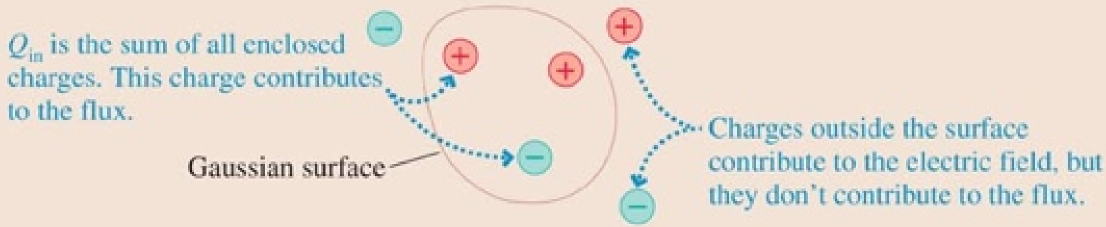
week 3

chapter 28 - Gauss's Law

Important Concepts

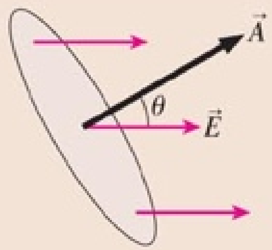
Charge creates the electric field that is responsible for the electric flux.



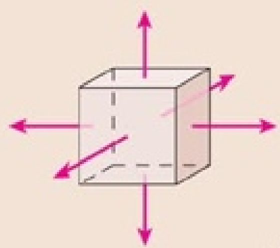
Flux is the amount of electric field passing through a surface of area A:

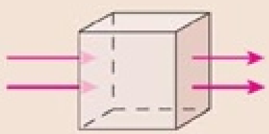
$$\Phi_e = \vec{E} \cdot \vec{A}$$

where $\vec{A}$ is the **area vector.**



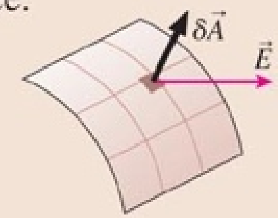
For closed surfaces:
A net flux in or out indicates that the surface encloses a net charge. Field lines through but with no *net* flux mean that the surface encloses no *net* charge.





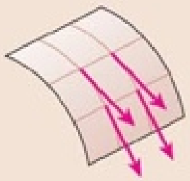
Surface integrals calculate the flux by summing the fluxes through many small pieces of the surface:

$$\Phi_e = \sum \vec{E} \cdot \delta\vec{A}$$
$$\rightarrow \int \vec{E} \cdot d\vec{A}$$



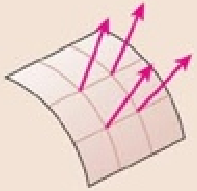
Two important situations:
If the electric field is everywhere tangent to the surface, then

$$\Phi_e = 0$$



If the electric field is everywhere perpendicular to the surface *and* has the same strength E at all points, then

$$\Phi_e = EA$$



General Principles

Gauss's Law

For any *closed* surface enclosing net charge Q_{in}, the net electric flux through the surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

The electric flux Φ_e is the same for *any* closed surface enclosing charge Q_{in}.

Symmetry

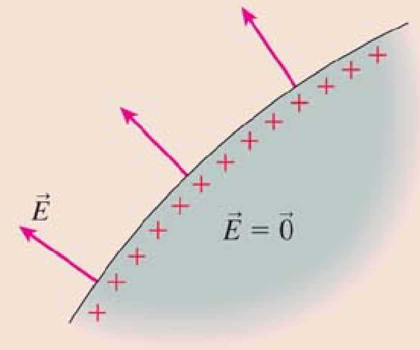
The symmetry of the electric field must match the symmetry of the charge distribution.

In practice, Φ_e is computable only if the symmetry of the Gaussian surface matches the symmetry of the charge distribution.

Applications

Conductors in electrostatic equilibrium

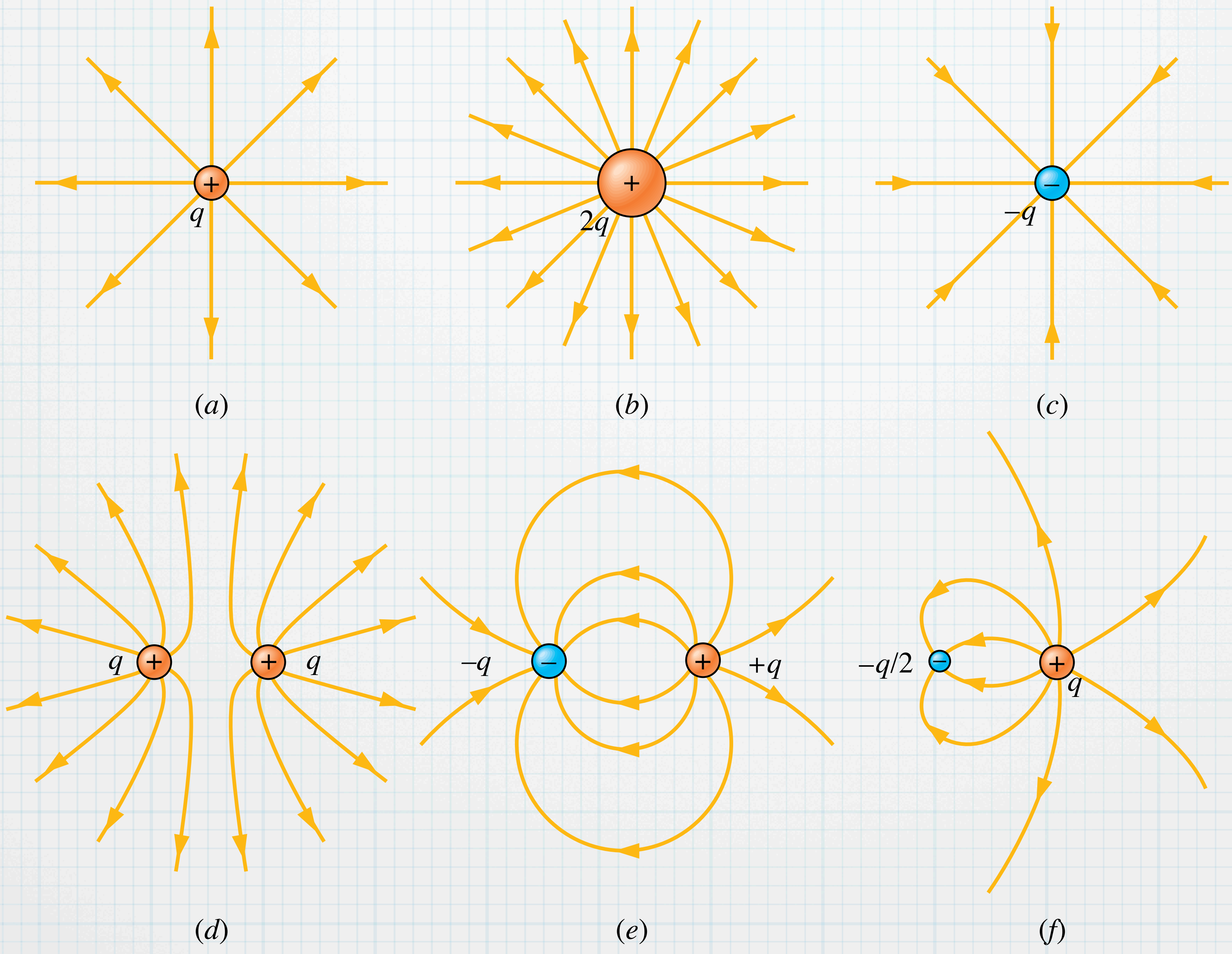
- The electric field is zero at all points within the conductor.
- Any excess charge resides entirely on the exterior surface.
- The external electric field is perpendicular to the surface and of magnitude η/ϵ_0, where η is the surface charge density.
- The electric field is zero inside any hole within a conductor unless there is a charge in the hole.



