week 10

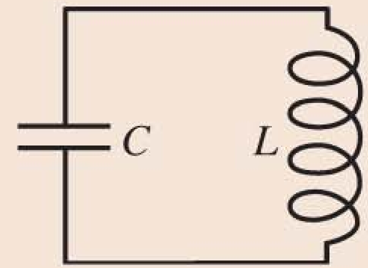
AC circuits

Applications

(leave for next week)

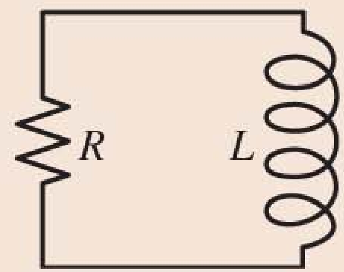
LC circuit

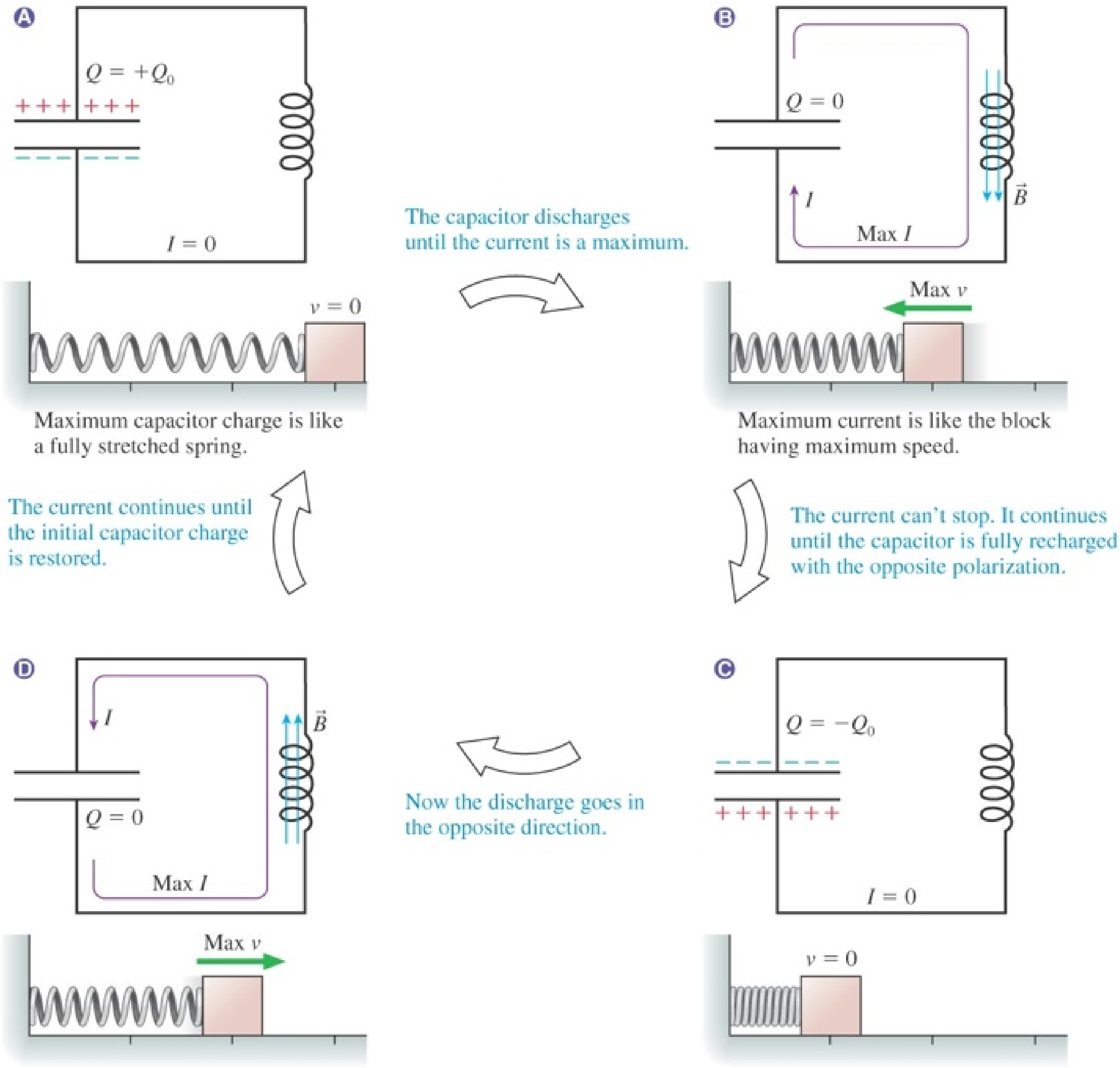
Oscillates at $\omega = \sqrt{\frac{1}{LC}}$



LR circuit

Exponential change with $\tau = \frac{L}{R}$

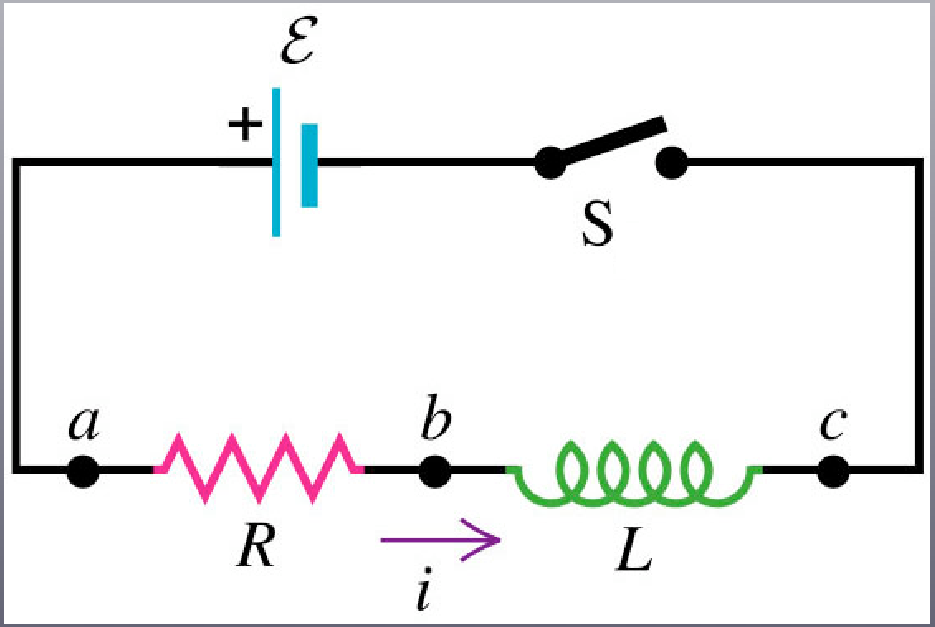



A
$Q = +Q_0$
$I = 0$
$v = 0$
Maximum capacitor charge is like a fully stretched spring.
The capacitor discharges until the current is a maximum.
B
$Q = 0$
I
Max I
$\vec{B}$
Max v
Maximum current is like the block having maximum speed.
The current can't stop. It continues until the capacitor is fully recharged with the opposite polarization.
C
$Q = -Q_0$
$I = 0$
$v = 0$
Now the discharge goes in the opposite direction.
D
I
$\vec{B}$
$Q = 0$
Max I
Max v
The current continues until the initial capacitor charge is restored.

An inductor (inductance L) and a resistor (resistance R) are connected to a source of emf as shown. When switch S is closed, a current begins to flow and grows until it reaches a final value.

The *final value of the current*

1. is directly proportional to both R and L
2. is directly proportional to R and inversely proportional to L
3. is inversely proportional to R and directly proportional to L
4. is inversely proportional to both R and L
5. is independent of L



An inductor (inductance L) and a resistor (resistance R) are connected to a source of emf as shown. When switch S is closed, a current begins to flow and grows until it reaches a final value.

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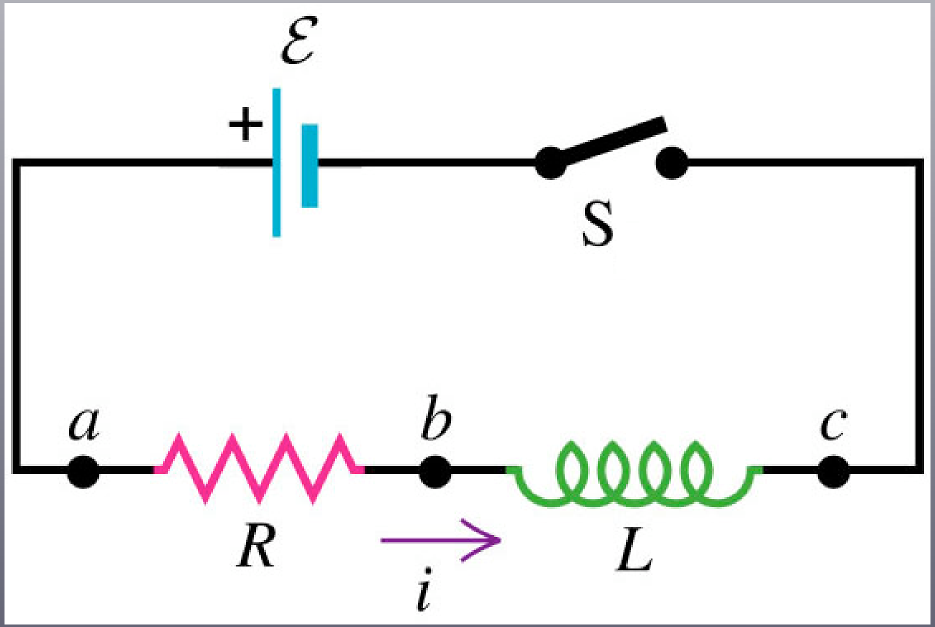
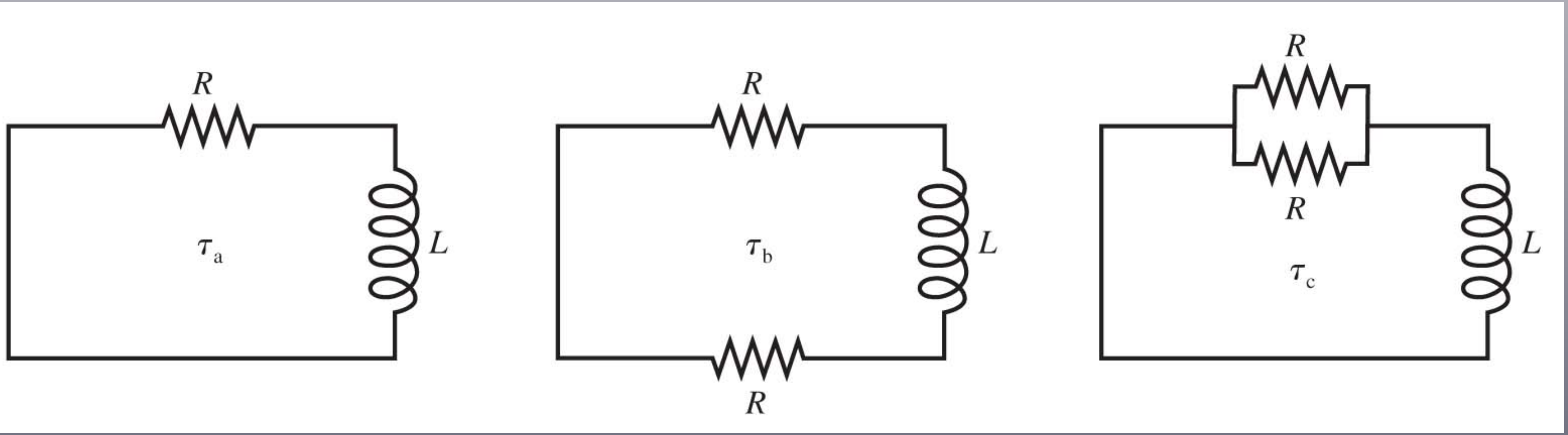


Rank in order, from largest to smallest, the time constants τ_a, τ_b, and τ_c of these three circuits.

