Week 2: Legged Robotics & Kinematics
AMR - Autonomous Mobile Robots

Marco Hutter

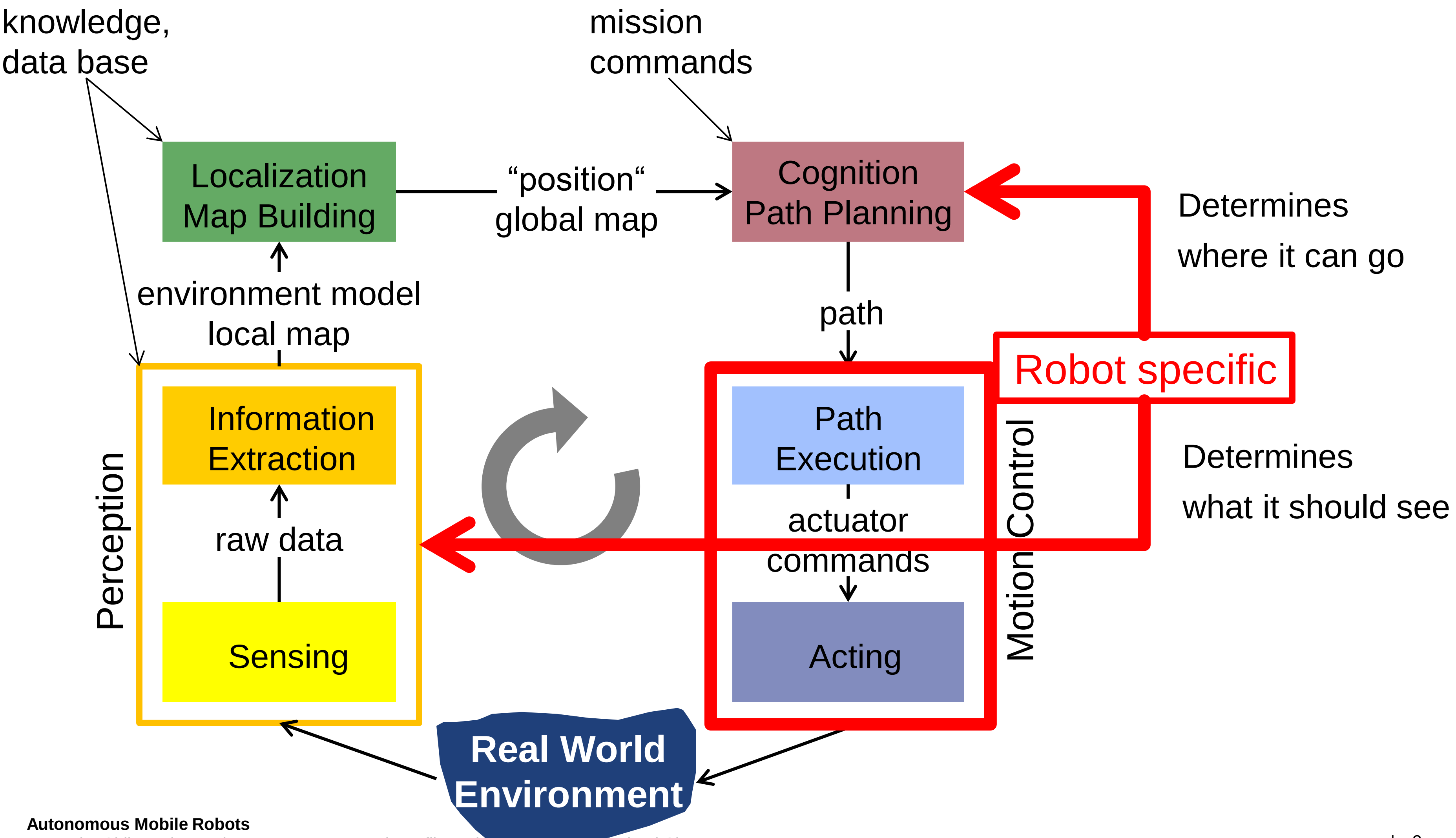
Margarita Chli, Paul Furgale, Martin Rufli, Davide Scaramuzza, Roland Siegwart

For additional content refer to the accompanying book
Introduction to Autonomous Mobile Robots, R. Siegwart, I. Nourbakhsh, D. Scaramuzza, The MIT Press, 2011.

AMRx
some admin points before starting

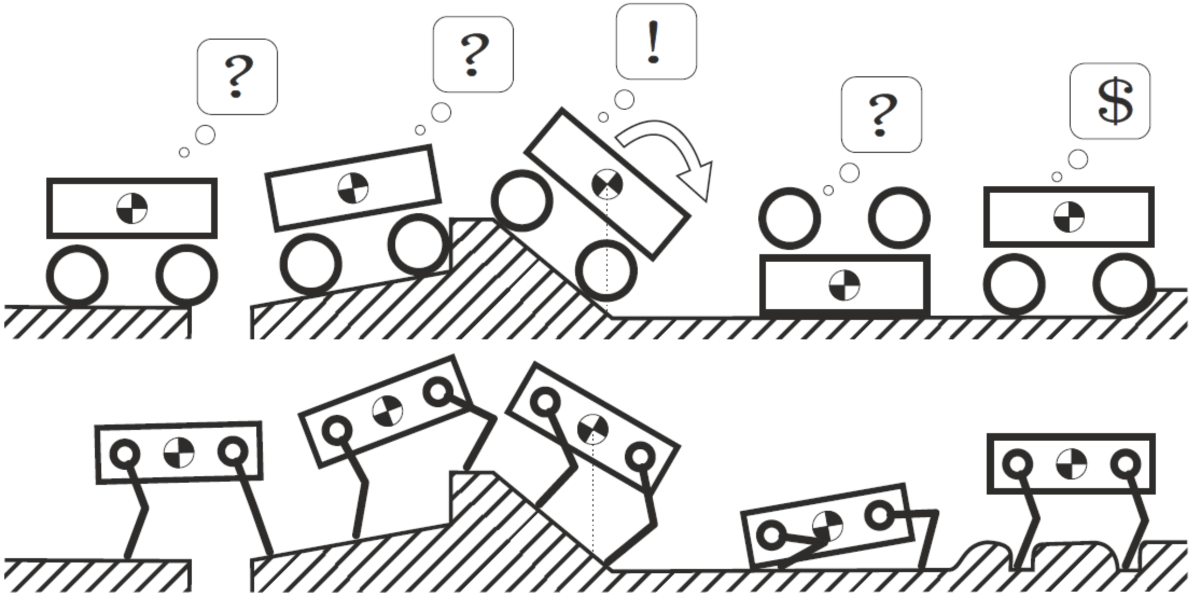
- Registration in edX and watch lecture segments 142 registrations
- Solving Problem Sets 31 worked on Problem Sets
  - they don't count for your grade
- Tell us about any questions you have 4 questions this week
  - Box at the end of the Problem Sets
  - Email to the lecturer
- Bring laptop along for the lectures (if available)
  - Short examples during lecture
  - Recapitulation of worked examples
- Flipped Classroom (you ask us, and we try to help)
  - Recordings are stopped (stupid questions don't impact your career)
  - Interrupt me whenever you have questions/comments

See-think-act cycle



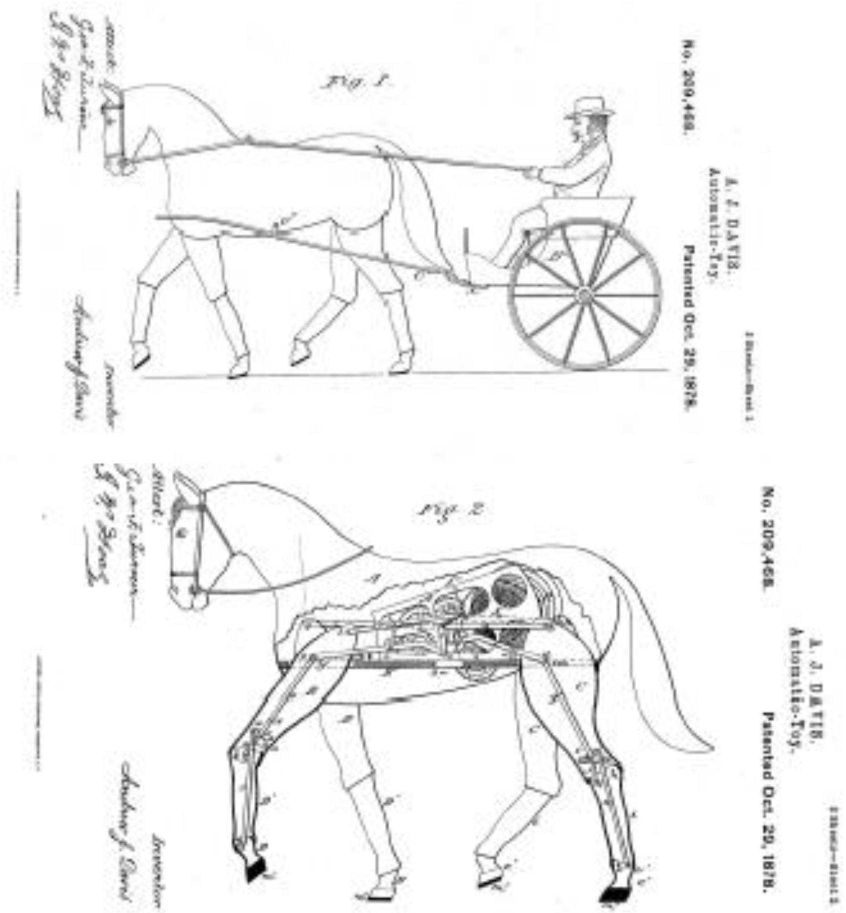
Why legged robots?