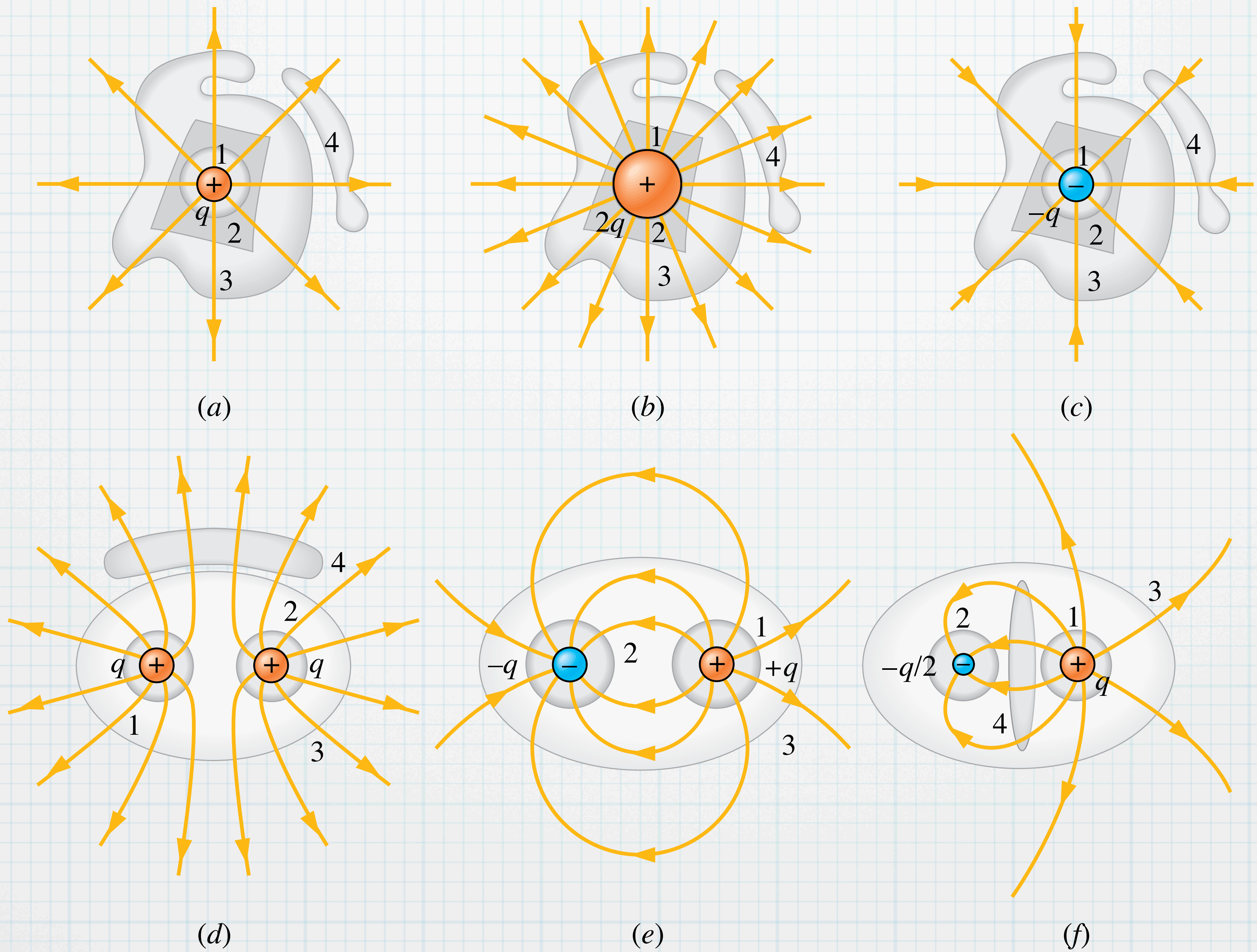
Terms and Notation

symmetric	electric flux, Φ_e	surface integral	screening
Gaussian surface	area vector, $\vec{A}$	Gauss's law	

Here is the central idea: recall field lines ...



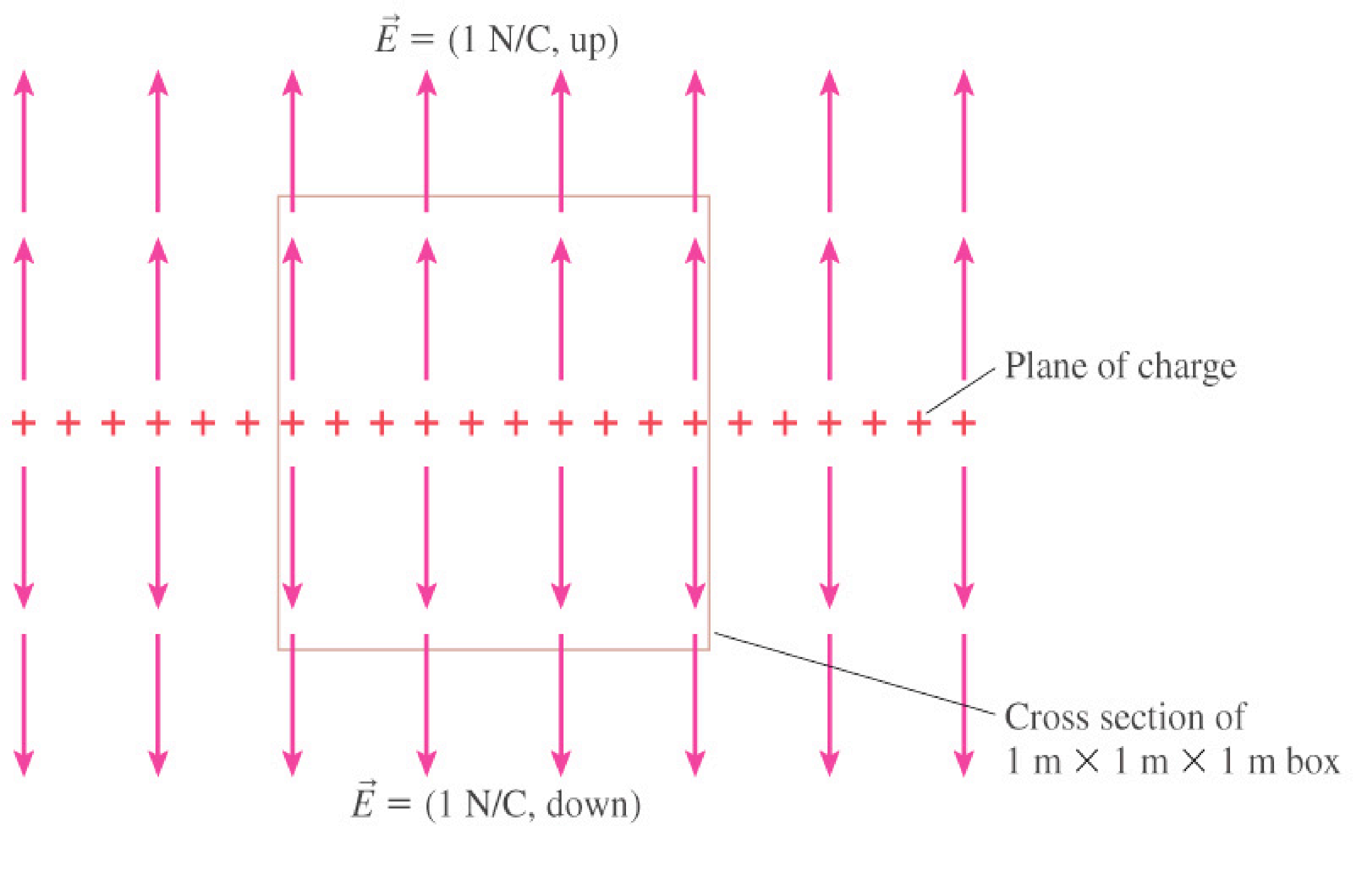
The number of electric field lines emerging from minus the number entering any closed surface is proportional to the charge enclosed



Electric flux just quantifies "the number of lines crossing the surface"

The total electric flux through this box is

1. 6 Nm^2/C.
2. 4 Nm^2/C.
3. 2 Nm^2/C.
4. 1 Nm^2/C.
5. 0 Nm^2/C.



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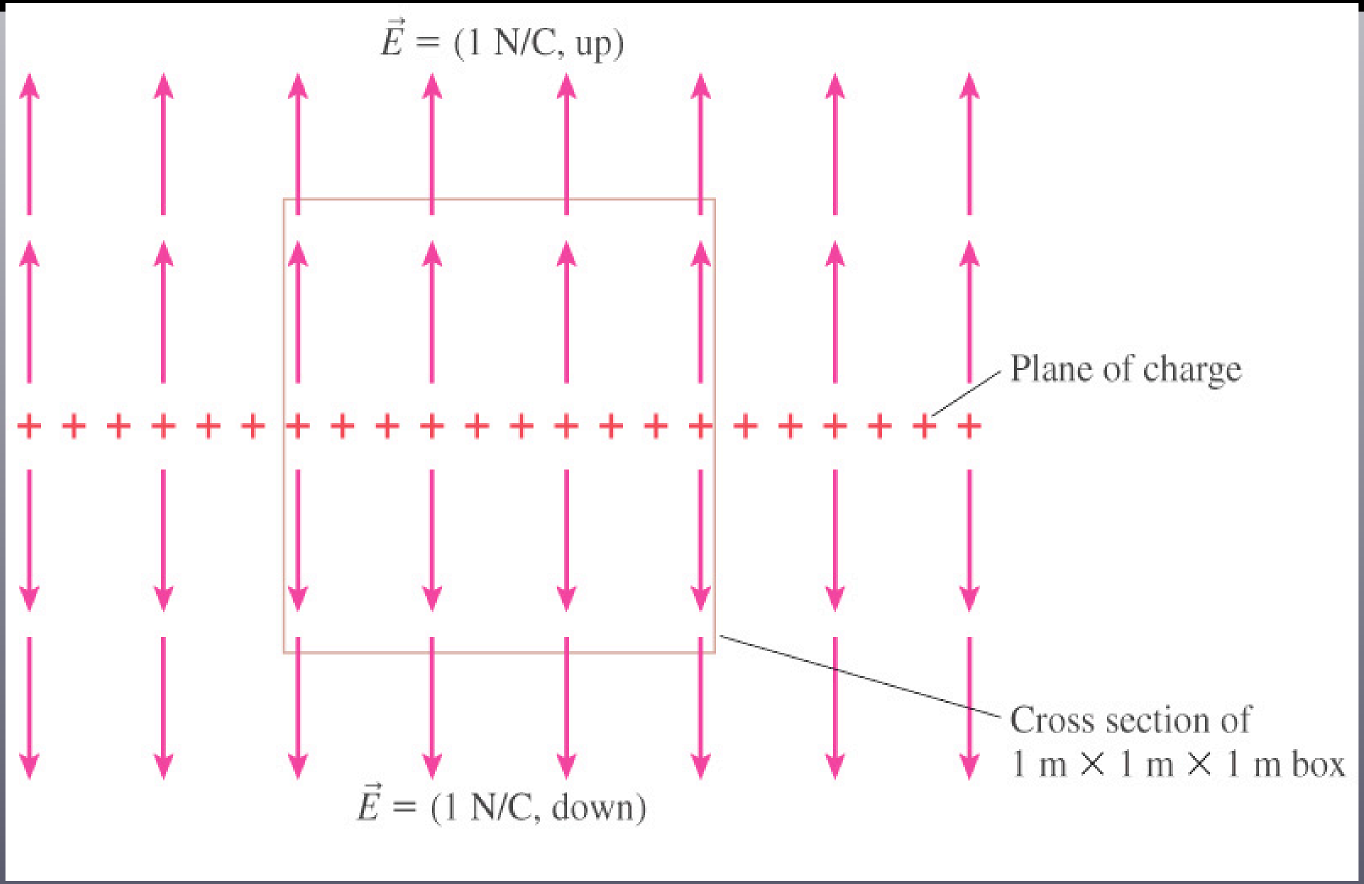
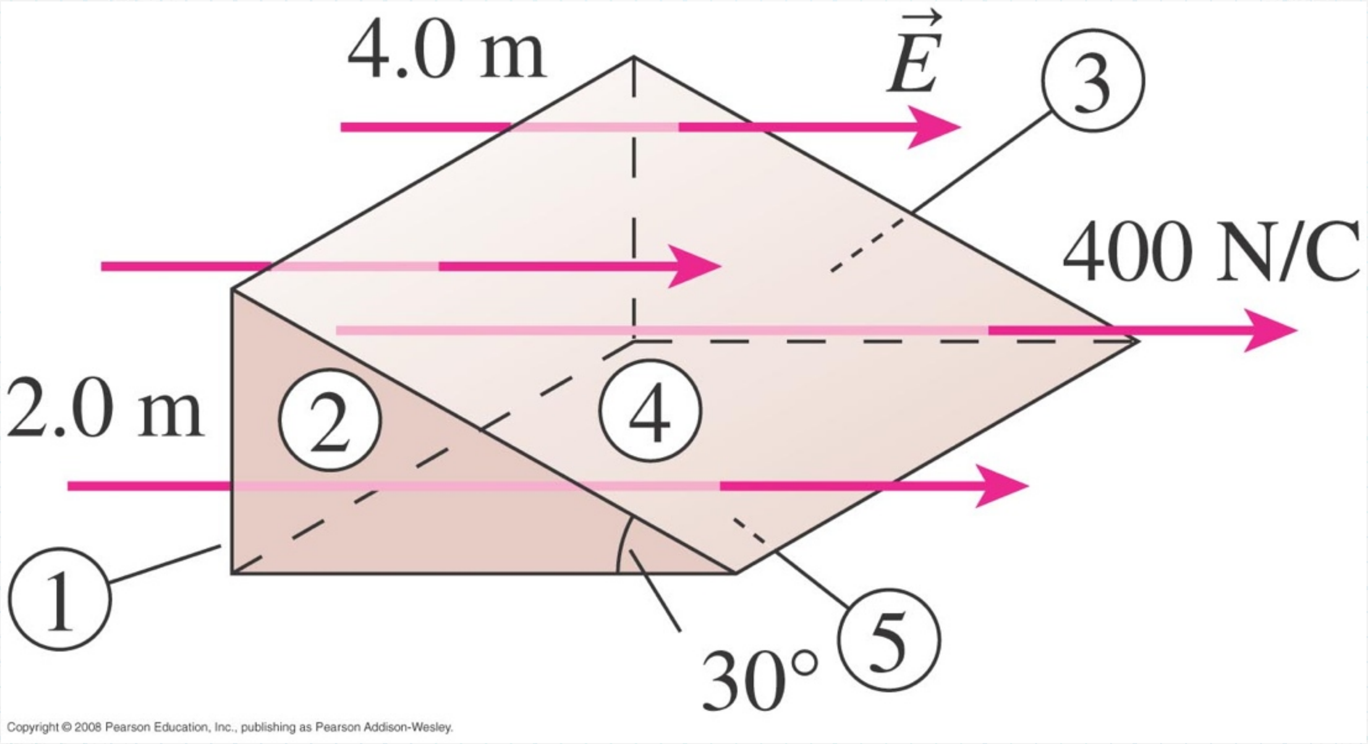