1. $\tau_a > \tau_b > \tau_c$
2. $\tau_b > \tau_a > \tau_c$
3. $\tau_b > \tau_c > \tau_a$
4. $\tau_c > \tau_a > \tau_b$
5. $\tau_c > \tau_b > \tau_a$



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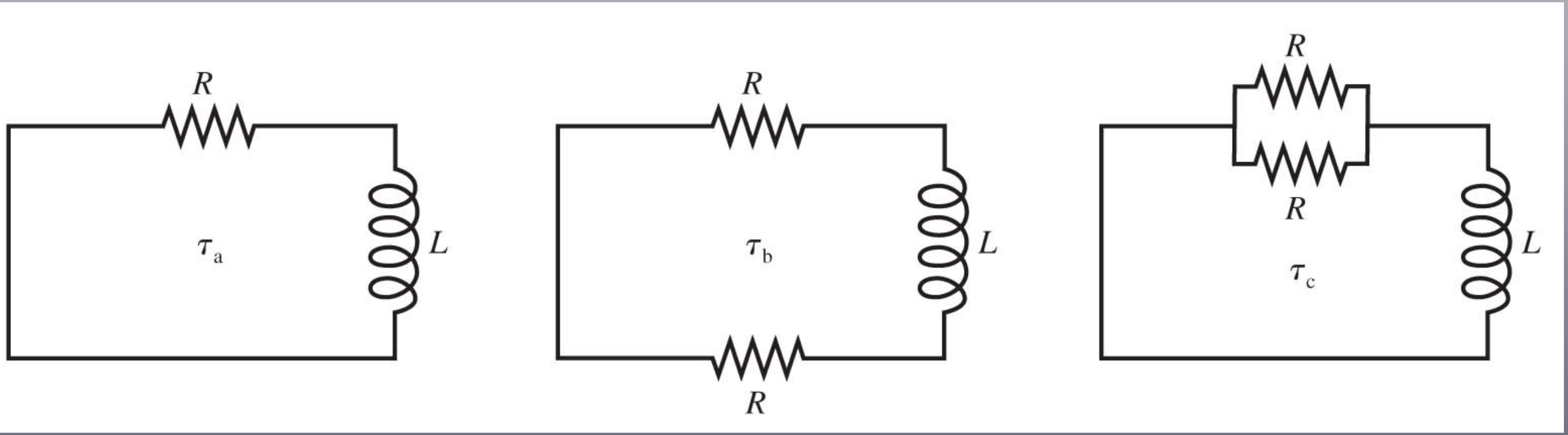
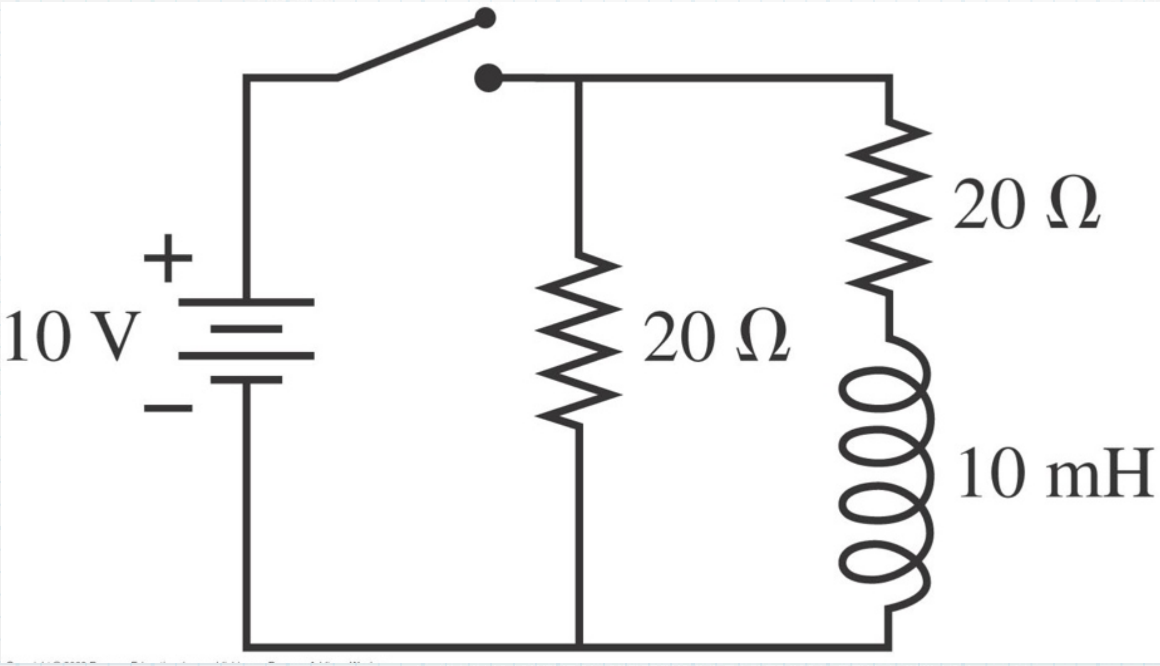


The switch in the figure has been open for a long time. It is closed at t = 0 s.
(a) What is the current through the battery immediately after the switch is closed?
(b) What is the current through the battery after the switch has bee closed for a long time?
(c) What is meant by a "long time" in (b)?

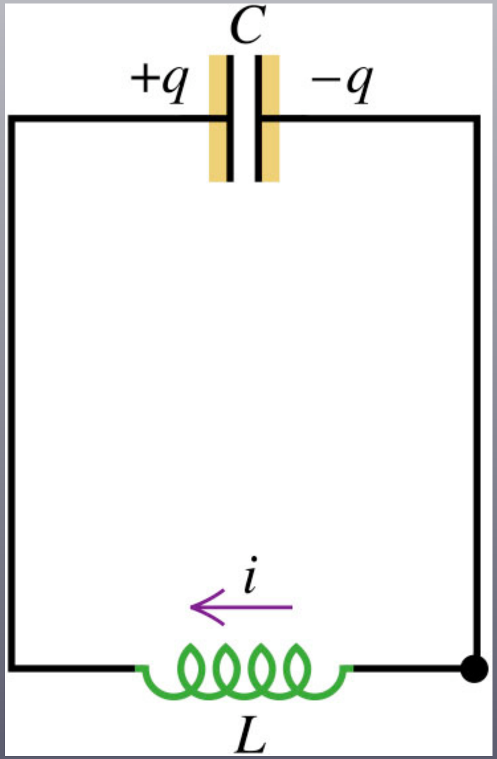


Ans: (a) 0.50 A (b) 1.0 A (c) a time much longer than 0.5 ms

An inductor (inductance L) and a capacitor (capacitance C) are connected as shown.

If the values of both L and C are doubled, what happens to the *time* required for the capacitor charge to oscillate through a complete cycle?

1. it becomes 4 times longer
2. it becomes twice as long
3. it is unchanged
4. it becomes 1/2 as long
5. it becomes 1/4 as long



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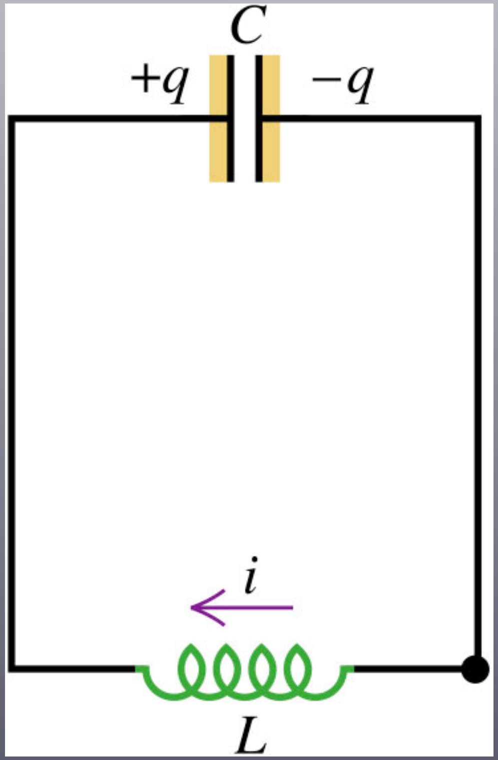
Recall the angular frequency is $\omega = \sqrt{(1/LC)}$. The period *T* satisfies $\omega T = 2\pi$, so $T = 2\pi \sqrt{(LC)}$.

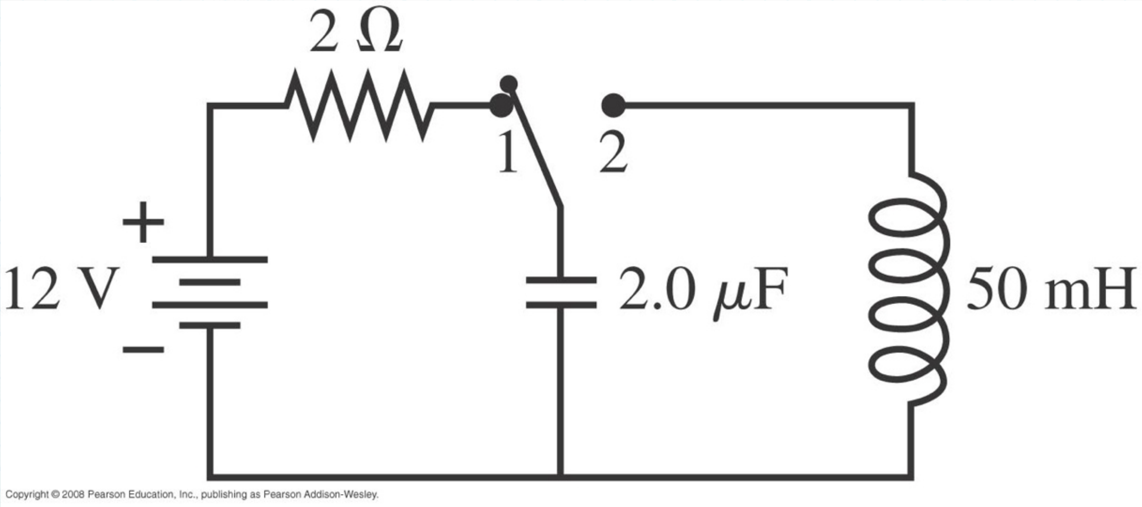
Therefore, doubling both *L* and *C* increases *T* by the square root of 2^2



34-76. The switch in the figure has been in position 1 for a long time. It is changed to position 2 at t=0s.

(a) What is the maximum current through the indictor?

(b) What is the first time at which the current is maximum?



Ans: (a) 76 mA; (b) T=2.0 ms, t_{max}=T/4=0.50 ms

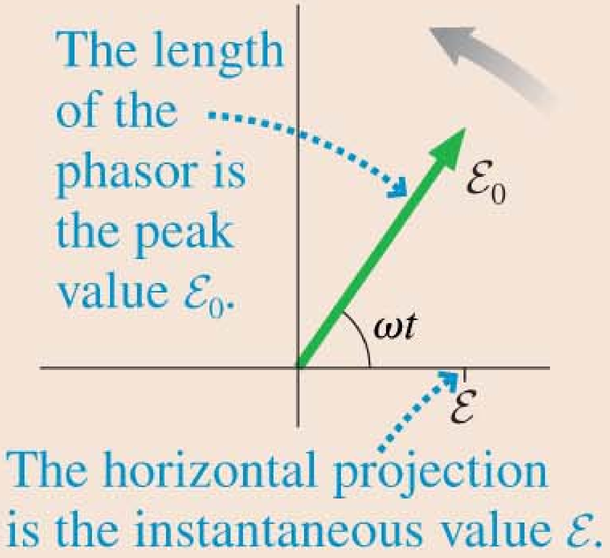
Important Concepts

AC circuits are driven by an emf

$$\mathcal{E} = \mathcal{E}_0 \cos \omega t$$

that oscillates with angular frequency $\omega = 2\pi f$.

Phasors can be used to represent the oscillating emf, current, and voltage.



Important Concepts

Basic circuit elements

Element	i and v	Resistance/ reactance	I and V	Power
Resistor	In phase	R is fixed	$V = IR$	$V_{rms}I_{rms}$
Capacitor	i leads v by 90°	$X_C = 1/\omega C$	$V = IX_C$	0
Inductor	i lags v by 90°	$X_L = \omega L$	$V = IX_L$	0

For many purposes, especially calculating power, the **root-mean-square** (rms) quantities

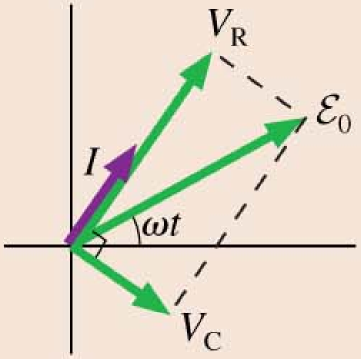
$$V_{rms} = V/\sqrt{2} \qquad I_{rms} = I/\sqrt{2} \qquad \mathcal{E}_{rms} = \mathcal{E}_0/\sqrt{2}$$

are equivalent to the corresponding DC quantities.