- Legged systems can overcome many obstacles

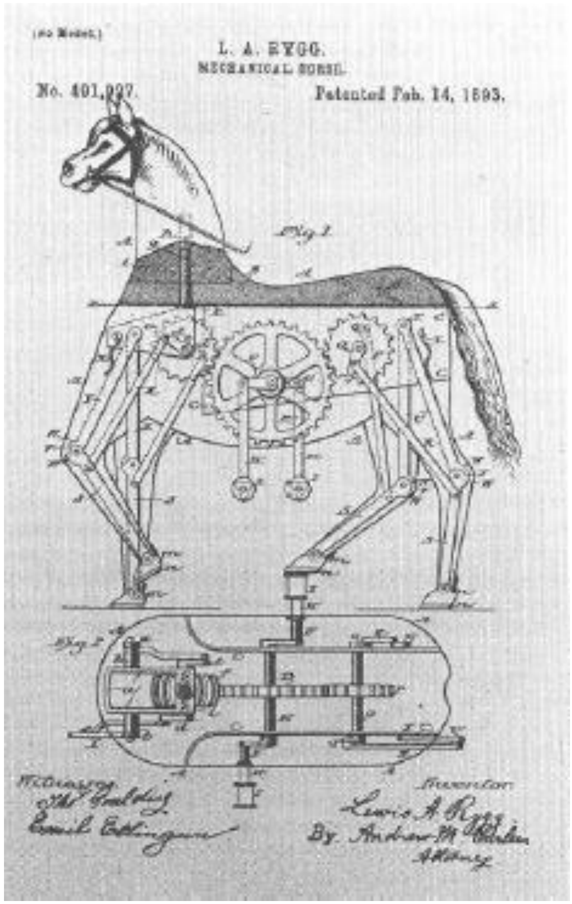


- But it is quite hard to achieve this since
  - many DOFs must be **controlled** in a coordinated way
  - the robot must **see** detailed elements of the terrain

History of Legged Robotics
Walking Mechanism – First patents



Davis, 1878



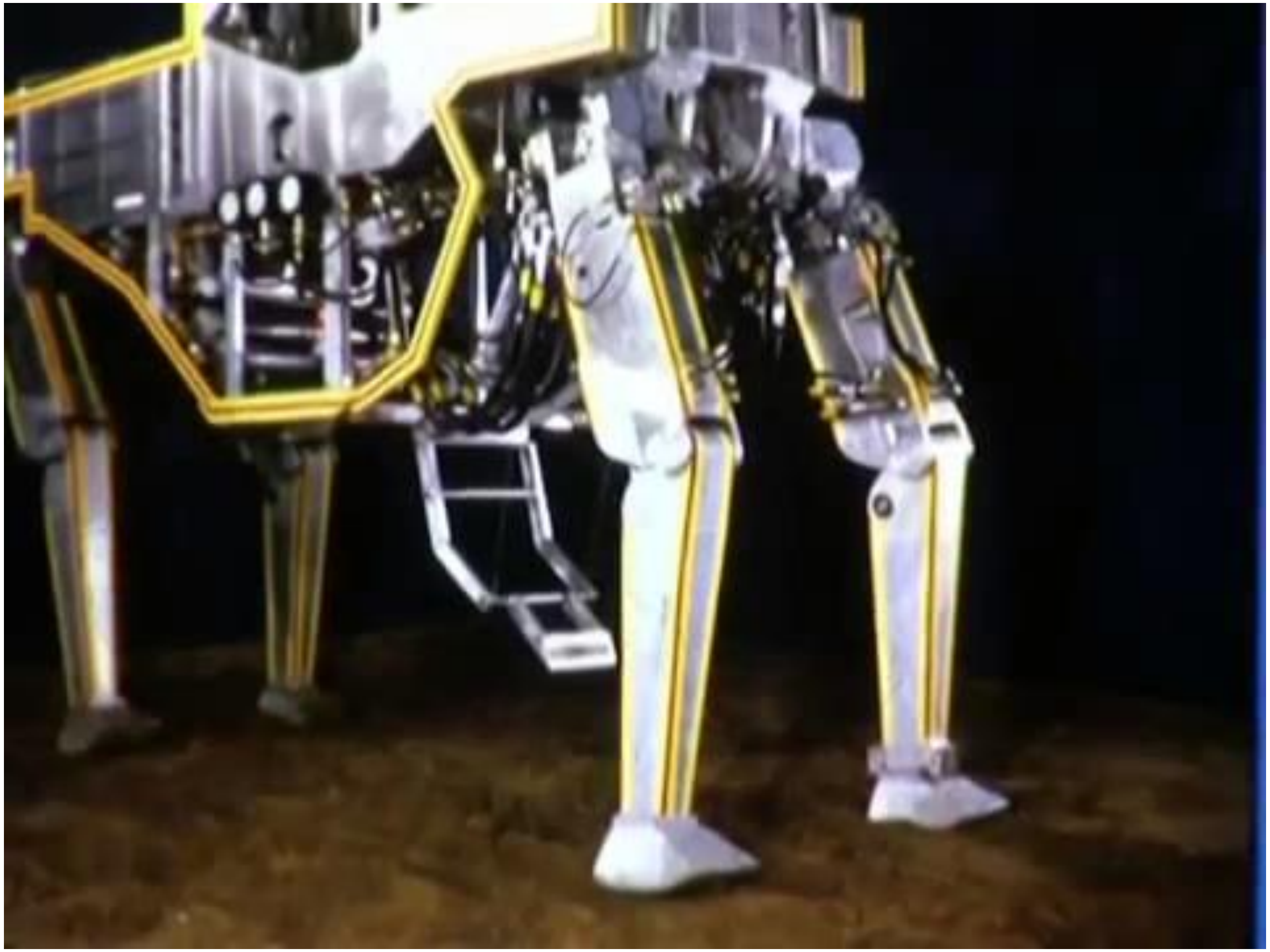
Rygg, 1893

History of Legged Robotics
Walking Mechanism – Theo Jansen

History of Legged Robotics

Walking Mechanism – Theo Jansen

History of Legged Robotics
GE Walking Truck – human controlled 4-ped



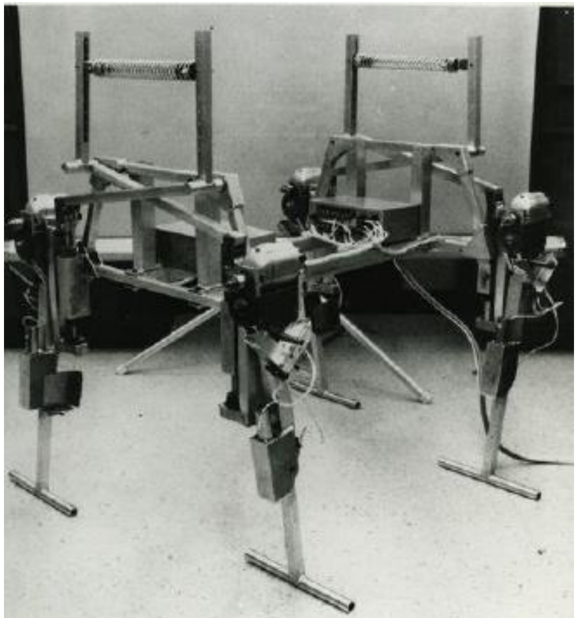
GE Truck, 1968

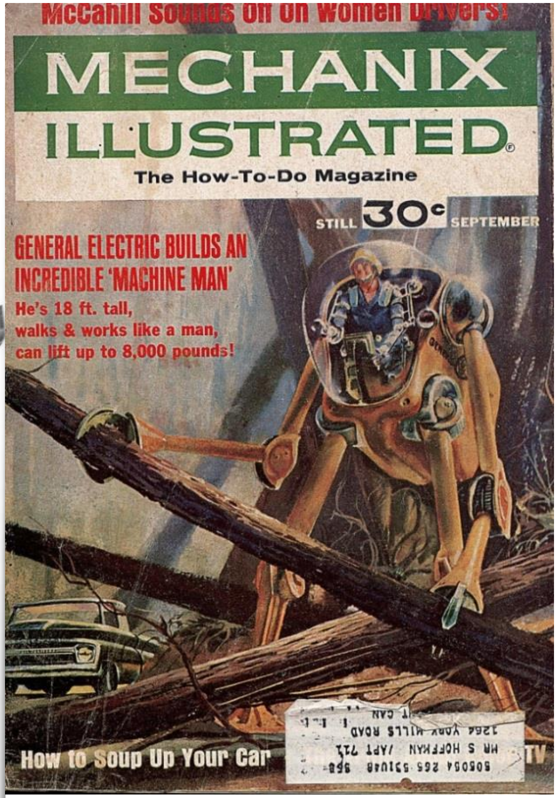
History of Legged Robotics
Walking Excavator – human controlled 4-ped

Menzi Muck 2015

History of Legged Robotics
Phony Pony, GE Hardiman and many more…

- More on http://cyberneticzoo.com/
  - steam-actuated humans
  - mechanical elephants
  - …


A. Frank & R. McGhee, 1966



History of Legged Robotics
Humanoid robots after 2000

- Honda Asimo

Toyota Humanoid

Great engineering, extremely dexterous

History of Legged Robotics
Humanoid robots after 2000

- Fukushima 2011

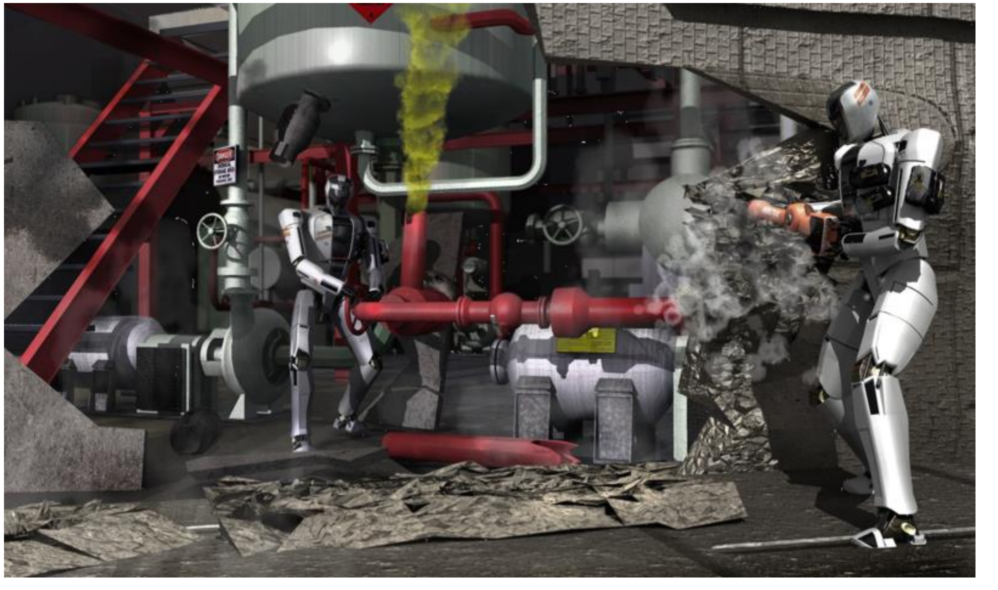
Great engineering, extremely dexterous but…

