What Can Music Theory Pedagogy Learn From Mathematics Pedagogy?

By Leigh VanHandel

Music theory fundamentals are often compared to the basics of mathematics; the skills involved in spelling intervals, scales, and chords are required to be as second-nature to musicians as skills such as addition, subtraction, multiplication and division are to anyone engaging in mathematics. The relationship between mathematics and music theory is an intuitive one as well: music theory consists of many instances of systematic thought that are closely related to mathematical principles; composers have long used the notion of algorithms in their compositions; and recent topics in transformational theory have borrowed from advanced mathematics.

However, the pedagogy of mathematics education is an immense field that has enjoyed much more research attention and funding than the pedagogy of music theory. Current research trends in mathematics pedagogy include incorporating elements of cognitive science and neuroscience in an effort to connect mind, brain, and education. If mathematics and music theory are related, as frequent comparisons and intuition may tell us, teachers of music theory may be able to apply findings in mathematics pedagogy to our own discipline. My intent in this discussion is to illustrate that there is likely a relationship between the cognitive processes involved in learning music theory fundamentals — pitches, intervals, scales, keys, and chords — and the cognitive processes involved in learning basic mathematical processes, and that understanding how students learn mathematics may help us teach music theory more efficiently.

What, other than intuition, can tell us that there is a relationship between mathematics and music theory fundamentals? The evidence that comes out of the literature of music education – specifically, that of identifying factors that may contribute to student musical achievement or success at the university level. Several studies have found significant relationships between measures of academic achievement, such as standardized tests or IQ tests, and achievement in music.[1] Two studies by Harrison focus on how

[1] For example, Roby (1962). Summaries of multiple studies on general intelligence and academic achievement are listed in Gordon (1968) and

multiple factors affect performance in freshman year music theory coursework, in order to determine which factors were the best predictors of performance based on grade.[2] The factors investigated included general achievement, as measured by scores on the verbal and math portions of the Scholastic Aptitude Test (SAT); overall academic achievement as measured by high school GPA; musical aptitude, as measured by performance on a standardized music aptitude test; musical experience prior to enrollment in college, including ensembles and private lessons; principal instrument; and gender. Her first study found that for a freshman year music theory course including four components (written work, aural skills, sight-singing, and keyboard skills), the best predictor of success in the course was performance on the math portion of the SAT exam. Her second article separated the four components of written, aural, sight-singing and keyboard skills and explored the relationship between the predictive factors and the four components; she also investigated whether the same factors were similarly predictive for both first- and second-semester classes. Her analysis determined that for the first semester, student scores on the SAT math test were significant predictors of performance for each of the four components of freshman music theory, and were the best overall predictor of performance in the written skills component. For the second semester the SAT math score was still the best predictor of performance in the written skills component, but was not a significant predictor of performance in the other three components of the curriculum (aural skills, sight-singing, and keyboard skills). Her conclusions include the observation that "[t]he significant relationship between scores on the math component of the SAT and grades in the written-work component might be explained by the structured thought processes required for both."[3] She notes in

Harrison (1996); full citations are in the bibliography.

[2] C. Harrison, "Predicting Music Theory Grades: The Relative Efficiency of Academic Ability, Music Experience, and Musical Aptitude," *Journal of Research in Music Education* 38, no. 2 (1990), 124–137; and C. Harrison, "Relationships between Grades in the Components of Freshman Music Theory and Selected Background Variables," *Journal of Research in Music Education* 38, no. 3 (1990), 175–186.

[3] Harrison, "Relationships between Grades in the Components of Freshman Music Theory and Selected Background Variables," 184.

a follow-up analysis that "it is reasonable to suspect that different forms of music achievement may require different skills and knowledge for success."[4]

Other studies support Harrison's findings that achievement in mathematics predicts success in the written components of a music fundamentals course, but does not predict performance on the aural skills components. Bahna-James' study of high school students in a performing arts school setting found significant correlations between student performance in mathematics classes (basic arithmetic, beginning algebra, geometry, trigonometry, and calculus) and written music theory fundamentals skills (focusing specifically on intervals, key signatures, and chords).[5] The correlation between performance in these mathematics classes and the aural skills activities of sight-singing and rhythmic dictation were mixed.[6] In a study by Schleuter focusing primarily on the predictive value of standardized music achievement tests, student SAT scores were found to have no significant relationship to grades in aural skills and sight-singing.[7] More recently, Jones and Bergee found a strong association between two non-musical factors, high school class rank percentile and score on the math portion of the American College Test (the ACT, a standardized test similar to the SAT), on student performance in the freshman written music

[4] C. Harrison, "Relationships between Grades in Music Theory for Nonmusic Majors and Selected Background Variables," *Journal of Research in Music Education* 44, no. 4 (1996): 350.

