vec{H}=0 \qquad \vec{\nabla}\times\vec{H}=\varepsilon_0\frac{\partial\vec{E}}{\partial t}+\frac{\partial\vec{P}}{\partial t}=\varepsilon\frac{\partial\vec{E}}{\partial t}$$

$$\vec{D}=\varepsilon_0\vec{E}+\vec{P}=\varepsilon\vec{E}$$

$$\vec{P}=\varepsilon_0\chi\vec{E}$$

The polarization is proportional to the field:

$$\vec{P}=\varepsilon_0\chi\vec{E}$$

This has the effect of simply changing the dielectric constant (refractive index n):

$$\varepsilon=\varepsilon_0\left(1+\chi\right)=n^2$$

Wave equations in a medium

The induced polarization in Maxwell's Equations yields another term in the wave equation:

$$\frac{\partial^2 E}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \qquad \frac{\partial^2 E}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2} = 0$$

This is the **Inhomogeneous Wave Equation**.

The polarization is the driving term for a new solution to this equation.

$$\frac{\partial^2 E}{\partial z^2} - \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \qquad \frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

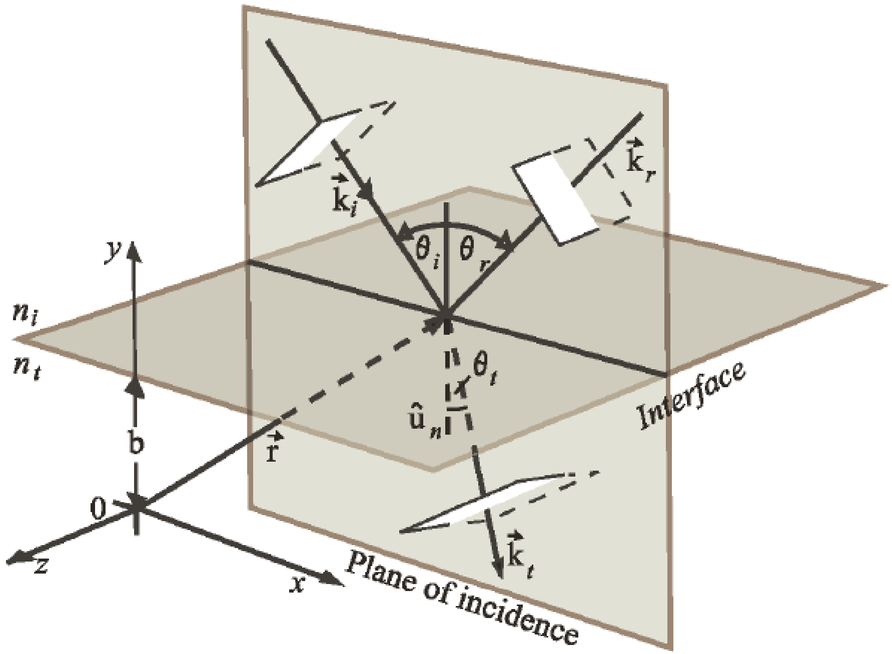
Homogeneous (Vacuum) Wave Equation

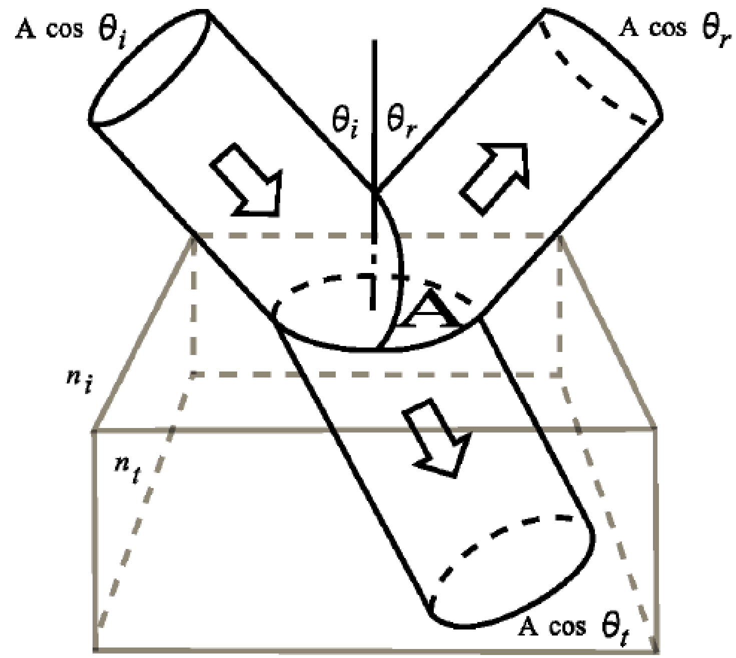
$$\begin{aligned}\mathbf{E}(z,t) &= \mathrm{Re}\{\mathbf{E}_0 e^{i(kz-\omega t)}\} \\ &= \tfrac{1}{2}\{\mathbf{E}_0 e^{i(kz-\omega t)} + \mathbf{E}_0^* e^{-i(kz-\omega t)}\} \\ &= |\mathbf{E}_0|\cos(kz-\omega t)\end{aligned}$$

$$\frac{c}{v} = n$$

Phase velocity

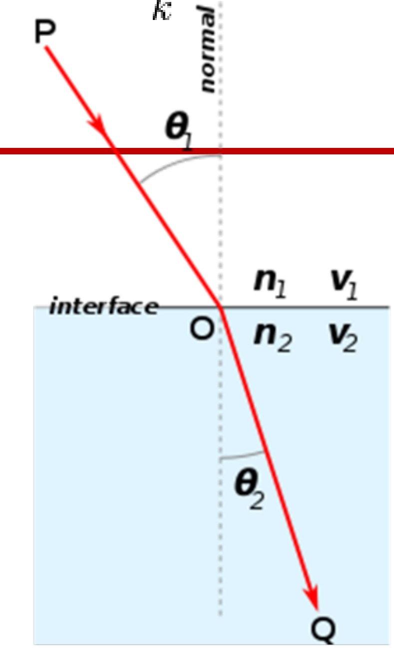
Reflection and Transmission @ dielectric interface





Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \ .$$

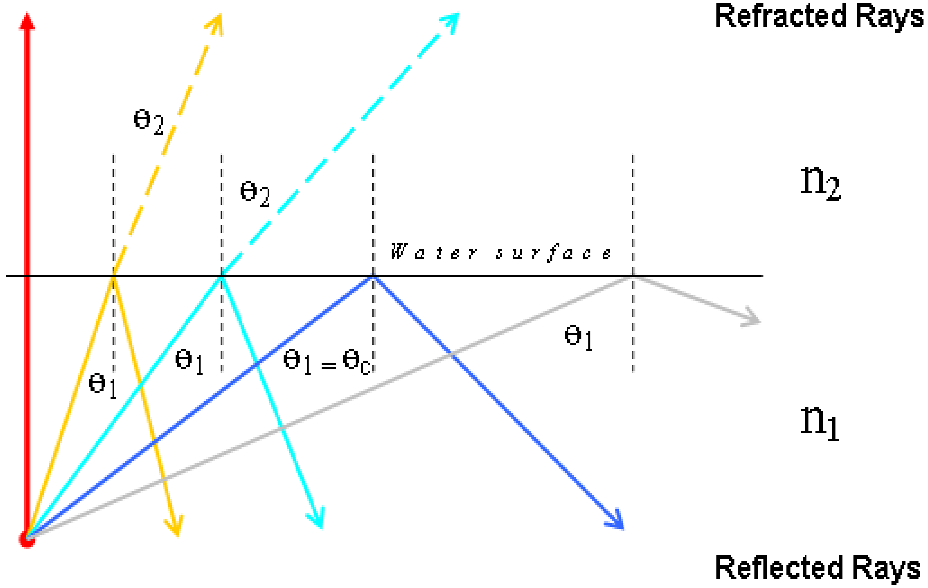


$k_{xRegion1} = k_{xRregion2}$

$n_1 k_0 \sin\theta_1 = n_2 k_0 \sin\theta_2$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$k_0 = \frac{2\pi}{\lambda_0} = \frac{\omega}{c}$$

