## Welcome to AP Calculus!

Get ready for a CHALLENGING, rigorous, intense, yet SUPER FUN year! I am so excited to be working with you next year, and I hope you are also looking forward to one of the most demanding, yet also most rewarding, experiences in your high school career!

Before we get started, there are two main points to consider about whether or not you'd like to stay, or to enroll, in AP Calc:

1. It will be one of the hardest (if not the) hardest class you will ever take in high school. You will be challenged beyond your limits (©), constantly, because the class meets every day.
2. You will succeed if you are committed to it. I will be there to help you every step of the way, and if you are putting in the effort, no matter how challenging the material gets, You WILL Make it and Be Successful!
3. I will tolerate ZERO complaints about work and if you think you will have a hard time with the constant demands of daily homework, I do NOT recommend this course. You WILL be ok if "math isn't your thing" as long as you're willing to do the work. If you're not willing to work hard though, AP Calc is not for you.

In that vein, it is important to start AP Calc with a solid foundation in Precalculus and Algebra II. Throughout the year we will come back and use concepts from other classes that you have learned. In order to prepare yourself, you MUST COMPLETE THIS SUMMER PACKET!

| Details and Q\&A about the packet: |  |
| :--- | :--- |
| Due Date: | The first Thursday/Friday of school (depends on your schedule A/B). |
| Counts For: | $\mathbf{5 0}$ points in the P \& A category |
| Late Work: | 10 points off for every day it is late. |
| Can't Print? | It's fine! Just do the work on your own paper, you can turn that in. |
| Assessment: | There will be an assessment on this packet on the first Thursday/Friday <br> of school. Make sure you watch the videos and take ownership of the <br> work, so that you can be ready for the class or assessment. |
| Need Help? | Email Ms. S. at ioana.stoica@k12. dc.gov; I will answer your emails over <br> the summer, and will be very happy to hear from students before August, <br> so don't be shy! You can also text me at 240-643-0059. <br> There will also be VIDEOS on each section of the packet by June 15 th on <br> Ms. Stoica's new YOUTUBE Channel $\odot$ <br> https://www.youtube.com/channel/UC5gk05qabtryKWo4uFbtLRQ or <br> for short: $\underline{\text { https://goo.gl/8S44X6 }}$ |

## DO NOT WAIT Until mid-August to Complete this Packet!

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## Part 1 Notes \& Examples: Linear Functions

| Concept: Point- <br> Slope Format <br> for Linear <br> Functions | You might be familiar with the slope-intercept form of a line, $\mathbf{y}=\mathbf{m x}+\mathbf{b}$. In AP Calculus, you <br> will use the point-slope form $99 \%$ of the time: |
| :--- | :--- |
| Sxample 1: | Q: Write the equation for a line with a slope of 2 passing through the point $(-1,3)$. |
| Example 2: | Q: Write an equation for a line between the point $(2,-9)$ and $(-3,4)$. |


| Concept: <br>  <br> Vertical Lines | A horizontal line: |
| :--- | :--- |
|  | A vertical line: |
| Example 3: | Q: Write the equation horizontal and vertical lines passing through the point $(2,-7)$. |


| Concept: <br> Parallel and <br> Perpendicular/ <br> Normal Lines | Parallel lines: |
| :--- | :--- |
| Perpendicular or NORMAL lines: |  |
| Example 4: | Q: Write the equation for perpendicular and normal lines to $y=3 x+2$ passing through the point <br> $(5,8)$. |


| Concept: <br> Standard Form <br> for Lines | Another possible equation form for lines is the standard form: $\mathbf{a x}+\mathbf{b y}=\mathbf{c}$. This form isn't <br> terribly useful right away because it doesn't tell you the slope, or the y-intercept without further <br> algebra manipulation. |
| :--- | :--- |
| Example 5: | Q: A line has the equation $2 \mathrm{x}-7 \mathrm{y}=14$. What are the slope and y-intercept of this line? |

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## Part 1 Problem Set: Linear Functions

1. Find the equation of the following lines; first, in point-slope form; then, distribute and get the slopeintercept form (see Example 1).

| a.$\mathrm{m}=-7$, passing through point <br> $(-3,-7)$ | b. $\mathrm{m}=-1 / 2$, passing through <br> point $(2,-8)$ | c.$\mathrm{m}=2 / 3$, passing through point <br> $(-6,1 / 3)$ <br> point-slope form: |
| :--- | :--- | :--- |
| point-slope form: |  |  |
| slope-intercept form: | slope-intercept form: | slope-intercept form: |

2. Find the equation of the lines passing through the following points (see Example 2). You can use the form of your choice.