[5] T. Bahna-James, "The Relationship Between Mathematics and Music: Secondary School Student Perspectives," *The Journal of Negro Education* 60, no. 3 (1991), 477-485.

[6] The primary goal of Bahna-James' study was to illustrate that musically inclined students, such as the type attending an urban performing arts high school, tend to have a negative perception of their abilities in mathematics; her study focused more on the qualitative elements of how students felt about mathematics and whether the students believed there was a relationship between mathematics and music theory.

[7] S. Schleuter, "A Predictive Study of an Experimental College Version of the Musical Aptitude Profile with Certain Music Achievement of College Music Majors." *Psychology of Music* 11 no. 1 (1983), 32–36. Schleuter's study only included applied music, sight-singing, and ear training classes and did not include written music theory.

theory course.[8] These two factors outweighed any musical factor, including performing medium (instrument/voice) and any prior study of music theory fundamentals. As in Schleuter's study, Jones *et al.* found no significant direct relationship between the ACT-Math score and performance in aural skills classes.

If performance on standardized tests such as the math SAT or ACT is one of the best predictors of performance in written freshman-level music theory courses, it stands to reason that there is some kind of relationship between the cognitive processes used in solving the types of problems that appear on the math SAT or ACT and in the music theory curriculum. What are the elements of this relationship? What can music theorists learn from the vast amounts of mathematics education research, and how can we transfer that to the domain of music theory?

Mathematics and music share many similarities. In order to communicate about both mathematics and music, a form of representation is necessary – a system of notation. To be fluent in either mathematics or music requires fluency with the system of representation, and as fluency advances, the systems of representation and the concepts represented become increasingly more complicated and abstract. The basic mathematical functions (addition, subtraction, multiplication and division) are organized as number systems, and the regularities present in the system help children acquire the knowledge necessary within that system.[9] Similarly, the fundamentals of music theory can be organized systematically, and if students understand the system it can help them understand each piece of the system and to grasp the whole.

Performing mathematical computations requires the use of algorithms, or step-by-step instructions for completing the task; the same may be said for the methods for deriving intervals, scales, or chords. Algorithms depend on representation, and the selection of an algorithm in both mathematics and music requires a student to make decisions regarding the simplicity, efficiency,

[8] M. Rusty Jones and M. Bergee, "Elements Associated with Success in the First-Year Music Theory and Aural-Skills Curriculum," *Journal of Music Theory Pedagogy* 22 (2008), 93–118.

[9] J. Kilpatrick, *et al.*, (Eds.), *Adding It Up: Helping Children Learn Mathematics.* National Research Council: Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. (Washington DC: National Academy Press, 2001), 2.

and precision of the algorithm.[10] Basic mathematical processes and music theory fundamentals also both require the development of expert understanding. Experts typically understand meaningful patterns of information rather than isolated, unconnected bits of knowledge.[11] In order to develop expert knowledge, students must be able to create an organized hierarchical structure for the domain; this will help them make connections between material and will also aid in effective retrieval of information. The need to develop an expert understanding of the material is one of the primary challenges in the freshman written music theory curriculum. Students need to be able to calculate intervals, scales, keys, and chords quickly and accurately in order to progress through the class, as well as to succeed in classes that engage with more abstract theoretical concepts. It may be possible to extrapolate from the extensive research within the literature on mathematics education on developing an expert understanding of mathematical fundamentals and apply some of the findings to the written music theory classroom.