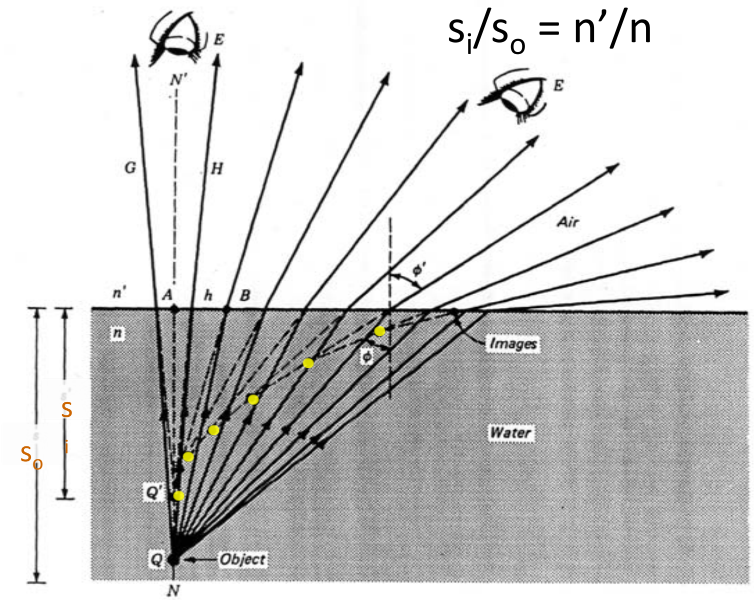


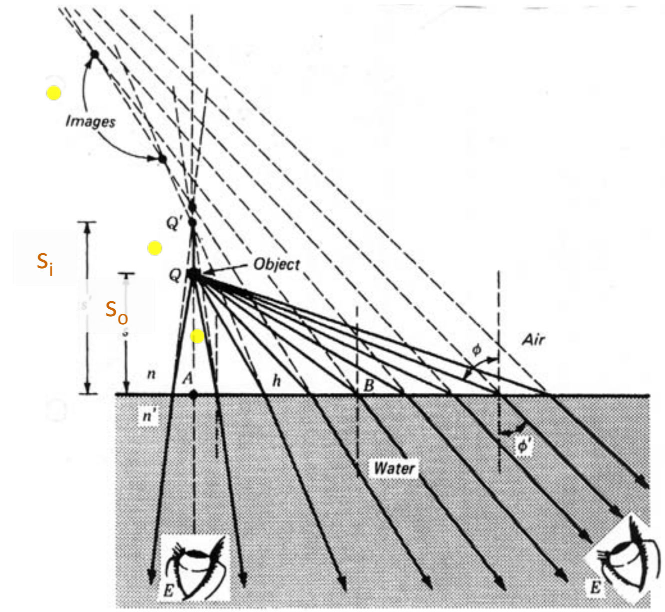
$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = 1.333 \cdot 0.766 = 1.021,$$

$$\theta_{\text{crit}} = \arcsin\left(\frac{n_2}{n_1}\sin\theta_2\right) = \arcsin\frac{n_2}{n_1} = 48.6°.$$

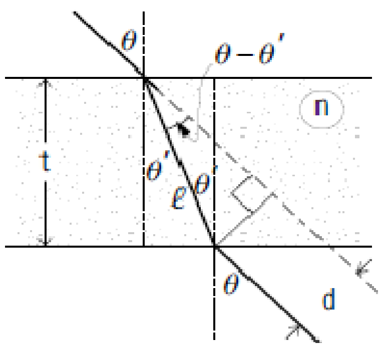
The fish problem …

$s_i/s_o = n'/n$



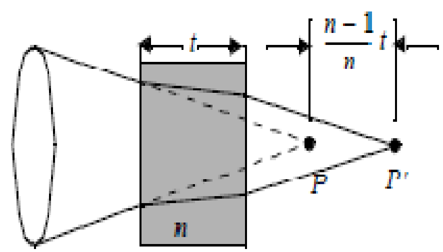


Displacement by a glass plate



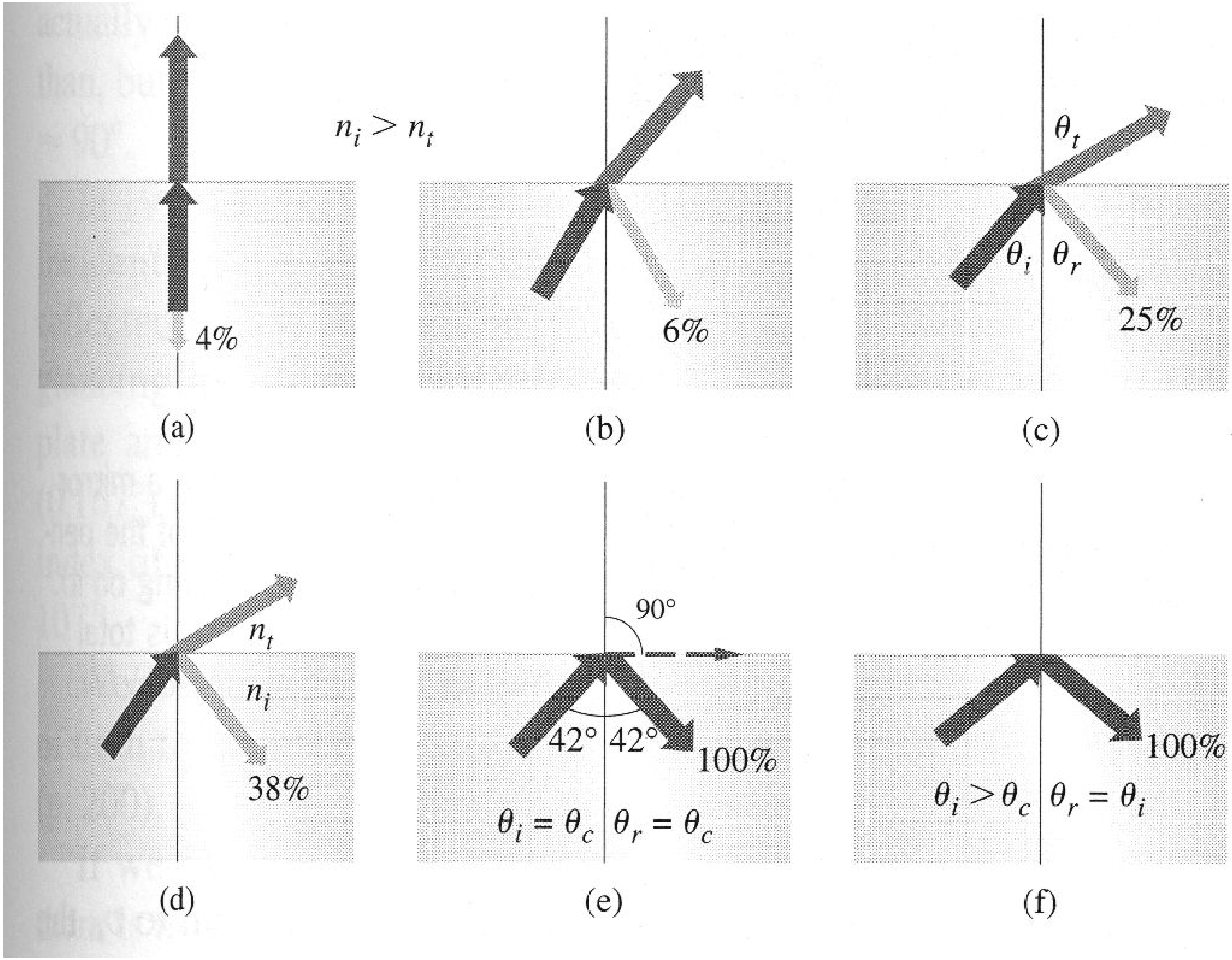
$$d = \frac{t \sin\left(\theta - \theta'\right)}{\cos\left(\theta'\right)}$$

Plane parallel plate placed in between a lens and its focus:



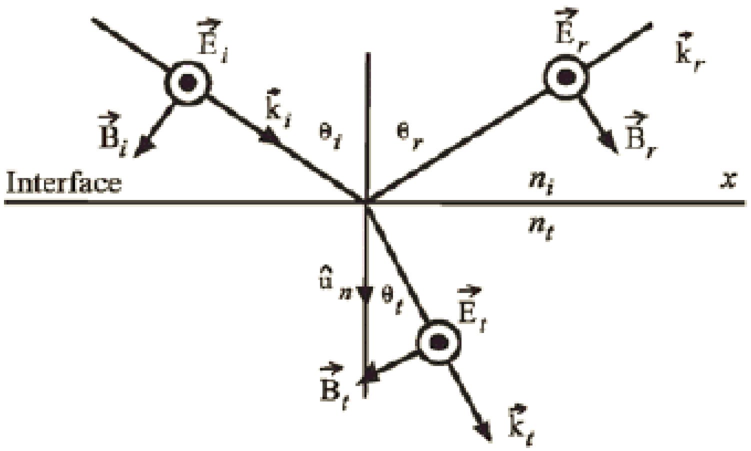
A simple calculation based on the paraxial approximation shows that the focus is displaced by amount $\frac{n-1}{n}t$. However, at steeper incidence angles, the focal shift becomes a function of the incidence angle, which leads to spherical aberration.

Reflection and refraction at a flat dielectric interface



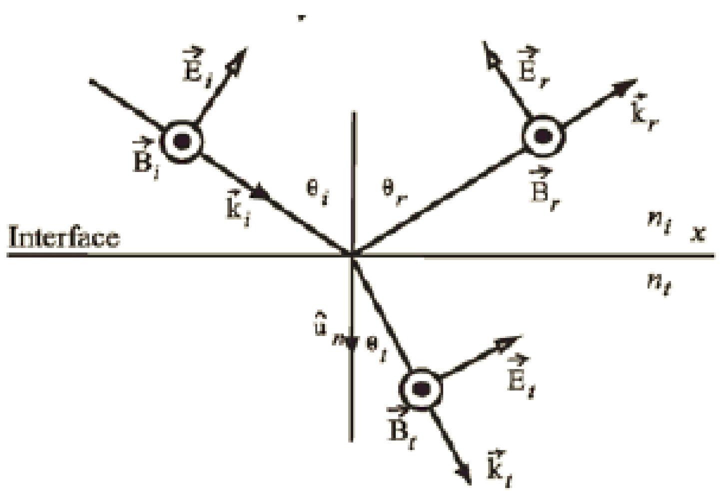
Reflection and Transmission (Fresnel's equations)

Can be deduced from the application of boundary conditions of EM waves.



$$r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$



$$r_\parallel = \left(\frac{E_{0r}}{E_{0i}}\right)_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

$$t_\parallel = \left(\frac{E_{0t}}{E_{0i}}\right)_\parallel = \frac{2n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