Key Skills

Phasor diagrams

- Start with a phasor (v or i) common to two or more circuit elements.
- The sum of instantaneous quantities is vector addition.
- Use the Pythagorean theorem to relate peak quantities.



For an *RC* circuit, shown here,

$$v_R + v_C = \mathcal{E}$$

$$V_R^2 + V_C^2 = \mathcal{E}_0^2$$

Key Skills

Kirchhoff's laws

Loop law The sum of the potential differences around a loop is zero.

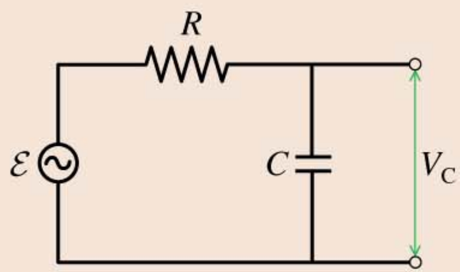
Junction law The sum of currents entering a junction equals the sum leaving the junction.

Instantaneous and peak quantities

Instantaneous quantities v and i generally obey different relationships than peak quantities V and I.

Applications

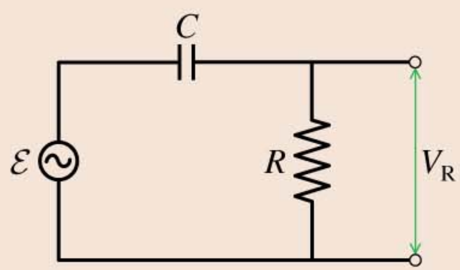
RC filter circuits



$$V_C = \mathcal{E}_0 X_C / \sqrt{R^2 + X_C^2}$$

$$V_C \to \mathcal{E}_0 \text{ as } \omega \to 0$$

A **low-pass filter** transmits low frequencies and blocks high frequencies.



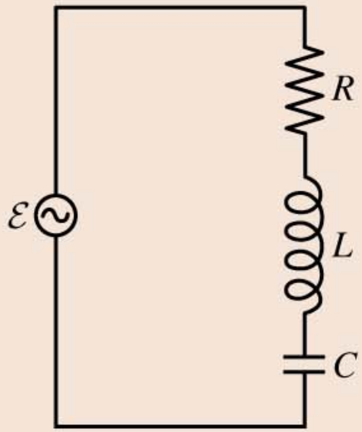
$$V_R = \mathcal{E}_0 R / \sqrt{R^2 + X_C^2}$$

$$V_R \to \mathcal{E}_0 \text{ as } \omega \to \infty$$

A **high-pass filter** transmits high frequencies and blocks low frequencies.

Applications

Series *RLC* circuits



$I = \mathcal{E}_0/Z$ where Z is the **impedance**

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$V_R = IR \qquad V_L = IX_L \qquad V_C = IX_C$$

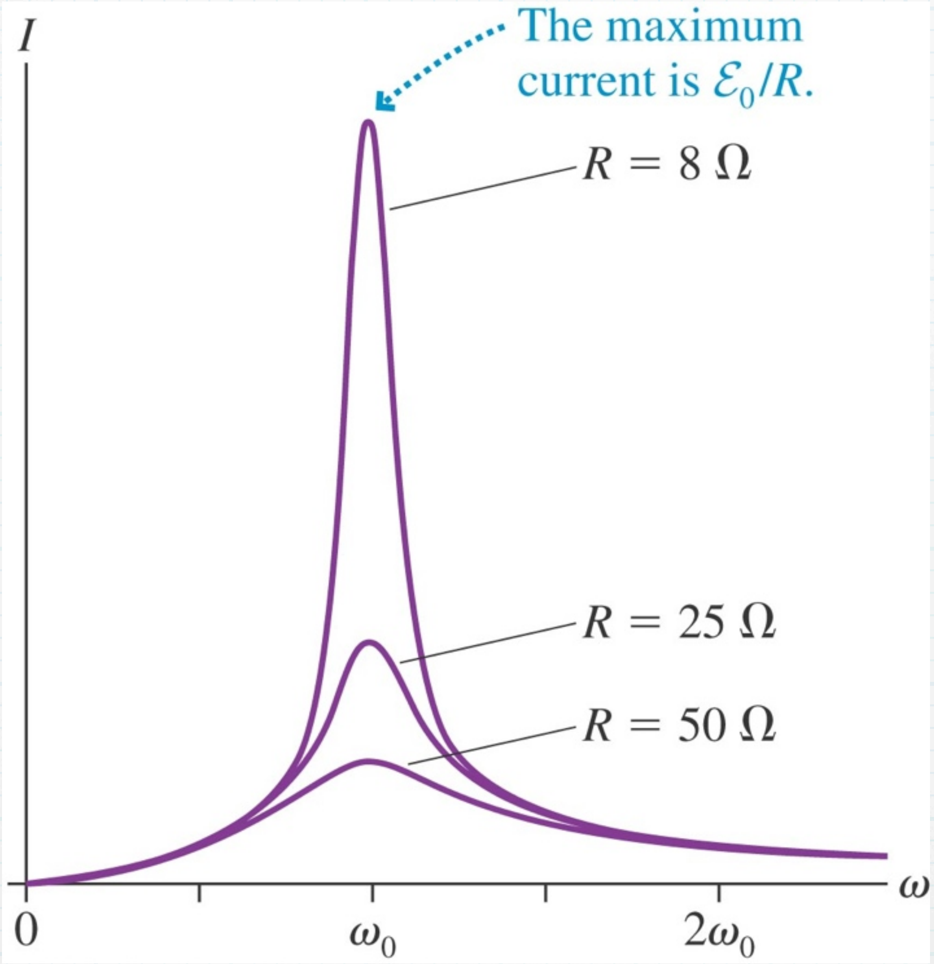
When $\omega = \omega_0 = 1/\sqrt{LC}$ (the **resonance frequency**), the current in the circuit is a maximum $I_{max} = \mathcal{E}_0/R$.

In general, the current i lags behind $\mathcal{E}$ by the **phase angle** $\phi = \tan^{-1}\big((X_L - X_C)/R\big)$.

The power supplied by the emf is $P_{source} = I_{rms}\mathcal{E}_{rms}\cos\phi$, where $\cos\phi$ is called the **power factor.**

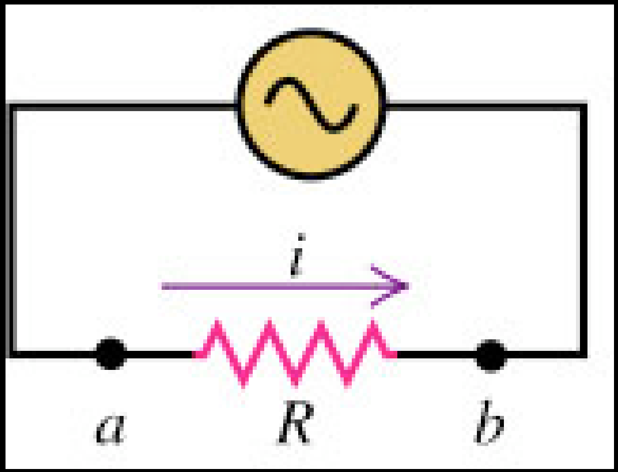
The power lost in the resistor is $P_R = I_{rms}V_{rms} = (I_{rms})^2R$.

Resonance

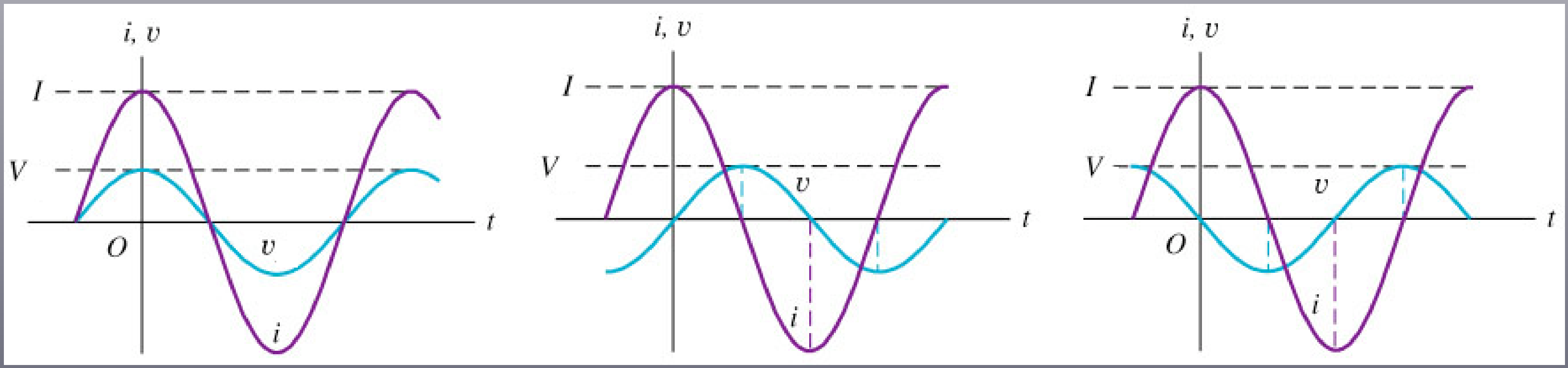


I
The maximum current is $\mathcal{E}_0/R$.
$R = 8\ \Omega$
$R = 25\ \Omega$
$R = 50\ \Omega$
ω
0
ω_0
$2\omega_0$

A resistor is connected across an ac source as shown. Which graph correctly shows the instantaneous current through the resistor and the instantaneous voltage $v = v_a - v_b$ across the resistor?



(current "*i*" in purple, voltage "*v*" in blue)



1.

2.

3.

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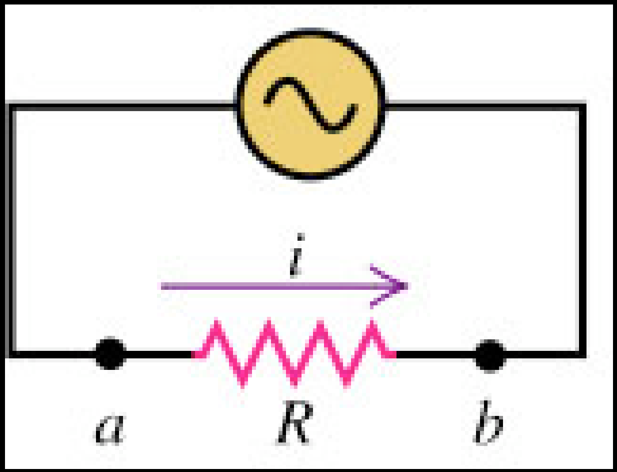

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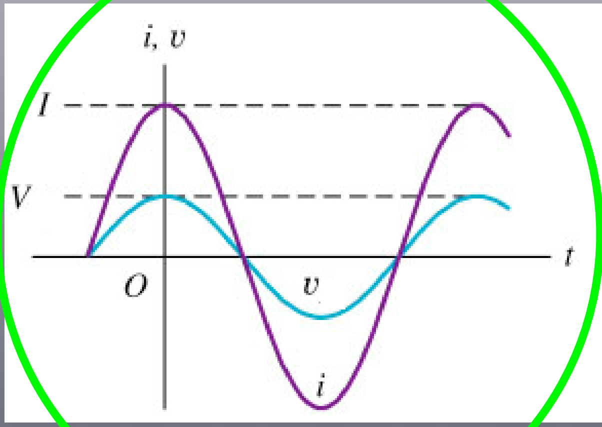
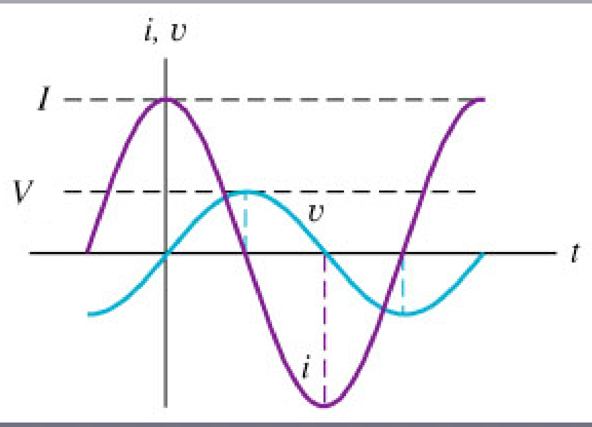
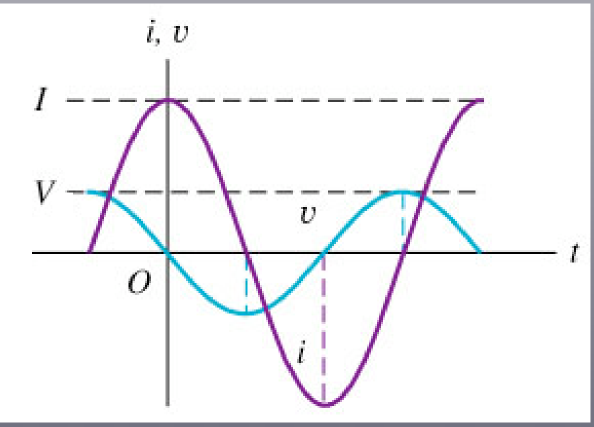