When encouraging students to develop the necessary fluency with theory fundamentals, theory instructors frequently use analogies to language and mathematics. Requests to recognize the representation system are often related to language, with questions such as, "If you had to spend time identifying each letter in the word 'the', how quickly would you be able to read?" Similarly, students are frequently directed to memorize elements such as intervals or key signatures via an analogy to mathematics: "At some point in your life, you had to count on your fingers to solve 4 + 3; now you just 'know' it."

How *do* students learn to solve 4 + 3, and how do they reach the point where they just "know" it?[12] And, more importantly, how can

[10] Kilpatrick et al., *Adding It Up,* 103.

[11] J. Bransford, et al., (Eds.) *How People Learn: Brain, Mind, Experience and School.* National Research Council: Committee on Developments in the Science and Learning, Committee on Learning Research and Educational Practice, and Commission on Behavioral and Social Sciences and Education. (Washington, DC: National Academy Press, 2000), Chapter 2.

[12] Much of the research on acquisition of the fundamentals of mathematics involves children, as that is when they are presented with the material (addition, subtraction, multiplication, division). It is common for research in mathematics education to extrapolate the

knowing how students learn to solve 4 + 3 help teach us about how students learn the fundamentals of music theory, and about how we can teach the material more effectively?

Many music theory instructors rely on the drill and test method of instruction, believing that exposing students to the material repeatedly, and holding the student responsible for its acquisition through testing, will result in knowledge. However, as far back as 1935 mathematicians were realizing that drill, absent other types of reinforcement, leads to little if any growth in quantitative thinking.[13] The timed test on concepts not yet mastered can also have detrimental effects on the student's disposition to the material.[14] Students will not develop more mature and efficient ways of working with the material based on drill alone; they must be instructed how to develop more efficient methods.

Mathematics educators have learned that acquiring proficiency with basic mathematics skills requires much more than rote memorization. The current general consensus is that students learn or develop increasingly advanced and abstract systems and methods for generating answers, and are able to choose adaptively among strategies depending on the mathematical context and their ability level. After working through these systems, many are eventually able to use immediate recall as their predominant method for simple arithmetic skills. This theory has its roots in Piaget's notion of discrete stages or steps of thought. In that model, each step produces a new type of understanding by building on the previous set of knowledge. According to this model, a student uses one strategy to solve problems, switches to a more advanced strategy at some point and abandons the first strategy,

findings of studies on children to those of adult learners, especially in studies involving remedial or fundamental math courses at the collegiate level. In addition, much literature on best practices in mathematics teaching is also generated by studies at the K–12 level and extended to collegiate mathematics teaching practice. (Speer et al., 2010 point this out, while also acknowledging that there are differences at the collegiate level in teacher, student, and in teaching practice; they call for there to be more research on best practices in collegiate mathematics education.)

[13] W. Brownell and C. Chazal, "The Effects of Premature Drill in Third-Grade Arithmetic," *Journal of Educational Research* 29, no. 1 (1935), 17–28.

[14] Kilpatrick et al., *Adding It Up,* 193.

and later may move to an even more advanced strategy.[15] Recent educational research, however, reveals that the relationship is more complex, and that students frequently use more than one strategy when presented with tasks and are able to readily switch between strategies, and that this is true not only of children but also through teenage years and into adulthood.[16]

An example of students using a variety of strategies to solve the same problem can be seen in the five common strategies children use for single-digit addition:

1. the sum strategy, in which a student asked to solve 4 + 3 will put up four fingers, then put up three more, and then count each finger from 1 to 7 to arrive at the answer;
2. the min strategy, in which a student would count up from the larger number the amount indicated by the smaller number ("4, … 5, 6, 7");
3. the decomposition strategy, in which the student would translate the problem into an easier form by relying on known information to generate the unknown ("4 + 2 is 6, so 4 + 3 is 7");
4. the retrieval strategy, in which the student is simply able to retrieve the answer from memory;
5. the guessing strategy, in which the student simply guesses at an answer.

Aside from *guessing,* these strategies are listed in increasing order of complexity and abstraction.

According to Siegler and Shipley, students use different strategies for different mathematics problems; in fact, students sometimes use different strategies for the *same* problem when presented on different days, and they do not always move from a

[15] R. Siegler, "Implications of Cognitive Science Research for Mathematics Education," in *A Research Companion to Principles and Standards for School Mathematics,* edited by J. Kilpatrick et al., (Reston, VA: National Council of Teachers of Mathematics, 2003), 219–233.