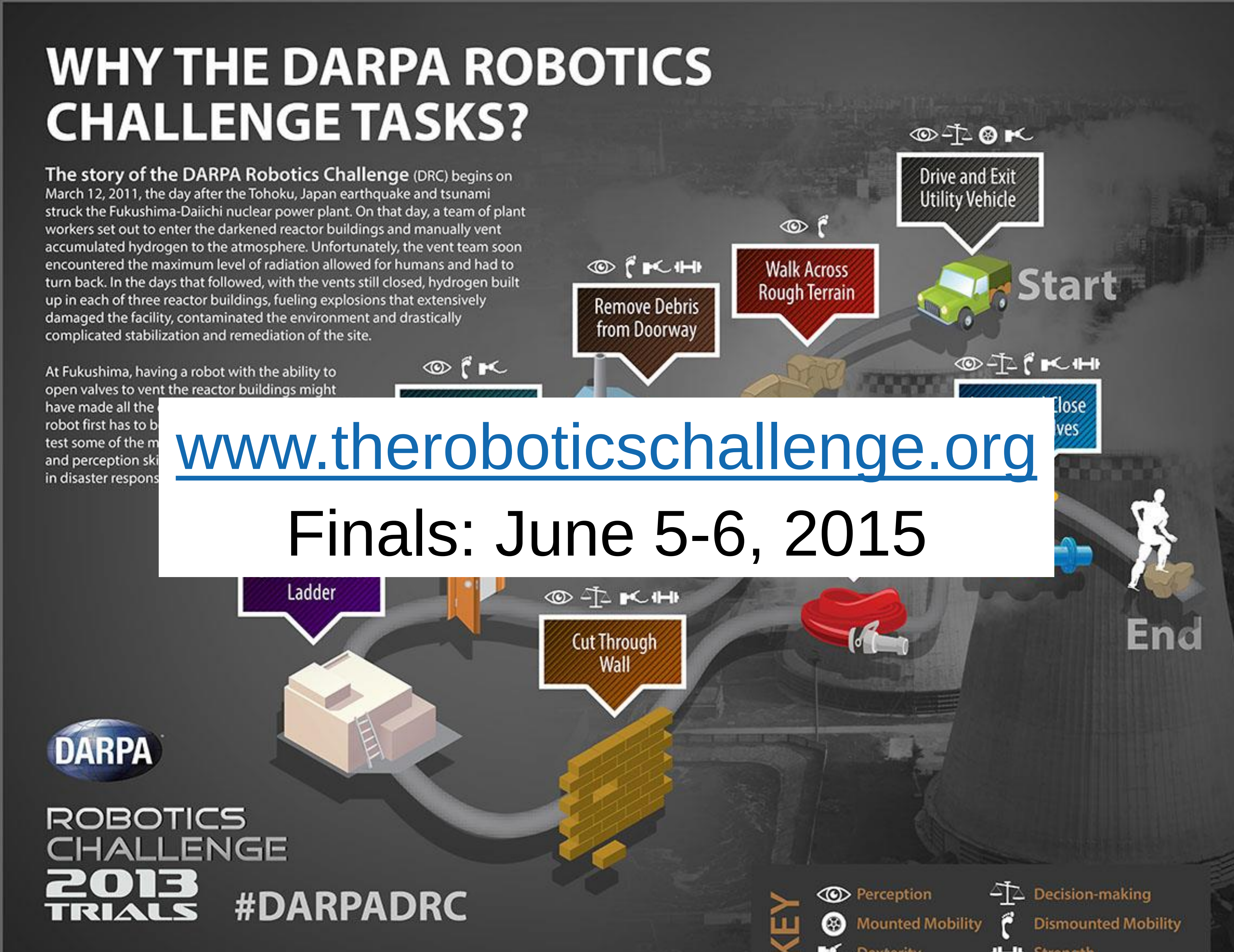
absolutely useless for real applications

History of Legged Robotics

Finally, a breakthrough?

- Fukushima 2011 => DARPA Robotics Challenge 2012



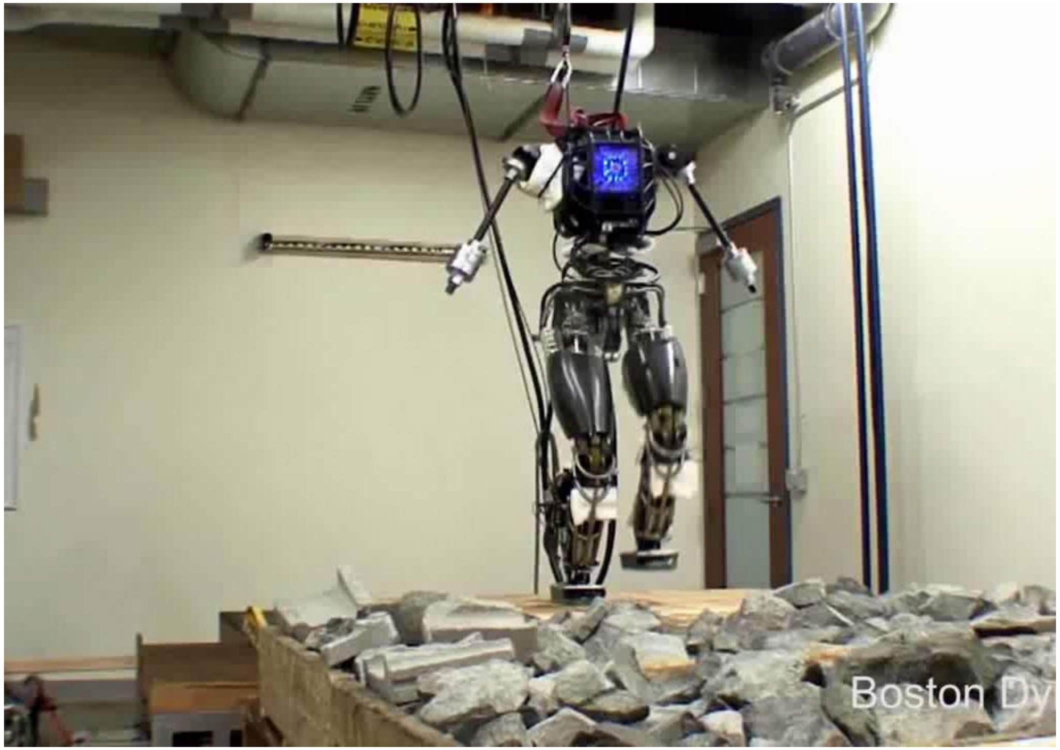


www.theroboticschallenge.org

Finals: June 5-6, 2015

Legged Robotics Today
Finally, a breakthrough?

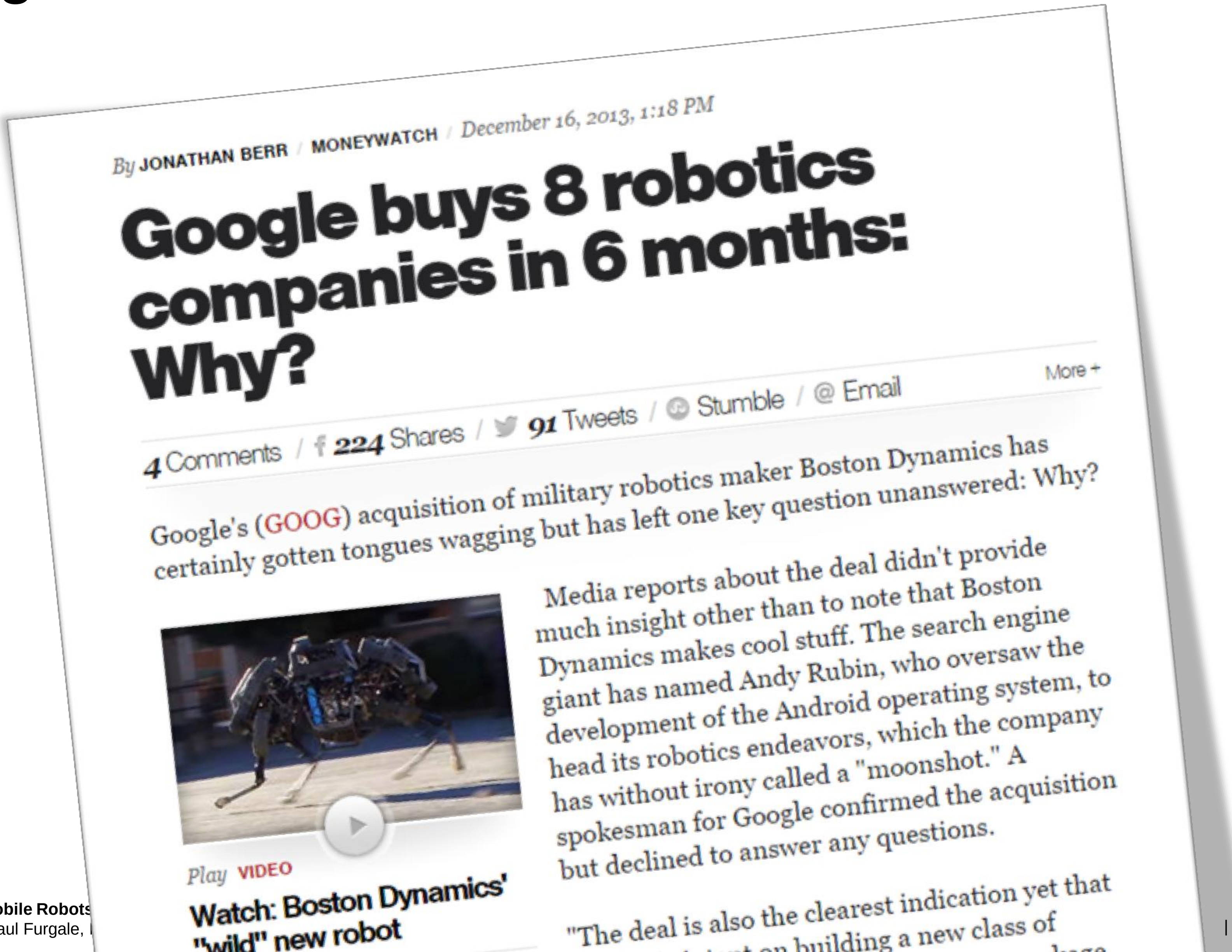
- Most prominent example: Boston Dynamics
  - Atlas: the most advanced humanoid robot in DRC



Legged Robotics Today
Finally, a breakthrough?

- Most prominent example: Boston Dynamics
  - BigDog and its successors

Legged Robotics Tomorrow


By JONATHAN BERR / MONEYWATCH / December 16, 2013, 1:18 PM

Google buys 8 robotics companies in 6 months: Why?

4 Comments / 224 Shares / 91 Tweets / Stumble / @ Email More +

Google's (GOOG) acquisition of military robotics maker Boston Dynamics has certainly gotten tongues wagging but has left one key question unanswered: Why?

Play VIDEO
Watch: Boston Dynamics' "wild" new robot

Media reports about the deal didn't provide much insight other than to note that Boston Dynamics makes cool stuff. The search engine giant has named Andy Rubin, who oversaw the development of the Android operating system, to head its robotics endeavors, which the company has without irony called a "moonshot." A spokesman for Google confirmed the acquisition but declined to answer any questions.

"The deal is also the clearest indication yet that ... on building a new class of ...