Normal Incidence

$$r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$r_\parallel = \left(\frac{E_{0r}}{E_{0i}}\right)_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

$$t_\parallel = \left(\frac{E_{0t}}{E_{0i}}\right)_\parallel = \frac{2n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

Note: independent of polarization

$\theta_i = 0$ and $\theta_t = 0$ →

$$r_\perp = r_\parallel = \frac{n_t - n_i}{n_t + n_i}$$

$$t_\perp = t_\parallel = \frac{2n_i}{n_t + n_i}$$

$$R_\perp = R_\parallel = \left(\frac{n_t - n_i}{n_t + n_i}\right)^2$$

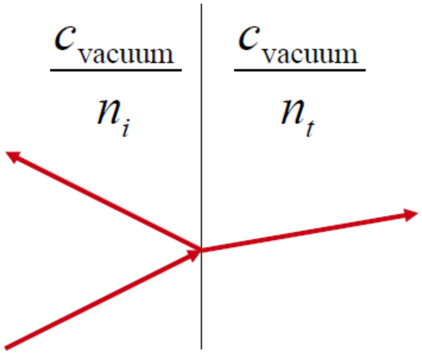
$$T_\perp = T_\parallel = \frac{4n_t n_i}{(n_t + n_i)^2}$$

Reflection and Transmission of Energy @ dielectric interfaces

Recall Poynting vector definition:

$$\|\mathbf{S}\| = c\varepsilon_0 \|\mathbf{E}\|^2$$

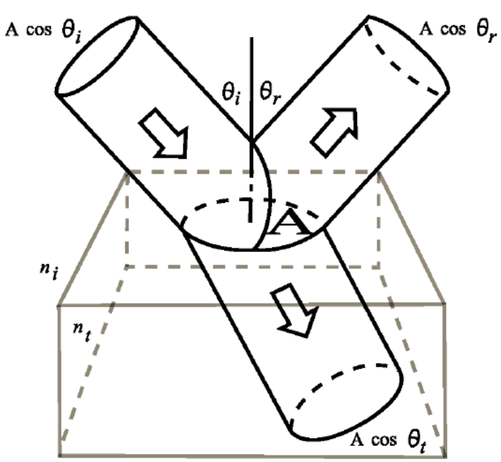
different on the two sides of the interface

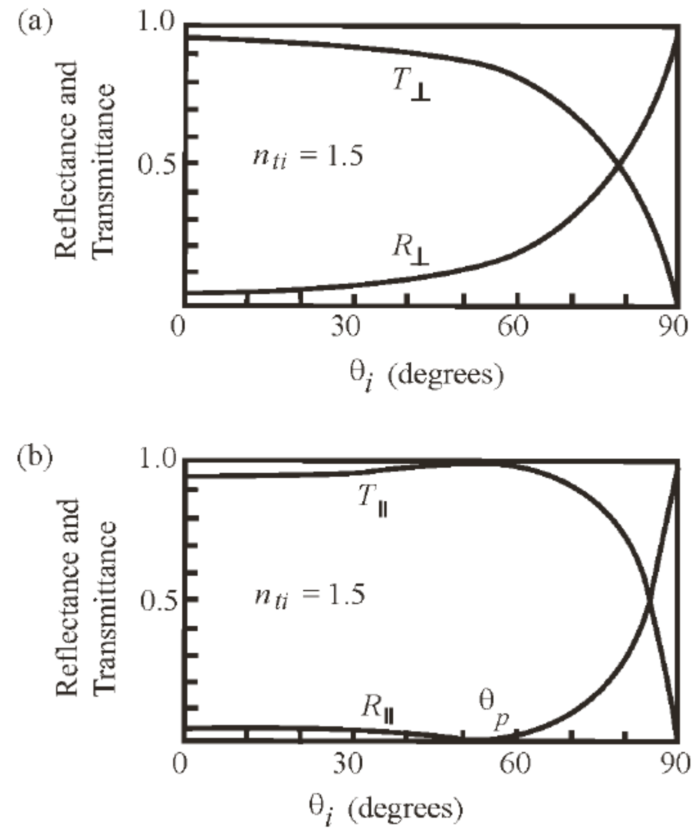


$$R = \left(\frac{E_{0r}}{E_{0i}}\right)^2 = r^2$$

$$T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i}\left(\frac{E_{0t}}{E_{0i}}\right)^2 = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t^2$$

$$R + T = 1, \text{ i.e. } r^2 + \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t^2 = 1$$





Prisms

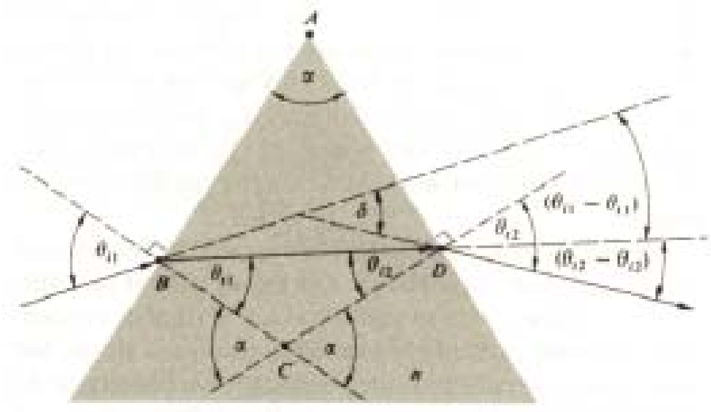


Figure 5.56 Geometry of a dispersing prism.

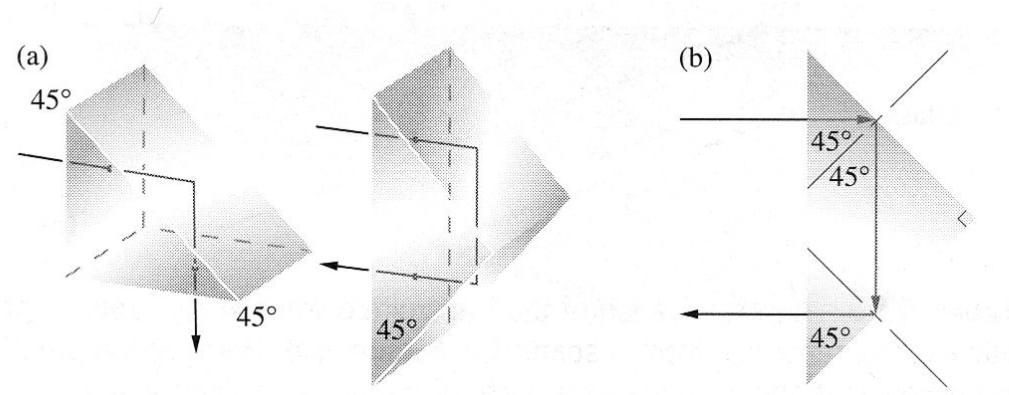


Figure 4.52 Total internal reflection.

The total deviation:

$$\delta=(\theta_{i1}-\theta_{t1})+(\theta_{t2}-\theta_{i2})$$

Since $\alpha=\theta_{t1}+\theta_{i2}$,

$$\delta=\theta_{i1}+\theta_{t2}-\alpha$$

From Snell's law

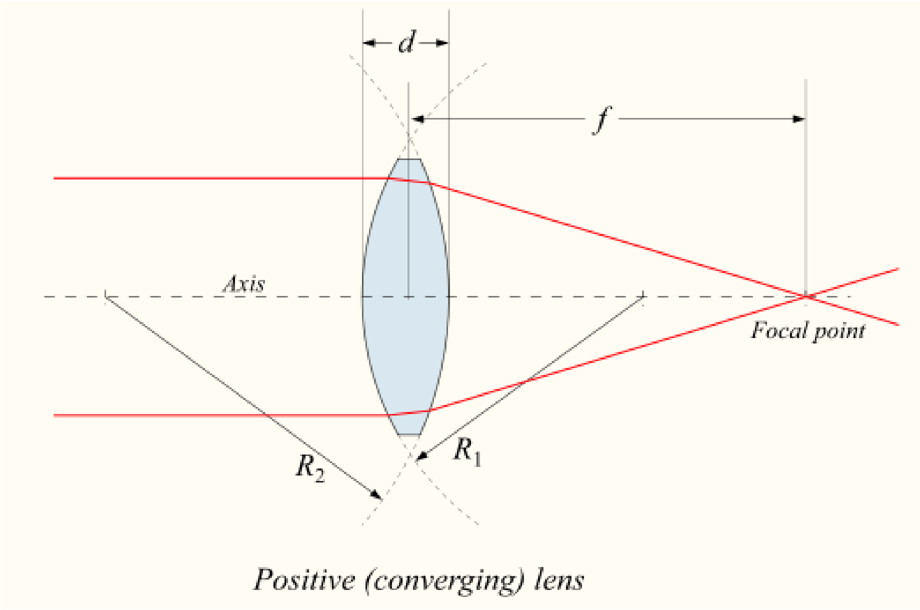
$$\theta_{t2}=\sin^{-1}(n\sin\theta_{i2})=\sin^{-1}[(\sin\alpha)(n^2-\sin^2\theta_{i1})^{1/2}-\sin\theta_{i1}\cos\alpha]$$

The deviation is then

$$\delta=\theta_{i1}+\sin^{-1}[(\sin\alpha)(n^2-\sin^2\theta_{i1})^{1/2}-\sin\theta_{i1}\cos\alpha]-\alpha$$

δ Increases with n. For visible light, n increases as frequency increases. Therefore, blue light deviates more than red light.