3. Determine the equation of a line passing through the point $(5,-3)$ with an undefined slope and with a zero slope (see Example 3).

Undefined Slope Line Eq:
Zero Slope Line Eq:
4. Determine the equation of a line passing through the point $(-2,0)$ with an undefined slope and with a zero slope (see Example 3).

Undefined Slope Line Eq:
Zero Slope Line Eq:

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5. Find the equation of a line passing through the point $(2,8)$ and parallel to the line $y=\frac{2}{3} x-1$; then write an equation for a line passing through this point that is normal (see Example 4).

Parallel Line:
Normal Line:
6. Find the equation of a line passing through the point $(-3,6)$ and parallel to the line $y=-\frac{1}{2}+6$; then write an equation for a line passing through this point that is normal (see Example 4).

Parallel Line:
Normal Line:
7. Find the equation of a line with an $x$-intercept $(2,0)$ and a $y$-intercept $(0,3)$ (see Example 2).
8. Write equations of the line through the given point that are parallel and normal to the given line; note that you may have to convert the standard equation into a point-slope equation to determine the slope first (see Examples 4 and 5).

| a. Point: $(5,-3)$ |  |  |
| :--- | :--- | :--- |
| Line: $x+y=4$ | b. Point: $(-6,2)$ <br> Line: $5 \mathrm{x}+2 \mathrm{y}=7$ | c.Point: (-3, -4$)$ <br> Line: $\mathrm{y}=-2$ (think carefully <br> about the slopes here!) <br> Parallel Line: <br> Perpendicular/Normal Line: |
| Parallel Line: | Parallel Line: |  |
|  |  | Perpendicular/Normal Line: |

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9. Find the equation of a line perpendicular to the $y$ - axis passing through the point $(4,7)$ (think about whether this will be a vertical or horizontal line, then see Example 3).
10. Find the equation of a line perpendicular to the $x$ - axis passing through the point $(4,7)$ (think about whether this will be a vertical or horizontal line, then see Example 3).
11. Write an equation of the line containing $(4,-2)$ and parallel to the line containing the points $(-1,4)$ and $(2,3)$.
12. Write an equation of the line containing $(-4,2)$ and normal to the line containing the points $(-4,1)$ and $(-3,2)$.
13. Sketch the following lines; you may use any method of your choice to graph these, and remember, a line is defined by TWO points, you don't need more than that on your sketch!
a. $y=-2 x-2$

b. $x-2 y=8$

c. $y-1=3(x+3)$

d. $\mathrm{x}=3$ and $\mathrm{y}=5$ (plot both and label)


## Part 2 Notes \& Examples: Factoring

| Concept: What is factoring? | Factoring is perhaps the single most important skill from algebra that you will use ALL year in AP Calculus. It refers to taking a quantity or expression and re-writing it as a multiplication problem. <br> You can factor numbers: $81=3 \cdot 3 \cdot 3 \cdot 3=3^{4}$ <br> You can also factor expressions: $x^{2}+x-2=(x-1)(x+2)$ <br> Note that when you are factoring expressions, if you attempt to go "backwards" and distribute the right hand side above, you will get back what you started with: $(x-1)(x+2)=x^{2}-x+2 x-2=x^{2}+x-2$ <br> * Note that the order of distribution does not matter. This is in fact the commutative property of algebra: when you move terms around that are added, the order doesn't matter, meaning, forget "FOIL". FOIL gives students the wrong impression that you have to do first, then outer, then inner, then last. Not at all!!! |
| :---: | :---: |
| Example 6 | Factoring Out Common Terms: <br> Q: Factor $5 x^{5}+15 x^{4}-20 x^{3}$ as much as possible. |
| Example 7 | "Regular" Factoring: <br> Q: Factor $x^{2}+2 x-8$ and also $x^{4}+2 x^{2}-8$ as much as possible. |
| Example 8: | Difference of Squares: <br> Q: Factor $25 x^{4}-9$ as much as possible. <br> * Note: there is NO rule for a SUM of perfect squares. You cannot factor sums of perfect squares. |
| Example 9: | Combo of Different Techniques: <br> Q: Factor $2 x^{5}-32 x$ as much as possible. <br> Hint: You always want to start by factoring out common terms: |