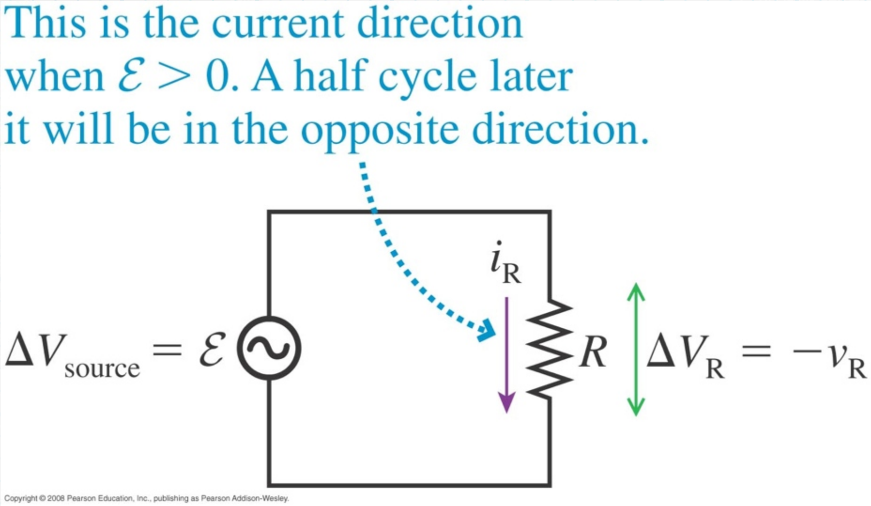
1.



2.



3.


This is the current direction
when $\mathcal{E} > 0$. A half cycle later
it will be in the opposite direction.
i_R
$\Delta V_{source} = \mathcal{E}$
R
$\Delta V_R = -v_R$
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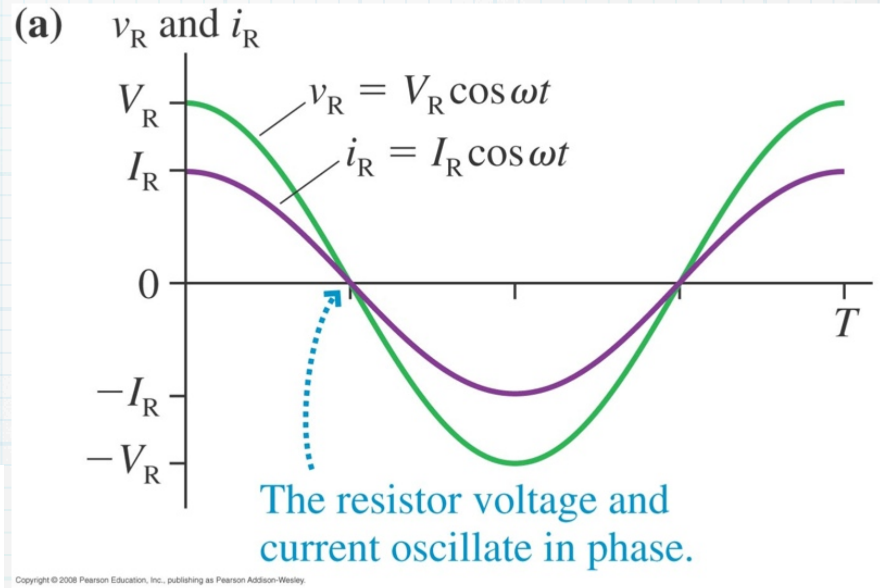
(a)
v_R and i_R
V_R
I_R
0
$-I_R$
$-V_R$
$v_R = V_R \cos \omega t$
$i_R = I_R \cos \omega t$
t
T
The resistor voltage and
current oscillate in phase.
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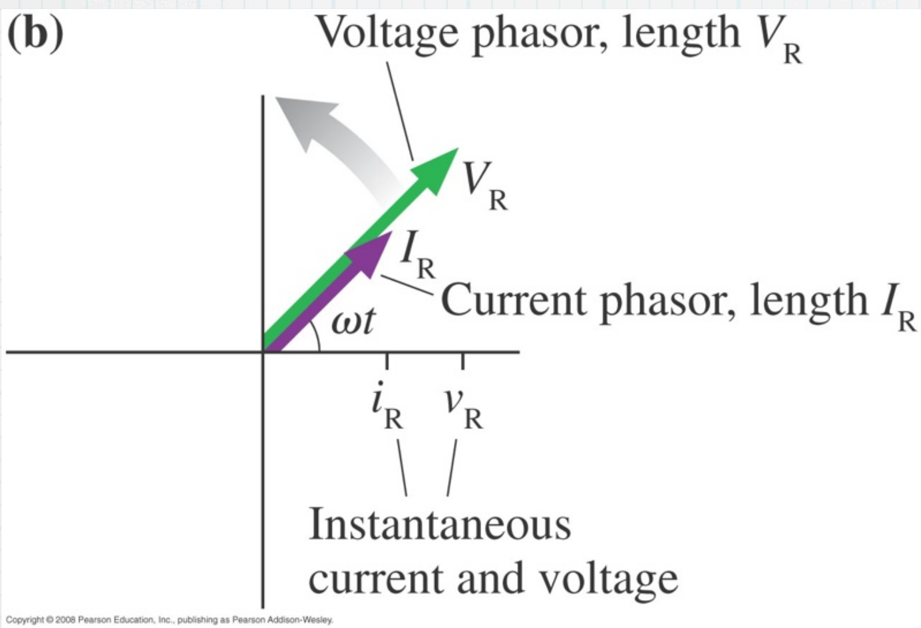
(b)
Voltage phasor, length V_R
V_R
I_R
Current phasor, length I_R
ωt
i_R
v_R
Instantaneous
current and voltage
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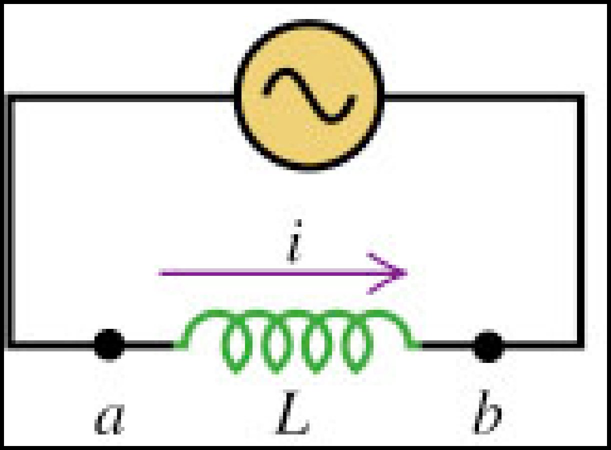
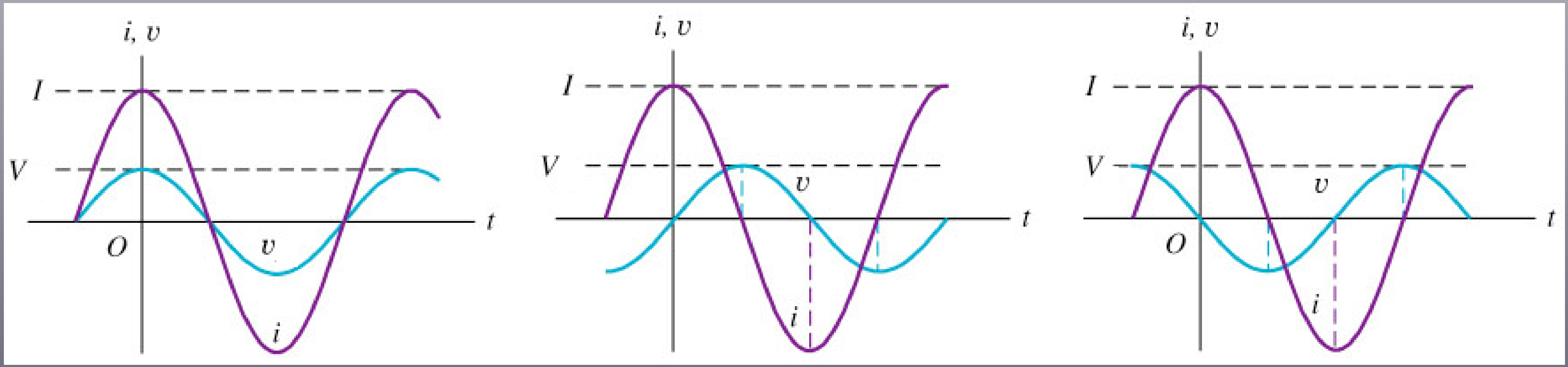
An inductor is connected across an ac source as shown. Which graph correctly shows the instantaneous current through the inductor and the instantaneous voltage $v = v_a - v_b$ across the inductor?



(current "*i*" in purple, voltage "*v*" in blue)



1. 2. 3.

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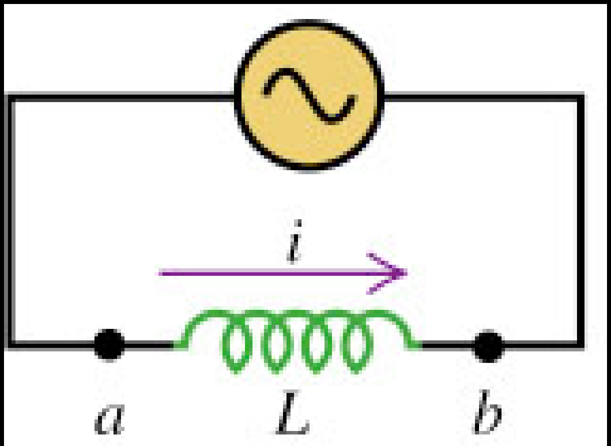
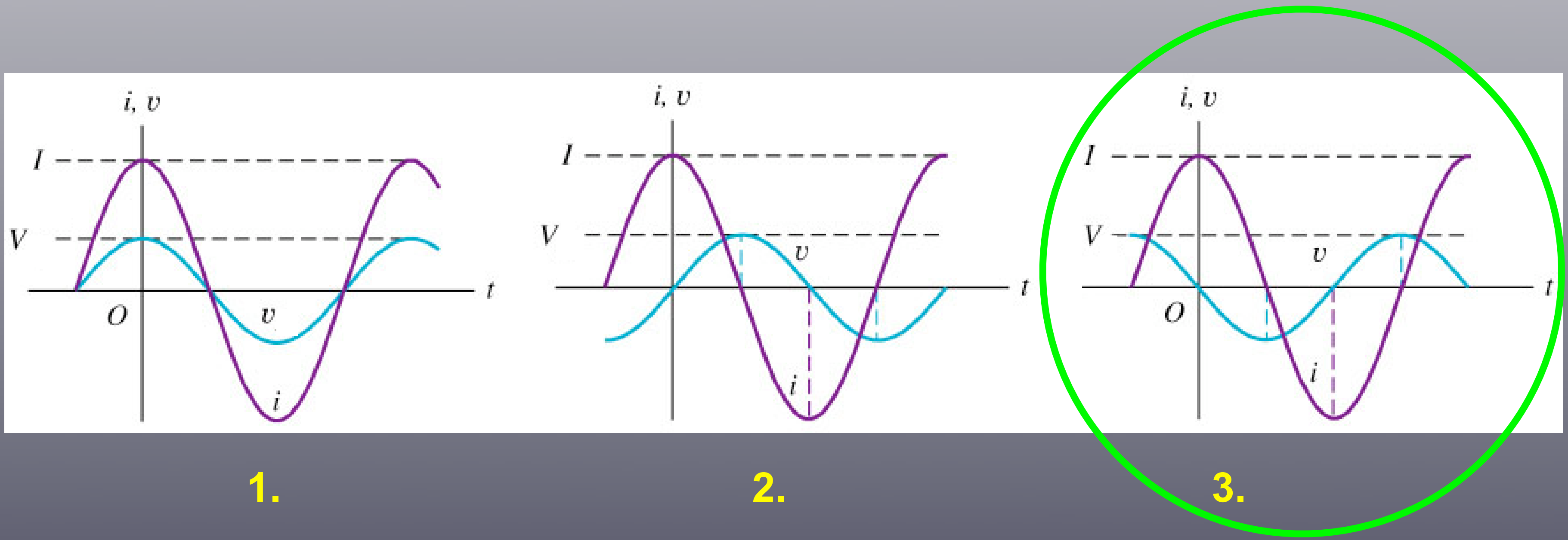


