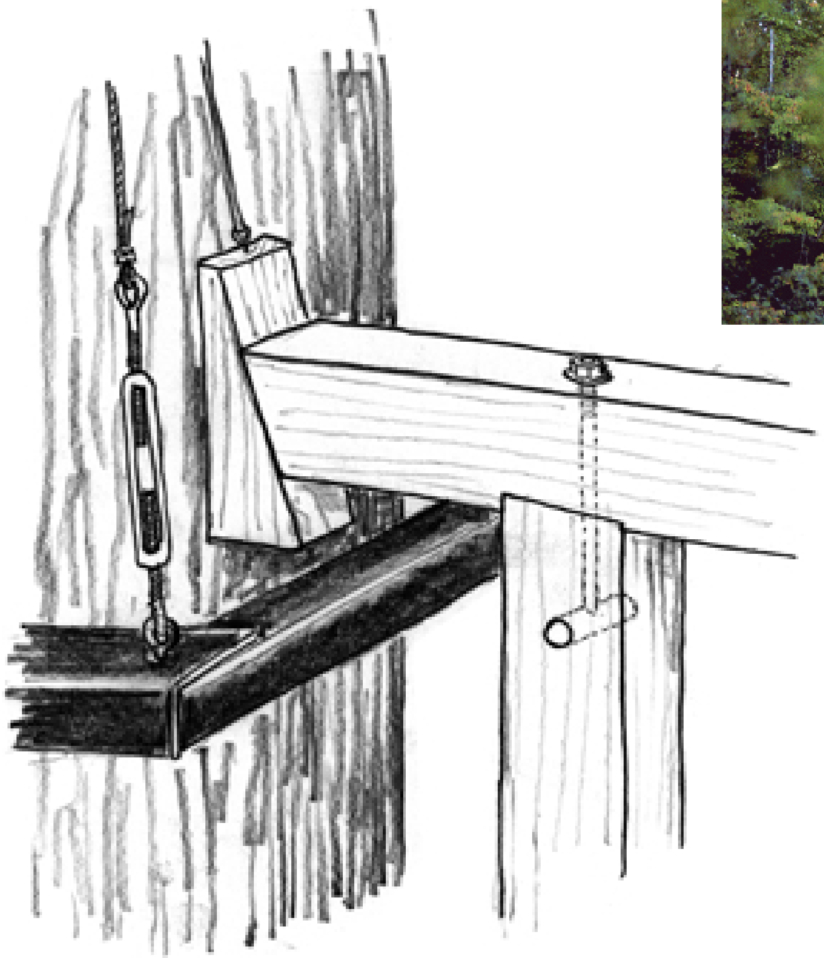
WEDGE FRICTION



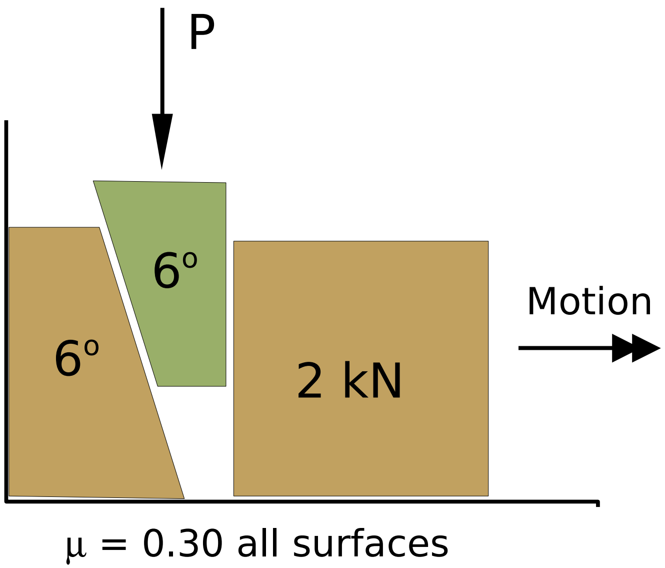
Truss and Collar Wedge detail for use in treehouse construction

Wedge Friction

- Wedge friction is a special application of ramp friction. Most wedge problems have two ramps that must be solve. Not all surfaces have the same coefficient of friction.
- The typical wedge problem tends to be rather tedious in its solution. All wedge problems can be solved using analytical methods. However, it is generally easier to solve these problems using trigonometric methods. This is especially true if the surfaces are not perpendicular.
- Perhaps the biggest mistake a student makes in solving this type of problem is determining the direction of impending motion for all surfaces concerned. This is addressed in the following examples; but make sure you take time to ensure direction of motion is correct. Direction of motion is not an assumption that can be proved or disproved. If your direction of motion is wrong, your solution will be wrong.

