

# Welcome to AP Calculus !

## Summer Assignment for AP Calculus AB and BC North Gwinnett High School

[Diane\\_Shumway@gwinnett.k12.ga.us](mailto:Diane_Shumway@gwinnett.k12.ga.us)  
[Carolyn\\_Renner@gwinnett.k12.ga.us](mailto:Carolyn_Renner@gwinnett.k12.ga.us)  
[Ganesan\\_Sowdayya@gwinnett.k12.ga.us](mailto:Ganesan_Sowdayya@gwinnett.k12.ga.us)

### Required Book:

**Barron's AP Calculus** 8+ version, by Shirley O. Hockett and David Bock

Please purchase this book. Throughout the school year your instructor will assist you in using this book as a study guide and preparation for the AP Exam.

This packet is meant to help you review some of the mathematics that lead up to calculus. Of course, all mathematics up until this point has simply been a foundation for calculus, but these are the most important topics. Since the focus of this class must be the actual content of the AP test, this packet is meant only to be a review and not an in-depth course of study. If you find yourself weak in any of these areas, make sure to review them and strengthen your understanding before August.

### *Do I really need to know the stuff in this packet?*

Yes, you do. These skills will appear on various assignments and assessments throughout the course. Make sure you have mastered them before school starts.

We are looking forward to meeting you and sharing an exciting, challenging, and rewarding experience together as we learn and prepare for the AP Calculus Exam!

- The North Gwinnett High School Calculus Team

**You are expected to be able to do all of these questions**  
**WITHOUT a calculator except**  
**section XXI) Using a graphing calculator**

**I) Simplify the following fractions:**

1)  $\frac{1}{x} + \frac{1}{y} =$

2)  $\frac{1}{x} + \frac{1}{x^2} =$

3)  $\frac{\frac{1}{x} + 1}{\frac{1}{x}} =$

4)  $\frac{\frac{x}{x+y}}{x} =$

5)  $\frac{\frac{1}{x+h} + \frac{1}{x}}{x} =$

**II) Factor each expression:**

6)  $x^2 - 16$

7)  $x^2 - x - 6$

8)  $6x^2 - x - 2$

9)  $4x^3 - 19x^2 - 5x$

10)  $x^2 + 9$

11)  $x^4 - 13x^2 - 30$

12)  $x^3 + 27$

13)  $x^3 - 8$

14)  $(2x-3)^3(x+1) + (x-3)(2x-3)^2$

15)  $(3x-2)^{-4}(x+3) + (x+3)^2(3x-2)^{-3}$

III) Solve the following equations/inequalities for  $x$ :

16)  $x^2 + 5x - 24 = 0$

17)  $x^2 - 9 = 5$

18)  $3x^2 - 5x - 2 = 0$

19)  $x^2 - 4x = 0$

20)  $(x-1)(x^2 - 11x + 30) = 0$

21)  $\sqrt{x+1} = 41$

22)  $\frac{y}{x+1} = \frac{z}{x}$

23)  $\sqrt[3]{x+1} - 4 = -1$

24)  $x^{-2} = \frac{1}{9}$

25)  $2\sqrt{x} = x - 3$

26)  $\frac{8+x}{x} - 5 = 0$

27)  $x^{-1} = -3$

28)  $x^{\frac{4}{3}} = 81$

29)  $3x^2 - 6x - 24 \leq 0$

30)  $\frac{2x-1}{(x+2)(x^2+3)} = 0$

31)  $x^3 - 2x^2 - 5x + 10 = 0$

IV) *Are the following expressions equal to  $\ln 4$  ?*

32)  $2 \ln 2$

33)  $\frac{\ln 8}{\ln 2}$

34)  $\ln 8 - \ln 2$

35)  $\ln 4 + \ln 1$

36)  $\ln 4 \cdot \ln 1$

36)  $(\ln 2)^2$

V) *Write an equation of a line based on the given information:*

37) Find the equation of the line that has a slope of 5 and passes through the point (3, -4).

38) Find the equation of the line that passes through the points (4, 1) and (3, -2).

39) Find the equation of the line that passes through the points (-2, 1) and is parallel to the line  $4x + 2y = -1$ .

40) Find the equation of the line that has a slope of 0 and passes through the point (-5, 1).