Find the electric fluxes Φ_1 to Φ_5 for surfaces 1 through 5 in the figure.

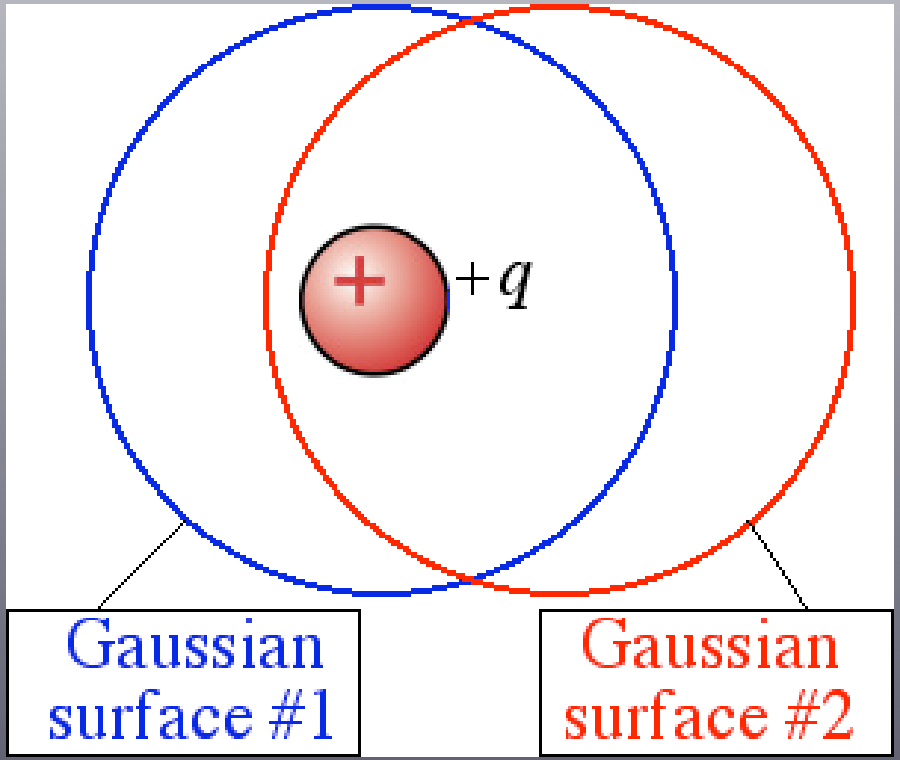


Ans: -3200 Nm^2/C, 0, 0, 0, +3200 Nm^2/C

A spherical Gaussian surface (#1) encloses and is centered on a point charge +q. A second spherical Gaussian surface (#2) of the same size also encloses the charge but is not centered on it.

Compared to the electric flux through surface #1, the flux through surface #2 is

1. greater
2. the same
3. less, but not zero
4. zero
5. not enough information given to decide



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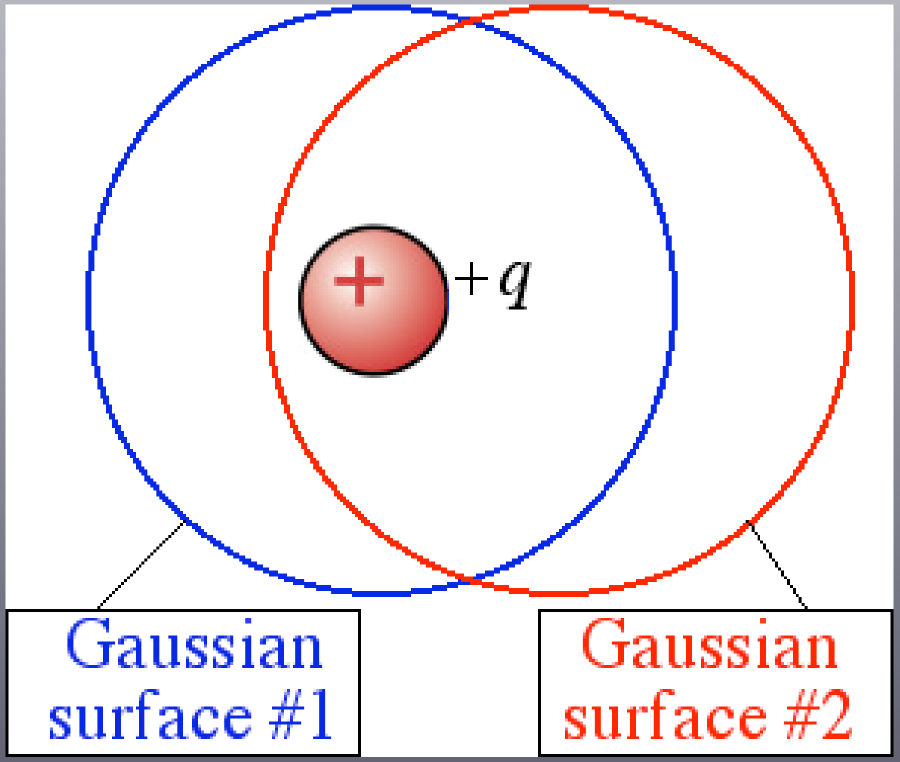
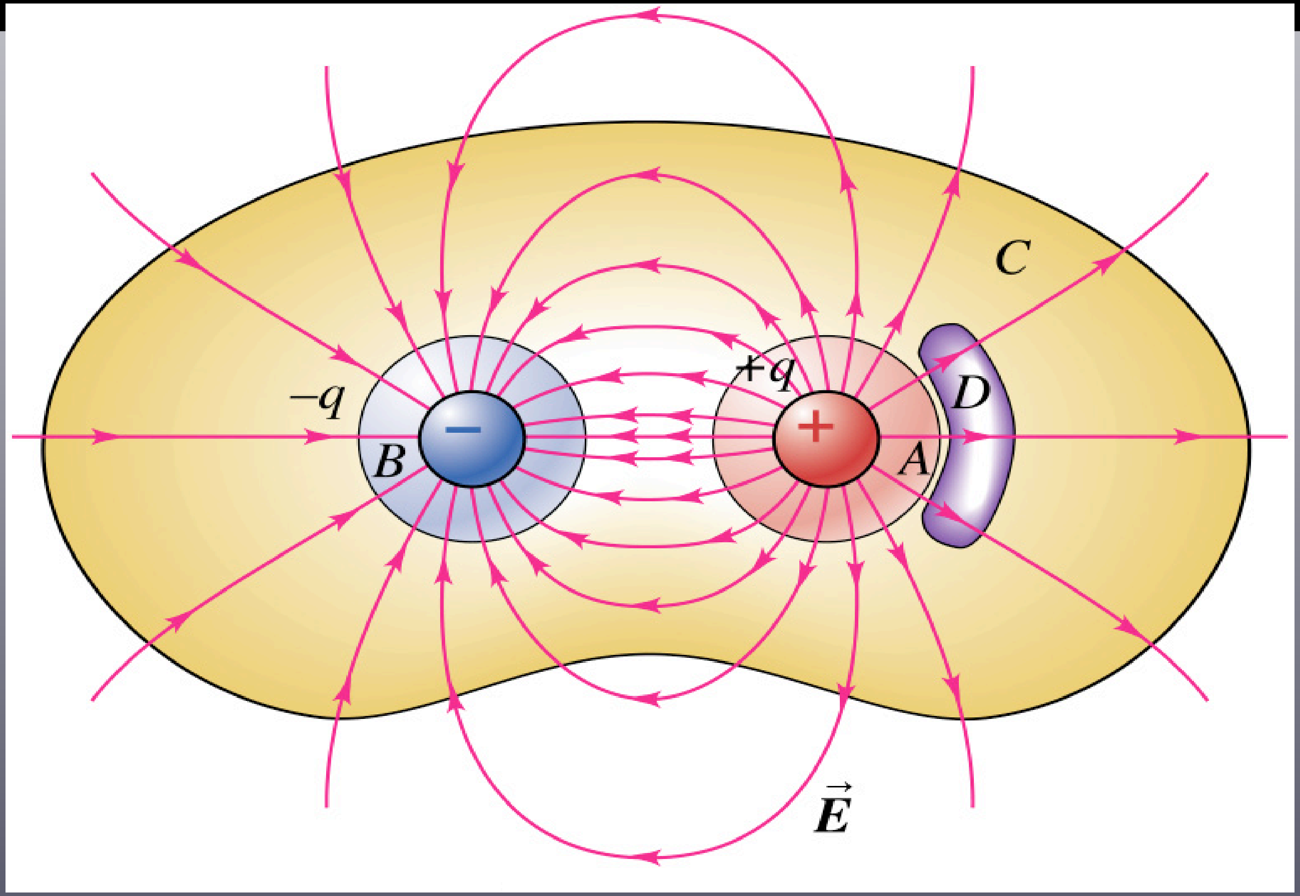