Thin Lens Imaging



Paraxial approximation

$$\sin(\theta) \approx \tan(\theta) \approx \theta$$

$$\cos(\theta) \approx 1$$

See Hecht Ch. 5 and review the following Equations. **"Sign" convention :**
(See Hecht Table 5.1, Fig. 5.12, Table 5.2)

$$\frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_i}$$

$$x_0 x_i = f^2$$

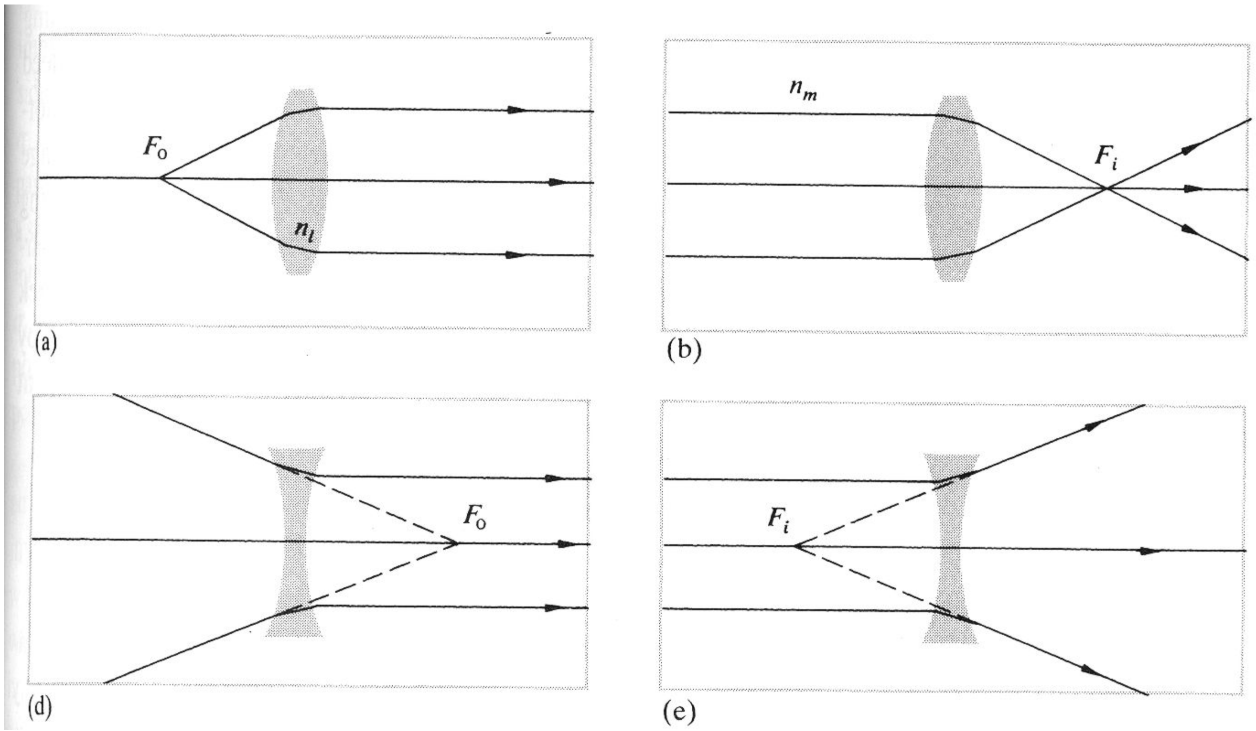
$$M_T \equiv \frac{y_i}{y_0} = -\frac{s_i}{s_o}$$

$$M_L \equiv \frac{dx_i}{dx_0} = -\frac{f^2}{x_0^2}$$

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right],$$ Thick lens

$$\frac{1}{f} \approx (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right].$$ $n=n_l/n_m$ Thin lens (d is negligible)

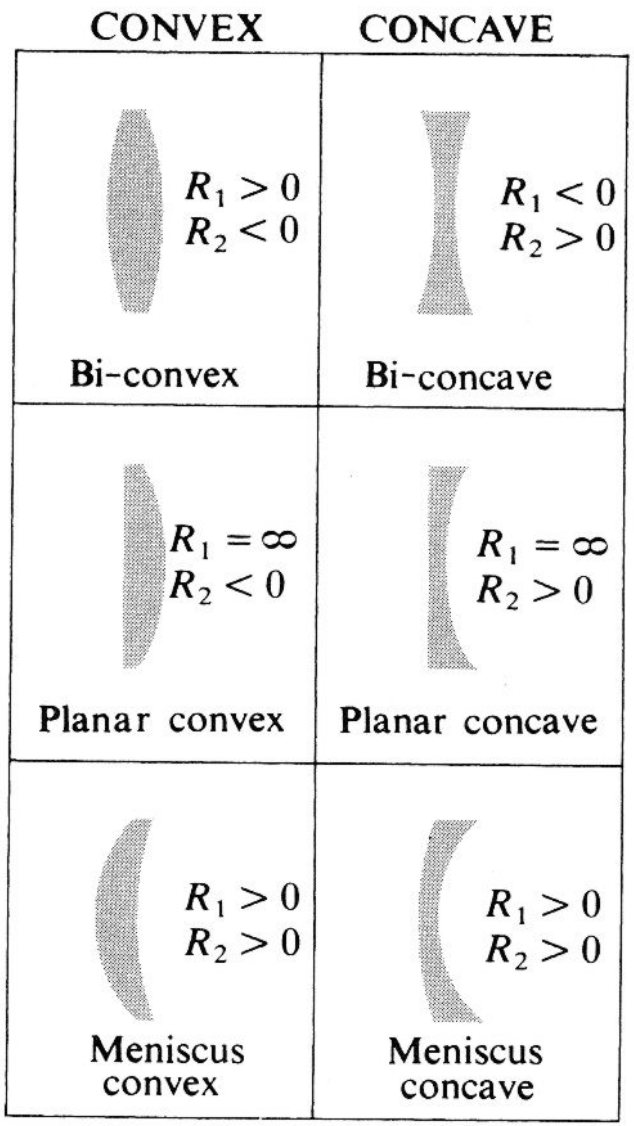
Object **Image**



Bi-convex

Bi-concave

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{(n_l-n_m)}{n_m}\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(n_{lm}-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=\frac{1}{f}$$



Positive Negative

$$\frac{n_m}{s_o} + \frac{n_m}{s_i} = (n_l - n_m)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

n_l is the lens index, n_m is the medium index

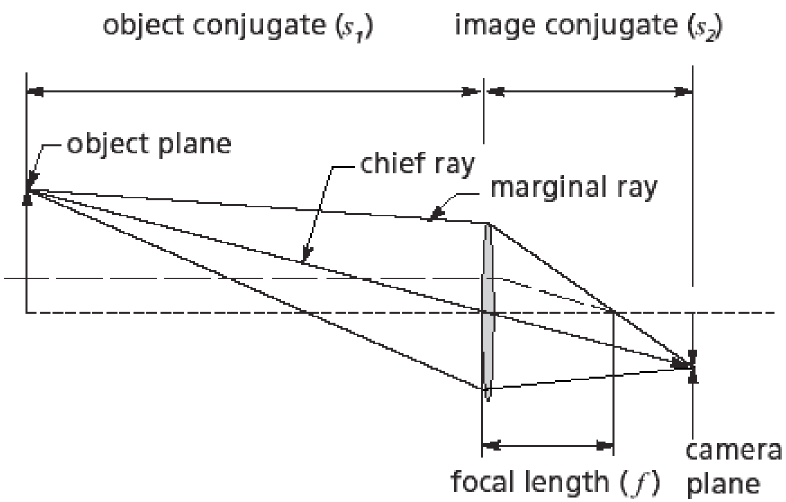
$$\frac{1}{s_o} + \frac{1}{s_i} = (n_{lm} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{1}{f}$$

A ray propagates from left to right, passing through surface 1, then surface 2

If $R_1 < R_2$, and R_1, R_2 >0, [ ] > 0 (convex)

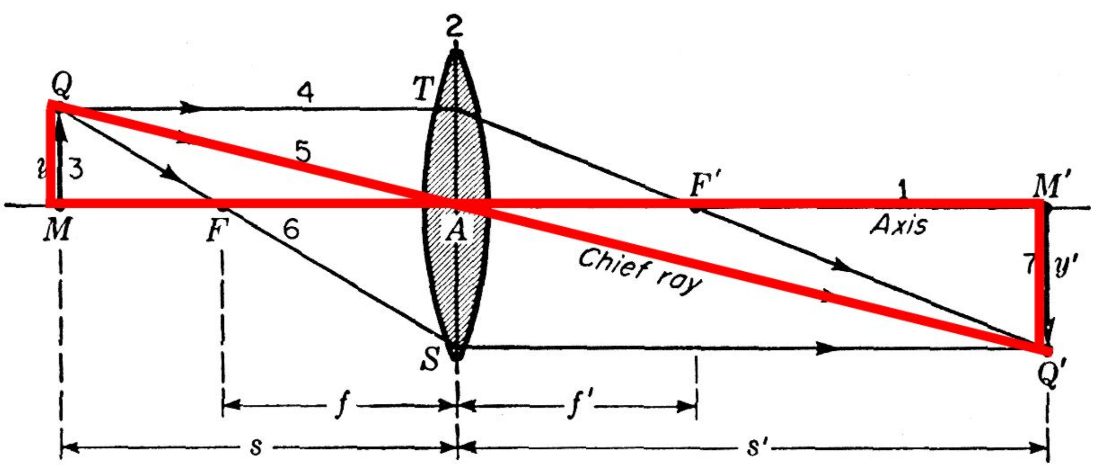
If $R_1 > R_2$, and R_1, R_2 >0, [ ] < 0 (concave)