Legged Robotics

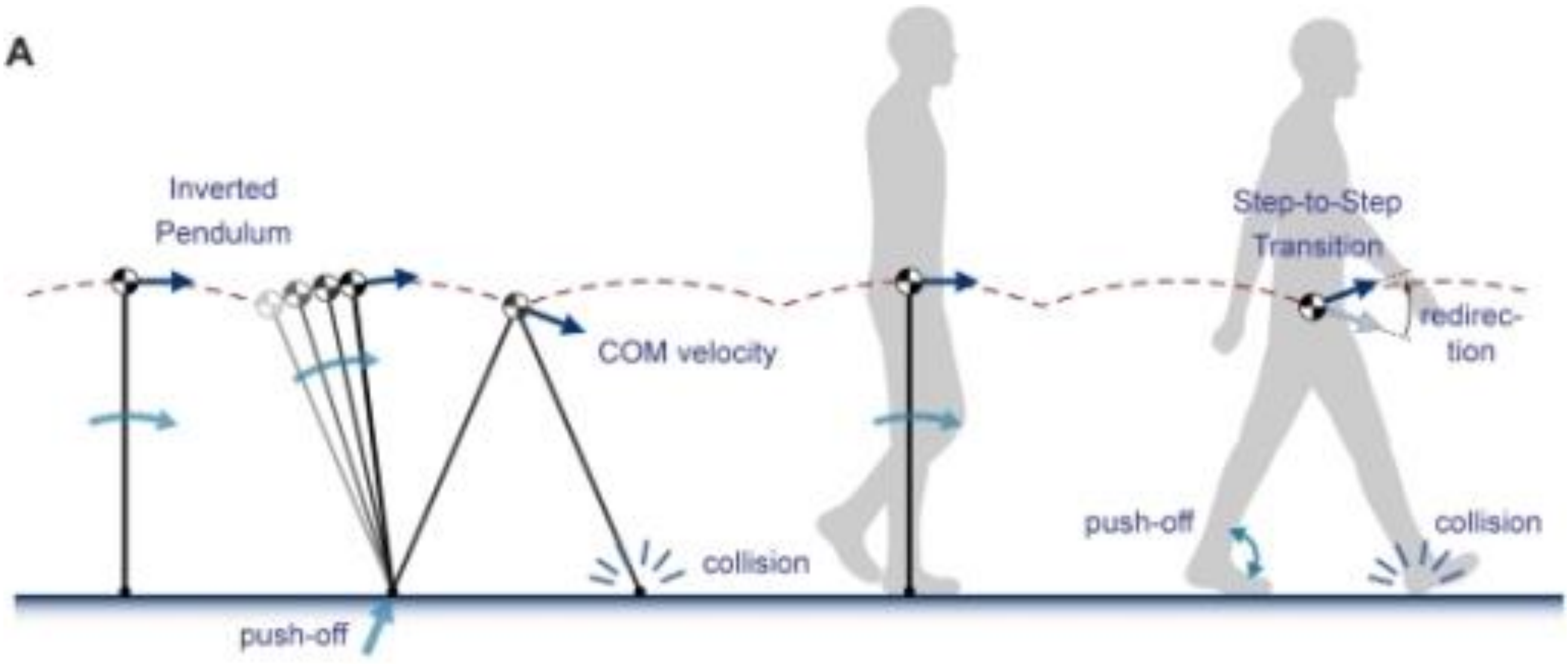
Where are we really and what are the challenges?

- Understanding
- Stability
- Control

of Locomotion

Static walking principles
Inverted Pendulum

- Static walking can be represented by inverted pendulum

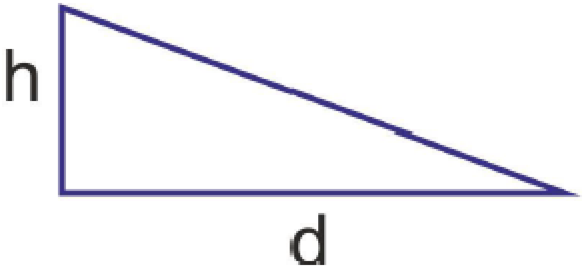


Static walking principles
Inverted Pendulum

- Static walking can be represented by inverted pendulum
- Exploit this in so-called passive dynamic walkers

Energetically very efficient

$$COT = \frac{E_{used}}{m \cdot g \cdot d} = \frac{m \cdot g \cdot h}{m \cdot g \cdot d} = \frac{h}{d}$$



Static walking principles
Inverted Pendulum

- Static walking can be represented by inverted pendulum
- Exploit this in so-called passive dynamic walkers
- Add small actuation to walk on flat ground

Cornell Ranger

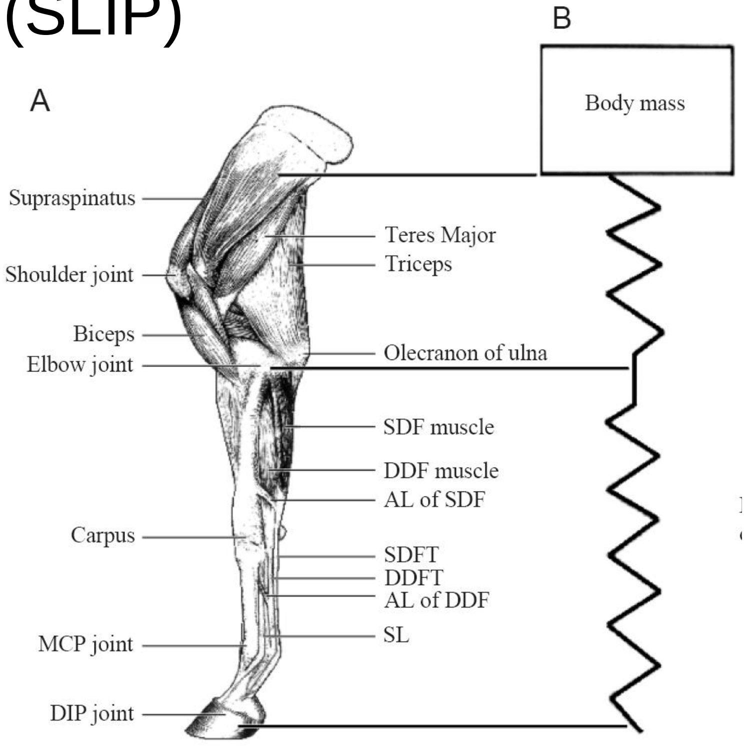
Total distance:	65.24 km
Total time:	30:49:02
Power:	16.0 W
COT:	0.28

Dynamic locomotion
Leg Structure

Dynamic locomotion
Leg Structure

- Leg during running is NOT and inverted pendulum
- Spring loaded inverted pendulum (SLIP)
  - ☑ are robust against collisions
  - ☑ can better handle uncertainties
  - ☑ can temporarily store energy
  - ☑ reduce peak power

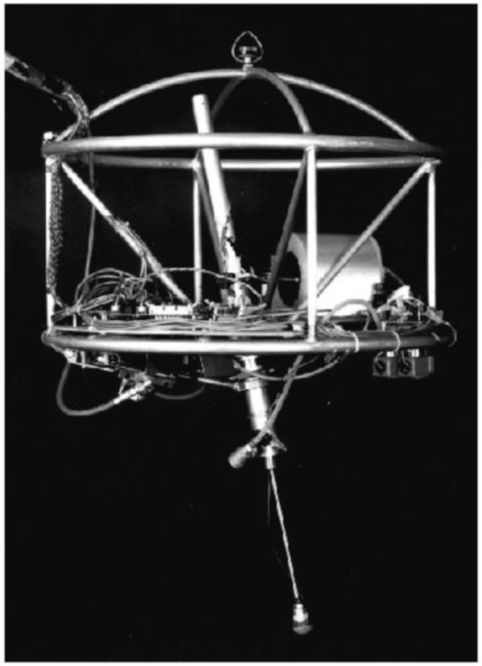
  [Alexander 1988,1990,2002,2003]



McGuigan & Wilson, 2003 – *J. Exp. Bio.*

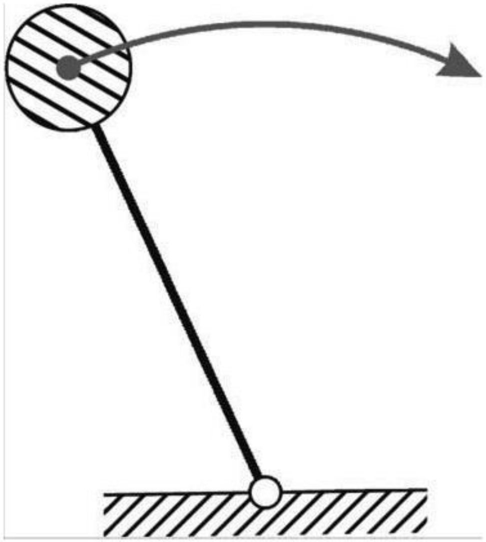
Dynamic Locomotion
SLIP principles in robotics

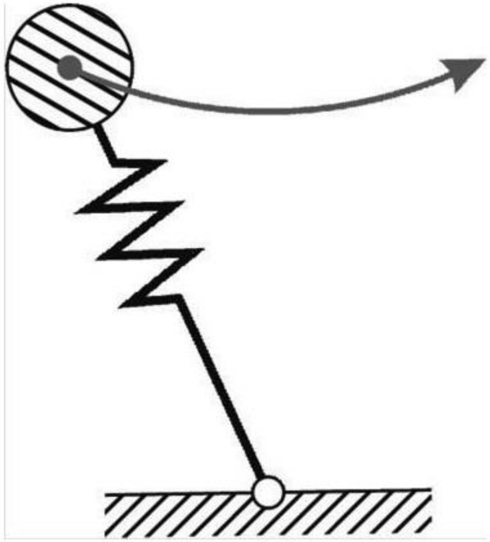
- Early Raibert hoppers (MIT leg lab) [1983]
  - Pneumatic piston
  - Hydraulic leg "angle" orientation



Understanding Locomotion
Summary

- Static locomotion (IP) ⇔ Dynamic Locomotion (SLIP)