| Concept: using <br> factorizations <br> to solve <br> equations. | Zero is a very cool and special number! The multiplicative property of zero tells us that any <br> number multiplied by 0 is 0! Also, if you are multiplying numbers and you get 0, one of the <br> numbers must have been 0! |
| :--- | :--- |
| For example, if $a \cdot b \cdot c \cdot d \cdot e \cdot f=0$, that means at least one of the numbers a, b, c, d, e, or f |  |
| was zero! We use this property to solve factored equations. ANY POLYNOMIAL, non-linear |  |
| equation in AP Calculus will be solved through this method. |  |$|$| Factoring Out Common Terms: |
| :--- |
| Q: Solve the equation $2 x^{5}-32 x=0$. |

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## Part 2 Problem Set: Factoring

14. Factor the following expressions. See Examples 6-9.
1) $x^{2}+x-30$
2) $x^{2}-8 x+16$
3) $x^{2}+7 x+12$
4) $x^{2}-13 x+40$
5) $x^{2}-5 x-24$
6) $x^{2}+3 x-180$
7) $x^{2}+2 x y-3 y^{2}$
8) $x^{2}-5 x-36$
9) $x^{2}-5 x+4$
10) $x^{2}+14 x+45$
11) $\mathrm{x}^{2}+4 \mathrm{x}-21$
12) $x^{2}-10 x y-11 y^{2}$
13) $7 x^{2}+22 x+3$
14) $2 x^{2}-5 x y-12 y^{2}$
15) $4 \mathrm{x}^{2}+4 \mathrm{x}-35$
16) $2 \mathrm{x}^{2}-26 \mathrm{x}+72$
17) $3 x^{2}+2 x-8$
18) $18-27 \mathrm{x}-5 \mathrm{x}^{2}$
19) $28 x^{2}+60 x-25$
20) $48 x^{2}+22 x-15$
21) $18 \mathrm{x}^{2}+9 \mathrm{x}+1$
22) $6 x^{2}+7 x+2$
23) 

$15 \mathrm{x}^{2}-\mathrm{x}-2$
24) $15 \mathrm{x}^{2}-16 \mathrm{x}+4$
25)

$$
45 x^{4}-50 x^{3}+5 x^{5}
$$

26) $12 x^{2}+3 x^{3}+12 x$
15. Factor the following using "common term" and "difference of squares" factoring. See Example 8.
1) $9 x^{2}-1$
2) $4 n^{2}-49$
3) $36 k^{2}-1$
4) $p^{2}-36$
5) $2 x^{2}-18$
6) $196 n^{2}-144$
7) $180 m^{2}-5$
8) $294 r^{2}-150$

## 16. Solve the following equations by factoring. See example 10.

1) $(k+1)(k-5)=0$
2) $(a+1)(a+2)=0$
3) $(4 k+5)(k+1)=0$
4) $(2 m+3)(4 m+3)=0$
5) $x^{2}-11 x+19=-5$
6) $n^{2}+7 n+15=5$

## Part 3 Notes \& Examples: Factoring Applications

| Concept: <br> Rational <br> function <br> discontinuities. | We cannot divide by zero! <br> Therefore, if the denominator of a function is ever zero, the function is undefined, or in other <br> words, it has a "discontinuity". To find discontinuities, determine the x-values for which the <br> denominator is zero. |
| :--- | :--- |
| Example 11 | Q: Determine the discontinuities of the following function: $f(x)=\frac{x-1}{x^{2}-16}$. State the domain. |


| Concept: Points <br> of intersection. | To determine if two functions intersect, we need to set them equal to each other and then solve <br> for the missing variable. If finding x, don't forget to plug in for y at the end; if finding y, plug in <br> y to find x ! |
| :--- | :--- |
| Example 12 | Q: Find the points of intersection of $x^{2}+y=6$ and $x+y=4$. |