Imaging by a single thin lens

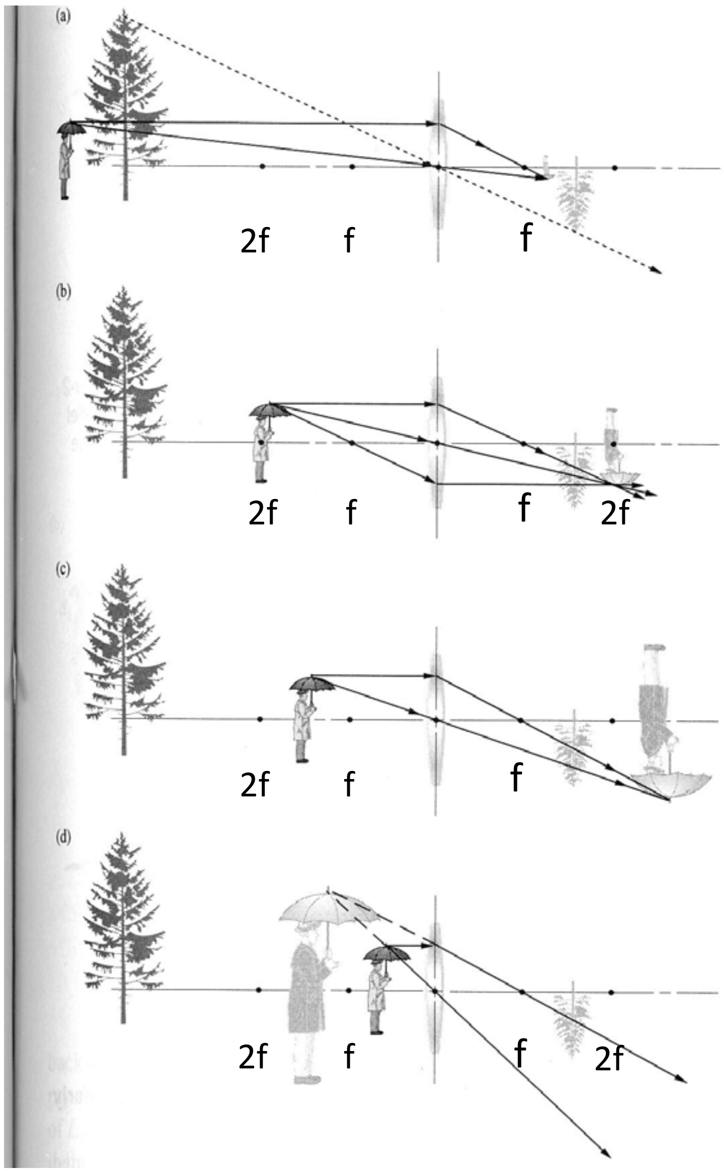


Ray tracing s > f

Positive lens: real, inverted image



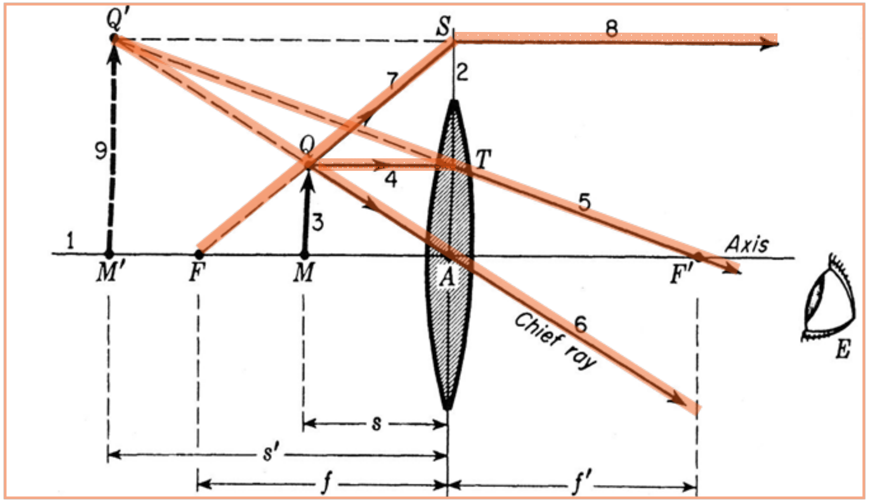
magnification $M_T = y'/y = -s'/s = -s_i/s_o$



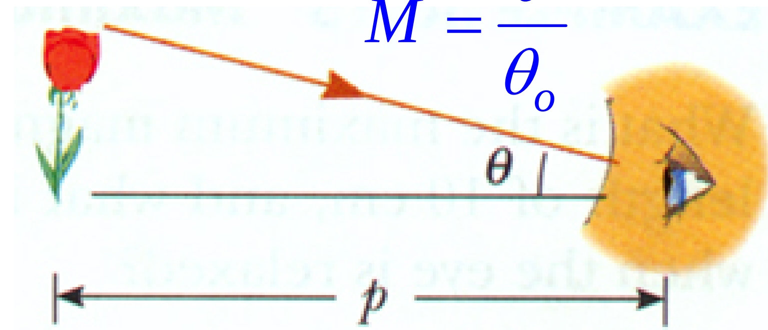
Magnifying glass

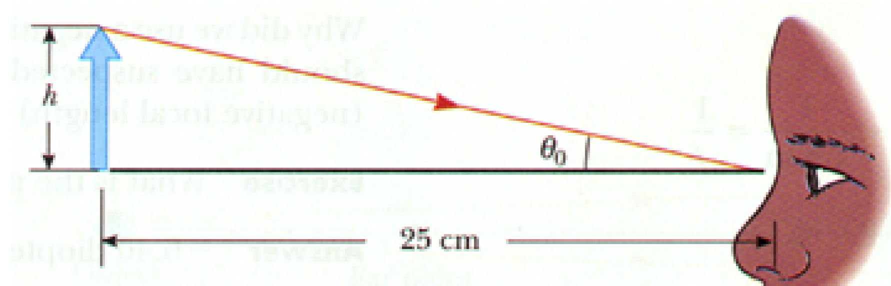
Ray tracing s < f

Positive lens: virtual, erect image

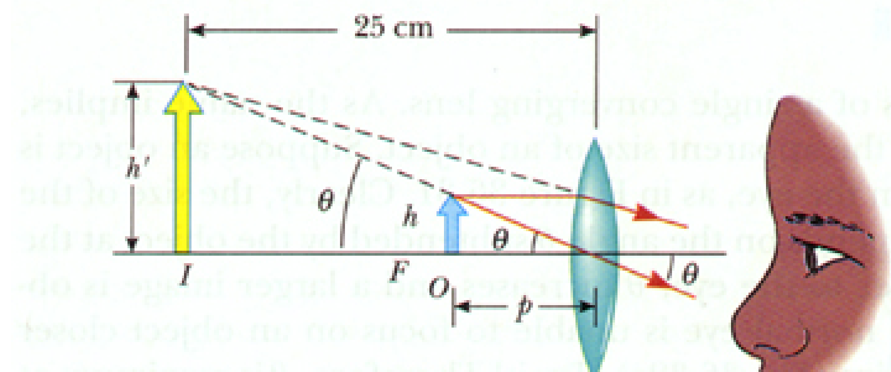


$$M = \frac{\theta}{\theta_o}$$





(a)



(b)