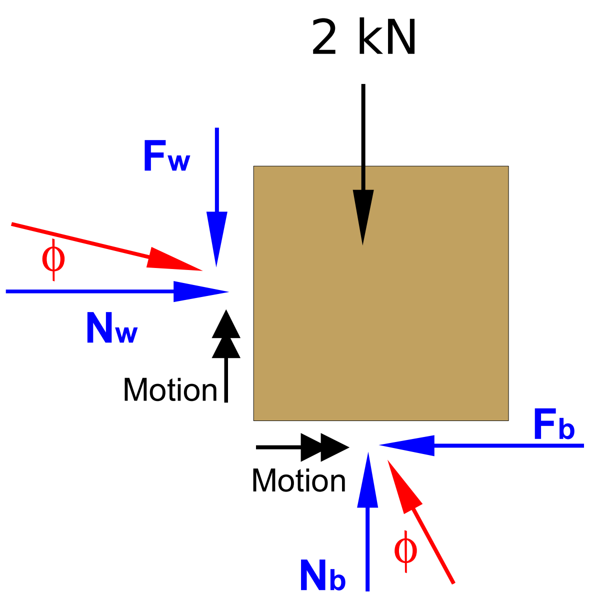
Example 12 - Wedges

- In the problem shown below, a force 'P' is applied in a downward direction to a wedge. The wedge bears against a stationary plane to the left with a 6º angle to the vertical. The right plane of the wedge is vertical. The wedge is used to move a block 2000 N to the right. All surfaces have a coefficient of static friction of 0.30. Find 'P'.



Example 12 - Wedges

- Start with the known weight. The FBD is shown. The direction of the frictional force on the bottom of the block is easily determined; it opposes the direction of impending motion which is to the right.
- However, since the block does not move vertically, how does one determine the direction of frictional force on that surface?
- The easiest way to do that is look at the direction of motion of the mating surface. Since the wedge is moving downward, the force of friction on the wedge is upward. However, Newton's third law applies. The reaction (F_w) on the surface of the mass is downward as shown.



Example 12 - Wedges

- When solving analytically, make sure you organize your relationships so you can keep track

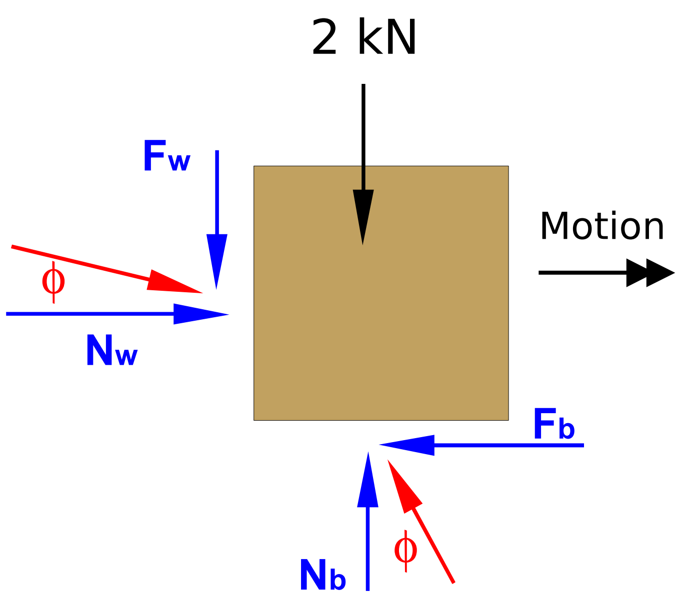
$$\mu = \tan(\phi) = 0.30 = \frac{F_w}{N_w} = \frac{F_b}{N_b}$$

Thus $F_w = 0.30 \cdot N_w$
$F_b = 0.30 \cdot N_b$

$$\Sigma F_y = -2000 + N_b - F_w$$
$$\Sigma F_x = -F_b + N_w$$

substituting:

$$\Sigma F_y = 0 = -2000 + N_b - 0.30\,N_w$$
$$\Sigma F_x = 0 = \quad -0.30\,N_b + \quad N_w$$