Two point charges, $+q$ (in red) and $-q$ (in blue), are arranged as shown.

Through which closed surface(s) is the net electric flux equal to zero?

1. surface A
2. surface B
3. surface C
4. surface D
5. both surface C and surface D



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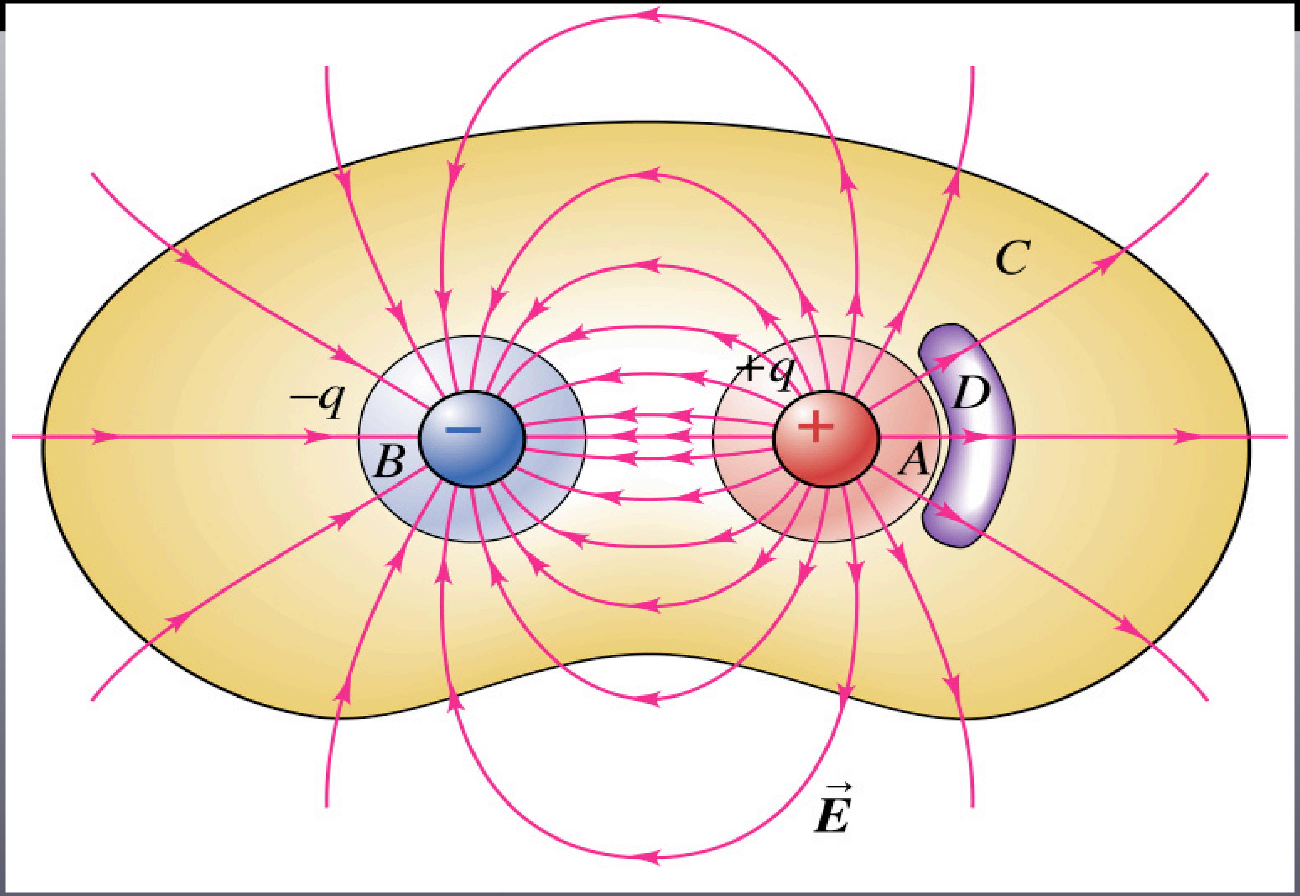
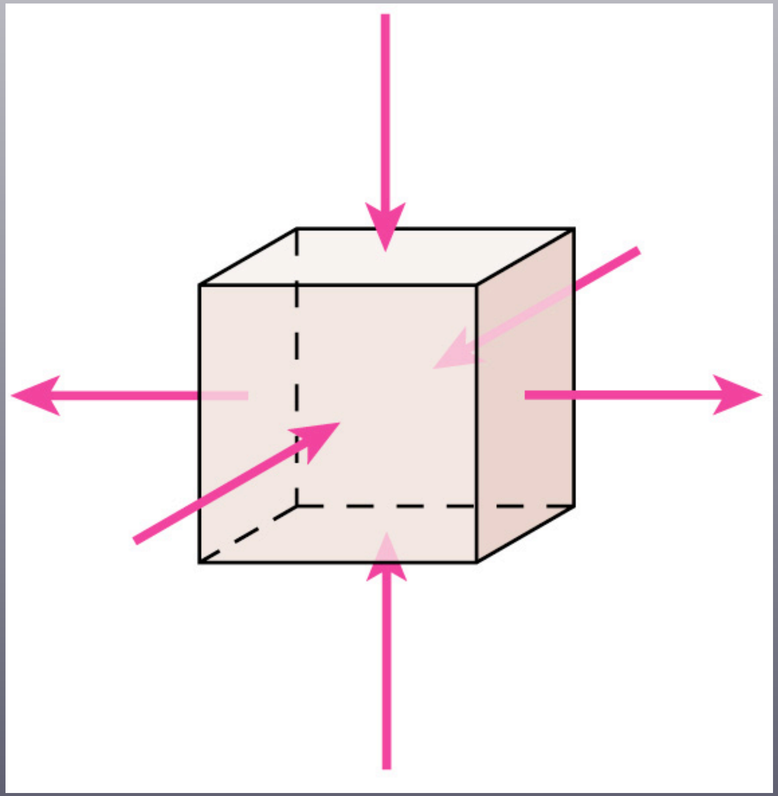


This box contains

1. a net positive charge.
2. a net negative charge.
3. a negative charge.
4. a positive charge.
5. no net charge.