Magnifying glass

$$M = 1 + \frac{25cm}{f}$$ For reading distance

$$M = \frac{25cm}{f}$$ For infinite image

Negative lens – ray tracing & virtual image

Summary : Real and Virtual Images

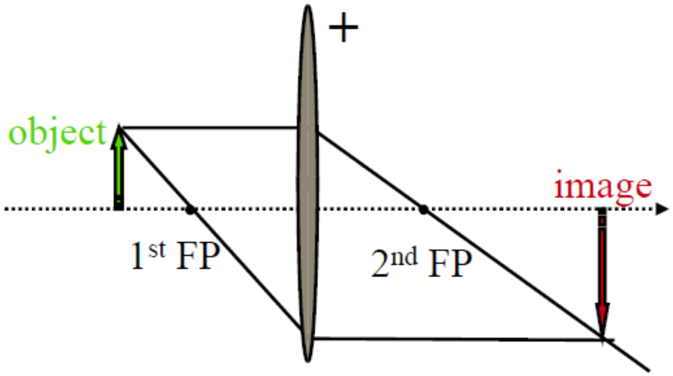


image: real & inverted; $M_T<0$

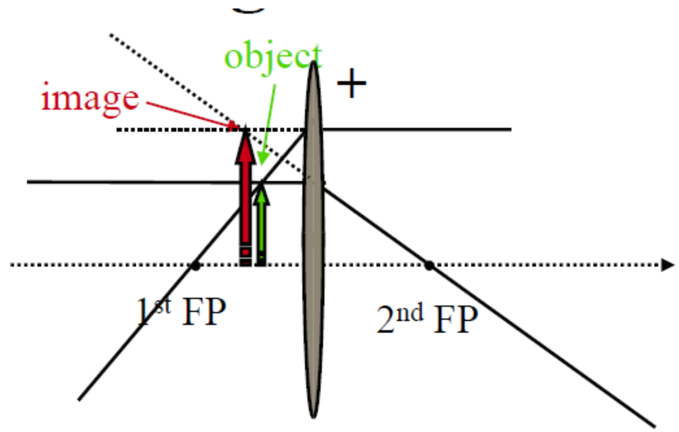


image: virtual & erect; $M_T>1$

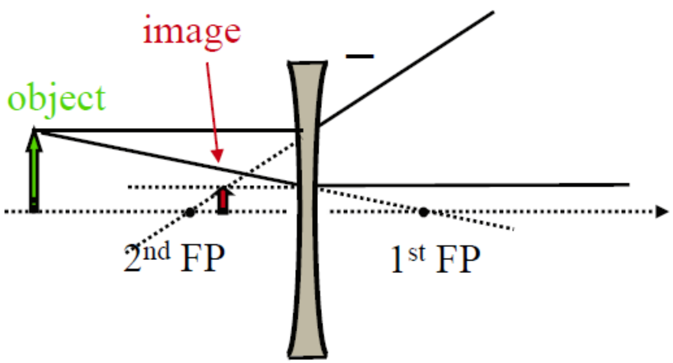


image: virtual & erect; $0<M_T<1$

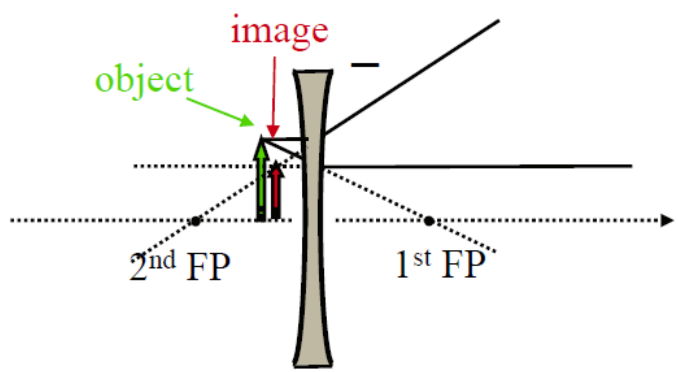
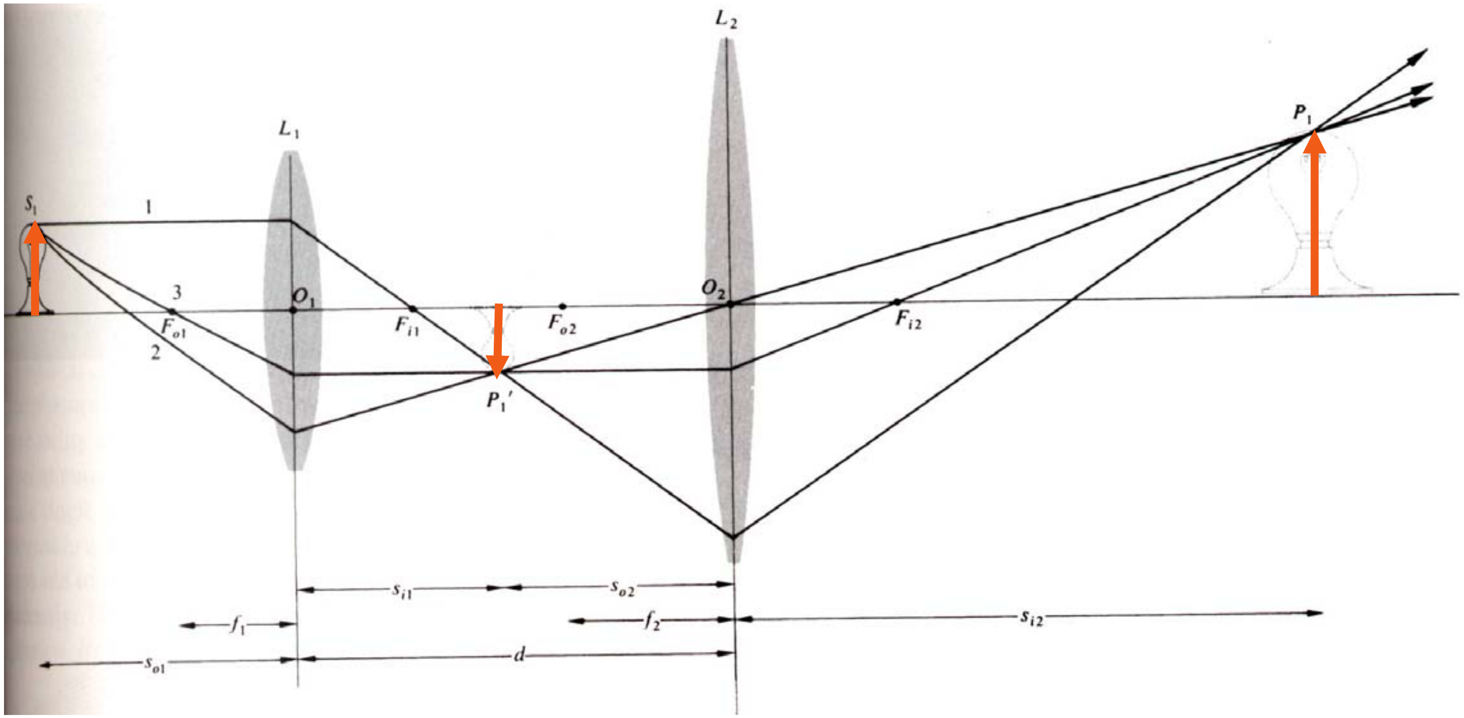


image: virtual & erect; $0<M_T<1$

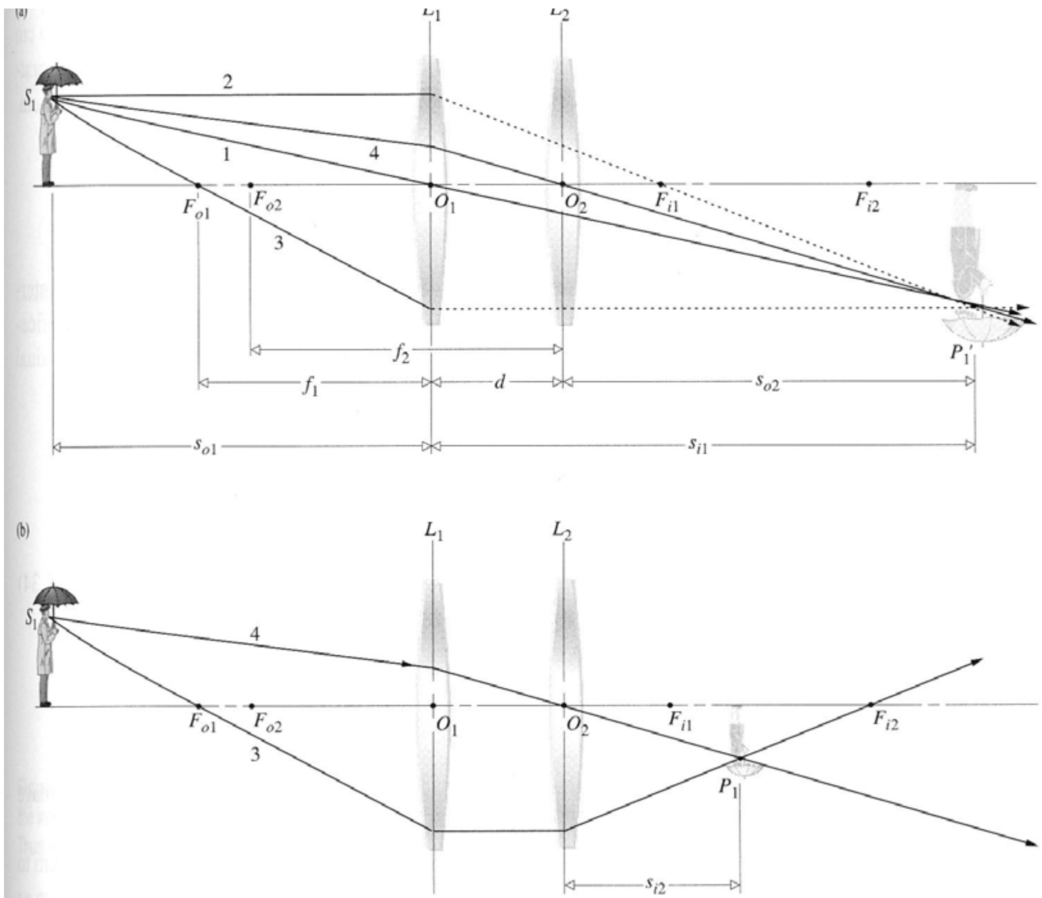
Lens Combination



1. Form intermediate image from L_1 at P’.
2. Use intermediate image at P’ as input to L_2.

Ray tracing – two lens combination

Lens distance d < f, i.e. f_1 or f_2



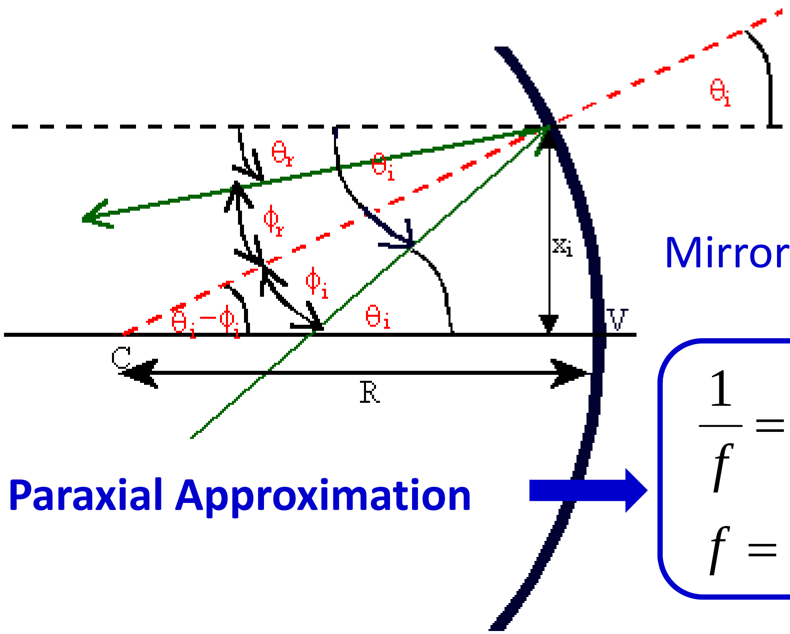
1. Draw rays 2 & 3 without the presence of L_2. Form intermediate image at P'.