[16] R. Siegler and C. Shipley, "Variation, Selection, and Cognitive Change," in *Developing Cognitive Competence: New Approaches to Process Modeling*, edited by T. Simon and G. Halford, (Hillsdale, NJ: Lawrence Erlbaum Associates, 1995): 31–76. Also Siegler, "Implications of Cognitive Science Research," and references within both articles.

less advanced strategy to a more advanced strategy.[17] Siegler and Shipley refer to this phenomenon as *adaptive strategy*; students will choose the strategy they feel is necessary in order to complete the task. If a student is able to determine the answer via retrieval, they frequently will, as that is usually the fastest and most accurate method; however, if they are uncertain about the answer, or if it is not available for retrieval, they will adaptively choose among the other 'backup' strategies for verification or for deriving the solution.

Research into student strategies on mathematical tasks reveal that there are four dimensions of strategic competence: (a) which strategies are used, (b) when each strategy is used, (c) how each strategy is executed, and (d) how strategies are chosen.[18] A change in any one of these dimensions can result in overall improvements in student speed and accuracy at a task. If students gradually develop the ability to retrieve answers to single-digit addition problems, and develop confidence in that ability, they will rely less and less on the backup strategies, but may still resort to using the backup strategies on difficult problems or on problems where they are uncertain about the retrieved answer.

However, research also shows that students are reluctant to adopt new strategies unless they see an immediate benefit to doing so or are presented with a situation that virtually requires it. For example, if a student who predominantly relies on the *sum* strategy has been introduced to the *min* strategy, they will likely continue to use the *sum* strategy unless they are given a problem such as "22 + 3", where the preferred strategy has obvious deficits. Unless presented with experiences that illustrate the advantages of a new strategy, students tend to hesitate to adopt the new strategy and favor older, more familiar strategies.

Siegler and Shipley cite multiple studies illustrating that strategy diversity and adaptive strategy is used in the acquisition of other skills and in other cultures, and is used by adults as well. These concepts can be applied to music fundamentals instruction. As an analogue to simple addition tasks, we can consider the typical processes and strategies students encounter when they learn to write and/or identify intervals.

[17] Ibid., 33–34.

[18] P. Lemaire and R. Siegler, "Four Aspects of Strategic Change: Contributions to Children's Learning of Multiplication," *Journal of Experimental Psychology-General* 124, no. 1 (1995), 83–96; Siegler and Shipley, *op. cit*.

The generally accepted first step in teaching intervals is to teach students to calculate or identify the generic size of the interval without regard for quality – a fourth, a fifth and so on. The first strategy beginning theory students typically employ is to count note names as if they were counting numbers, albeit in a mod–7 system using letter names instead of numbers; e.g., a fifth above A would be calculated by 'counting' "A, B, C, D, E," and a third above B would be counted as "B, C, D." This strategy is roughly analogous to the *sum* or *min* method in mathematics; the student is identifying a starting point and systematically counting to arrive at the solution.

The second strategy often employed is a visual one – trying to get students to recognize what a third or a sixth looks like when represented on the staff. This can relate to the *decomposition* or *retrieval* strategy; when presented with a fifth, the student might be able to instantly recognize it from its visual appearance; if the student is presented with a sixth, they might remember what a fifth looks like and realize that the interval is one line or space larger than that. As with strategies in mathematics, some students may make the jump to this strategy automatically; other students may require that it be presented to them explicitly.

Next, when the concept of interval quality is introduced, there are multiple mental strategies that students can use to spell or identify requested intervals, listed in order from least efficient/desirable to most:[19]

1. Students can memorize the number of half steps or whole steps in a given interval and spell the interval by counting;
2. Students can use a scale-based method for determining intervals; they can imagine that the bottom note of the interval is the tonic degree of a scale, and can use their knowledge about scales and keys to determine the answer;
3. Students can memorize certain pieces of information and use those in a strategy similar to the *decomposition* strategy, where they relate their known information to an unknown. For example, if a student has memorized that C4 to E4 is a major third, they will be able to determine that C4 to E♭4 is a minor third because it is a half step smaller;

[19] Note that I am specifically considering *mental* strategies here; there are certainly other strategies that students use, including kinesthetic strategies.