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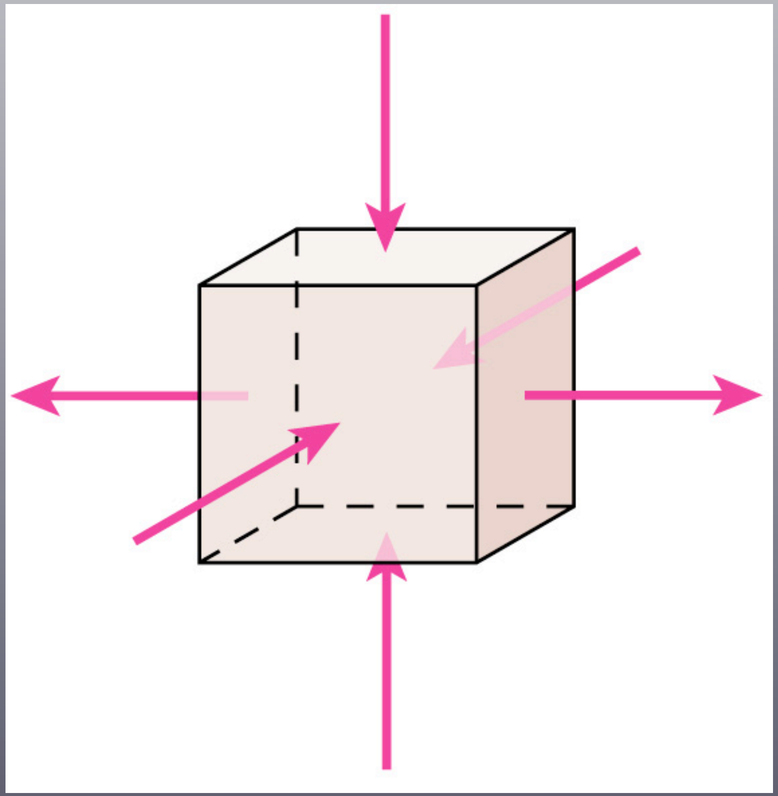
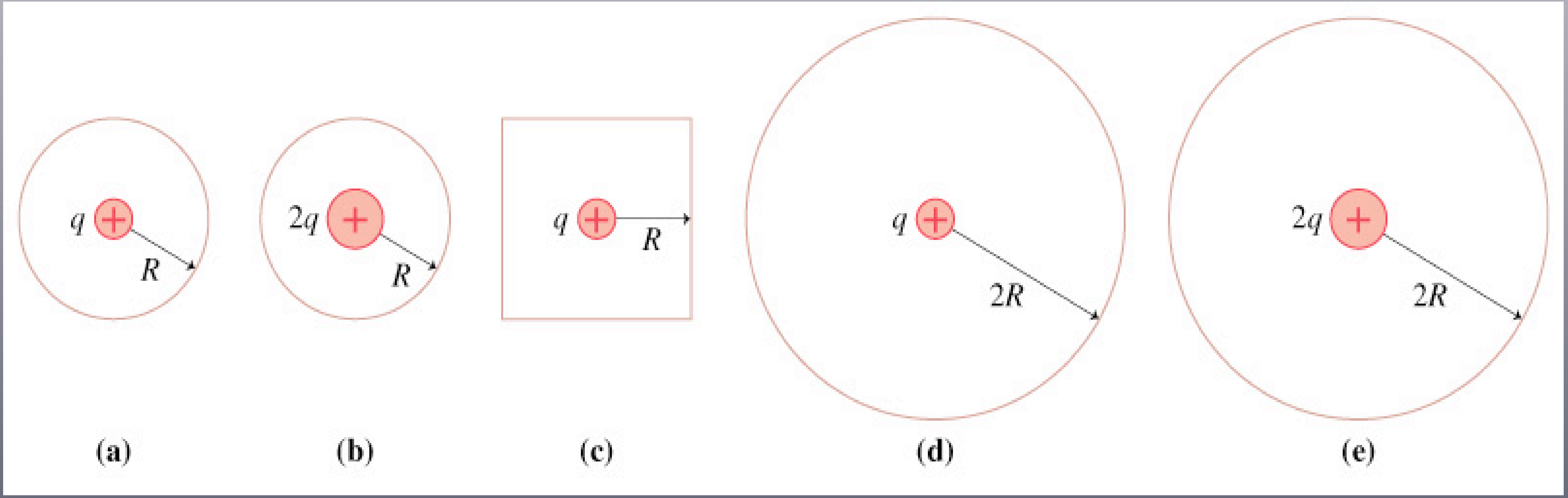


These are two-dimensional cross sections through three-dimensional closed spheres and a cube. Rank order, from largest to smallest, the electric fluxes Φ_a to Φ_e through surfaces a to e.

1. $\Phi_a > \Phi_c > \Phi_b > \Phi_d > \Phi_e$
2. $\Phi_b = \Phi_e > \Phi_a = \Phi_c = \Phi_d$
3. $\Phi_e > \Phi_d > \Phi_b > \Phi_c > \Phi_a$
4. $\Phi_b > \Phi_a > \Phi_c > \Phi_e > \Phi_d$
5. $\Phi_d = \Phi_e > \Phi_c > \Phi_a = \Phi_b$



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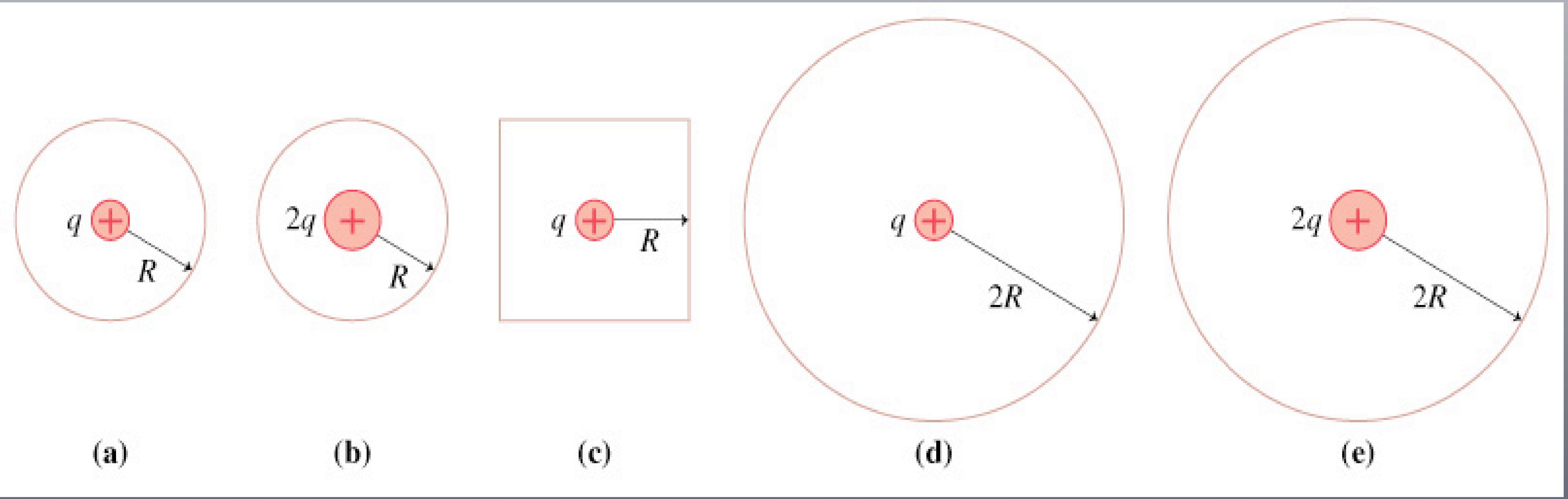
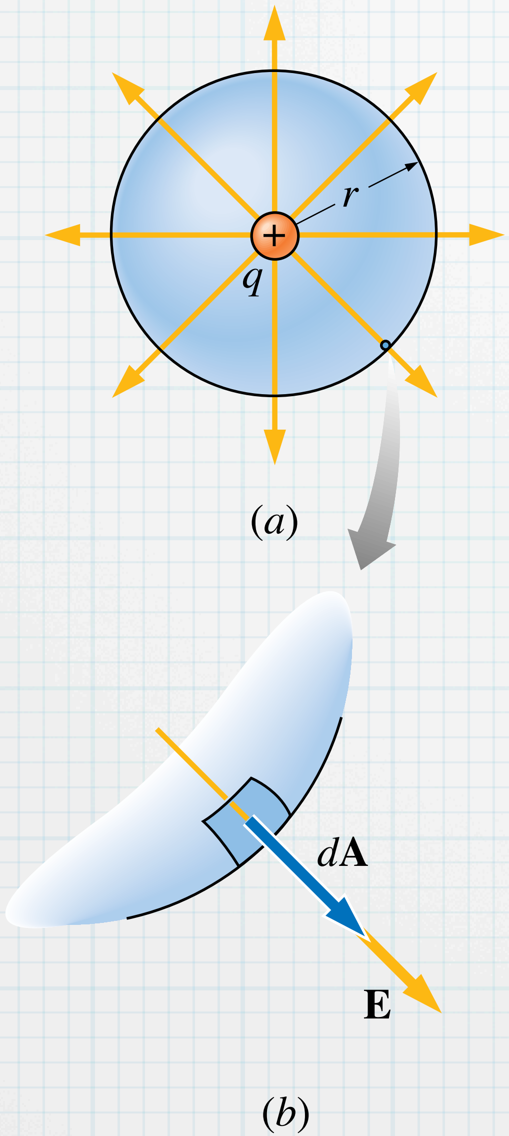


Field of point charge, from Gauss's Law $\Phi = q/\epsilon_0$



Spherical symmetry:
1. Field points radially out
2. Magnitude of field is the same at any point a fixed distance r away from charge

Use surface=sphere of radius r

$$\begin{aligned}\Phi &= \oint \vec{E}\cdot d\vec{A} \\ &= \oint E(r)dA \\ &= E(r)\oint dA \\ &= 4\pi r^2 E(r)\end{aligned}$$

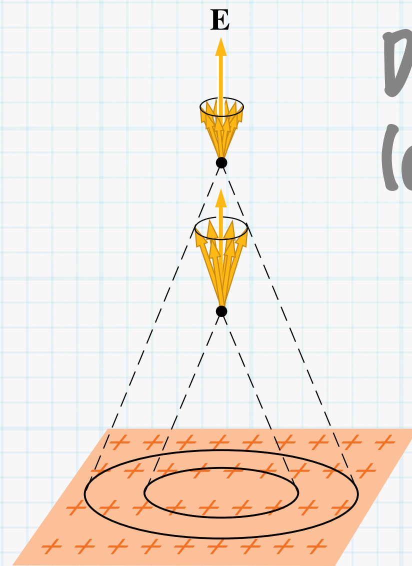
$$4\pi r^2 E(r) = q/\epsilon_0$$

$$\Rightarrow E(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

Note Can also go backwards: given $E(r)$ compute Φ, check Gauss's law

Simple case:

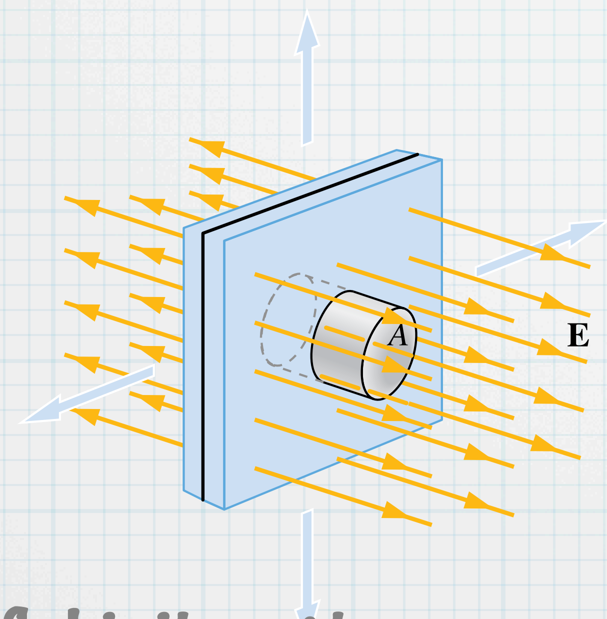
-plane symmetry



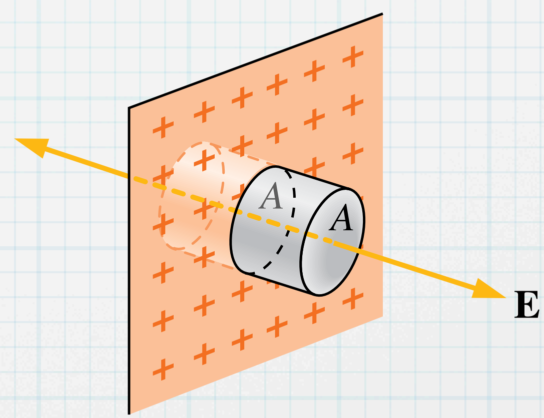
Done earlier, by integration (disk as R $\rightarrow \infty$). Obtained:

$$E = \frac{\sigma}{2\epsilon_0}$$

check this answer: (COB)



field direction
pillbox



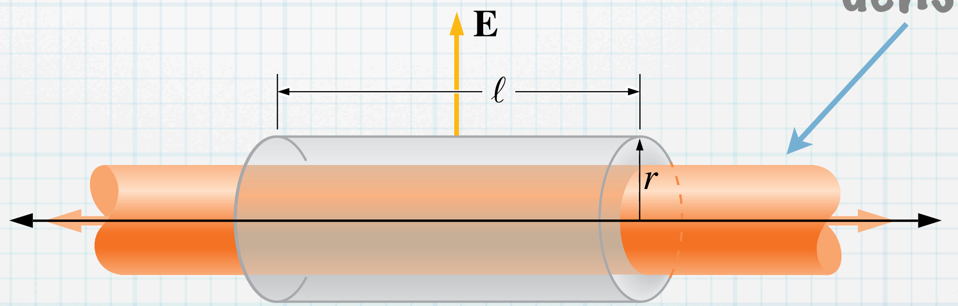
charge enclosed

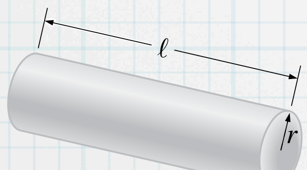
$$\sigma A$$

Another simple case: cylindrical symmetry

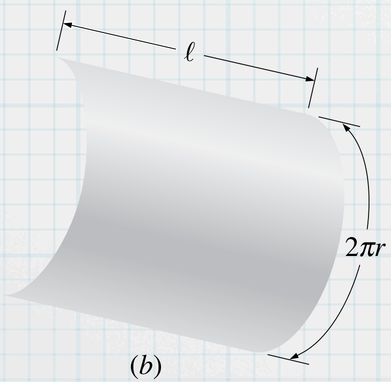
-rotations about axis
-translations along axis
-reflections on any plane normal to axis
-reflections on any plane parallel to and containing axis

linear charge density λ





(a)



(b)

Gaussian "pill box," area is

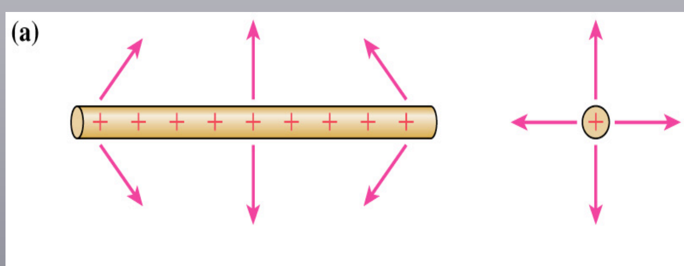
$$(2\pi r)\cdot \ell$$

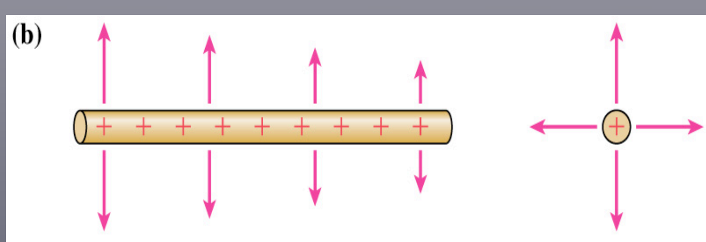
(COB)

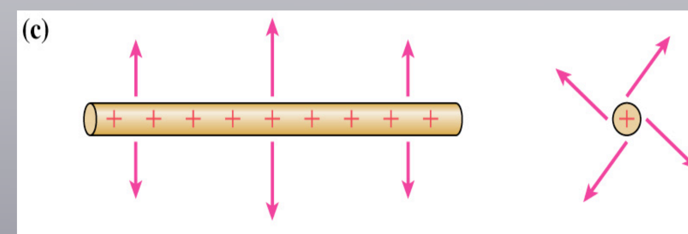
$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

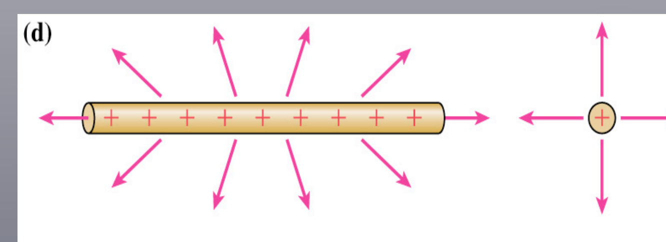
A uniformly charged rod has a finite length L. The rod is symmetric under rotations about the axis and under reflection in any plane containing the axis. It is not symmetric under translations or under reflections in a plane perpendicular to the axis other than the plane that bisects the rod. Which field shape or shapes match the symmetry of the rod?

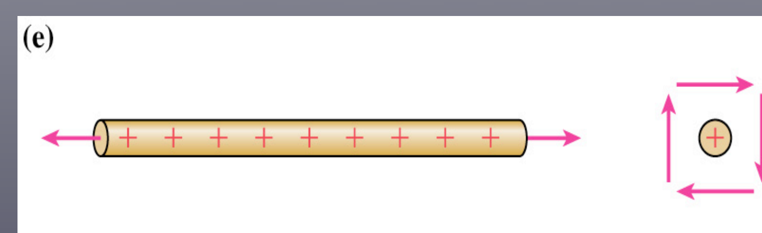
1. c and e
2. a and d
3. e only
4. b only
5. none of the above











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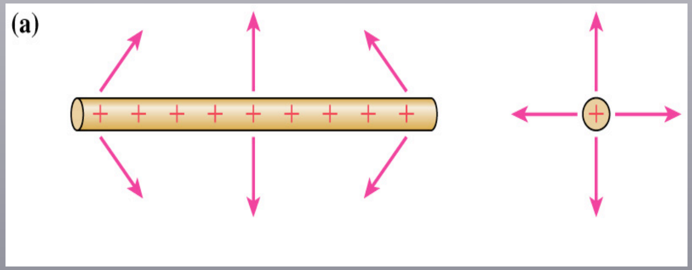

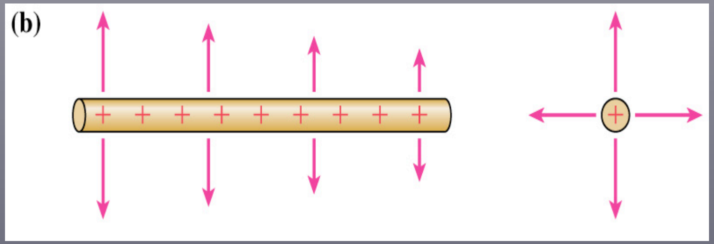

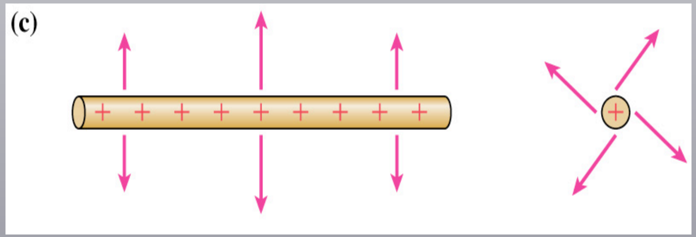

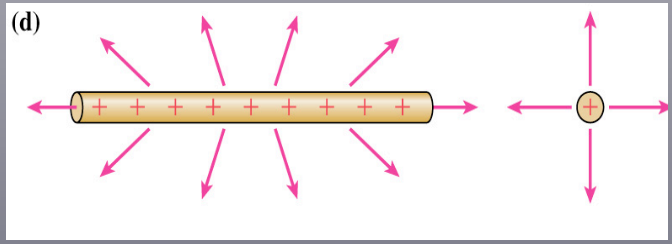

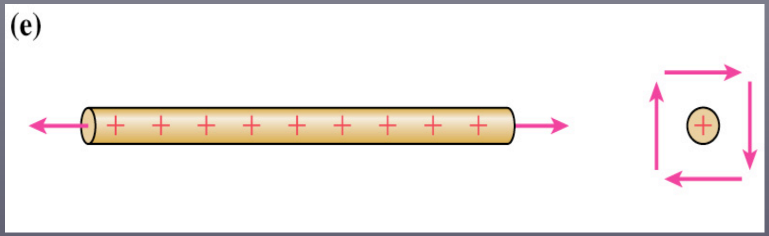

Which Gaussian surface would allow you to use Gauss's law to determine the electric field outside a uniformly charged cube?