2. Draw ray 4 through O_2. This is unchanged after L_2 is inserted.

3. Draw ray 3 in the presence of L_2 so that it passes through F_{i2}. Image appears at P_1 .

Figure 5.28 Two thin lenses separated by a distance smaller than either focal length.

Spherical Mirror



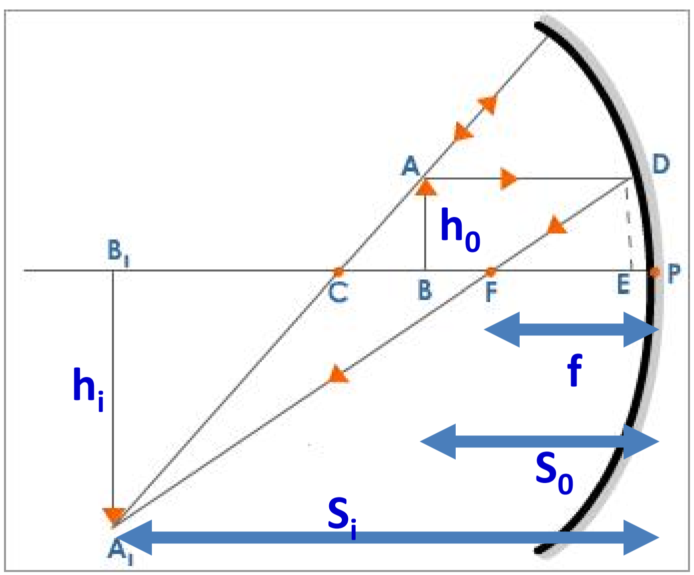
Mirror Formula

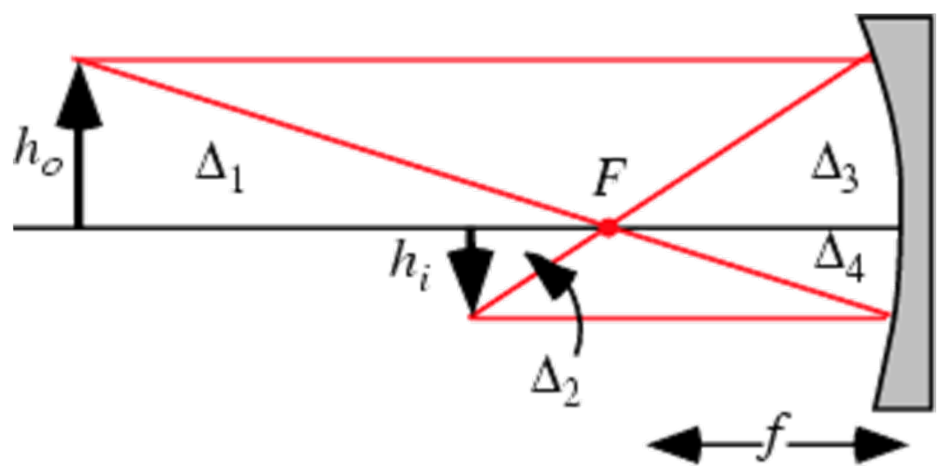
Paraxial Approximation

$$\frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_i}$$

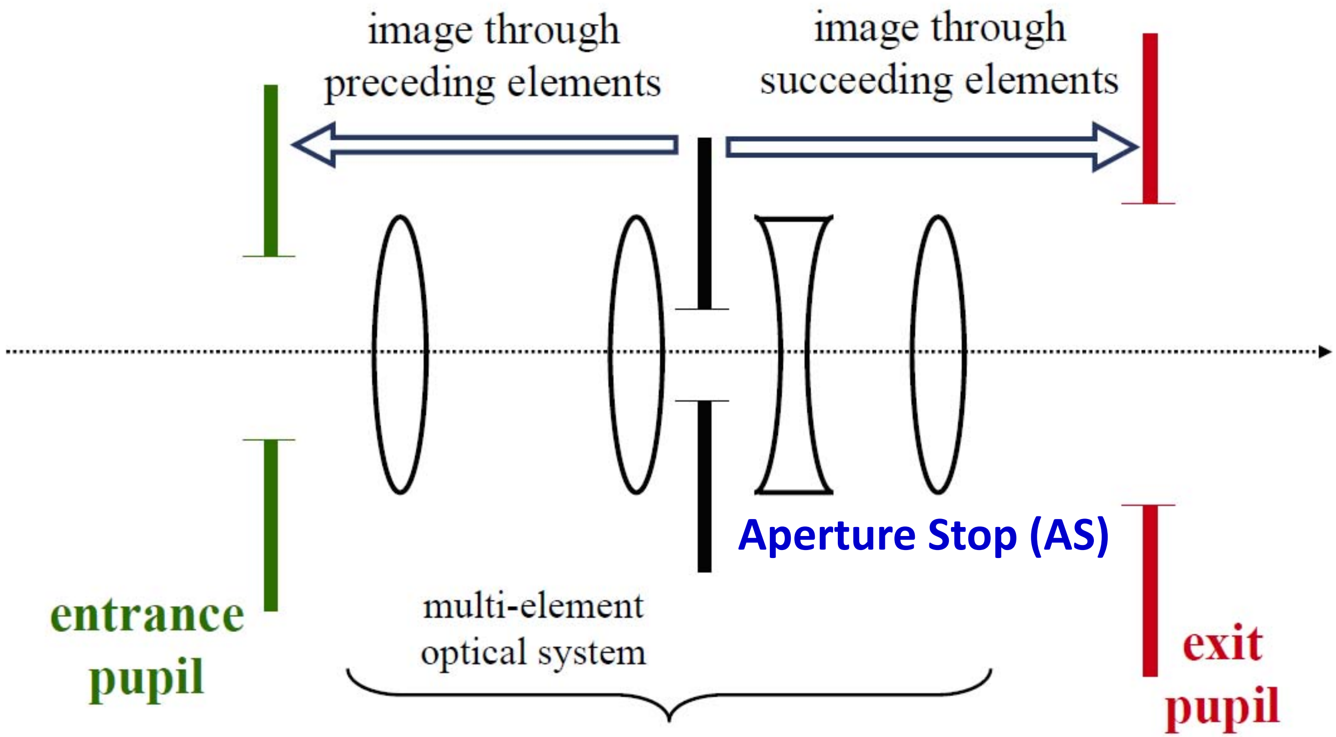
$$f = R/2$$

See Hecht Table 5.4 for sign convention





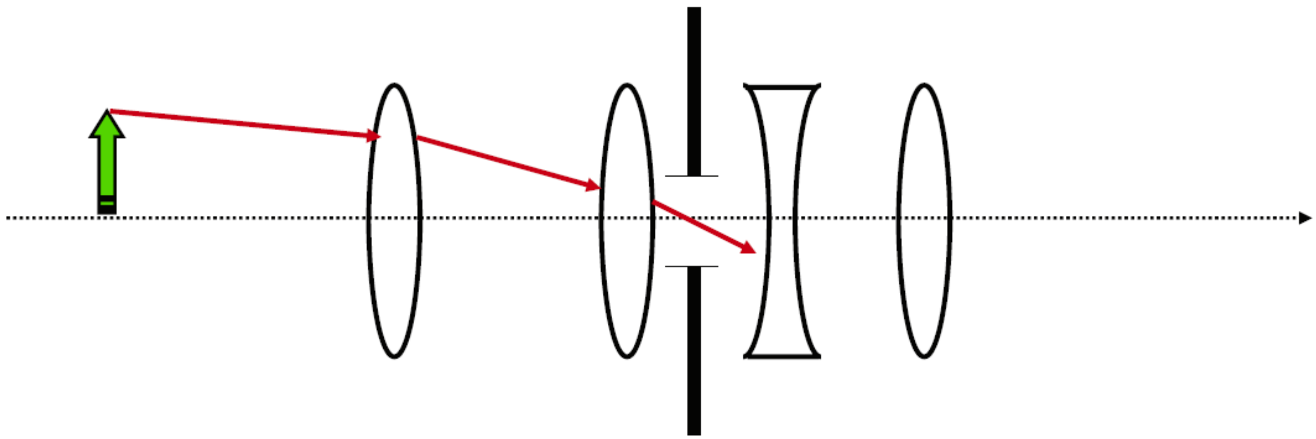
Aperture Stop and Entrance & Exit Pupil



The **aperture stop** (AS) is defined to be the stop or lens ring, which physically limits the solid angle of rays passing through the system from an **on-axis** object point. The aperture stop limits the brightness of an image.