4. Immediate recall – as above, related to decomposition, since decomposition requires that students have an immediately available referent to work with.

As with the strategies for single-digit addition, these strategies are all available for students to use as they are developing their fluency with the material. However, each of these strategies has strengths and weaknesses that both the student and the instructor need to consider.

As with counting letter names, the half step/whole step strategy is analogous to the *sum* or *min* system for addition, as it requires a step-by-step counting process from a starting point to determine the ending point. The strength of this system is that it is algorithmic – when executed correctly, the system will result in the correct answer. However, this system is undesirable due to a number of serious weaknesses. One such weakness is the number of points of potential failure in the system – students may misremember how many half steps are in a Perfect 5th, for example, or may end up with an enharmonic equivalent (C♯–A♭ rather than C♯–G♯), or may simply miscount. A second weakness is that this system tends to be extremely slow, especially for larger intervals. Because of these major problems with this method, many instructors do not use this strategy at all; however, some instructors do use this system, and some textbooks teach it explicitly. Even if an instructor does not teach or encourage this method, a student may have encountered it elsewhere or may even develop it on their own.

The scale-based method has elements of both the *sum* or *min* strategy and of *decomposition.* The advantage of the scale-based method for spelling intervals is that it builds upon existing knowledge; students can rely upon their knowledge of scales and keys to determine the answer. However, there are several disadvantages. First, student knowledge of scales and keys is typically still developing, so relying on that knowledge may be uncomfortable for some students, or may lead to incorrect answers based on mistakes in student's understanding of scales and keys. Second, even if students have mastered key signatures, there are notes that do not appear as tonics on the major or minor circle of fifths, such as D𝄪; this means the student who relies solely on this system will be unable to calculate the interval, or will have to calculate it by spelling the scale using whole steps and half steps. Third, students may make mistakes by thinking about minor scales

and keys instead of major scales and keys, or vice versa.[20] Fourth, the scale-based strategy works better for finding intervals above a given note than finding intervals below a given note.

The third strategy is analogous to the strategy of *decomposition* in that students are able to retrieve certain familiar pieces of information and use those as a reference to generate answers to unfamiliar questions. A student who has memorized that C–E is a Major 3rd can easily derive the answer when asked for a Major 3rd above C♯, C𝄪, C♭, or C♭♭. Students can determine their own references; some may choose to memorize the 'white-key' intervals, and others may choose to memorize common intervals. This strategy contains elements of the *retrieval* method, discussed next, as it requires there to be some piece of information readily available to be retrieved. The strengths of this method are that it is relatively quick, as it relies on an element of retrieval, and that there are fewer steps in the process where a student can make an error.

The fourth strategy, *retrieval,* is the optimal strategy; it is the quickest method and, once developed, the least prone to errors. In this strategy students are able to recall quickly, efficiently, and accurately that, for example, E to C♯ is a Major 6th, and that F to A♭ is a minor 3rd. This strategy does not require that students memorize all possible intervals; rather, students tend to memorize frequently encountered intervals and are able to immediately recall those. When presented with an interval they don't have available for immediate recall, students will utilize a different strategy.

Anecdotal evidence points to a combination of *retrieval* and *decomposition* as the preferred method for expert musicians to spell intervals. In a study conducted by Allen Winold, expert musicians were asked to describe their thought process when asked to calculate intervals; the results indicated that they either "just knew" the answer or, for more difficult intervals, would switch to a strategy similar to the scale method and/or decomposition to determine the answer. Unfortunately, Winold's study was never

[20] Some instructors have their students think about major scales for major intervals and natural minor scales for minor intervals. (Students frequently generate this method on their own, as well.) A potential problem with this strategy is that the interval from $\hat{1}$ to $\hat{2}$ is a Major 2nd in both the major and minor scales, and this causes a common error for students who are instructed to think of intervals in this way. Other instructors use only the major scales, and have students generate minor, Augmented and diminished intervals from the major scale intervals.