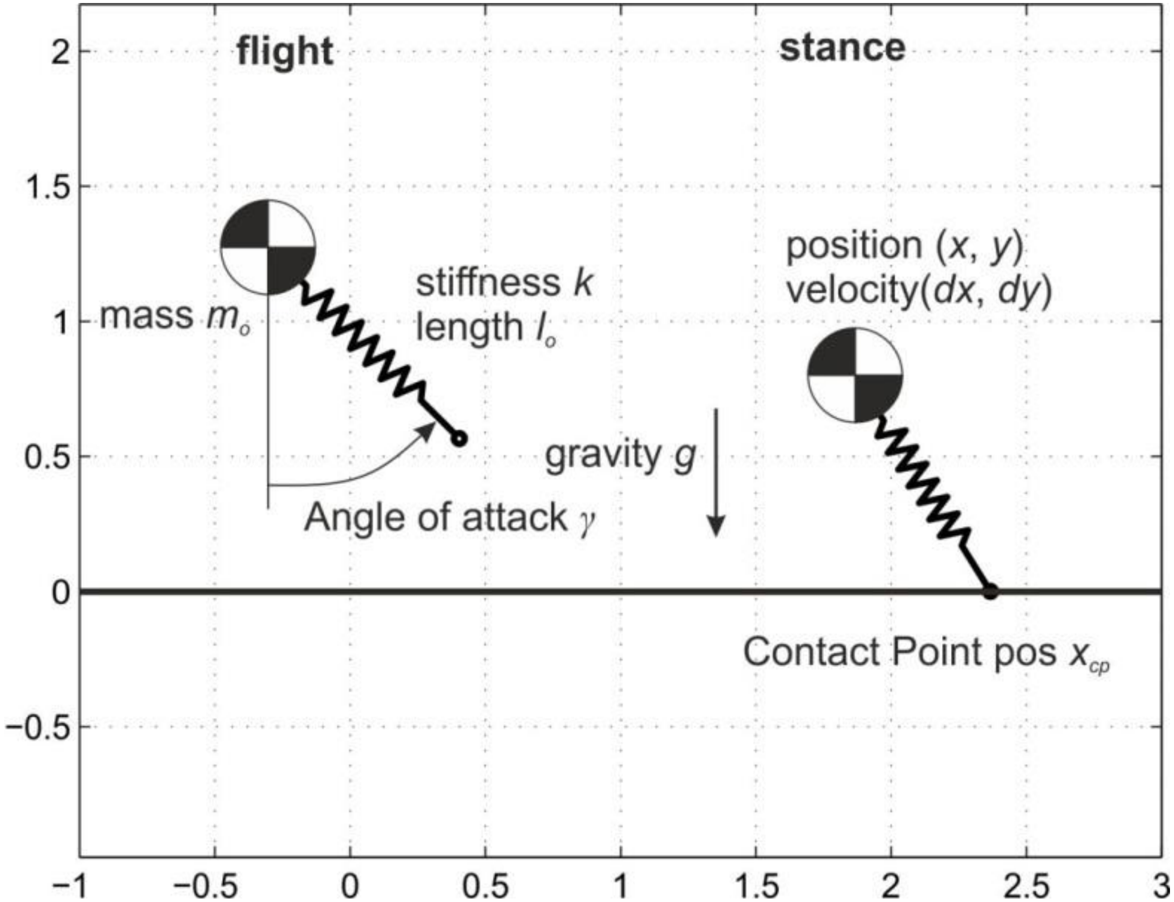




- How to determine stability?
- How to control (actively stabilize)?

Dynamic Locomotion
Stability Analysis

Discuss the edX limit-cycle movie



- Point mass: $\mathbf{q} = \begin{bmatrix} x \\ y \end{bmatrix}$
- Equation of Motion:
  - flight

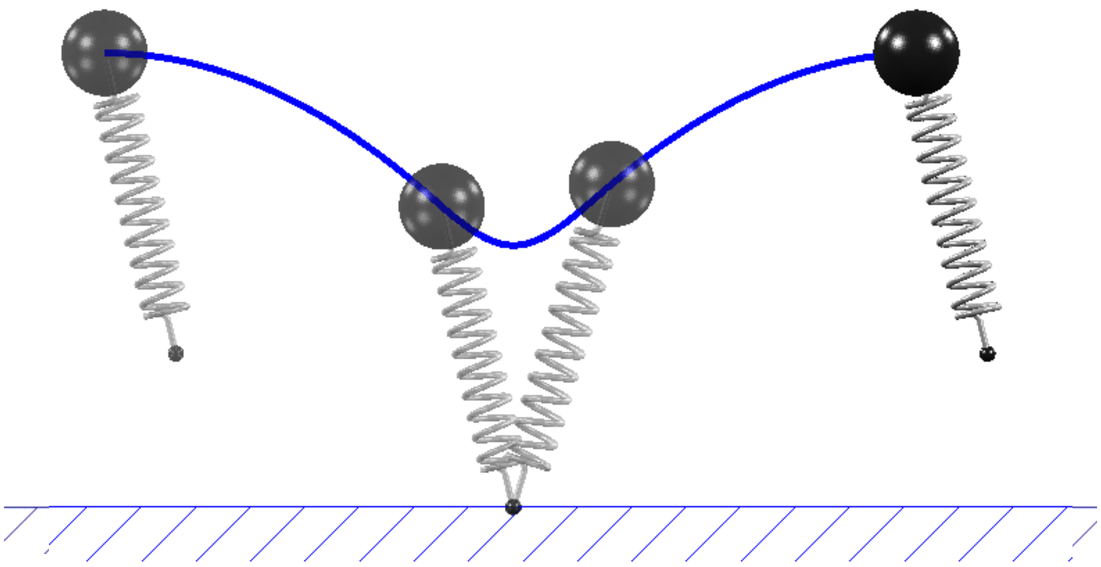
$$\ddot{\mathbf{q}} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 \\ -g \end{bmatrix}$$

  - stance

$$\ddot{\mathbf{q}} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} \frac{-F_{spring}^{x}}{m_o} \\ \frac{F_{spring}^{y}}{m_o} - g \end{bmatrix}$$

$$\mathbf{F}_{spring} = f(\gamma, k, l)$$

Stability of Locomotion
Limit Cycle Analysis



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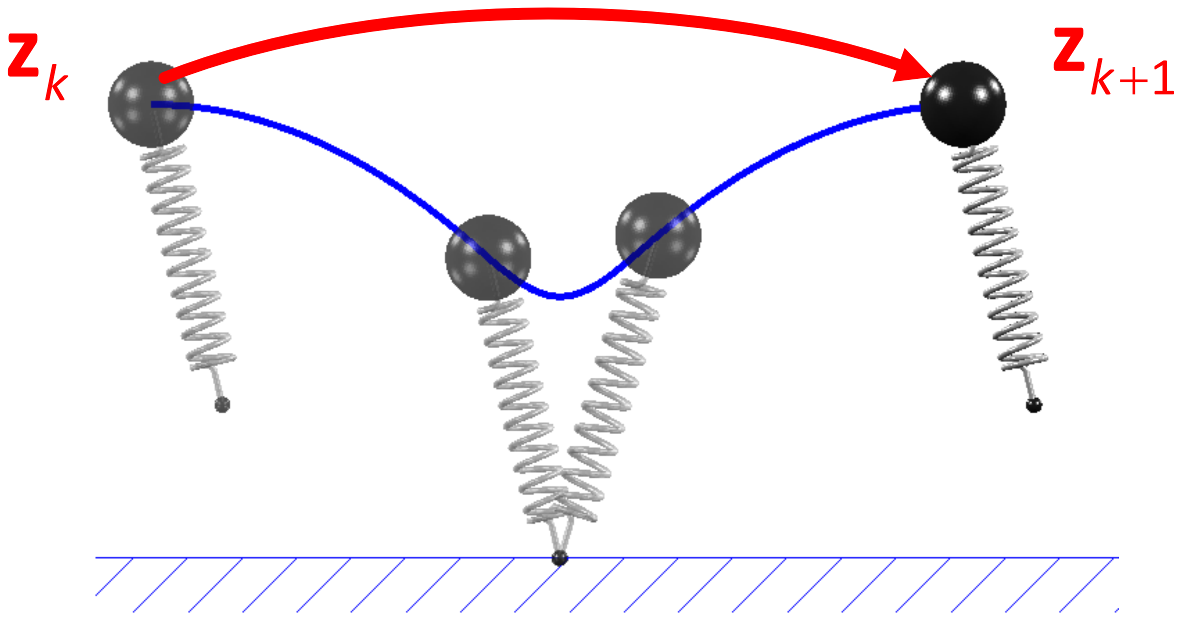
$$\ddot{\mathbf{q}} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} \dfrac{-F^x_{spring}}{m_o} \\ \dfrac{F^y_{spring}}{m_o} - g \end{bmatrix}$$

$$\mathbf{F}_{spring} = f\left(\gamma, k, l\right)$$

Stability of Locomotion
Limit Cycle Analysis



- Point mass: $\mathbf{q} = \begin{bmatrix} x \\ y \end{bmatrix}$
- System state

$$\mathbf{z} = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} \xrightarrow{\text{analysis at apex}} \mathbf{z} = \begin{bmatrix} y \\ \dot{x} \end{bmatrix}$$

- How does the state propagate from one apex to the next?

Stability of Locomotion
Limit Cycle Analysis

- Given the Poincaré mapping from one apex to the next

$$\mathbf{z}_{k+1} = P(\mathbf{z}_k)$$

  - P includes differential equations that cannot be analytically solved!

- Analyze periodic stability of a fixed point $\mathbf{z}^* = P(\mathbf{z}^*)$

- How can we do this? Linearization…

$$\mathbf{z}_{k+1} = \mathbf{z}^* + \Delta\mathbf{z}_{k+1} = P(\mathbf{z}^* + \Delta\mathbf{z}_k) = \underbrace{P(\mathbf{z}^*)}_{\mathbf{z}^*} + \left.\frac{\partial P}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^*} \cdot \Delta\mathbf{z}_k + \underbrace{\left.\frac{\partial^2 P}{\partial \mathbf{x}^2}\right|_{\mathbf{x}=\mathbf{x}^*} \cdot \Delta\mathbf{z}_k^2}_{\approx 0} + \ldots$$

$$\Delta\mathbf{z}_{k+1} = \frac{\partial P}{\partial \mathbf{z}} \Delta\mathbf{z}_k = \mathbf{\Phi}\Delta\mathbf{z}_k$$

?

Stability of Locomotion
Numerical Limit Cycle Analysis

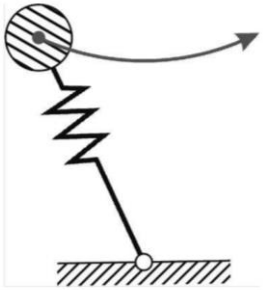
- Find the linearization of the Poincaré map around fix-point

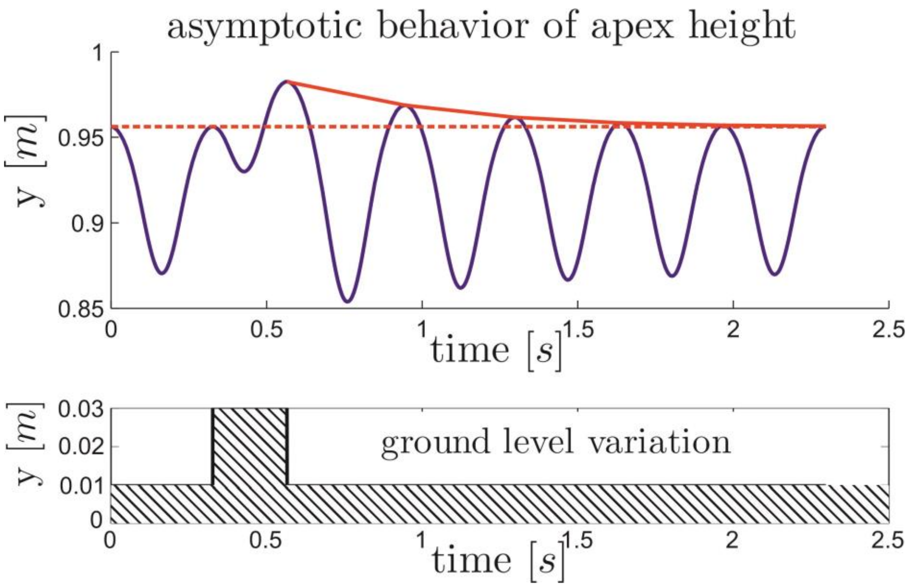
$$\begin{bmatrix} \Delta y_{k+1} \\ \Delta \dot{x}_{k+1} \end{bmatrix} = \left.\frac{\partial P}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^*} \cdot \begin{bmatrix} \Delta y_k \\ \Delta \dot{x}_k \end{bmatrix}$$