1. A cube whose center coincides with the center of the charged cube and which has parallel faces.
2. A sphere whose center coincides with the center of the charged cube.
3. Neither 1 nor 2.
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For which of the following charge distributions would Gauss's law not be useful for calculating the electric field?

1. a uniformly charged sphere of radius R
2. a spherical shell of radius R with charge uniformly distributed over its surface
3. a right circular cylinder of radius R and height h with charge uniformly distributed over its surface
4. an infinitely long circular cylinder of radius R with charge uniformly distributed over its surface
5. Gauss's law would be useful for finding the electric field in all of these case

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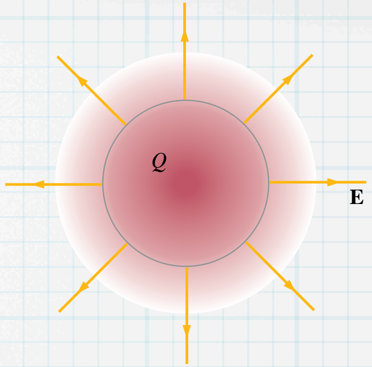
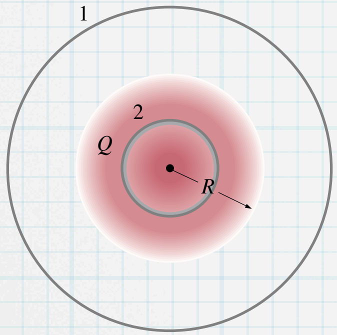
Using Gauss's Law: uniformly charged sphere
Total charge Q, radius R





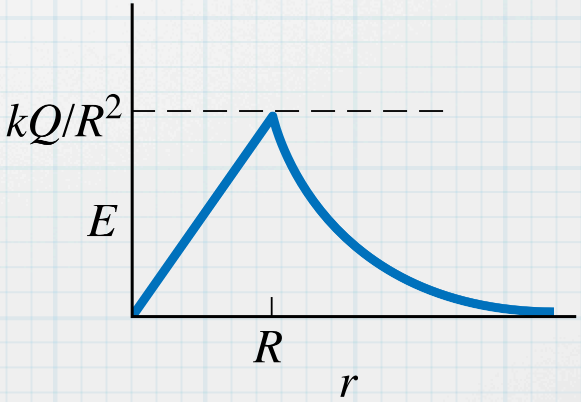
Use spherical symmetry as before

1. Same as for point charge, enclosed charge is Q

$$E(r) = k\frac{Q}{r^2} \quad (r > R)$$

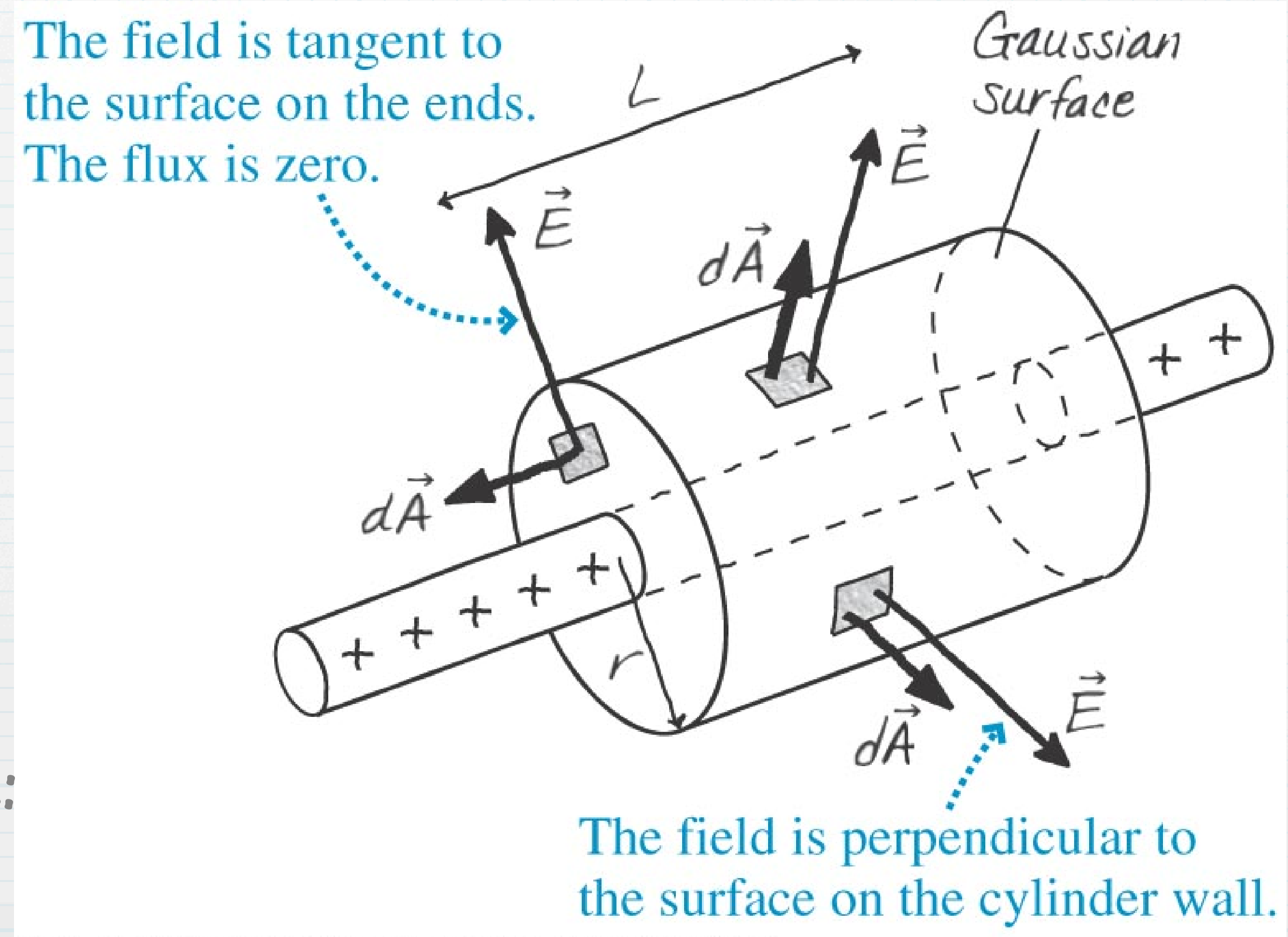
2. For r<R (COB)

$$E(r) = kQ\frac{r}{R^3}$$



Problem: determine the magnitude of the electric field a distance r away from the axis of a uniformly charged infinite cylindrical rod of radius R carrying charge density ρ.

For $r > R$ compare your answer to the expression obtained last week for the finite line of charge Q uniformly distributed over its length L in the limit as $L \to \infty$ and $Q \to 0$ keeping $\lambda = Q/L$ fixed.



Ans:

$$E(r) = \frac{1}{2\epsilon_0}\rho r, \quad r < R$$

$$E(r) = \frac{1}{2\epsilon_0}\frac{R^2\rho}{r}, \quad r > R$$

Relation to line of charge:

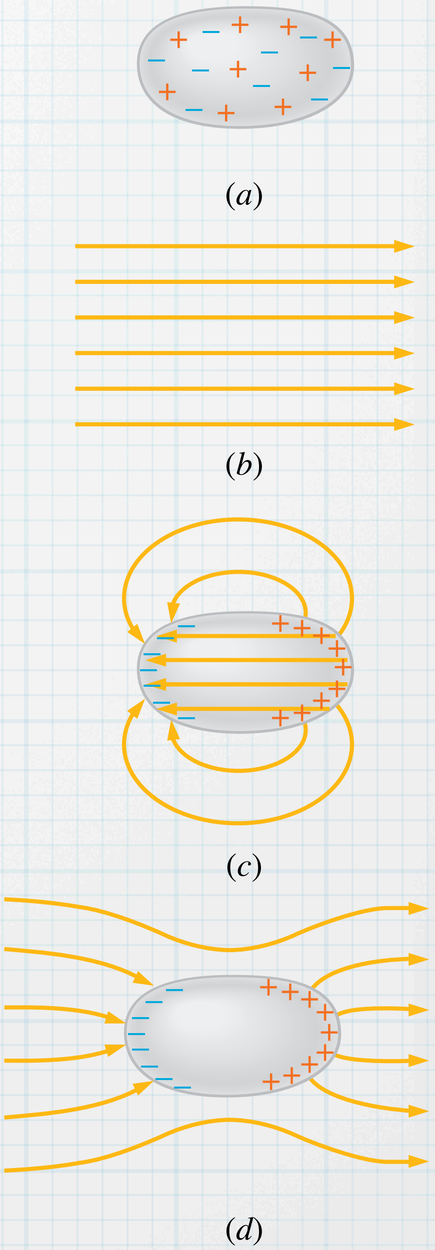
$$\lambda = \pi R^2 \rho$$

Gauss's Law and Conductors

Electrostatic Equilibrium

-static = no motion

-charges experience zero net force

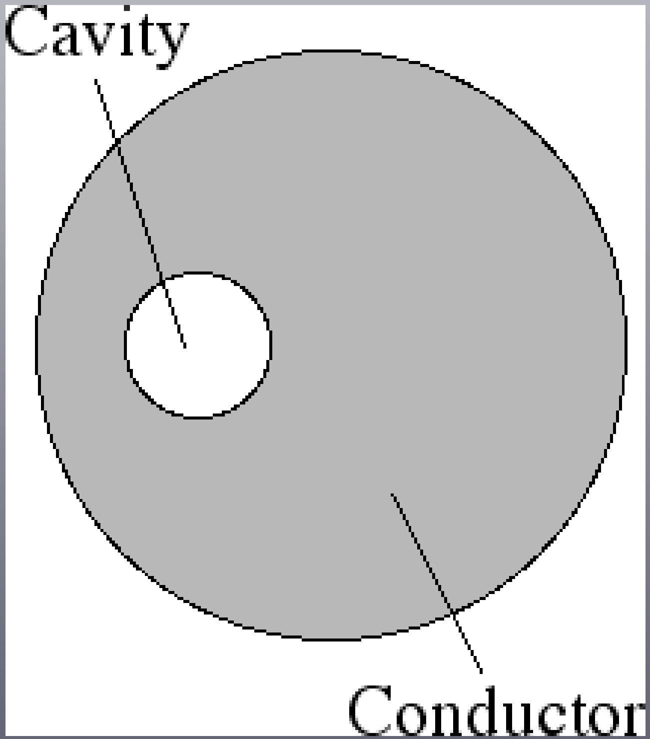


The electric field is zero inside a conductor in electrostatic equilibrium

A solid spherical conductor has a spherical cavity in its interior. The cavity is *not* centered on the center of the conductor.