## Part 3 Problem Set: Factoring Applications

17. Find the discontinuities and the domain of the following functions. See Example 11.

| a. $f(x)=\frac{(x+3)\left(x^{2}+x-30\right)}{(x+5)\left(x^{2}+5 x+6\right)}$ | b. $f(x)=\frac{2+e^{x}}{2 x^{2}-18}$ | c. $f(x)=\frac{x^{2}+x-30}{x^{3}+7 x^{2}+10 x}$ |
| :--- | :--- | :--- |
| Discontinuities: | Discontinuities: | Discontinuities: |
| Domain: | Domain: | Domain: |

18. Find the point(s) of intersection of the following functions. See Example 12.

| a. $2 x+y=21$ and $2 x=y-x^{2}$ | b.$y+3=x^{3}-20 x$ <br> and $5 x-y=3$ | c.$x=3-y^{2}$ and $3 y=x-1$ <br> (solve for x first, then you will <br> have an equation in terms of y ) <br>  <br> Point(s) of intersection (x, y): <br> Point(s) of intersection (x, y): | Point(s) of intersection (x, y): |
| :--- | :--- | :--- | :--- |

## Part 4 Notes \& Examples: Rationalization

| Concept: <br> Rationalization. | You have encountered rationalizing in the context of simplifying fractions; you may have <br> learned that a fraction in the form $\frac{2}{\sqrt{3}}$ is not considered simplified if a root is present in the <br> denominator. To rationalize, you multiply by $\frac{\sqrt{3}}{\sqrt{3}}$, obtaining: $\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}=\frac{2 \sqrt{3}}{3}$. <br> You can also rationalize expressions, not just numbers. To do this, you will multiply by the <br> conjugate of a radical expression. A conjugate is just the original expression with the "sign" <br> switched (from pos to neg or neg to positive). For example, the conjugate of $\sqrt{x+3}-x$ is <br> $\sqrt{x+3}+x$. Notice only the sign on the outside of the root switches, not the one inside! |
| :--- | :--- |
| Example 13 | Q: Rationalize the following: $\frac{x-\sqrt{3 x+4}}{x-4}$ |

## Part 4 Problem Set: Rationalization

## 19. Rationalize the following. See Example 13.

a. $\frac{\sqrt{x}-2}{x-4}$ b. $\frac{\sqrt{2 x+22}-4}{x+3} \quad$ c. $\frac{5-\sqrt{x}}{x-25}$

## Part 5 Notes \& Examples: Local Limits

| Example 14 | The limit of a function exists even when the function itself does not, as long as the right and left <br> Graphical <br> Limits | at that point are equal (the function "meets up"). |
| :--- | :--- | :--- |
| $x \rightarrow 0$ |  |  |
| b) $\lim _{x \rightarrow 1} f(x)=$ |  |  |

## 20. Graphical Limits. See Example 14.



Using the above graph, find each of the following (You should assume that $\mathrm{y}=0$ is a horizontal asymptote and $x=-4$ is a vertical asymptote):

1) $f(-2)=$ $\qquad$ 2) $\lim _{x \rightarrow-2^{+}} f(x)=$ $\qquad$ 3) $\lim _{x \rightarrow-2} f(x)=$
2) $\lim _{x \rightarrow-1^{+}} f(x)=$ $\qquad$
3) $\lim _{x \rightarrow-1^{-}} f(x)=$ $\qquad$
4) $\lim _{x \rightarrow-1} f(x)=$ $\qquad$
5) $\lim _{x \rightarrow 1^{+}} f(x)=$ $\qquad$
6) $\lim _{x \rightarrow 1^{-}} f(x)=$ $\qquad$
7) $\lim _{x \rightarrow 1} f(x)=$ $\qquad$
8) $f(3)=$ $\qquad$
9) $\lim _{x \rightarrow 3^{+}} f(x)=$ $\qquad$
10) $\lim _{x \rightarrow 3^{-}} f(x)=$ $\qquad$
11) $\lim _{x \rightarrow 3} f(x)=$ $\qquad$
12) $\lim _{x \rightarrow-4^{+}} f(x)=$ $\qquad$
13) $\lim _{x \rightarrow \infty} f(x)=$ $\qquad$
14) $f(1)=$ $\qquad$
15) $\lim _{x \rightarrow-3} f(x)=$ $\qquad$
16) $f(-4)=$ $\qquad$