(current "*i*" in purple, voltage "*v*" in blue)



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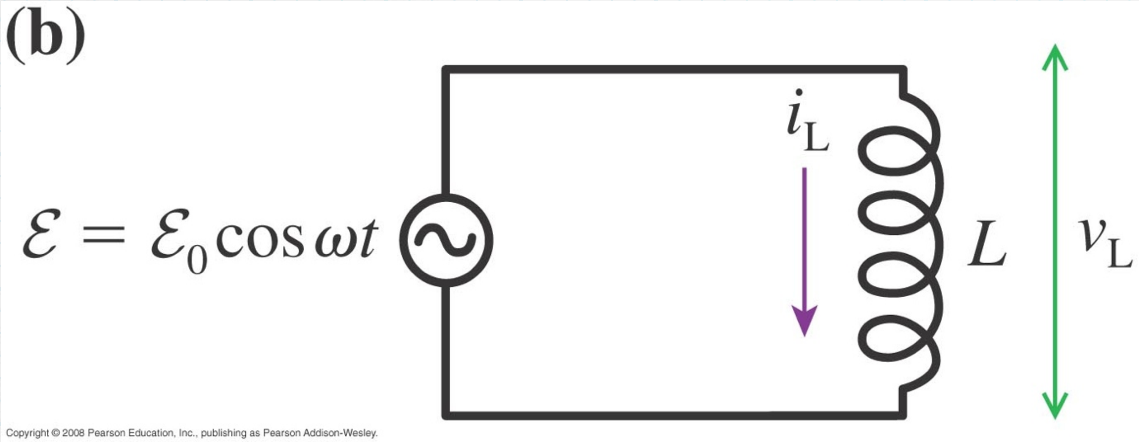
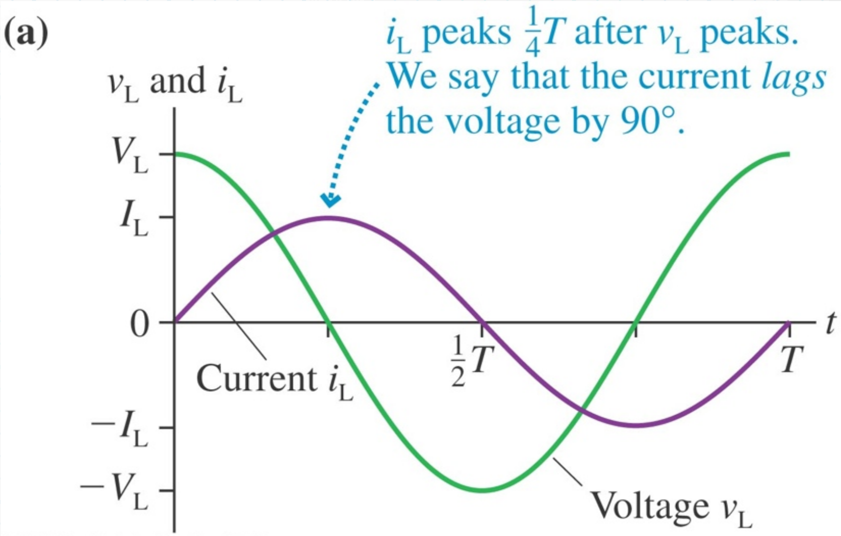
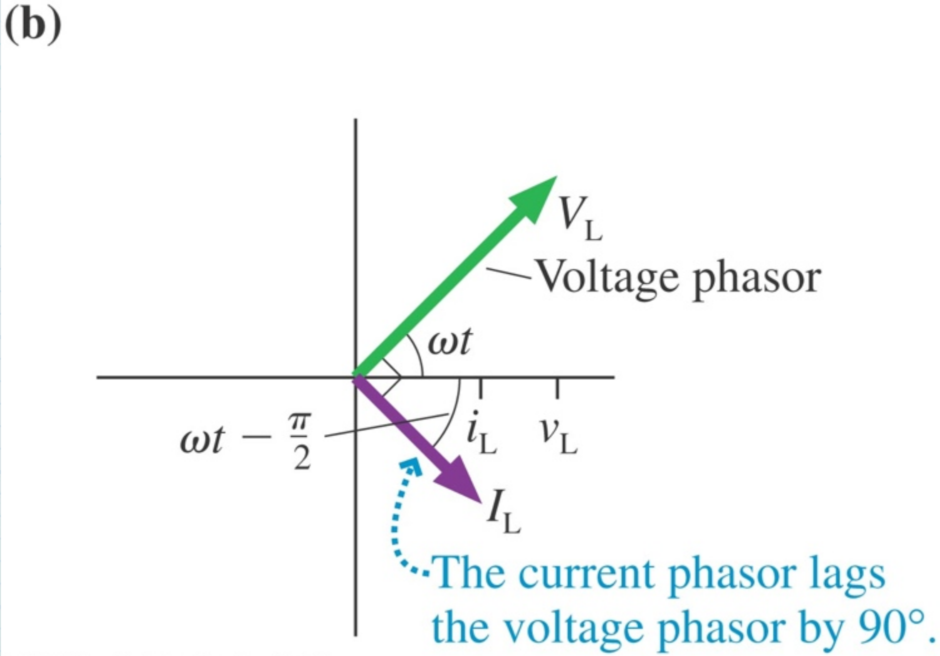


(b)
$\mathcal{E} = \mathcal{E}_0 \cos \omega t$
i_L
L
v_L

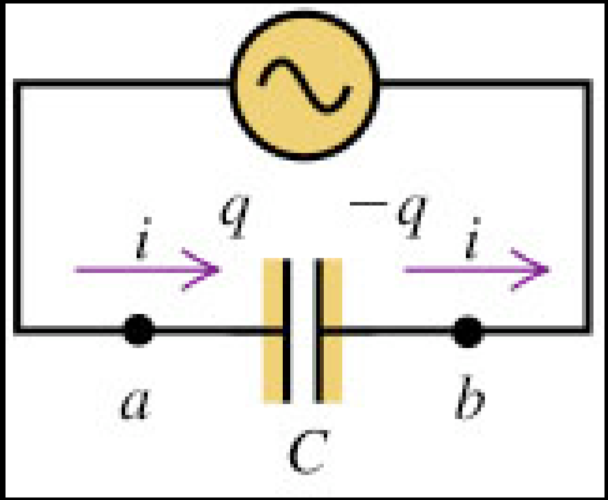


(a)
i_L peaks $\frac{1}{4}T$ after v_L peaks. We say that the current *lags* the voltage by 90°.
v_L and i_L
V_L
I_L
0
t
$\frac{1}{2}T$
T
Current i_L
$-I_L$
$-V_L$
Voltage v_L

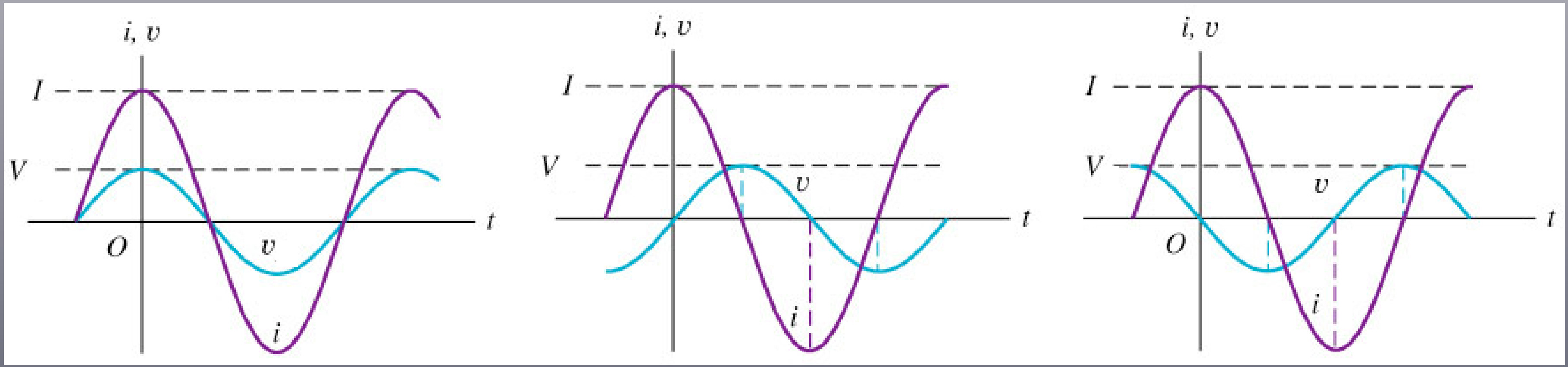


(b)
V_L
Voltage phasor
ωt
$\omega t - \frac{\pi}{2}$
i_L
v_L
I_L
The current phasor lags the voltage phasor by 90°.

An capacitor is connected across an ac source as shown. Which graph correctly shows the instantaneous current through the capacitor and the instantaneous voltage $v = v_a - v_b$ across the capacitor?



(current "*i*" in purple, voltage "*v*" in blue)



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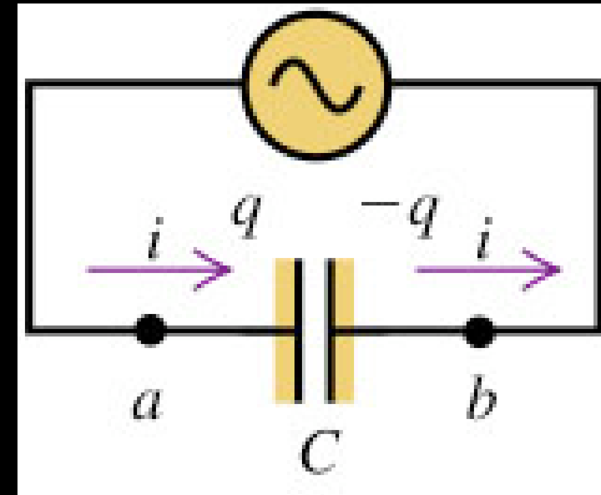
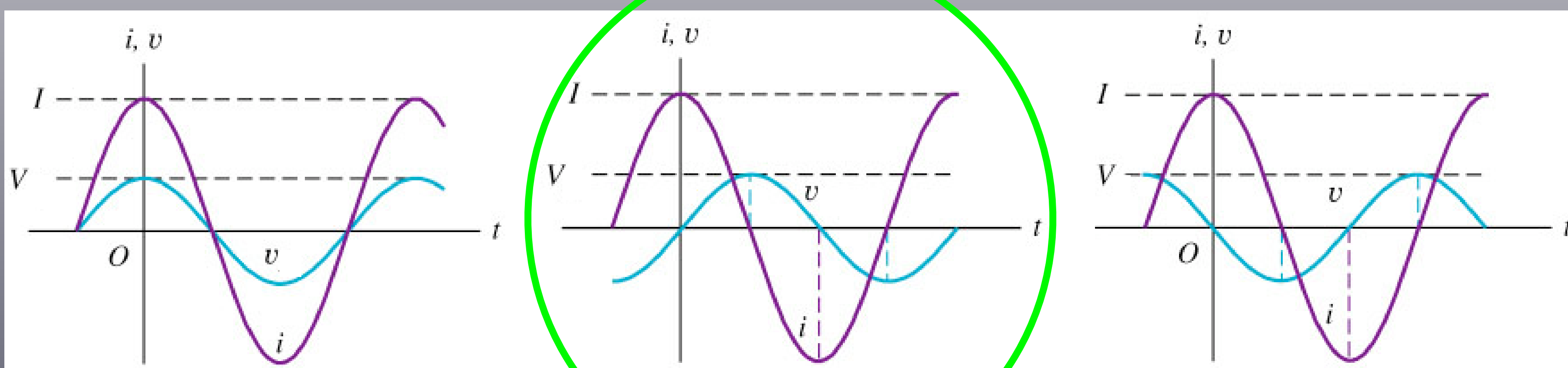