published; it would be an excellent experiment to replicate, as it would be an important insight into whether these two strategies are the optimal and most common strategies for expert musicians, and would verify the notion of adaptive strategies at use in the domain of music theory fundamentals.[21]

Our goal as music theory instructors is to move students from the inefficient strategies to the efficient strategies to facilitate more accurate and immediate knowledge. However, since research illustrates that students are generally unwilling to adopt new strategies unless they are clearly shown that their current strategy is unsuitable, how can we help students who tend to rely on the labor- and cognition-intensive strategies of counting whole steps and half steps or scale-based systems to move towards using the quicker, more abstract method of decomposition or the ultimate goal of retrieval? Siegler and Shipley propose a model for illustrating how students develop their own adaptive strategies, and how they select which strategy to use.[22] In the model, each time a student uses a strategy, they accumulate knowledge about its speed and accuracy in terms of global usage, usage in problems with particular features, usage in individual problems, and its novelty, which is considered a strength based on Piaget's observation that students "are often interested in exercising newly acquired cognitive capabilities."[23] The novelty wears off with each subsequent use of a strategy, but the student gains critical information about the strategy's usefulness, speed, and accuracy, so the student is able to better gauge when to use that strategy in the future. Siegler and Shipley also factor in overall success rate of a strategy and recent success rate, surmising that both continued and recent successes with a particular strategy will encourage a student to incorporate that strategy into their adaptive strategy.

However, as mentioned earlier, students often will not adopt a new strategy unless they see a benefit to using it. Therefore, if a student is presented with a new strategy for spelling intervals

[21] Personal communication, Allen Winold, November 5, 2010. In public presentations of this material, I have asked audiences of expert musicians to calculate a difficult interval (one not likely to be available for immediate retrieval) and have asked them to describe their mental process in calculating the interval. Indeed, most reported using some version of the scale-based or decomposition method.

[22] Siegler and Shipley, *op. cit.*

[23] Ibid., p. 55.

above a given note, but is not given the opportunity to explicitly and repeatedly use that strategy successfully, the likelihood is that a student will not adopt that strategy. If a student is missing a critical piece of information necessary for success with a strategy – say, for example, if a student is weak at scales or keys – the student will be unlikely to adopt that strategy for calculating intervals and will instead likely rely on the more reliable (to them) method of counting half steps and whole steps. To convince a student to move away from a low-level strategy, they need to be explicitly shown that another strategy is more effective and more efficient. To do this, use exercises that directly address the weaknesses of the method. For example, if students are still using the counting method, it will take much longer for them to calculate answers for larger intervals. Students who are repeatedly presented with large or compound intervals will realize that the low-level counting strategy is not their most effective option, and that using another strategy would be to their benefit.

Similarly, to get students to move away from the scale-based method, use exercises that exploit the weaknesses in that method. If a student is asked to calculate a Major 6th above D♯, the student using the scale-based method will be unable to do so without incorporating an element of the *decomposition* strategy; they must first figure out the interval above a note they "know," and must use that information to find out the answer to the original question. Getting students to understand and meta-cognate about their thought process can help them to understand that if they were able to retrieve the D–B interval immediately, they would be able to answer the question in one step instead of two (step 1: immediately recall D-B is a Major 6th; step 2: raise D to D♯, raise B to B♯, so a Major 6th above D♯ is B♯), and that doing so would reduce the potential for error. Other types of questions that can reinforce the move to the decomposition strategy would be requiring students to find the requested interval below a given note, where the scale-based method is at a disadvantage, or the increased use of all interval types, including diminished and Augmented intervals. Again, a student can solve a question asking for an Augmented 5th above C♯ in fewer steps if they can call upon knowledge of a reference interval of the Perfect 5th from C–G; they can move from C–G to C♯–G♯, and then to C♯–G𝄪.[24]

[24] Obviously, there are multiple paths to the same solution even within the same strategy, depending on what is available for the student to