Example 12 - Wedges

Solving by elimination

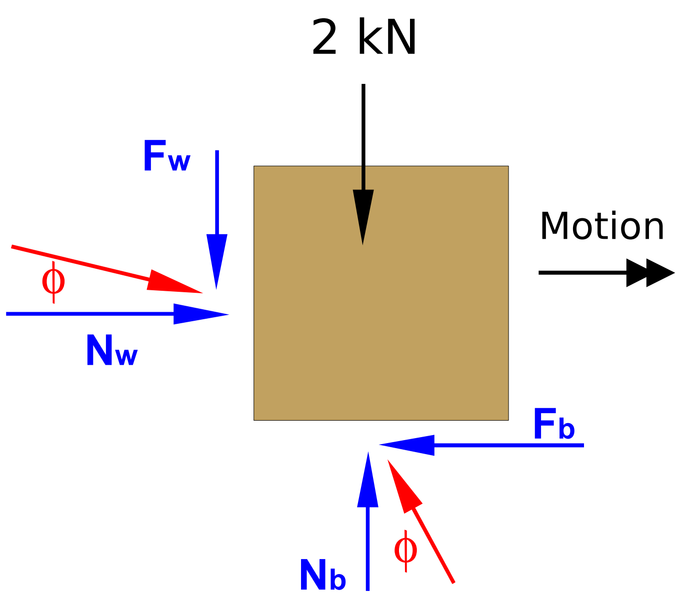
Divide ΣF_x by 0.30 then add to ΣF_y

$$\begin{aligned}\Sigma F_y &= 0 = -2000 + N_b - 0.30\,N_w \\ \Sigma F_x &= 0 = \qquad\quad - N_b + 3.3\bar{3}\,N_w \\ &\hline \\ &\quad -2000 + 0 + 3.0\bar{3}\,N_w\end{aligned}$$

$$\boxed{N_w = 659\,N \rightarrow}$$

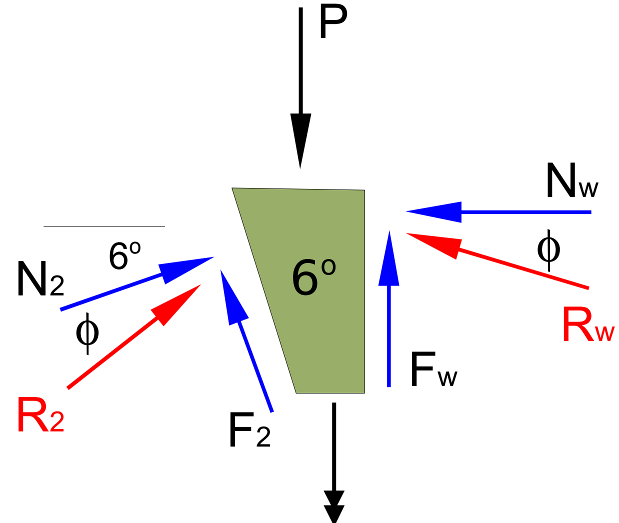
Substituting:

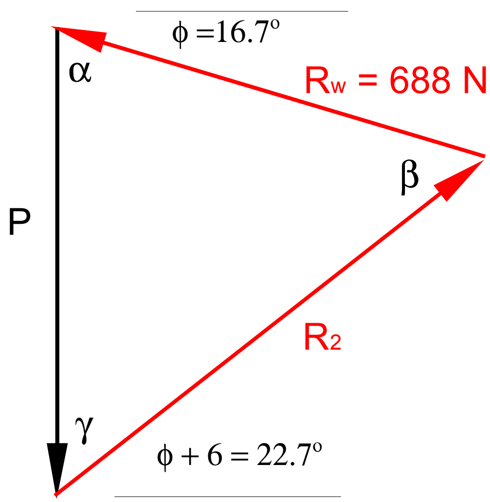
$$\boxed{\begin{aligned}F_w &= 198\,N \downarrow \\ F_b &= 659\,N \leftarrow \\ N_b &= 2197\,N \uparrow\end{aligned}}$$



Example 12 - Wedges

- Now develop a FBD of the wedge. We will solve this FBD trigonometrically
  - Use the pythagorean theorem to find R_w
  - Determine friction angle ϕ
  - Develop the force triangle using known values
  - Solve with the law of sines