41) Find the equation of the line that passes through the origin and is perpendicular to the line  $3x + 4y = -7$ .

42) Find the equation of the line that has an undefined slope and passes through the point (4, -5).

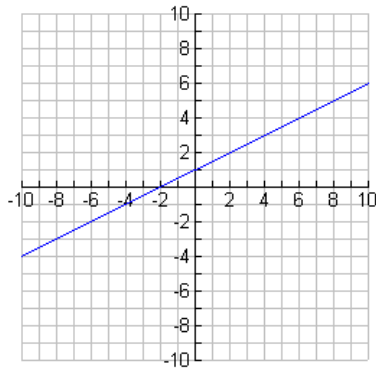
VI) *Intercepts:*

43) Find the  $x$  and  $y$  intercepts of  $x^2 + y^2 = 9$ .

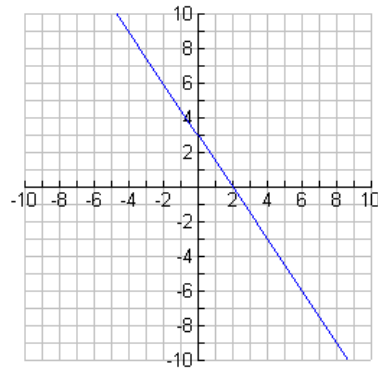
- 44) Find the equation of the line that has an  $x$ -intercept of 5 and a  $y$ -intercept of 3.

VII) Write the equation for the following graphs:

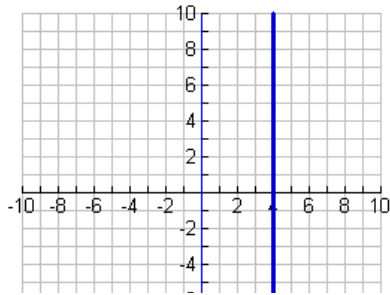
45)



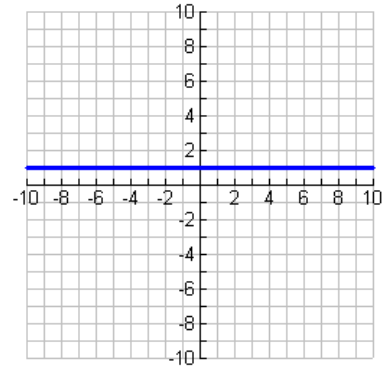
46)



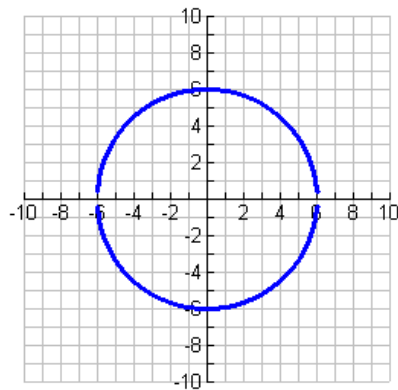
47)



48)



49)



VIII) Given the slope, sketch the following lines:

- 50) Sketch a line with a slope of 2.  
51) Sketch a line with a slope of  $\frac{1}{2}$ .  
52) Sketch a line with a slope of -2.  
53) Sketch a line with a slope of  $-\frac{1}{2}$ .

**IX) Sketch the following graphs:**

- |                           |                            |
|---------------------------|----------------------------|
| 54) $f(x) = 3x + 1$       | 55) $f(x) = x^2$           |
| 56) $f(x) =  x $          | 57) $f(x) = x^3$           |
| 58) $x = 3$               | 59) $y = -4$               |
| 60) $f(x) = \ln x$        | 61) $f(x) = \sqrt{x}$      |
| 62) $f(x) = \frac{1}{x}$  | 63) $f(x) = \frac{1}{x^2}$ |
| 64) $f(x) =  x + 1 $      | 65) $f(x) = x^2 + 2x - 3$  |
| 66) $f(x) = x^3 + 1$      | 67) $f(x) = (x + 1)^2$     |
| 68) $f(x) = -x^2 + 1$     | 69) $f(x) = (x + 1)^{1/3}$ |
| 70) $f(x) = x^{2/3}$      | 71) $f(x) = e^x$           |
| 72) $f(x) = -\frac{1}{x}$ | 73) $x^2 + y^2 = 25$       |