Research shows that students learn most effectively when they are allowed to choose the strategy they use to solve problems, and that as they develop knowledge and confidence in the more advanced strategies, the less sophisticated strategies tend to be left behind. In addition, students who adaptively utilize more strategies tend to learn the material better, because they learn to determine what strategy is most effective for which type of problem.[25] Therefore, the most effective pedagogical approach to teaching a skill such as intervals may be to gradually introduce various desirable strategies, allowing the students time to determine for themselves the advantages and disadvantages of each strategy.[26] As instruction on the topic progresses, however, providing problems of increasing difficulty and having students meta-cognate about the strategies they use may help convince the student still using a more inefficient strategy that moving to a more advanced strategy will ultimately benefit them.[27]

In my fundamentals classroom, I have found it more helpful to work through fewer exercises with the class as a whole, but to have students describe their process for getting the answer to each exercise to the rest of the class. A sample conversation might go something like this:

Instructor: Student #1, what is a minor 3rd above F?
Student #1: A♭.
Instructor: Good! Can you tell us how you got that answer?
Student #1: I just knew it.
Instructor: Okay, excellent! Now, what if you didn't "just know" it? How could you have found the answer?
Student #1: I could have thought of a major 3rd above F, which would be A, and then lowered the A by a half step to make it a minor interval.
Instructor: Good. Can anyone else think of a way to find this interval?
Student #2: I thought of the F minor scale, and I know that A♭ is $\hat{3}$ in the F minor scale.
Instructor: Good! Any other ways?
Student #3: I thought of the F *major* scale instead, and knew that A was $\hat{3}$ in F major, so F to A is a major third, so then I just lowered the A to A♭.

In this hypothetical conversation, the instructor could then highlight the fact that "just knowing," or immediate retrieval, is the fastest and most efficient method for calculating the answer. Students who repeatedly see their classmates using more efficient and accurate strategies successfully, and who are given the opportunity to practice those strategies themselves, are more likely to adopt the higher-level strategies. The instructor could also take the time to review prior information; for example, in the sample conversation above, the instructor could have asked Student #2 how she knew A♭ was $\hat{3}$ in F minor, providing an opportunity to review scales and/or key signatures.

Siegler and Shipley point out that strategy choice may have some basis in personality types.[28] Their study revealed three types of students: the *good* students, the *not-so-good* students, and the *perfectionists.* The *good* students were accurate and likely to rely on retrieval for answers; as expected, the *not-so-good* students were less accurate and relied on retrieval less often than the *good* students. The *perfectionists,* however, were as accurate as the *good* students, but they relied on retrieval even less often than the *not-so-good* students; instead, they relied on backup strategies to either arrive at the answer or to verify the answer they retrieved, slowing them down.

Students need to be both accurate and fluent with concepts. If a music theory student is slow but accurate in their interval spelling, they may be falling into Siegler and Shipley's *perfectionist* category; they may be able to retrieve answers, but for whatever reason, they do not trust their retrieval abilities and are verifying the answers

recall; if a student can call upon the knowledge that C♯–G♯ is a P5,then they only have to make one alteration to come up with the C♯–G𝄪 Augmented 5th.

[25] R. Siegler, "Implications of Cognitive Science Research for Mathematics Education," and references contained therein.

[26] As previously mentioned, many instructors do not wish to teach the counting half steps/whole steps strategy, and for good reasons. Because of the many flaws in that strategy, I would advocate that it is not necessary (and may even be detrimental) to include that low-level strategy in instruction, but teachers should be aware that students may have already learned it elsewhere or may derive it on their own.

[27] This instructional strategy is obviously not limited to intervals; it can be used throughout the curriculum.

[28] Siegler and Shipley, *op. cit.*

by using a slower, less efficient backup strategy. These students should be encouraged to rely on their retrieval abilities and should be given an opportunity to demonstrate to themselves that the higher-level strategies can be accurate and effective for them.

Understanding the adaptive strategy model can help the music theory fundamentals instructor teach elements of music theory fundamentals more effectively. It requires the instructor to reflect on what strategies students use, and which are the desired or most optimal strategy. In addition, the instructor must present material such that it gradually builds the student's skills while simultaneously reinforcing the development of higher-level and more abstract strategies.

However, if this is all that the music theory instructor does, at the end of the process students will have a good grasp of the fundamentals of music theory, but will not necessarily have an overall conception of how these different elements relate to one another. The overall goal of music theory fundamentals instruction should be to create an interconnected set of abstracted representations of fundamentals concepts; doing so helps students create a larger schemata or domain in which the skills and information reinforce one another and support additional, more abstract learning. The more a student understands how scales, intervals, triads, and seventh chords are related, the easier it will be for them to understand the overall concept of tonality and how more advanced concepts such as secondary dominants, tonicization, and modulation work within the system.