  - Choose $\Delta \mathbf{z}_k = \begin{bmatrix} \Delta y_k \\ \Delta \dot{x}_k \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} h$, with a small h
  - Simulate until the next apex, starting from $\begin{bmatrix} y_{k+1} \\ \dot{x}_{k+1} \end{bmatrix} = P\left(\mathbf{z}^* + \begin{bmatrix} 1 \\ 0 \end{bmatrix} h \right)$
  - Calculate $\left.\frac{\partial P}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^*} = \begin{bmatrix} \frac{y_{k+1} - y^*}{h} & * \\ \frac{\dot{x}_{k+1} - \dot{x}^*}{h} & * \end{bmatrix}$
  - Do the same thing for $\Delta \mathbf{z}_k = \begin{bmatrix} \Delta y_k \\ \Delta \dot{x}_k \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} h$

Stability of Locomotion
Numerical Limit Cycle Analysis

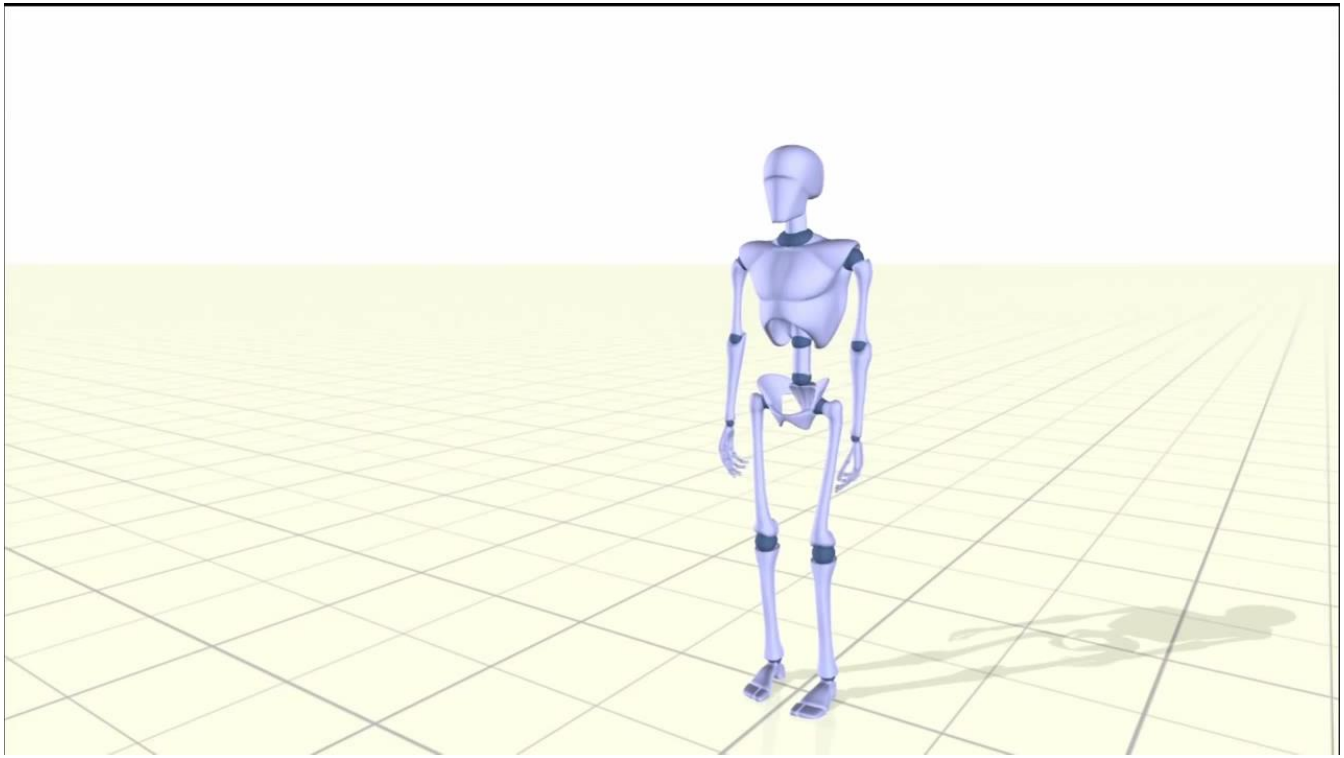
- The previous evaluation results in a 2x2 matrix $\left.\frac{\partial P}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^*}$
- Eigenvalue analysis:
  - System is energy conservative: $\lambda_1 = 1$
  - System can be
    - stable $|\lambda_2| < 1$
    - unstable: $|\lambda_2| > 1$





Stability and Control of Locomotion
Example of a biped

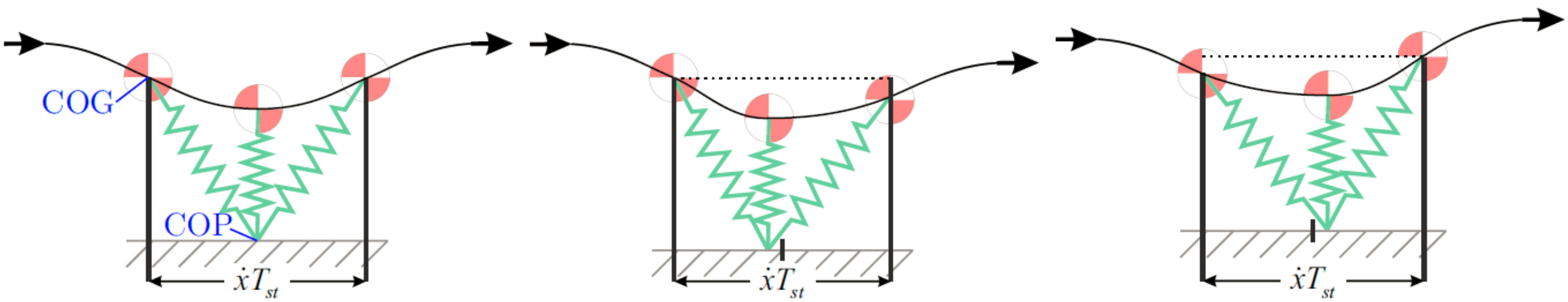
- External disturbances lead to instability



[Coros 2012]

Stability and Control of Locomotion
Example of a biped

- External disturbances lead to instability
- Foot step control for fall recovery



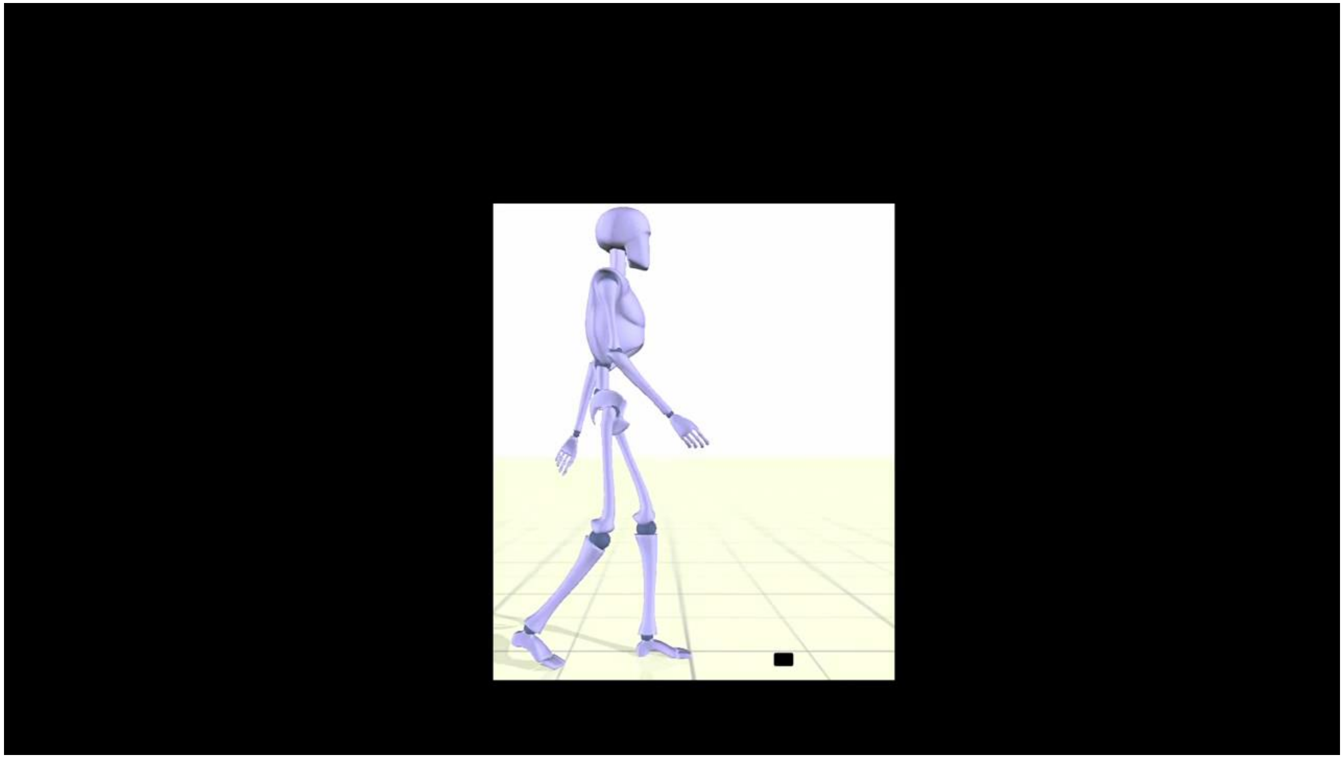
(a) Constant speed (b) Accelerate (c) Decelerate

FF FB

$$\mathbf{r}_F = \boxed{\frac{1}{2}\dot{\mathbf{r}}_{HC,des}T_{st}} + \boxed{k_R^{FB}\left(\dot{\mathbf{r}}_{HC,des} - \dot{\mathbf{r}}_{HC}\right)\sqrt{h_{HC}}}$$