(current “*i*” in purple, voltage “*v*” in blue)



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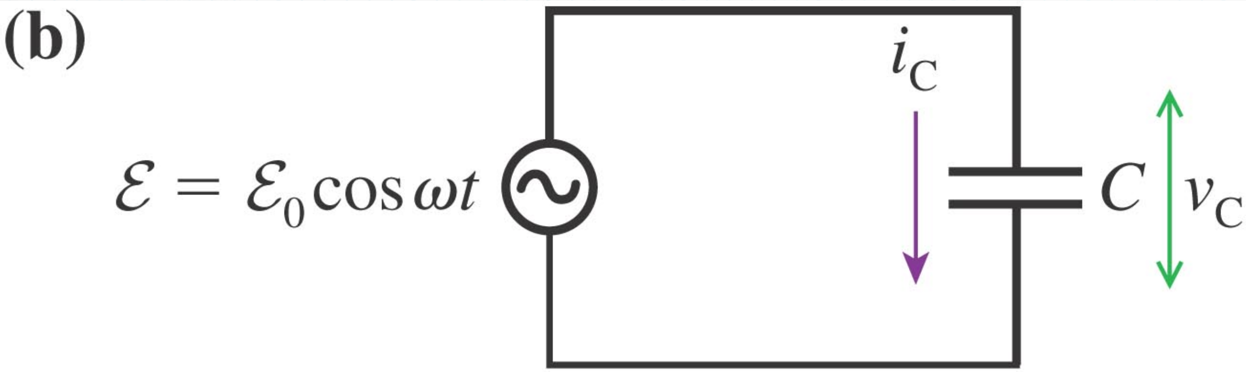
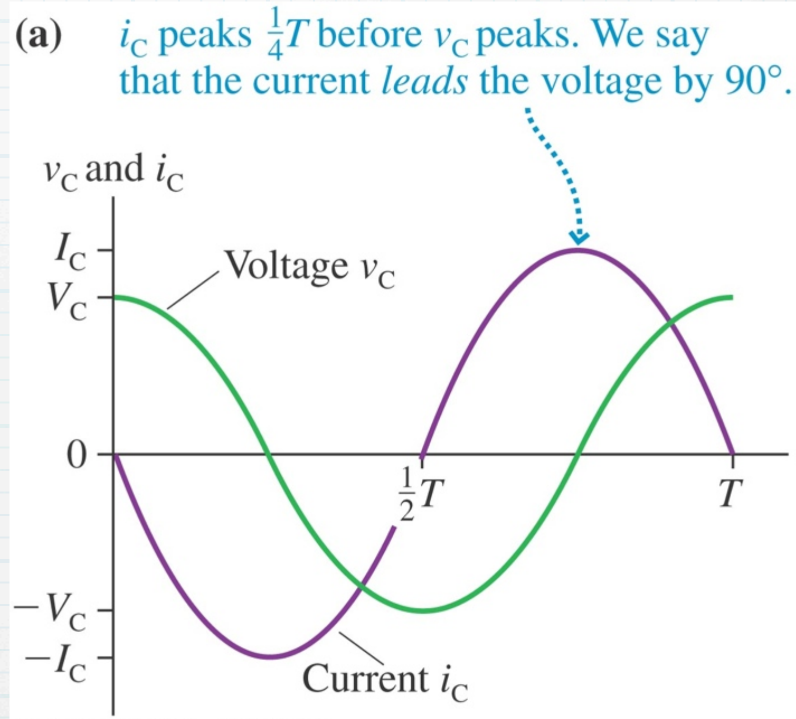
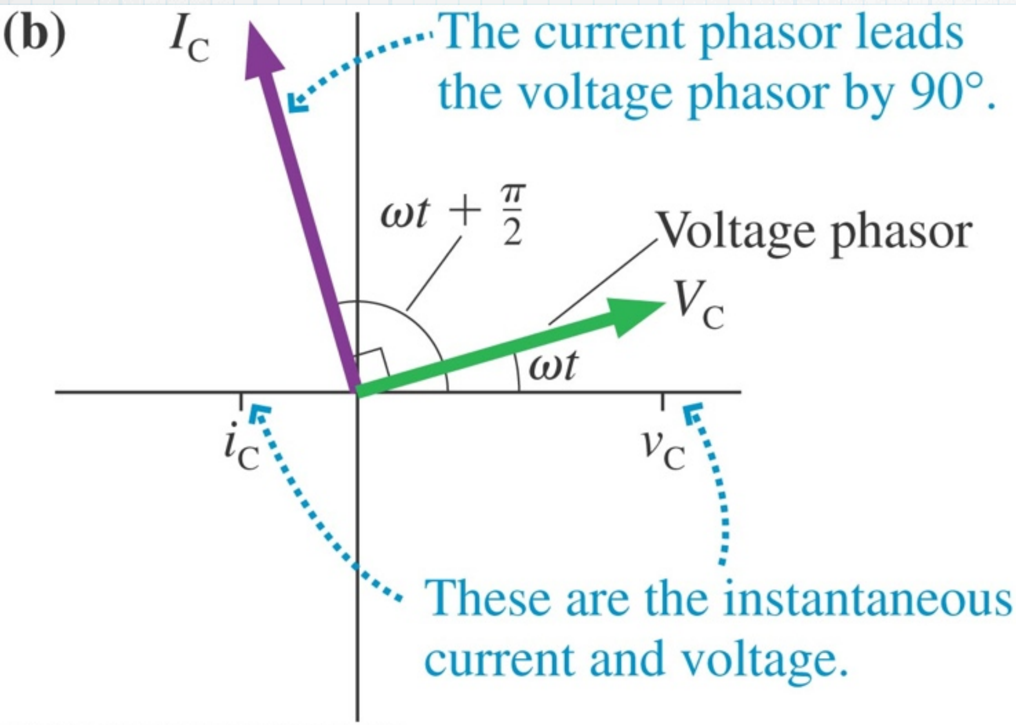
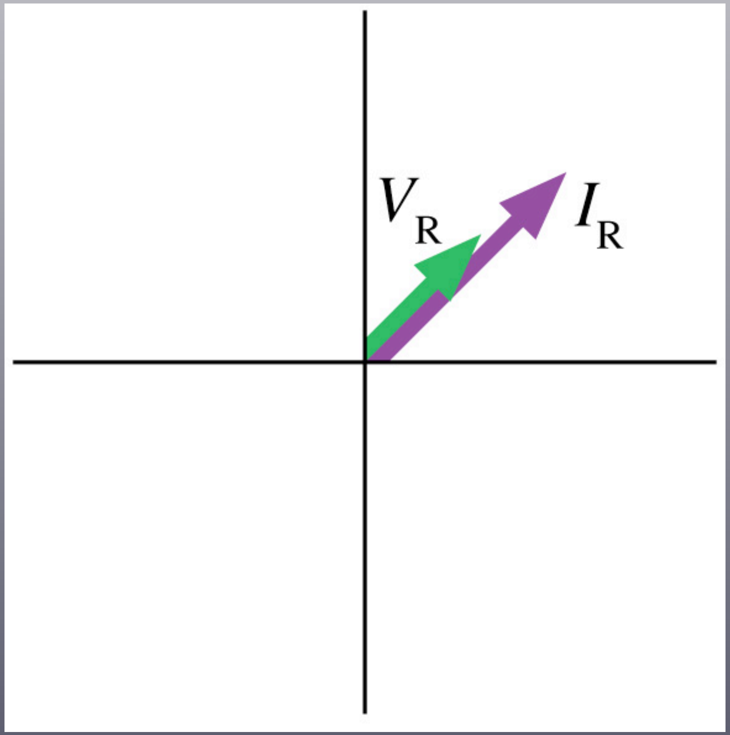

(b)
i_C
$\mathcal{E} = \mathcal{E}_0 \cos \omega t$
C
v_C


(a)
i_C peaks $\frac{1}{4}T$ before v_C peaks. We say that the current *leads* the voltage by 90°.
v_C and i_C
I_C
V_C
0
$-V_C$
$-I_C$
Voltage v_C
Current i_C
$\frac{1}{2}T$
T
t


(b)
I_C
The current phasor leads the voltage phasor by 90°.
$\omega t + \frac{\pi}{2}$
Voltage phasor
V_C
ωt
i_C
v_C
These are the instantaneous current and voltage.

The resistor whose voltage and current phasors are shown here has resistance R

1. > 1 Ω.
2. < 1 Ω.
3. It's not possible to tell.



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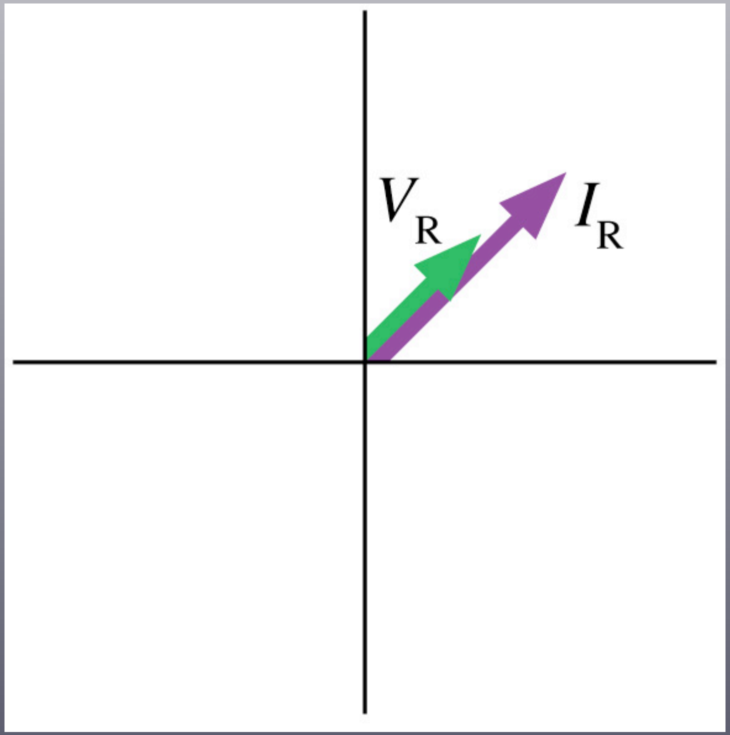
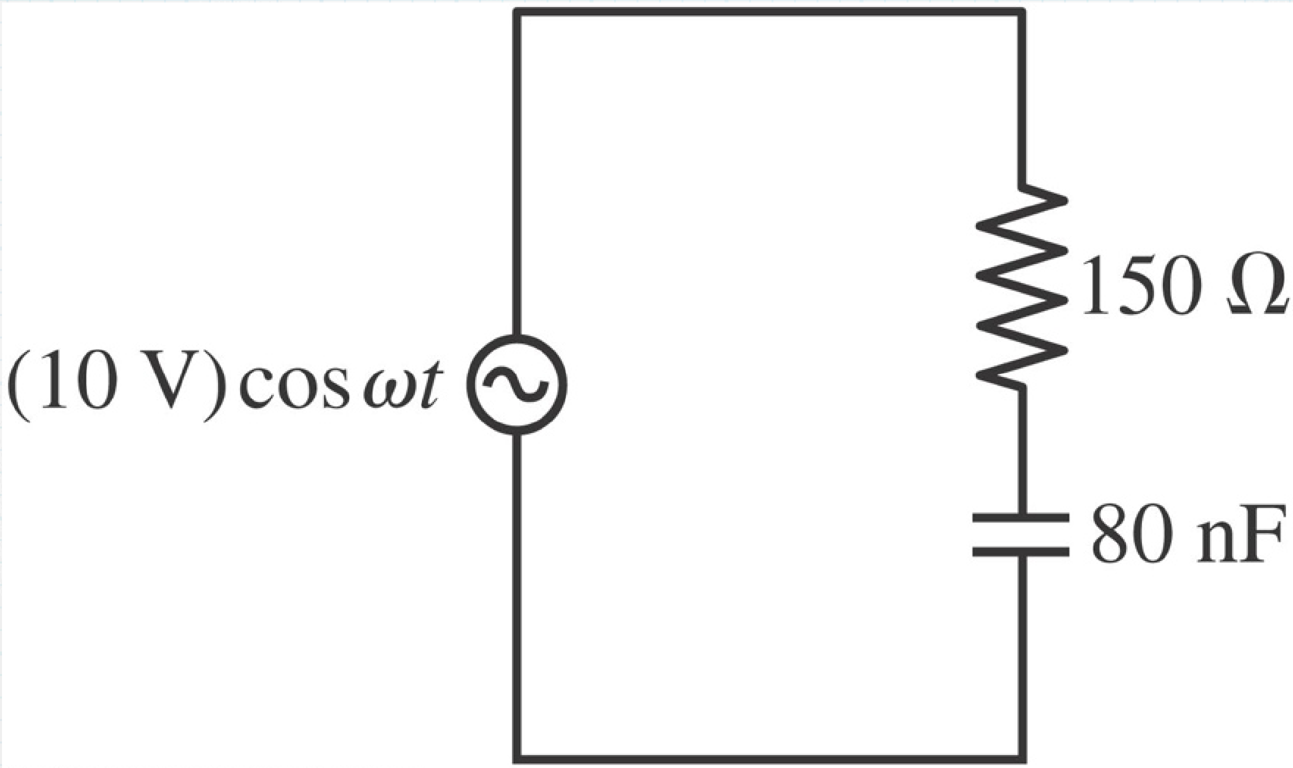