## 21. Algebraic Limits. See Example 15.

| 1. $\lim _{\mathrm{x} \rightarrow 6} \frac{\mathrm{x}-6}{\mathrm{x}^{2}-36}$ | 2. $\lim _{x \rightarrow 2} \frac{x^{2}-4 x+4}{x^{2}+x-6}$ | 3. $\lim _{x \rightarrow-3} \frac{x+3}{x^{2}+2 x-3}$ |
| :---: | :---: | :---: |
| 4. <br> $\lim _{x \rightarrow 3} \frac{x^{2}-7 x+12}{x-3}$ | 5. $\lim _{x \rightarrow-5} \frac{x^{2}+3 x-10}{x+5}$ | 6. $\lim _{x \rightarrow 2} \frac{x^{4}-16}{x-2}$ |
| 7. $\lim _{x \rightarrow 7} \frac{x^{2}+x-56}{x^{2}-13 x+42}$ | 8. <br> $\lim _{x \rightarrow 0} \frac{10 x}{20 x^{2}+15 x}$ | 9. <br> $\lim _{x \rightarrow 1 / 3} \frac{6 x^{2}+x-1}{1-3 x}$ |
| 10. $\lim _{x \rightarrow-2} \frac{x^{3}+3 x^{2}+2 x}{x+2}$ | 11. $\lim _{x \rightarrow 3} \frac{2 x^{2}-5 x-3}{x-3}$ | 12. $\lim _{x \rightarrow 5} \frac{2 x^{2}-5 x-25}{x-5}=$ |

Part 6 Notes \& Examples: Trig Functions \& Trig Arithmetic

| Concept: <br> Radians and <br> Degrees. | Radians and degrees are different units for measuring angles, similar to kilograms and pounds: <br> they measure the same quantity. One full rotation (a circle) is defined as having $\mathbf{3 6 0} \mathbf{0}^{\circ}$ or $\mathbf{2 \pi}$ <br> radians. <br> If $360^{\circ}=2 \pi$ radians, then $\mathbf{1 8 0}$ <br> you will use radians exclusively. Say goodbye to degrees for good! |
| :--- | :--- |
| Example 16 | Converting from Degrees to Radians - multiply your degree value by $\frac{\boldsymbol{\pi}}{\mathbf{1 8 0}}$ <br> Q: Convert $225^{\circ}$ to radians. |
| Example $\mathbf{1 7}$ | Converting from Radians to Degrees - multiply your radian value by $\frac{\mathbf{1 8 0}}{\boldsymbol{\pi}}$ <br> Q: Convert $\frac{4 \pi}{3}$ radians to degrees. |


| Concept: Trig <br> functions using special <br> triangles and unit <br> circle. | Remember SOH - CAH - TOA. The other trig functions you will need to know are: <br> Cosecant (csc) - the reciprocal of sin; instead of O/H, it's H/O <br> Secant (sec) - the reciprocal of cos; instead of A/H, it's H/A <br> Cotangent (cot) - the reciprocal of tan; instead of O/A, it's A/O. <br> You will need to memorize the special triangles 45-45-90 and 30-60-90 and you will <br> need to be able to do all trig work BY HAND, no calculators allowed. In AP Calc, at <br> least 50\% of our work will be done without a calculator. <br> $30-60-90$ Triangle: |
| :--- | :--- |
| Example $\mathbf{1 8}$ | Us-45-90 Triangle: |
| 1 | Q: Find all trig function values for the angle $\frac{3 \pi}{4}$. |
| 1 |  |

$\qquad$


| Concept: Using <br> Unit Circle. | You can use the coordinates of the unit circle to find trig functions <br> for angles that cannot be used to draw triangles, such as 90,180, <br> 270, and 360 degrees. The $\mathbf{x}$-coordinate is the $\cos ($ angle $)$, and the <br> y-coordinate is the sin(angle). |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Example 20 | Q: Use the unit circle to find all trig values for $\pi$ radians. |
| You will also need the following trig identities: |  |