Stability and Control of Locomotion
Example of a biped

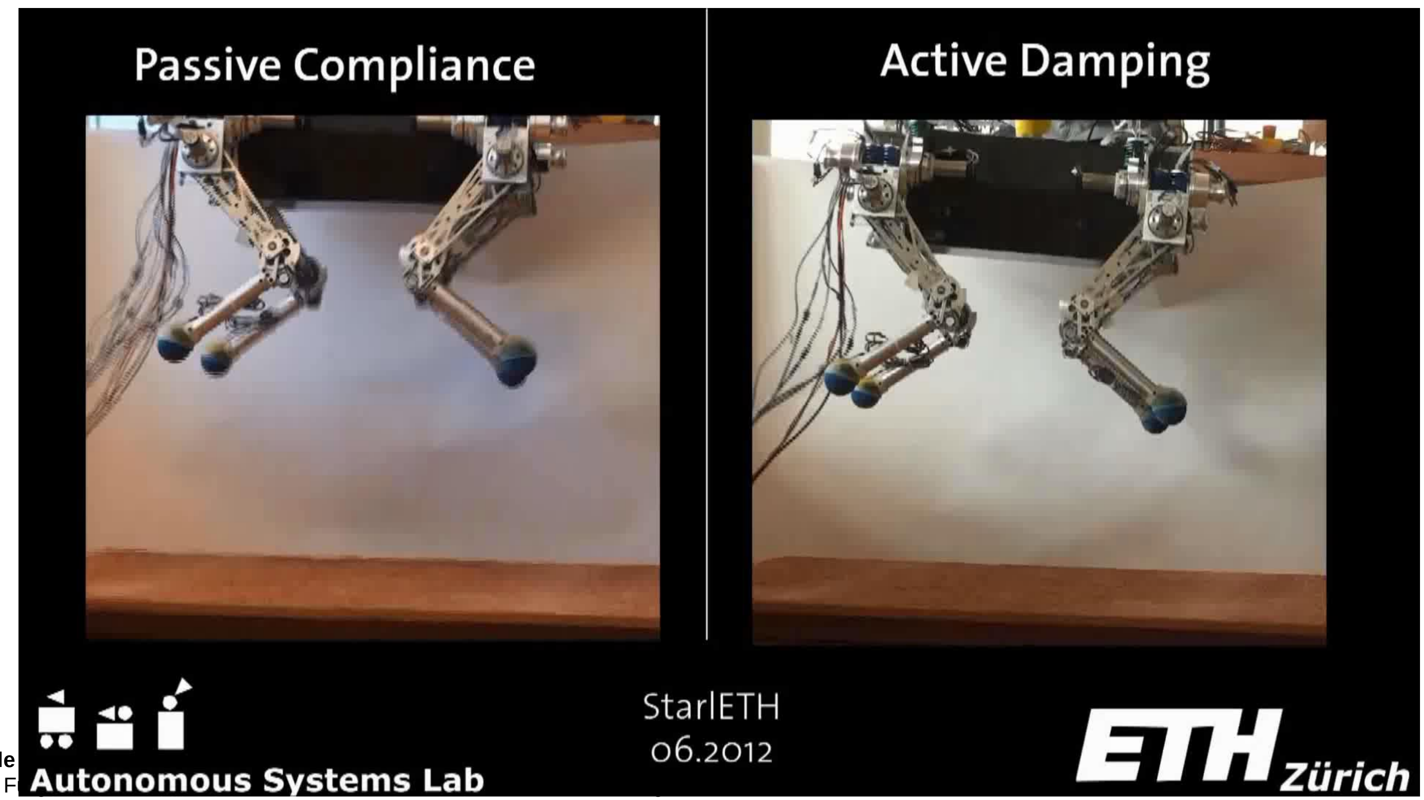
- External disturbances lead to instability
- Foot step control for fall recovery



[Coros 2012]

Legged Robotics
more information and projects

- More information online: http://leggedrobotics.ethz.ch/
- Semester or Master Thesis: http://www.asl.ethz.ch



Week 2: Kinematics
AMR - Autonomous Mobile Robots

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For additional content refer to the accompanying book
Introduction to Autonomous Mobile Robots, R. Siegwart, I. Nourbakhsh, D. Scaramuzza, The MIT Press, 2011.

Summary

- Basics of kinematic description of MBS
  - Translations, rotations, homogeneous transformations
  - Rigid body kinematics formulation
  - Velocities (also in moving CS)

- Application of kinematics
  - Generalized coordinates and Jacobians
  - (Inverse) (Differential) Kinematics

- Some remarks
  - Notation based on Mechanics 3, Techn. Dyn., MBS Dyn. (Glocker)
  - book is not always consistent / complete (historical reasons)

Kinematics
tricks and tools

- Setting up kinematics for large systems can be painful
  - You have to know how it works
  - Apply tools to simplify your life
  - Use MATLAB

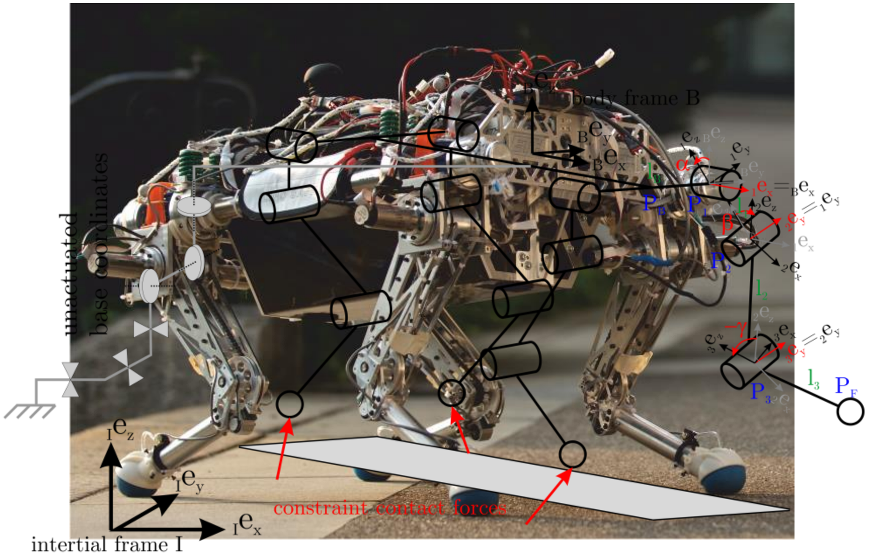
```
syms alpha beta gamma real
q=[alpha,beta,gamma]';
% defining position vectors
r=[ -sin(beta + gamma) - sin(beta);...
  sin(alpha)*(cos(beta + gamma) + cos(beta) + 1) + 1;...
  -cos(alpha)*(cos(beta + gamma) + cos(beta) + 1)];
% taking Jacobians
J = jacobian(r,q);
% simplifying analytical expressions
J = simplify(J);
```

Kinematics
tricks and tools

- Setting up kinematics for large systems can be painful
  - You have to know how it works
  - Apply tools to simplify your life
  - Use MATLAB
- ProNEu
  - https://bitbucket.org/ethz-asl-lr/c_proneu
  - System kinematics and dynamics

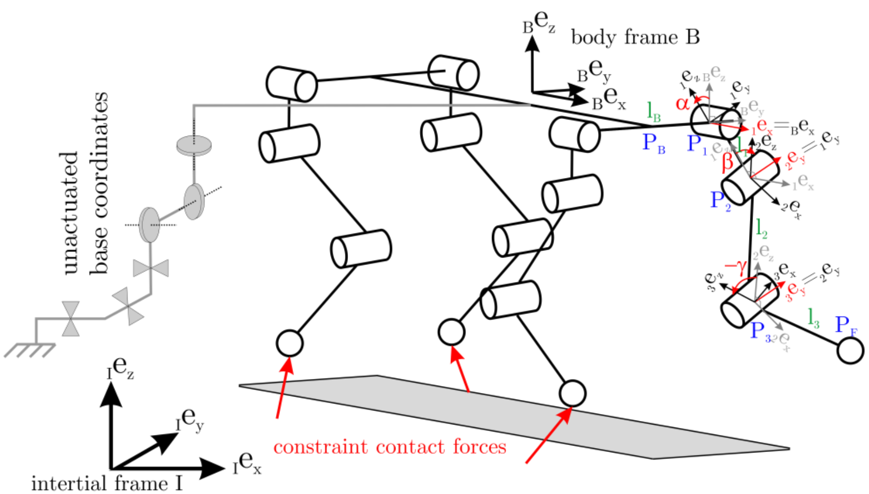
```
% Body 3
i = 3;
body(i).param.m = m3;
body(i).param.B_Th = sym(diag([Th3_xx Th3_yy Th3_zz]));
body(i).param.B_r_COG = sym([0;0;s3]);
body(i).cs.P_r_PO = sym([0;0;l2]);
body(i).cs.A_PB = eulerToRotMat_A_IB(0,q3,0);
body(i).tree.parent = 2;
```

Kinematics of floating base / mobile systems



- Quadrupedal robot
- Static walking
- 3 legs in stance [NR 1,2,3]
- 1 in swing [NR 4]

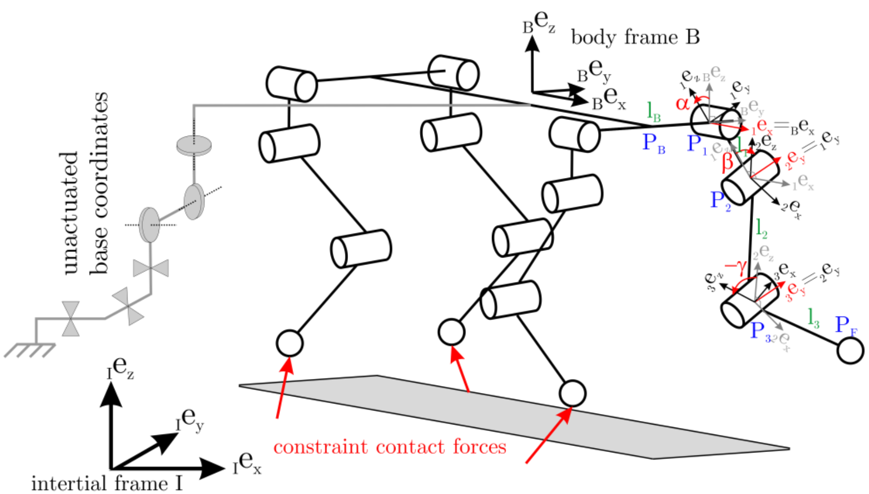
Kinematics of floating base / mobile systems



1. How many generalized coordinates?
2. How many base coordinates?
3. How many actuated joint coordinates?
4. How many contact constraints?