A capacitor is connected to a 15 kHz oscillator. The peak current is 65 mA when the rms voltage is 6.0 V.
What is the value of the capacitance C?

Ans: 81 nF

In the figure, what are V_R and V_C (the peak potential differences across the resistor and the capacitor) if the emf frequency is 10 kHz?



Ans: V_R = 6.0 V, V_C = 8.0 V

A 500 μH inductor is connected across an AC generator that produces a peak voltage of 5.0 V.

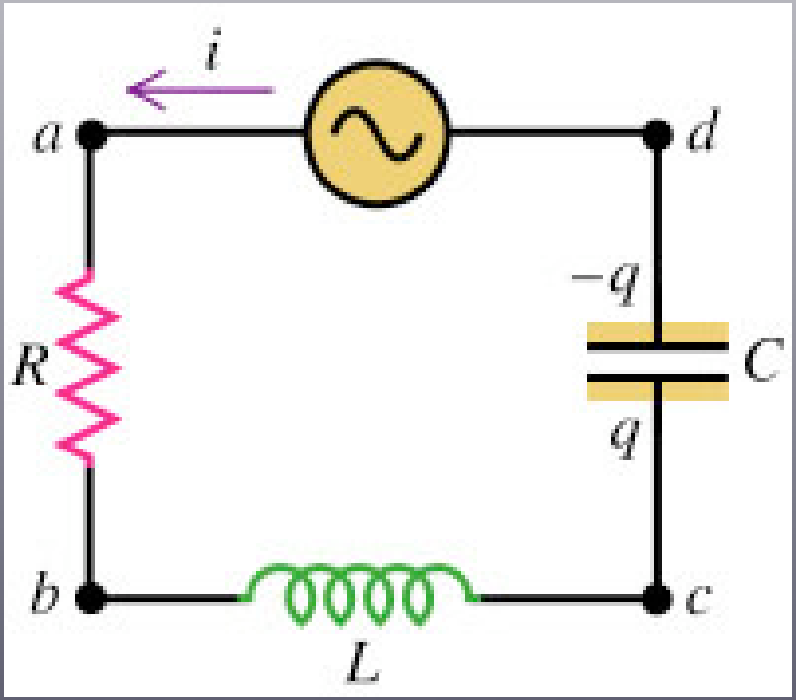
(a) For what frequency is the peak current 50 mA?

(b) What is the instantaneous value of the emf at the instant when $i_L = I_L$ (that is, when the instantaneous current equals the peak current)?

Ans: (a) 32 kHz, (b) 0

An L-R-C series circuit as shown is operating at its resonant frequency. At this frequency, how are the values of the capacitive reactance X_C, the inductive reactance X_L, and the resistance R related to each other?

1. $X_L = R$; X_C can have any value
2. $X_C = R$; X_L can have any value
3. $X_C = X_L$; R can have any value
4. $X_C = X_L = R$
5. none of the above



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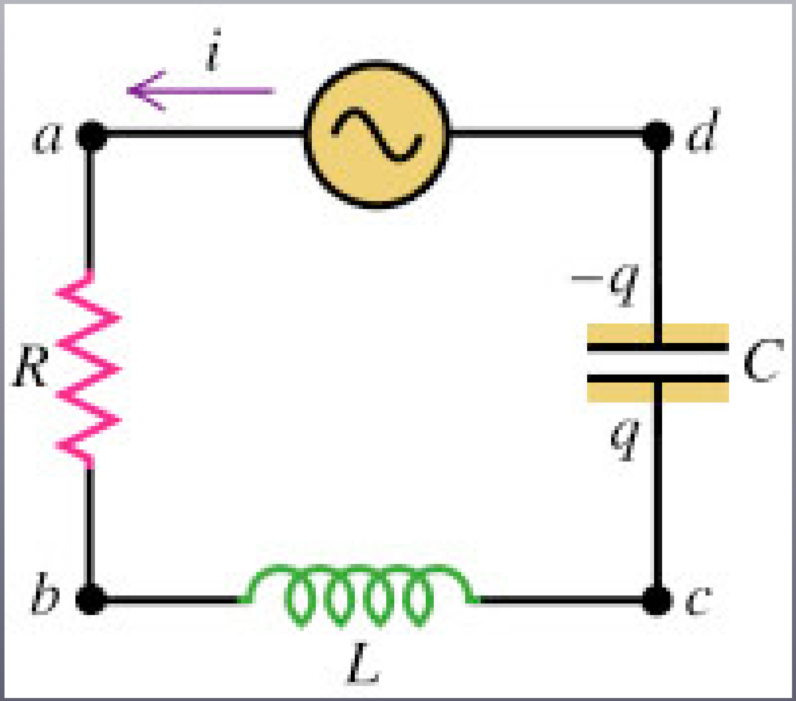
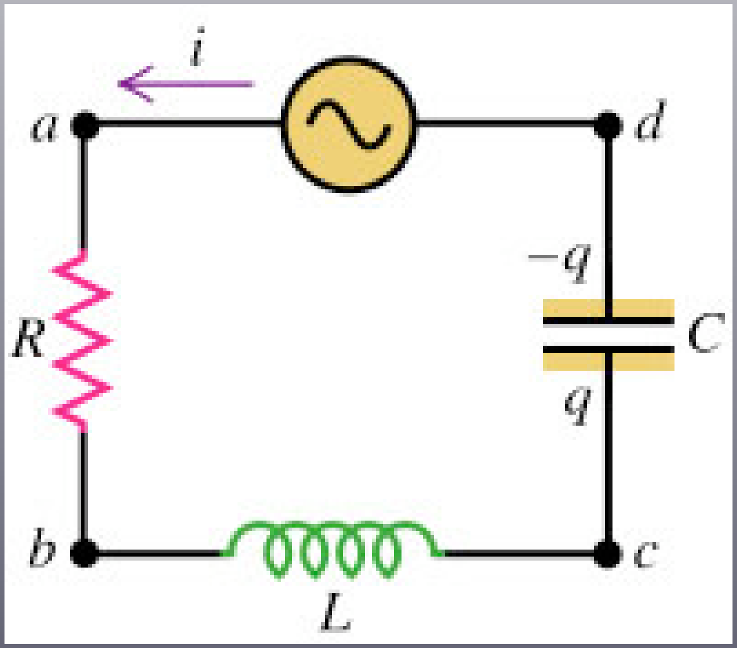


In an *L-R-C* series circuit as shown, the current has a very small amplitude if the emf oscillates at a very high frequency. Which circuit element causes this behavior?

1. the resistor R
2. the inductor L
3. the capacitor C
4. misleading question — the current actually has a very large amplitude if the frequency is very high



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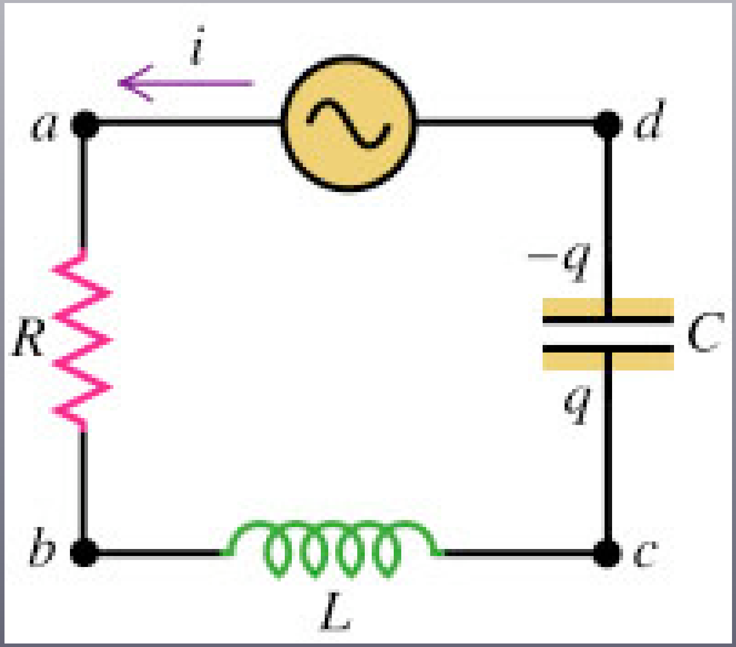
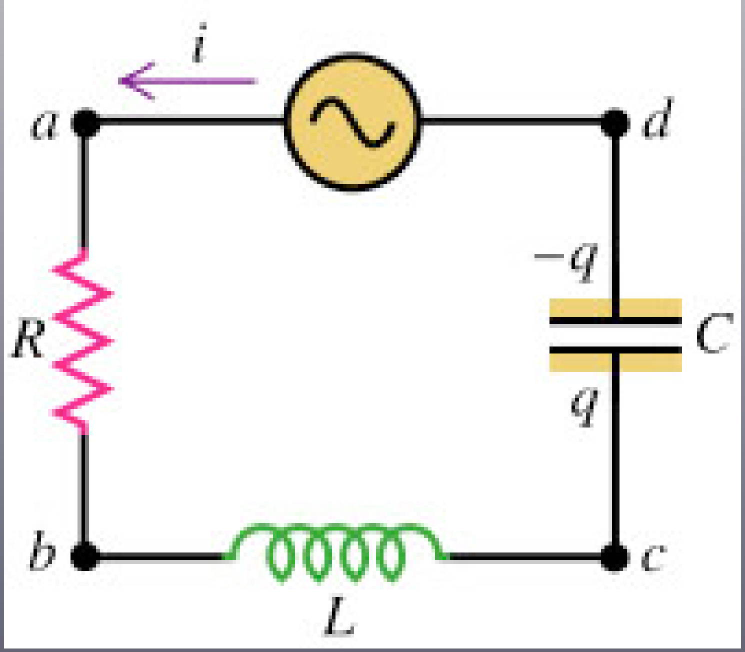


In an L-R-C series circuit as shown, there is a phase angle between the instantaneous current through the circuit and the instantaneous voltage v_{ad} across the entire circuit. For what value of the phase angle is the greatest power delivered to the resistor?