## Part 6 Problem Set: Trig Functions \& Trig Arithmetic

## 21. Convert the following degree measures to radians. See Example 16.

| a. $210^{\circ}$ | b. $330^{\circ}$ | c. $270^{\circ}$ | d. $120^{\circ}$ |
| :--- | :--- | :--- | :--- |
|  |  |  |  |

22. Convert the following radian measures to degrees. See Example 17.

| a. $\frac{\pi}{12}$ | b. $\frac{3 \pi}{4}$ | c. $\frac{7 \pi}{6}$ | d. $\frac{5 \pi}{4}$ |
| :--- | :--- | :--- | :--- |

If you do not understand something, it is YOUR RESPONSIBILITY to email Ms. Stoica or google it/look it up on Khan Academy! Also see Ms. Stoica's Youtube Channel for Visuals/Videos of the Examples \& More Tips, coming June 15 ${ }^{\text {th }}$ !
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23. Use the 45-45-90 triangle to find all trig function values for the given angles. Show your drawing and identify the six functions like in Example 18.

| a. $\frac{5 \pi}{4}$ | b. $\frac{7 \pi}{4}$ |
| :--- | :--- |
| Sketch with Special Triangle: | Sketch with Special Triangle: |
|  |  |
| Trig Functions: | Trig Functions: |
|  |  |

24. Use the 30-60-90 triangle to find all trig function values for the given angles. Show your drawing and identify the six functions like in Example 19.

| a. $\frac{7 \pi}{6}$ | b. $\frac{2 \pi}{3}$ |
| :--- | :--- |
| Sketch with Special Triangle: | Sketch with Special Triangle: |
|  |  |
| Trig Functions: | Trig Functions: |
|  |  |
| c. $\frac{5 \pi}{3}$ |  |
| Sketch with Special Triangle: |  |

25. Use the Unit Circle to find all trig function values for the given angles. Follow Example 20.

| a. $\frac{\pi}{2}$ radians | b. $\frac{3 \pi}{2}$ radians | c. 0 radians |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |

## Part 7 Notes \& Examples: Log and Exponent Properties \& Equations

| Concept: <br> Exponential Properties. | You should be familiar with the basic properties of exponential functions: $\left(x^{2}\right)^{3}=\quad x^{2} x^{3}=\quad \frac{x^{2}}{x^{3}}=\quad \frac{x^{3}}{x^{2}}=$ <br> More important, in AP Calculus you will often be asked to evaluate expressions like $\frac{2}{\sqrt[3]{8}}$ by hand, without a calculator. So let's take a look at more advanced properties of exponentials: <br> - Negative exponents switch numerator/denominator: $x^{-2}=\quad$ and $\frac{1}{x^{-2}}=$ <br> - Fractional exponents are really roots: $x^{1 / 2}=\quad, x^{1 / 3}=\quad, x^{1 / 4}=$ <br> - You can combine all rules of exponents into one problem: $x^{-2 / 3}=$ |
| :---: | :---: |
| Example 21 | Q: Evaluate $2^{-3}$ without a calculator. Leave your answer as a fraction. |
| Example 22 | Q: Evaluate 81/4 without a calculator. |
| Example 23 | Q: Evaluate $32^{-3 / 5}$ without a calculator. Leave your answer as a fraction. |


| Concept: | You will need to know the following $\log$ properties in order to condense or expand logarithms. |
| :--- | :--- |
| Logarithmic | - Additive property: $\log (\mathrm{a})+\log (\mathrm{b})=$ |
| Properties. | - Subtraction property: $\log (\mathrm{a})-\log (\mathrm{b})=$ |
|  | - Exponent Property: $\operatorname{alog}(\mathrm{b})=$ |