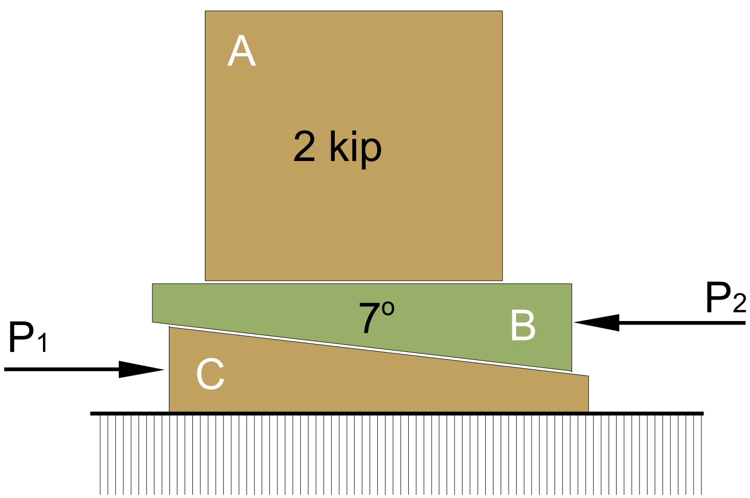
$$\phi = \tan^{-1}(0.3) = 16.7^o$$
$$\alpha = 90 - \phi = 73.3^o$$
$$\gamma = 90 - (\phi + 6) = 67.3^o$$
$$\beta = 180 - 73.3 - 67.3 = 39.4^o$$

$$\frac{P}{\sin(39.4^o)} = \frac{688}{\sin(67.3^o)}$$

$$\therefore \quad P = 473\, N \downarrow$$

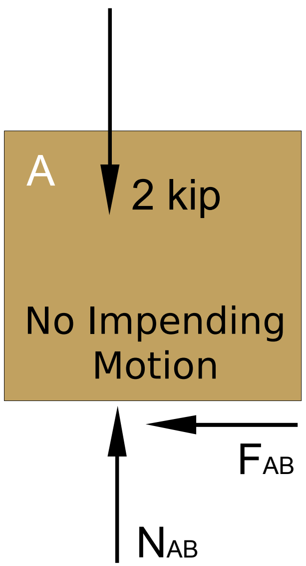
Example 13 - Wedges

- Determine the smallest values of forces P_1 and P_2 required to raise Block A while preventing Block A from moving horizontally. The coefficient of static friction for all surfaces of contact is 0.3. The weight of wedges B and C is negligible compared to the weight of Block A.



Example 13 - Wedges

- Start with a FBD of Block A



Equations of equilibrium for Block A

$$\Sigma F_x = 0: \quad -F_{AB} = 0$$

$$\therefore F_{AB} = 0$$

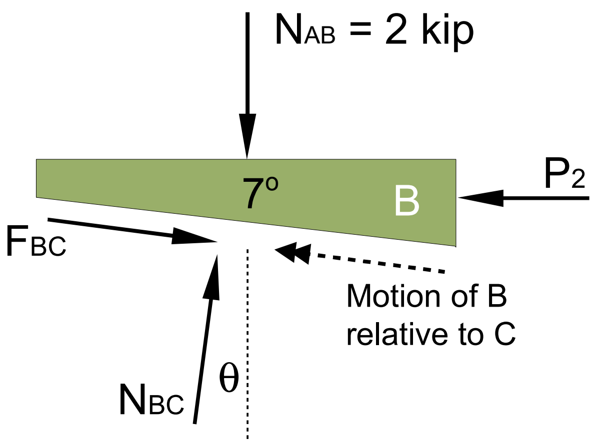
$$\Sigma F_y = 0: \quad -N_{AB} - 2\,kip = 0$$

$$\therefore N_{AB} = 2\,kip \uparrow$$

Alternatively, we know block 'A' does not move and we know there are no horizontal forces acting on the block. By inspection, it follows that force F_{AB} must be zero.

Example 13 - Wedges

- Construct a FBD of wedge B. Motion is to the left, so the force of friction is to the right and parallel to the bottom surface. The weight of Block A is shown as N_{AB}. By similar triangles, angle θ is 7°.



Equations of equilibrium for wedge B

$$\mu = 0.30 = \frac{F_{BC}}{N_{BC}} \qquad \therefore F_{BC} = 0.30 \cdot N_{BC}$$

$$\Sigma F_x = 0$$

$$F_{BC} \cdot \cos(7^o) + N_{BC} \cdot \sin(\theta) - P_2 = 0$$

$$\Sigma F_y = 0$$

$$-F_{BC} \cdot \sin(7^o) + N_{BC} \cdot \cos(\theta) - 2\,kip = 0$$

Example 13 - Wedges

Substituting $\theta = 7^o$ and $F_{BC} = 0.30 \cdot N_{BC}$, we have:

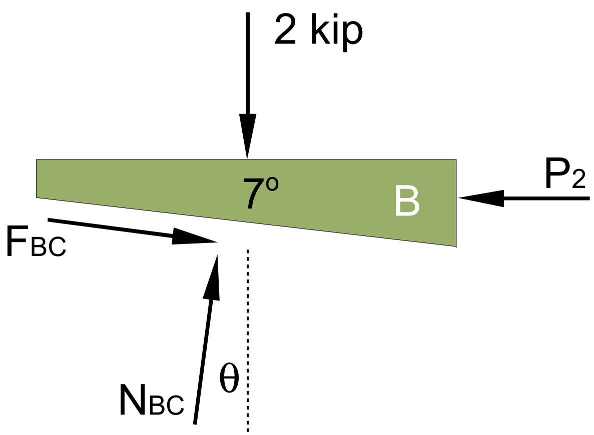
$$\Sigma F_x: \quad 0.30 \cdot N_{BC} \cdot \cos(7^o) + N_{BC} \cdot \sin(7^o) - P_2 = 0$$
$$\Sigma F_y: \quad -0.30 \cdot N_{BC} \cdot \sin(7^o) + N_{BC} \cdot \cos(7^o) - 2 = 0$$

<u>Simplifying</u>

$$\Sigma F_x: \quad 0.420 \cdot N_{BC} - P_2 = 0$$
$$\Sigma F_y: \quad 0.956 \cdot N_{BC} - 2 = 0$$

<u>Solving ΣF_y</u>

$$N_{BC} = \frac{2}{0.956} \quad \boxed{= 2.09\ kips}$$



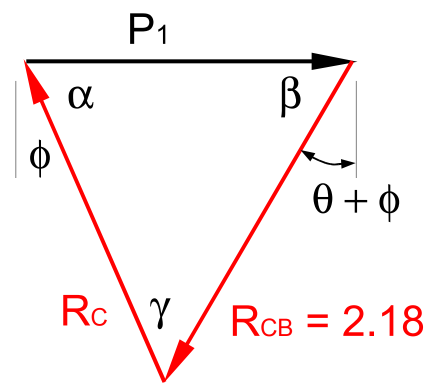
<u>Then substituting the result into ΣF_x</u>

$$P_2 = (0.420) \cdot (2.09) \quad \boxed{= 0.879\ kips}$$

Example 13 - Wedges

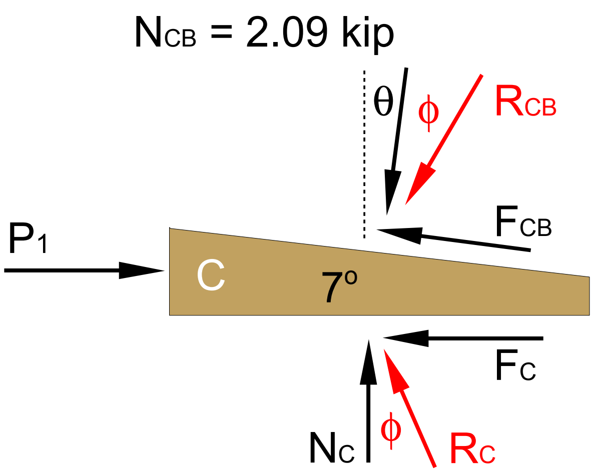
- Let's solve wedge C trigonometrically
- Constructing a FBD of wedge C, it is apparent we need F_{CB} to find R_{CB}

$$F_{CB} = \mu \cdot N_{CB} = 0.3 \cdot (2.09) = 0.627\,kips$$
$$R_{CB} = \sqrt{0.627^2 + 2.09^2} = 2.18\,kips$$



- We also need to know the friction angle

$$\alpha = \tan^{-1}(\mu) = 16.7^o$$

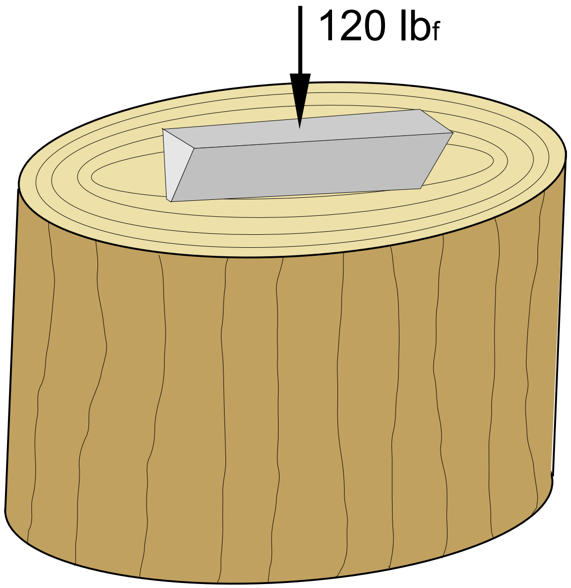


$$\alpha = 90 - \phi = 73.3^o$$
$$\beta = 90 - (\theta + \phi) = 66.3^o$$
$$\gamma = 180 - \alpha - \beta = 40.4^o$$
$$\frac{P_1}{\sin(40.4^o)} = \frac{2.18}{\sin(73.3^o)}$$