**X) Rewrite the following functions without absolute values:**

- |                  |                      |
|------------------|----------------------|
| 74) $f(x) =  x $ | 75) $f(x) =  x - 1 $ |
|------------------|----------------------|

**XI) Find the domain and range of each function:**

76)  $f(x) = x - 1$

77)  $f(x) = \frac{1}{x}$

78)  $f(x) = \frac{1}{x^2 + 1}$

79)  $f(x) = e^x$

80)  $f(x) = \sqrt{x - 4}$

81)  $f(x) = |x - 1| + 2$

82)  $f(x) = \ln x$

83)  $f(x) = \sqrt{x^2 - 3x - 4}$

84)  $f(x) = \frac{1}{x + 6} - 10$

**XII) Find the inverse of each function:**

85)  $f(x) = x + 3$

86)  $f(x) = \sqrt{x}$

87)  $f(x) = \frac{x}{x + 2}$

88)  $f(x) = \ln x$

**XIII) Find the compositions of the function if:**

$$f(x) = x^3 + 1, g(x) = x^2 - 2, \text{ and } j(x) = x + 3$$

89)  $f(2)$

90)  $f(j(x))$

91)  $f(j(2))$

92)  $g(g(x))$

93)  $f(x + h)$

94)  $\frac{f(x + h) - f(x)}{h}$

**XIV) Solve the simultaneous equations:**

**95a)**  $2x + 3y = 8$   
 $x + 2y = 5$

**95b)**  $y = x^2 + 2x + 9$   
 $7x + y = 19$

**96a)** The length,  $l$ , of a certain rectangle is twice the width,  $w$ . Write an equation for the perimeter the rectangle as a function of the width,  $w$ .

**96b)** If the area of the rectangle described above is 50 square feet, find the length and the width of the rectangle.

**XV) Intersection of curves:**

**97)** Find the point of intersection between the lines  $y = x + 1$  and  $3y - x = 5$ .

**98)** Find the point of intersection between the lines  $y = x + 7$  and the curve  $y = x^2 + 2x + 5$ . Also sketch the area between the graphs.

**XVI) What do the following mean?**

**99)** a graph is in the first quadrant

**100)**  $f(2) = 5$

**101)** an expression is a function

**102)** a zero of a function is 4

**103)**  $y$  is directly proportional to  $x$  (give an example)

**104)** the coefficient of the third term is 5 (give an example)

**105)** a function has only one root

**106)** a function is a polynomial

**107)** two triangles are similar

**108)** a function is even

**109)** a function is odd

**XVII) What are the following formulas?**



- 110) Quadratic formula
- 111) Pythagorean Theorem
- 112) the hypotenuse of a 45-45-90 isosceles right triangle with a leg of length  $x$ .
- 113) the hypotenuse of a 30-60-90 right triangle with shortest leg having a length of  $x$ .
- 114) the volume of a sphere
- 115) the volume of a cylinder
- 116) the volume of a cone
- 117) the volume of a box with a square base
- 118) the surface area of a sphere
- 119) the surface area of a cylinder with no top
- 120) the area of a triangle
- 121) the area of a trapezoid
- 122) the cross section through the center of a sphere
- 123) the volume of a prism that has an equilateral triangle with side of length  $x$  and height of length  $y$
- 124) area of an equilateral triangle in terms of the length of a side  $s$

**XVIII) Solve using similar triangles:**

- 125) A six foot man is standing 10 feet away from a 20 foot lamppost. What is the length of his shadow?
  
- 126) Water is dripping out of a conical figure that has a diameter of 8 inches and a height of 12 inches. When the depth of the water is 8 inches, what is the radius of the water?