1. zero
2. 90°
3. 180°
4. 270°
5. none of the above



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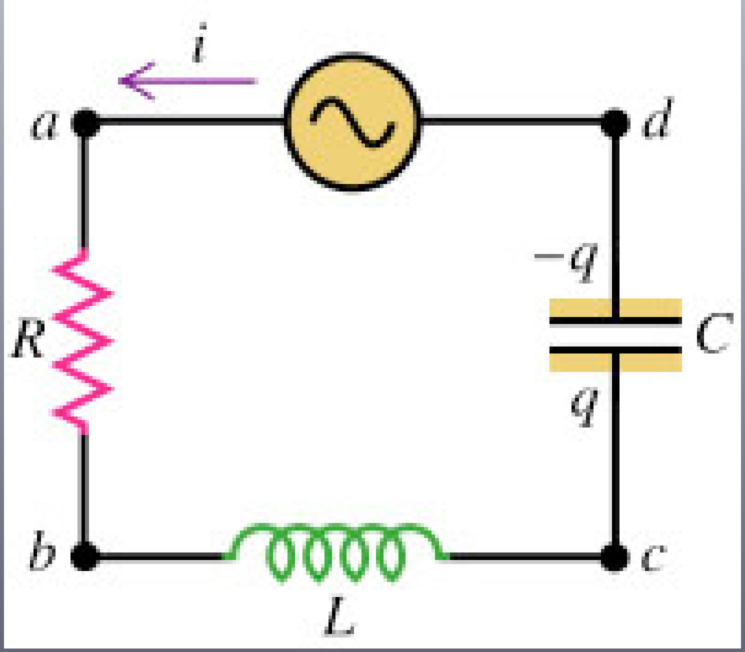

What inductor in series with a 100 Ω resistor and a 2.5 μF capacitor will give a resonance frequency of 1.0 kHz?

Ans: 10 mH

A series RLC circuit consists of a 50 Ω resistor, a 3.3 mH inductor, and a 480 nF capacitor. It is connected to an oscillator with a peak voltage of 5.0 V. Determine the impedance, the peak current, and the phase angle at frequencies 3000 Hz, 4000 Hz and 5000Hz.

Ans:

	$f = 3000$ Hz	$f = 4000$ Hz	$f = 5000$ Hz
Z (Ω)	70	50	62
I (A)	0.072	0.100	0.080
ϕ	–44°	0°	37°

A series RLC circuit has $V_C = 5.0$ V, $V_R = 7.0$ V, and $V_L = 9.0$ V. Is the frequency above, below or equal to the resonance frequency?

1. Above the resonance frequency
2. Below the resonance frequency
3. Equal to the resonance frequency

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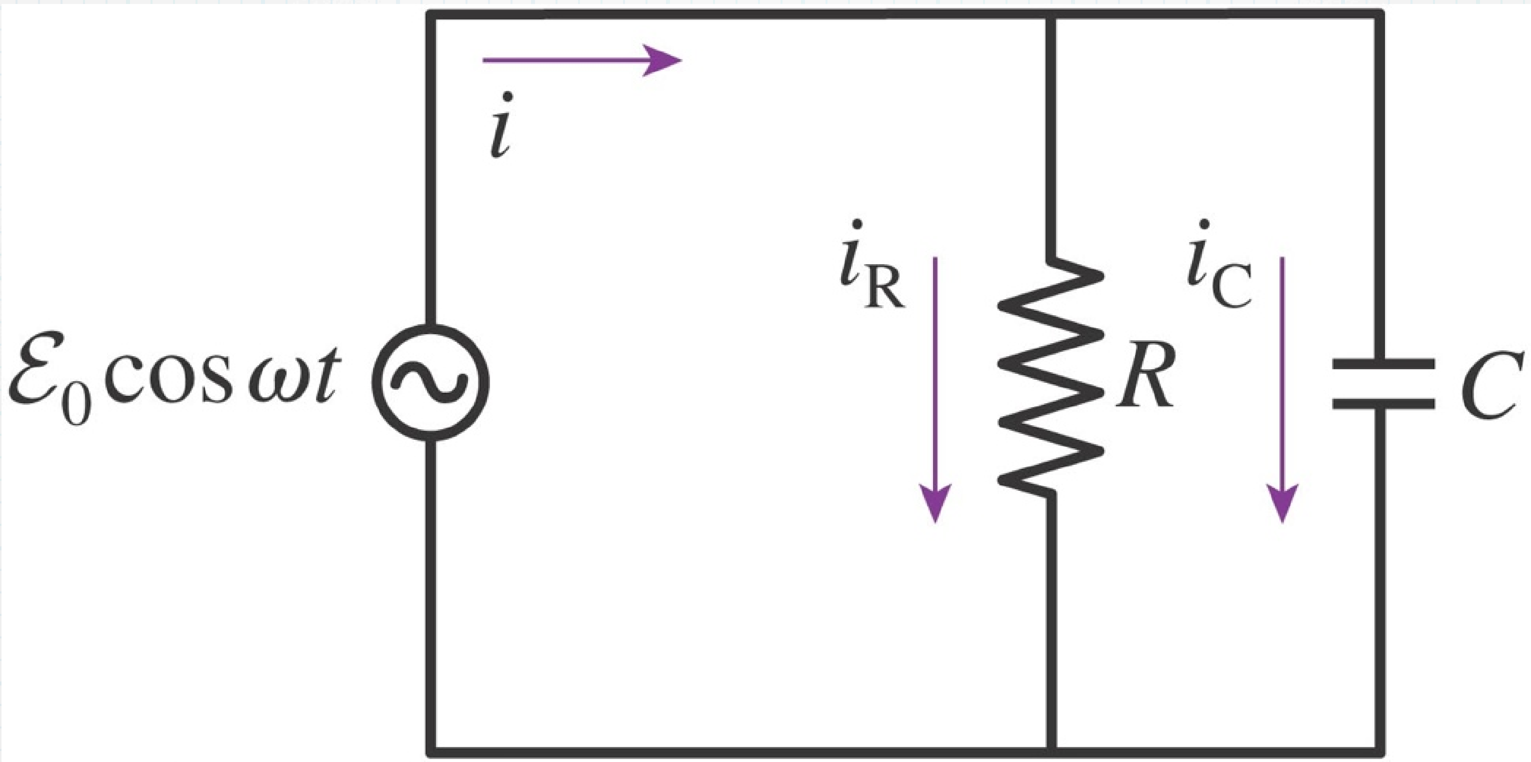
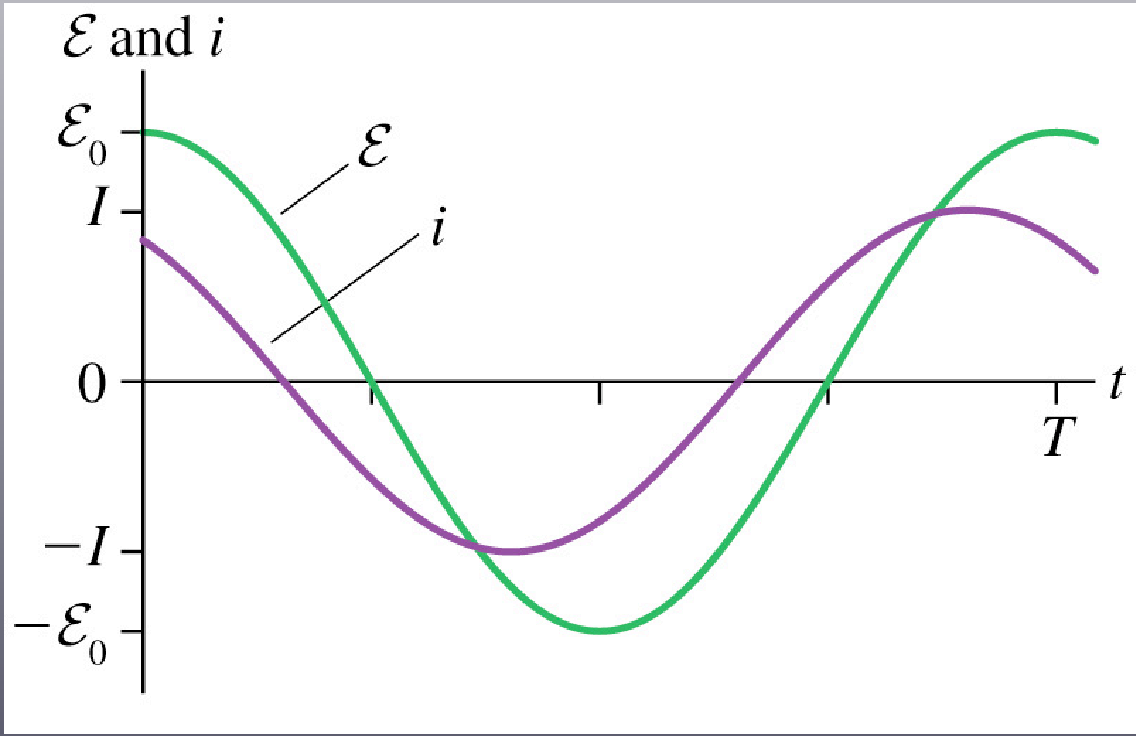
Here is a parallel RC circuit with an AC source. What are the peak currents I_C, I_R and I? What is the phase between the current i and the applied emf?



Follow-up: problems 36.70 and 36.71 in your textbook (all parallel or combination of series/parallel LRC circuits)

The emf and the current in a series RLC circuit oscillate as shown. Which of the following would increase the rate at which energy is supplied to the circuit?

1. Decrease $\mathcal{E}_0$
2. Increase L
3. Increase C
4. Decrease L



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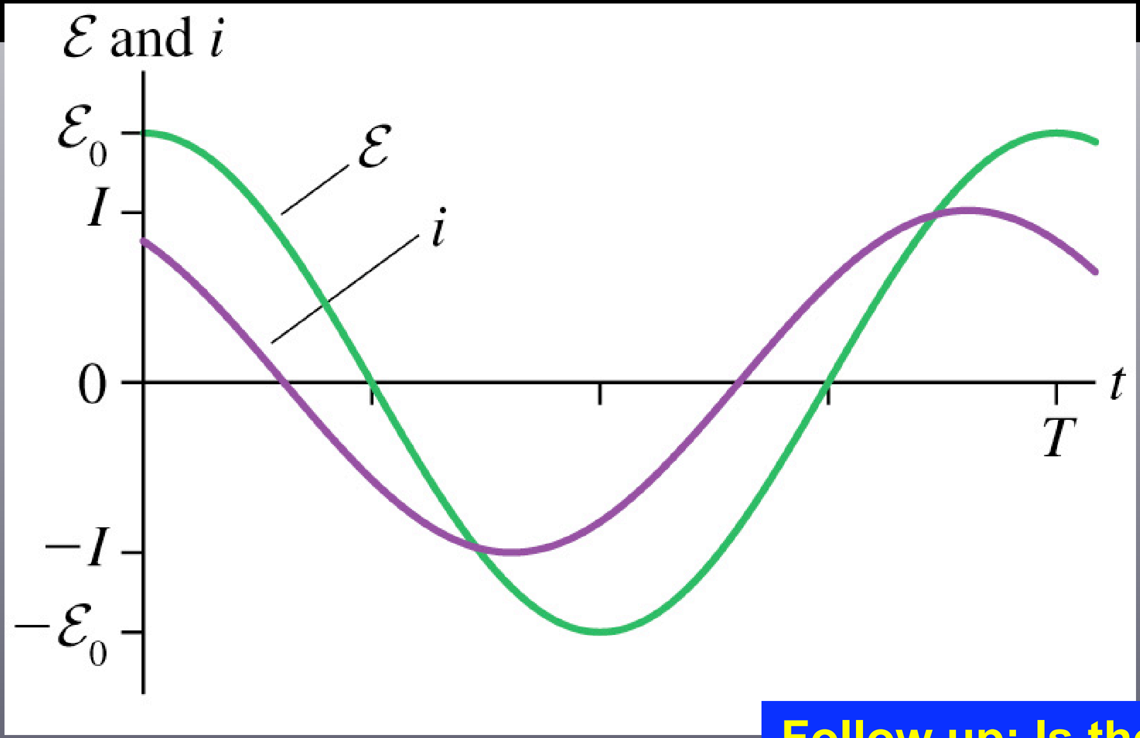

Follow up: Is there another way to increase the rate at which energy is supplied?

A motor attached to a 120 V/60 Hz power line draws an 8.0 A current. Its average energy dissipation is 800 W.

(a) What is the power factor?

(b) What is the rms resistor voltage?

(c) What is the motor's resistance?

(d) How much series capacitance needs to be added to increase the power factor to 1.0?

Ans: (a) 0.833 (b) 100 V (c) 12.5 Ω (d) 320 μF