| Concept: <br> Exponential <br> and <br> Logarithmic <br> Equations | Exponential and Logarithmic functions are inverses of each other. This means that, if you <br> compute the logarithm of an exponential, you will get the original expression back (as long as <br> they have the same base). Same if you compute the exponential of a logarithm. <br> For example, $\log _{3} 3^{x}=x$ and $3^{\log _{3} x}=x$ |
| :--- | :--- |
| Example 26 | Solving Exponential Equations Using Logs - you will need a calculator; the one on your <br> phone or a simple one online should be able to compute logs and exponents. You can <br> technically use a log with any base. For practical purposes, we will use base 10. <br> Q: Solve the following equation: $2^{x+1}=3^{x}$. |
| Example 27 | Solving Logarithmic Equations Using Exponentials - you will need a calculator; the one on <br> your phone or a simple one online should be able to compute logs and exponents. <br> Q: Solve the following equation: $\log _{3}(x+2)=4$ |

## Part 7 Problem Set: Log and Exponent Properties \& Equations

26. Evaluate the following expressions without a calculator. SHOW all work! Leave your answer as fractions or whole numbers (no decimals). See Examples 21-23.

| a. $4^{-3}$ | b. $\frac{2}{5^{-2}}$ | c. $2^{-4}$ | d. $\frac{-1}{3^{-4}}$ |
| :--- | :--- | :--- | :--- |
| e. $16^{\frac{1}{2}}$ | f. $8^{\frac{1}{3}}$ | g. $16^{-\frac{1}{4}}$ | h. $64^{-\frac{1}{6}}$ |


| e. $27^{-\frac{4}{3}}$ | f. $125^{-\frac{2}{3}}$ | f. $128^{-\frac{4}{7}}$ | h. $32^{-\frac{3}{5}}$ |
| :--- | :--- | :--- | :--- |
|  |  |  |  |

27. Expand the following logarithms. See Example 24.

| a. $\log (6 \cdot 11)$ | b. $\log \left(\frac{6}{11}\right)$ | c. $\log \left(\frac{6}{11}\right)^{3}$ | d. $\log \left(\frac{x^{2}}{y^{3}}\right)$ |
| :--- | :--- | :--- | :--- |
| e. $\log \left(x^{4} \cdot 11\right)$ | f. $\log \left(\frac{5 x}{3 y^{2}}\right)$ | g. $\log \left(\frac{y^{4}}{x}\right)$ | h. $\log \left(\frac{x^{2} y}{z^{7}}\right)$ |

28. Condense the following logarithms. See Example 25.

| a. $\log (6)-\log (4)+\log (2)$ | b. $\log (x)-2 \log (y)+$ <br> $\log (3)$ | c. $10 \log (x)-\mathrm{x} \log (2)$ | d. $\log (a)+\log (b)-\log (c)$ |
| :--- | :--- | :--- | :--- |
| e. $3 \log (2)-4 \log (2)$ | f. $2 \log (5)-3 \log (2)+$ <br> $4 \log (3)$ | g. $3 \log (5)+5 \log (3)$ | h. $\log (5)-\log (2)+$ <br> $\log (20)-\log (25)$ |

29. Solve the following exponential equations using logarithms. See Example 26.

| a. $5 \cdot 18^{6 x}=26$ | b. $2^{x-1}-5=5$ |
| :--- | :--- |
|  |  |


| c. $9^{x+10}+3=81$ | d. $11^{x-8}-5=54$ |
| :--- | :--- |
|  |  |
| e. $3 \cdot 6^{3 x+1}=21$ | f. $20^{-6 x}+6=55$ |

## 30. Solve the following logarithmic equations using exponentials. See Example 27.

| a. $\log _{2}(x+1)=4$ | b. $3 \log _{4}(2 x-1)-1=5$ | c. $\log _{2}(2 x)+\log _{2}(3)=3^{*}$ <br> *You will have to condense the logs first! |
| :--- | :--- | :--- |
|  |  |  |
| d. $2 \log _{4}(x)-\log _{4}(3)=2^{*}$ <br> *You will have to condense the logs first! | e. $\log _{8}(2)+\log _{8}\left(4 x^{2}\right)=1^{*}$ <br> *You will have to condense the $\operatorname{logss}^{*}$ first! | f. $\log _{10}(x+24)+\log _{10}(x-24)=2^{*}$ <br> *You will have to condense he logs first! At <br> the very end, you will have to FACTOR a <br> difference of squares. |