- Quadrupedal robot
- Static walking
- 3 legs in stance [NR 1,2,3]
- 1 in swing [NR 4]

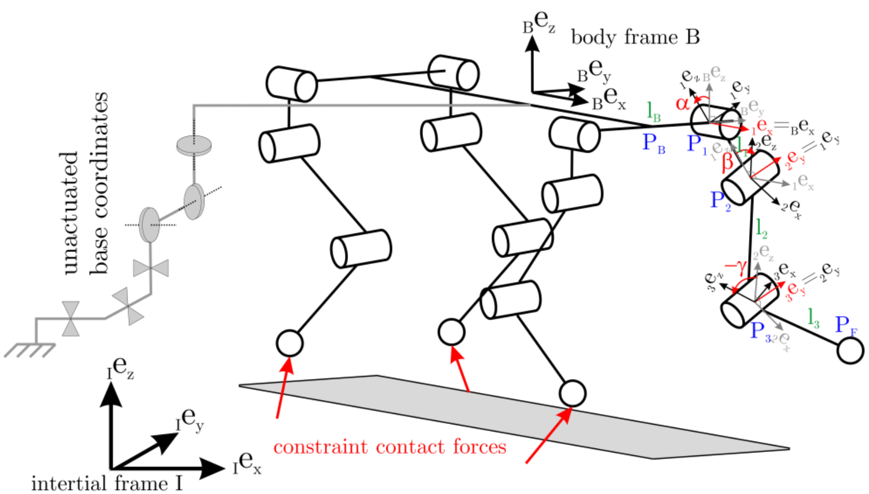
Differential Kinematics



5. Write down the contact constraint
6. How many DoFs remain adjustable?
7. Which DoFs remain adjustable?

- Quadrupedal robot
- Static walking
- 3 legs in stance [NR 1,2,3]
- 1 in swing [NR 4]

Inverse Differential Kinematics



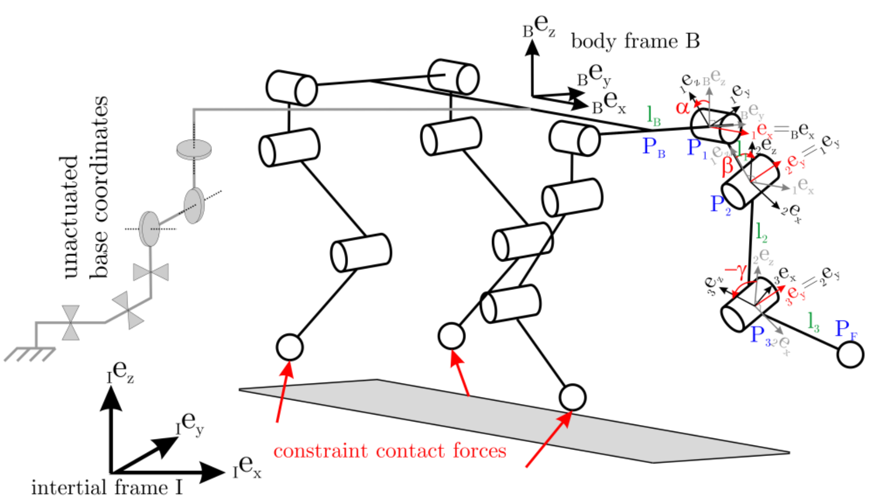
8. Given a desired swing velocity ${}_I\dot{\mathbf{r}}_{OP_{F4}}^{des}$ what is the generalized velocity?

$$\dot{\mathbf{q}} = f\left(\mathbf{q}, {}_I\dot{\mathbf{r}}_{OP4}^{des}\right)$$

9. Is it unique?
10. Is it possible to follow the desired swing trajectory without moving the joints of leg 4? How?

- Quadrupedal robot
- Static walking
- 3 legs in stance [NR 1,2,3]
- 1 in swing [NR 4]

Singularities



- There exist different formulations for "moving the foot (task1) while keeping the base position and orientation (task2)". Write down the solution for A and B!
- What is the difference?
- What happens in singular configs?

Task 1:

- contact constraints
- foot motion

$$\mathbf{J}_1 = \begin{bmatrix} {}_I\mathbf{J}_{OP1} \\ {}_I\mathbf{J}_{OP2} \\ {}_I\mathbf{J}_{OP3} \\ {}_I\mathbf{J}_{OP4} \end{bmatrix}, \mathbf{b}_1 = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ {}_I\dot{\mathbf{r}}_{OP4}^{des} \end{pmatrix}$$

Task 2:

- base position
- base orientation

$$\mathbf{J}_2 = \begin{bmatrix} {}_I\mathbf{J}_{OB} \\ {}_I\mathbf{J}_{BRot} \end{bmatrix}, \mathbf{b}_2 = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

A) Single Task

$$\dot{\mathbf{q}} = \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \end{bmatrix}^+ \begin{pmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{pmatrix} \qquad \min \left\| \mathbf{J} \cdot \dot{\mathbf{q}} - \mathbf{b} \right\|^2$$

B) Null-space motion

$$\dot{\mathbf{q}} = \mathbf{J}_1^+ \mathbf{b}_1 + \mathbf{N}_1 \mathbf{q}_0 \qquad \min \left\| \mathbf{J}_1 \cdot \dot{\mathbf{q}} - \mathbf{b}_1 \right\|^2$$

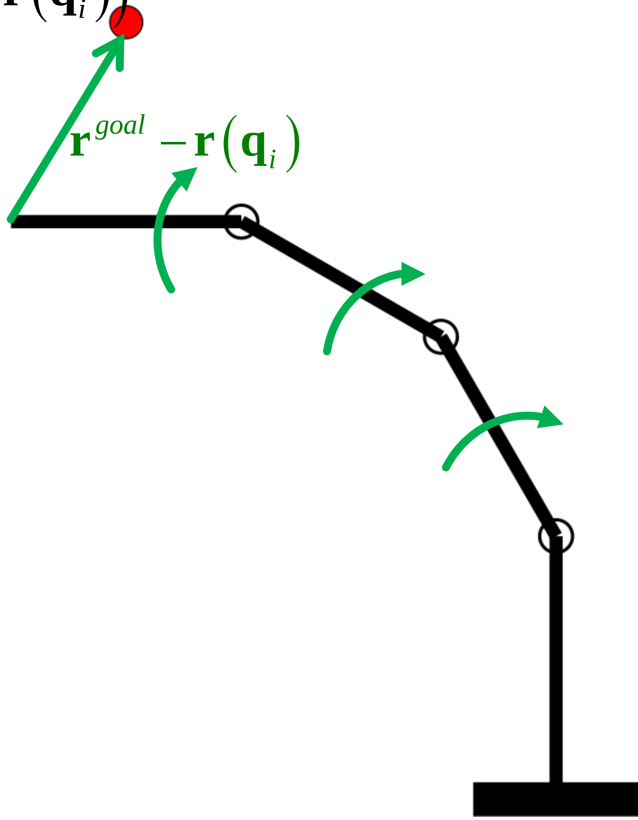
$$\mathbf{J}_2 \dot{\mathbf{q}} = \mathbf{J}_2 \left(\mathbf{J}_1^+ \mathbf{b}_1 + \mathbf{N}_1 \mathbf{q}_0 \right) = \mathbf{b}_2 \qquad \min \left\| \mathbf{J}_2 \cdot \dot{\mathbf{q}} - \mathbf{b}_2 \right\|^2$$

$$\mathbf{q}_0 = \left(\mathbf{J}_2 \mathbf{N}_1 \right)^+ \left(\mathbf{b}_2 + \mathbf{J}_2 \mathbf{J}_1^+ \mathbf{b}_1 \right) \qquad \text{s.t.} \quad \mathbf{J}_1 \cdot \dot{\mathbf{q}} - \mathbf{b}_1 = \mathbf{0}$$

Inverse Kinematics
Iterative Methods

- Presented method = Newton method

$$\mathbf{r} = \mathbf{r}(\mathbf{q}) \quad \rightarrow \quad \Delta\mathbf{r} = \mathbf{J} \cdot \Delta\mathbf{q} \quad \rightarrow \quad \mathbf{q}_{i+1} = \mathbf{q}_i + \mathbf{J}^+ \left(\mathbf{r}^{goal} - \mathbf{r}(\mathbf{q}_i)\right)$$



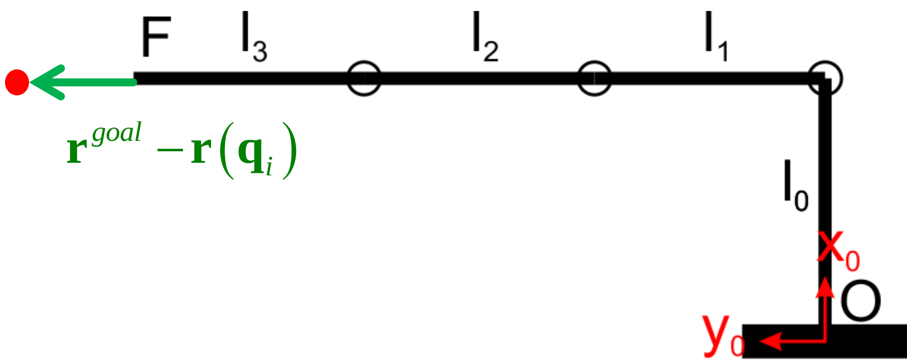
Inverse Kinematics
Iterative Methods

- Presented method = Newton method

$$\mathbf{r}=\mathbf{r}(\mathbf{q}) \quad \rightarrow \quad \Delta\mathbf{r}=\mathbf{J}\cdot\Delta\mathbf{q} \quad \rightarrow \quad \mathbf{q}_{i+1}=\mathbf{q}_i+\mathbf{J}^{+}\left(\mathbf{r}^{goal}-\mathbf{r}(\mathbf{q}_i)\right)$$

- What happens in singular positions? (target out of reach)
  - **J** becomes singular
  - Small $\Delta\mathbf{r}$ can lead to large $\Delta\mathbf{q}$

  } instable!!!



Inverse Kinematics
Iterative Methods

- Presented method = Newton method $\min \left\| \mathbf{J} \cdot \Delta\mathbf{q} - \Delta\mathbf{r} \right\|^2$

$$\mathbf{r} = \mathbf{r}(\mathbf{q}) \quad \rightarrow \quad \Delta\mathbf{r} = \mathbf{J} \cdot \Delta\mathbf{q} \quad \rightarrow \quad \mathbf{q}_{i+1} = \mathbf{q}_i + \mathbf{J}^+ \left(\mathbf{r}^{goal} - \mathbf{r}(\mathbf{q}_i) \right)$$

- Damping (Levenberg-Marquardt) $\min \left\| \mathbf{J} \cdot \Delta\mathbf{q} - \Delta\mathbf{r} \right\|^2 + \lambda^2 \left\| \Delta\mathbf{q} \right\|^2$

$$\Delta\mathbf{q} = \mathbf{J}^T \left(\mathbf{J}\mathbf{J}^T + \lambda^2 \mathbf{I} \right)^{-1} \cdot \Delta\mathbf{r}$$

in Problem Set 4, λ^2=0.1 works well!

- Jacobi-transposed (steepest decent)
  - Understand error as “force pulling to the goal”
  - Jacobi-transposed mapping $\tau = \mathbf{J}^T \cdot \mathbf{F}$
  - Linear scaling $\Delta\mathbf{q} = \alpha \mathbf{J}^T \cdot \Delta\mathbf{r}$
  - Possible choice for $\alpha = \dfrac{\Delta\mathbf{r}^T \mathbf{J}\mathbf{J}^T \Delta\mathbf{r}}{\Delta\mathbf{r}^T \mathbf{J}\mathbf{J}^T \mathbf{J}\mathbf{J}^T \Delta\mathbf{r}}$

in Problem Set 4, this α works well!

Introduction to Inverse Kinematics with Jacobian Transpose, Pseudoinverse and Damped Least Squares methods, **Samuel Buss,** 2009

Outlook

- Next week: Wheeled Systems and Kinematics

- Preparation
  - Watch online videos
  - Do the Problem Sets
  - Read book section
  - Make sure you know what you don't know and ask for it

- Introduction to vREP tomorrow 10.15 – 12 in HG G 1

The end