The **entrance pupil** of a system is **the image of the aperture stop as seen from an axial point on the object through those elements preceding the stop**. (Hecht p. 171)
The **exit pupil** of a system is **the image of the aperture stop as seen from an axial point on the image plane through the interposed lenses, if there is any**. (Hecht p. 172)

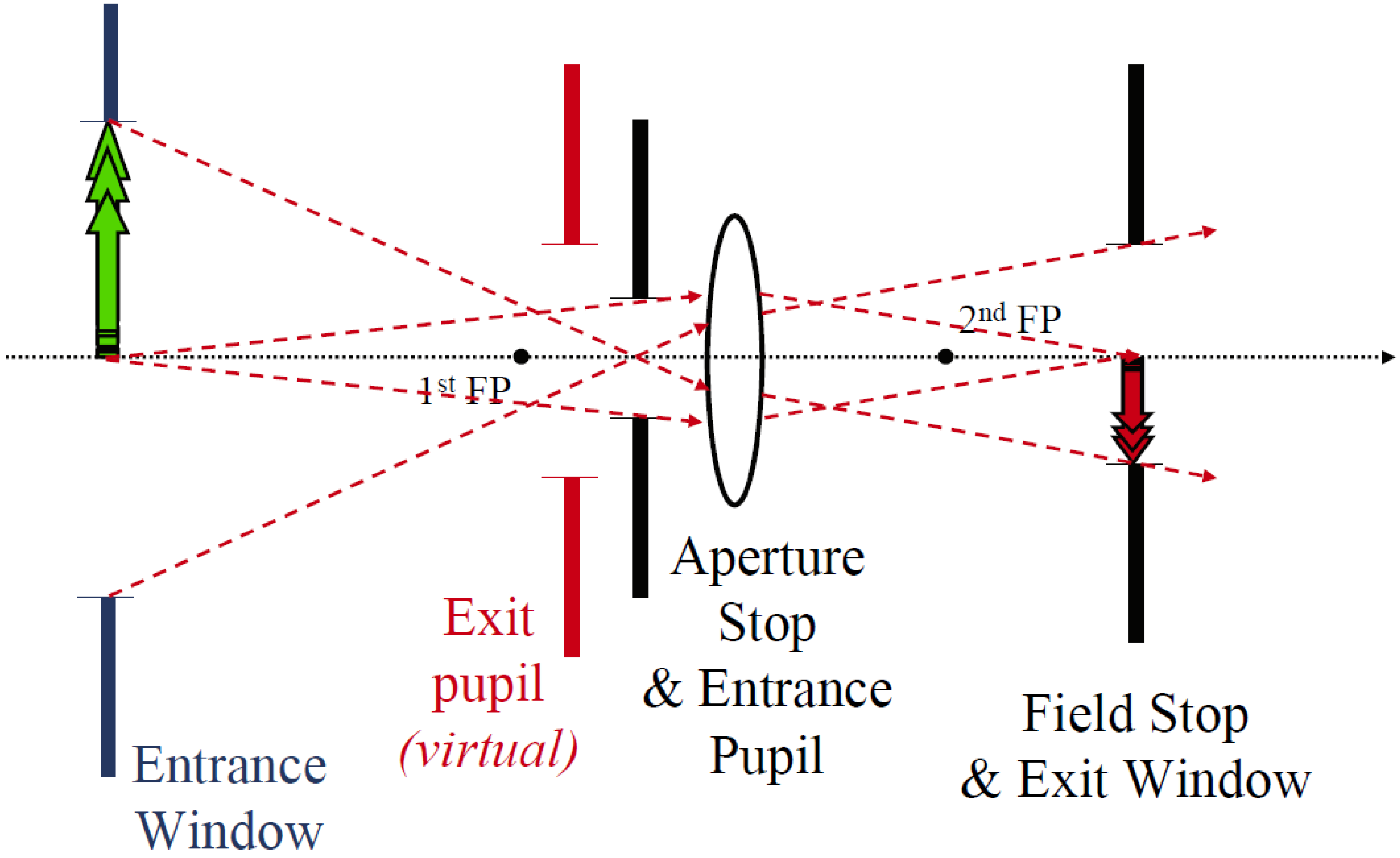
The Chief Ray



Starts from off-axis object,
Goes through the center of the Aperture

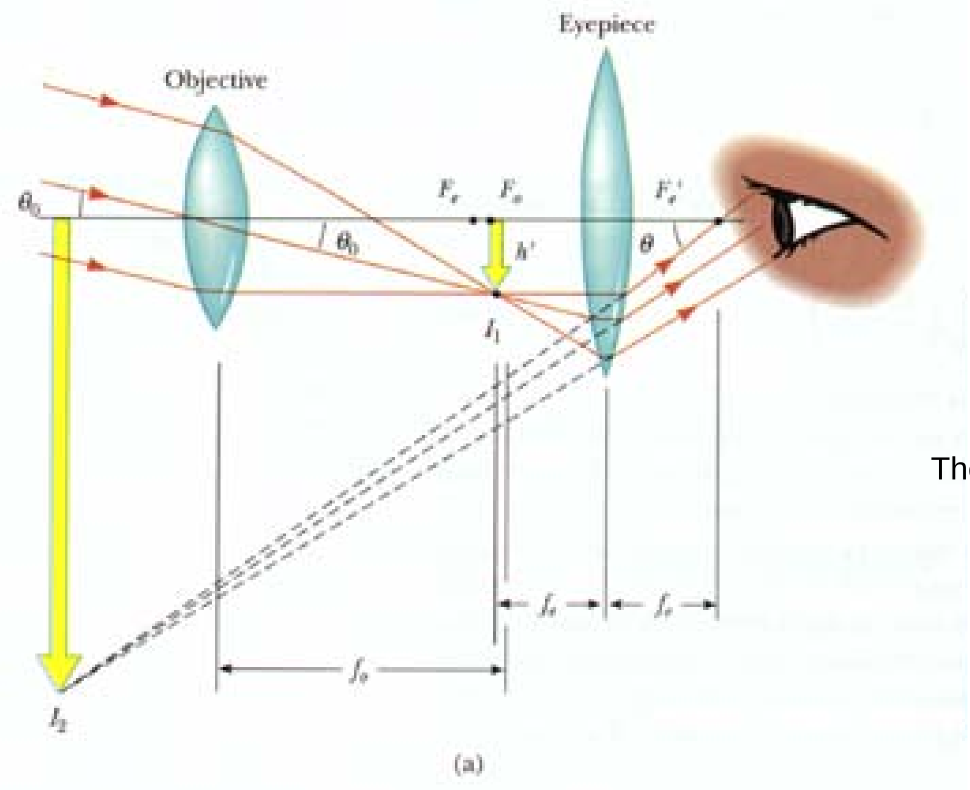
For an off-axis object, the chief ray (CR) is the ray that passes through the center of the aperture stop. Rays that pass through the edge of the aperture stop are marginal rays (MR).

Example II: Aperture Stop + Field Stop



Telescope

- Object is at infinity so image is at f
- Measure angular magnification
- Length of telescope light path is sum of focal lengths of objective and eyepiece



$$M = -\frac{f_o}{f_e}$$

$$\frac{CA_0}{CA_e} = \frac{s}{s'} = \frac{\theta'}{\theta} = M.$$

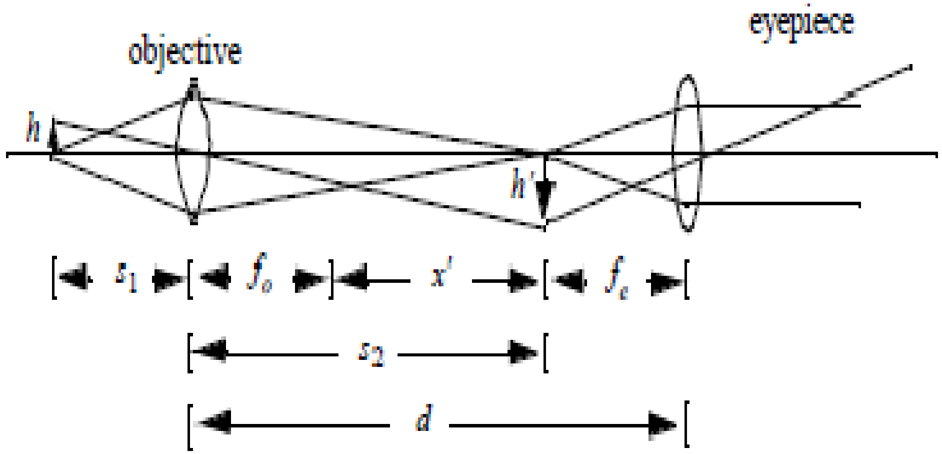
The **exit pupil** is the image of the aperture stop (AS).

Define CA_0 = entrance pupil clear aperture

CA_e= exit pupil clear aperture

From the diagram, it is clear that

Microscope



– The objective lens produces a real (inverted), magnified image of the object.

– The eyepiece re-images to a comfortable viewing distance and provides additional magnification.

- Magnification is product of lateral magnification of objective and angular magnification of eyepiece
- Note: Image is viewed at infinity

$$M_0 = \frac{h'}{h} = -\frac{s_2}{s_1} = \frac{-x'}{f_0}$$

$$M_e = \frac{25}{f_e}$$

$$M_{total} = M_0 \times M_e = \frac{-x'}{f_0} \cdot \frac{25}{f_e}$$