**XIX) Find the equations of the horizontal and vertical asymptotes of each function:**

127)  $y = \frac{1}{x-1}$

128)  $y = \frac{x^3}{x^3-1}$

**XX) Exponent Rules: Which of the following are true?**

129)  $x^0 = 1$

130)  $x^{-2} = \frac{1}{x^2}$

$$130) \sqrt{x+y} = \sqrt{x} + \sqrt{y}$$

$$131) x^5 \cdot x^3 = x^{15}$$

$$132) x^5 \cdot y^5 = (xy)^5$$

$$133) (x^3)^5 = x^8$$

$$134) x^{5-w} = \frac{x^5}{x^w}$$

$$135) x^{t+5} = (x^t)^5$$

$$136) \sqrt{\frac{9}{4}} = \frac{3}{2}$$

$$137) (4x)^{\frac{1}{2}} = 2x$$

$$138) \sqrt{\frac{1}{x}} = x^{-\frac{1}{2}}$$

$$138) \sqrt{x^2} = x$$

$$139) \sqrt{x^2 - 25} = x - 5$$

$$140) x^{\frac{4}{3}} = \sqrt[4]{x^3}$$

$$141) \left(x^{\frac{1}{2}} + y^{\frac{1}{2}}\right)^2 = x + y$$

$$142) x^{\frac{-2}{3}} = \frac{1}{\sqrt[3]{x^2}}$$

$$143) e^{\ln x^2} = x^2$$

$$143) \ln e^3 = 3$$

$$144) e^{2\ln 2 - \ln 5} = \frac{4}{5}$$

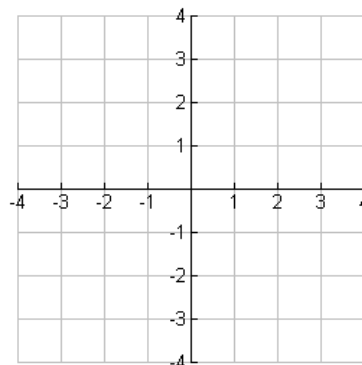
$$145) \ln x^2 = (\ln x)^2$$

$$146) \text{Expand using the properties of logarithms: } \ln \sqrt[3]{\frac{(3x+7)^4 (x+10)^3}{(5x-8)^2}}$$

147) Condense into a single logarithmic expression using the properties of logarithms:  $17 \ln x - \frac{2}{3} \ln(x^5 + 5)$

**XXI) Using the graphing calculator:**

148) Graph  $y = 0.1x^3 + 2x^2 - x - 3$  on the  $x$ - $y$  plane on the right:



149) Find the roots of the equation above.

150) Find the point of intersection for the graphs  $y = x^3 + x - 3$  and  $y = 2x + 4$ .

151) Find the maximum value for the graph  $f(x) = -x^4 + x - 4$ .

152) For the function in #151, find the intervals on which  $f(x)$  is increasing.

**XXII) What are the following trigonometric identities?**

153)  $\sec x =$

154)  $\csc x =$

155)  $\tan x =$

156)  $\cot x =$

157)  $\cos^2 x - 1 =$

158)  $\sec^2 x - 1 =$

159)  $\cot^2 x + 1 =$

**XXIII) Evaluate the following expressions:**

160)  $\sin\left(\frac{\pi}{6}\right)$

161)  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

162)  $\tan\left(\frac{7\pi}{6}\right)$

163)  $\cos(0)$

164)  $\cos\left(\frac{\pi}{4}\right)$

165)  $\csc\left(\frac{-5\pi}{6}\right)$

166)  $\sec(\pi)$

167)  $\cot\left(\frac{-\pi}{2}\right)$

168)  $\sin^{-1}\left(\frac{1}{2}\right)$

169)  $\tan\left(\frac{\pi}{2}\right)$

170)  $\sin^2\left(\frac{5\pi}{6}\right)$

171)  $\cot\left(\frac{2\pi}{3}\right)$

172)  $\sin\left(\frac{\pi}{2}\right)$

173)  $\cot^{-1}(-1)$

174)  $\sec\left(\frac{3\pi}{4}\right)$

175)  $\tan^{-1}(-1)$

176)  $\csc(\pi)$

177)  $\sec^2\left(\frac{\pi}{4}\right)$

**XXIV) Sketch one period of the following trigonometric graphs:**

178)  $y = \sin x$

179)  $y = \cos x$

180)  $y = \tan x$

181)  $y = \sec x$

182)  $y = \csc x$

183)  $y = \cot x$

**XXV) Solve the following trigonometric equations for the given domain:**

184)  $\sin x = \cos x$  on  $[0, 2\pi]$