If a positive charge is placed on the conductor, the electric field in the cavity

1. points generally toward the outer surface of the conductor
2. points generally away from the outer surface of the conductor
3. is zero
4. not enough information given to decide



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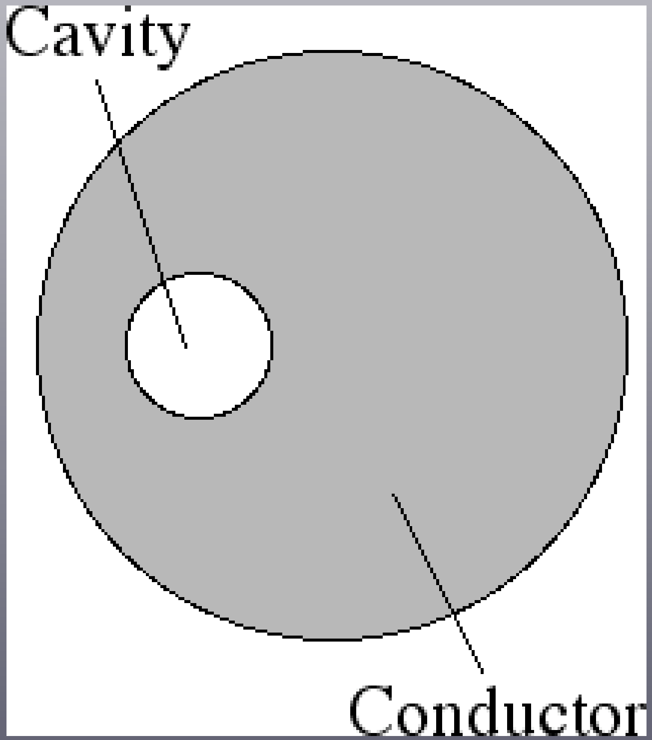
If we assume there was no charge in the cavity to begin with.

But (next slide)



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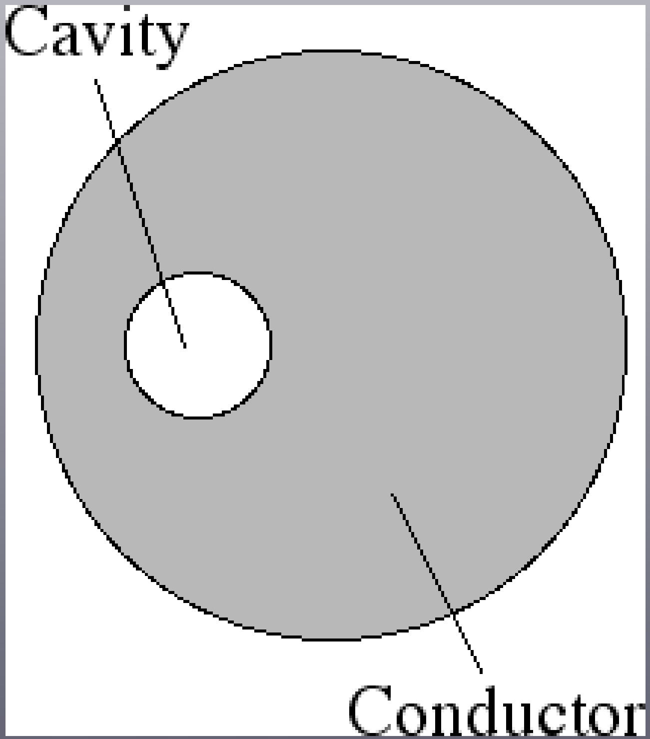
There could be a charge inside the cavity, maybe even a dipole...



A 250-nC point charge is placed at the center of an uncharged spherical conducting shell 20 cm in radius.

(a) What is the surface charge density on the outer surface of the shell?

(b) What is the electric field strength at the shell's outer surface?

Ans: (a) $0.50 \mu\mathrm{C/m^2}$

(b) $5.6 \times 10^4 \mathrm{N/C}$

There is a negative surface charge density in a certain region on the surface of a solid conductor.

Just beneath the surface of this region, the electric field

1. points outward, toward the surface of the conductor
2. points inward, away from the surface of the conductor
3. points parallel to the surface
4. is zero
5. not enough information given to decide

There is a negative surface charge density in a certain region on the surface of a solid conductor.

Just beneath the surface of this region, the electric field

1. points outward, toward the surface of the conductor
2. points inward, away from the surface of the conductor
3. points parallel to the surface
4. is zero
5. not enough information given to decide

This is really poorly worded: what is “beneath?” “outward?”

There is a negative surface charge density in a certain region on the surface of a solid conductor.

Inside the conductor, just by the surface of this region, the electric field

1. points outward, toward the surface of the conductor
2. points inward, away from the surface of the conductor
3. points parallel to the surface
4. is zero
5. not enough information given to decide

There is a negative surface charge density in a certain region on the surface of a solid conductor.

Inside the conductor, just by the surface of this region, the electric field

1. points outward, toward the surface of the conductor
2. points inward, away from the surface of the conductor
3. points parallel to the surface
4. is zero
5. not enough information given to decide

There is a negative surface charge density in a certain region on the surface of a solid conductor.

Outside the conductor, just by the surface of this region, the electric field

1. points outward, away from the surface of the conductor
2. points inward, toward the surface of the conductor
3. points parallel to the surface
4. is zero
5. not enough information given to decide

There is a negative surface charge density in a certain region on the surface of a solid conductor.

Outside the conductor, just by the surface of this region, the electric field

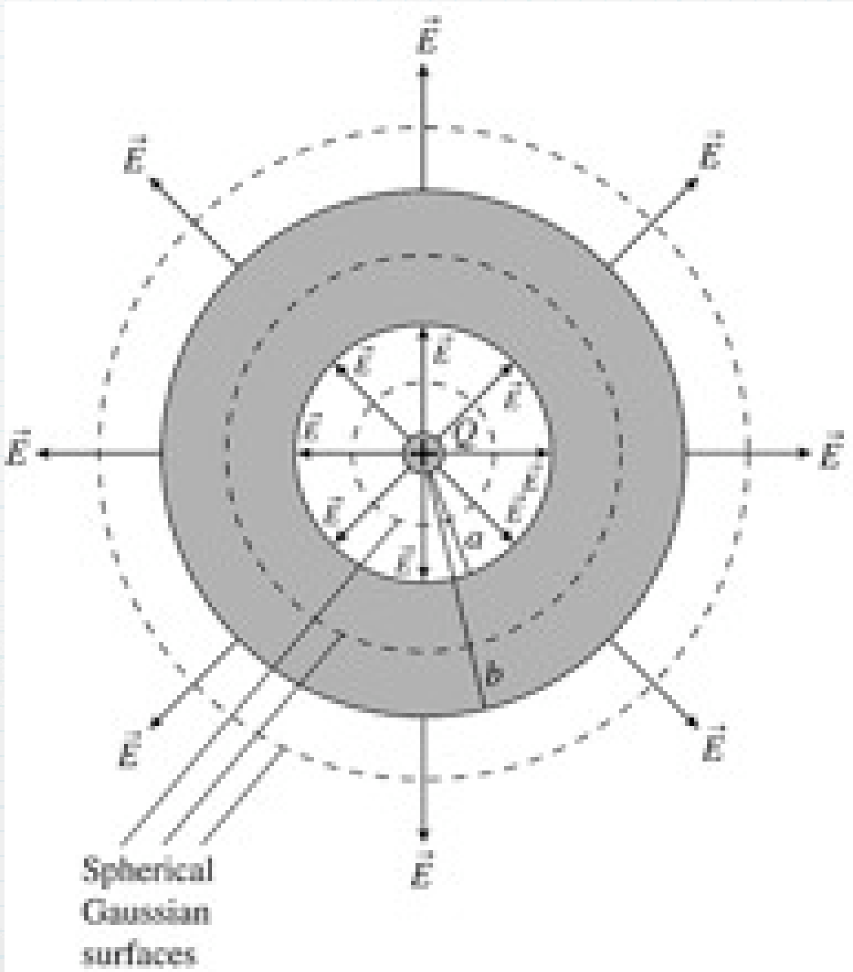
1. points outward, away from the surface of the conductor
2. points inward, toward the surface of the conductor
3. points parallel to the surface
4. is zero
5. not enough information given to decide

A hollow metal sphere has inner radius a and outer radius b. The hollow sphere has charge $+2Q$. A point charge $+Q$ sits at the center of the hollow sphere

(a) Determine the electric fields in the three regions $r \leq a$, $a < r < b$ and $r \geq b$.

(b) How much charge is on the inside surface of the hollow sphere?

(c) How much charge is on the exterior surface of the hollow sphere?



Ans:

(a) $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r^2}\hat{r}, \quad 0, \quad \vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{3Q}{r^2}\hat{r}$

(b) $-Q$

(c) $3Q$