$$\boxed{P_1 = 1.48\,kips}$$

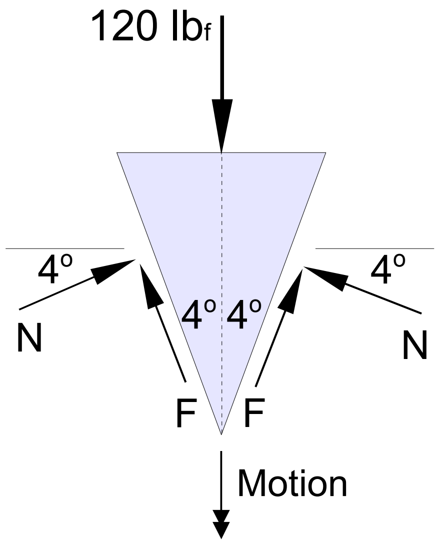
Example 14 - Wedges

- To split a log, a 120 lb force is applied to a wedge with an angle of 8°. Motion is impending. The coefficient of friction on all surfaces is 0.6.
  - Determine all forces acting on the wedge.
  - Will the wedge be forced out of the cut if the force is removed?



Example 14 - Wedges

- For motion to be impending, the vertical components of the friction forces and normal forces acting on the wedge must equal the applied force of 120 lb_f
- Develop the FBD for the wedge. Due to symmetry, the magnitudes of F and N on each side of the wedge are equal



$$\mu = 0.6 = \frac{F}{N} \qquad \therefore F = 0.6 \cdot N$$

$$\Sigma F_x = N_{xR} - N_{xL} = 0 \qquad \text{(not useful)}$$

$$\Sigma F_y = N \cdot \sin(4^o) \cdot 2 + F \cdot \cos(4^o) \cdot 2 - 120 = 0$$

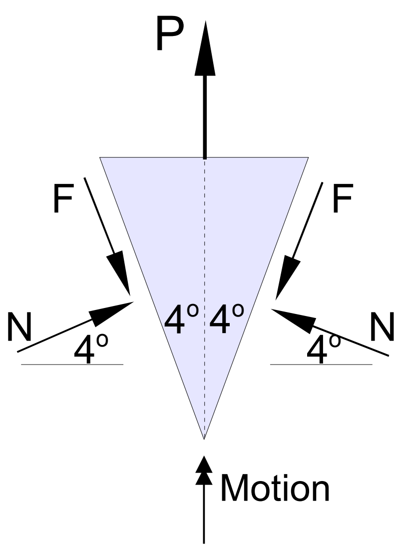
$$N \cdot \sin(4^o) \cdot 2 + 0.6 \cdot N \cdot \cos(4^o) \cdot 2 - 120 = 0$$

$$1.34 \cdot N = 120$$

$$\boxed{\begin{aligned} N &= 89.8\, lb_f \\ F &= 0.6 \cdot 89.8 = 54.1\, lb_f \end{aligned}}$$

Example 14 - Wedges

- For the wedge to be forced out...we know impending motion must occur when θ is less than friction angle ϕ = 30.96°. Since θ < ϕ, we know the wedge will not be forced from the cut.
- Alternatively, we only need to show the force required to cause the wedge to be forced from the cut is greater than 'P'...which is equal to zero in this case



$$\Sigma F_y = P + N\cdot\sin(4^o)\cdot 2 - F\cdot\cos(4^o)\cdot 2$$

$$\mu = 0.60 = \frac{F}{N} \qquad \therefore F = 0.60\cdot N$$

$$\Sigma F_y = P + 2\cdot N\cdot\sin(4^o) - 2\cdot 0.60\cdot N\cdot\cos(4^o)$$

$$P = 0.80\cdot N\cdot\sin(4^o) = 1.058\cdot N$$

This calculation indicates a value of P>0 must be applied to remove the wedge. Thus, it follows the wedge will not be forced from the cut when the 120 lb is removed.