One of the predominant paradigms in current research on mathematics pedagogy focuses on the role of conceptual vs. procedural knowledge and thinking. Procedural knowledge consists of the actions or steps taken to solve a problem; it emphasizes the *how* aspects of developing skills. Conceptual knowledge consists of an abstract understanding of principles and of the relationships and connections between pieces of knowledge in a given domain; it emphasizes the *why* aspects of knowledge.[29]

[29] A. Baroody, "The development of adaptive expertise and flexibility: The integration of conceptual and procedural knowledge," in *The Development of Arithmetic Concepts and Skills: Constructing Adaptive Expertise*, edited by A. Baroody et al. (Mahwah, NJ: Lawrence Erlbaum Associates, Inc., 2003), 1–34; B. Rittle-Johnson and K. Koedinger, "Iterating between lessons on concepts and procedures can improve mathematics knowledge," *The British Journal of Educational Psychology* 79, no. 3 (2009), 483–500; Siegler, *op. cit.*, p. 227.

In the domain of music theory fundamentals, an example of procedural knowledge would be the ability to spell intervals, scales, or chords, or to identify key signatures. It is possible to know how to do each one of these things without understanding how they are interconnected; procedural knowledge of the step-by-step processes can exist in isolation. Conceptual knowledge of music theory fundamentals, on the other hand, would consist of an understanding of how these elements interact to create the concept of the hierarchical system of tonality. For example, a student who understands that the tritone is a unique interval in a key will understand why chords containing that tritone are so strongly directed towards the tonic, rather than just having memorized that V^7 and vii^o are called dominant-functioning chords.

An ongoing discussion within mathematics pedagogy research regarding conceptual and procedural knowledge is whether one is more important to educational practice than the other. The literature is divided into skills-based educational theories, in which students learn skills before learning concepts, and concept-based educational theories, in which students learn concepts before learning skills. A third and more recent approach is to study how the two dimensions influence each other; the dimensions are not entirely discrete, since conceptual knowledge consists of connections made between knowledge within a domain, some of which may be procedural knowledge.

Recent studies in mathematics education have shown that conceptual and procedural knowledge appear to develop in an iterative fashion; that is, an increase of one type of knowledge leads to increases in the other.[30] These findings have implications for classroom practice, suggesting students presented with material that alternately focuses on concepts and procedures will learn material more efficiently and more thoroughly, and will be able to connect the procedural knowledge to the conceptual, rather than having them be distinct. One benefit of an effective integration of conceptual and procedural knowledge should be greater flexibility in the development and use of strategies; students who understand the conceptual reasons for using more abstract and optimal strategies will move towards using those strategies more quickly and effectively.

[30] Baroody, *op. cit.;* Rittle-Johnson et al., *op. cit.;* M. Schneider and E. Stern, "Conceptual and procedural knowledge of a mathematics problem: Their measurement and their causal interrelations," in *Proceedings of the 27th Annual Meeting of the Cognitive Science Society*, edited by B. Bara et al. (Mahwah, NJ: Lawrence Erlbaum Associates, Inc., 2005), 341–347.

What implications does the iterative approach have for educational practice in the domain of music theory fundamentals? The music theory instructor wants to make sure that the procedural elements of fundamental elements are connected to one another to create the more sophisticated and abstract conceptual knowledge of the system of tonality. One potential way to achieve this within the music theory fundamentals curriculum may be to constantly create conceptual relationships between the elements of fundamentals and the overall system of tonality.

For example, when students learn scales, the instructor could introduce the notion of function within context, and how an individual pitch-class can play different roles within different scales; a C has a much different function in the key of C Major, for example, than it does in D♭ Major. Exercises that ask students to identify all scales that contain a particular pitch class, or a pair of pitch classes, will help the students understand that the same entity or entities can take on a different meaning in a different context.

Similarly, when students learn intervals, it may be possible to help them contextualize intervals by not just relating them to scales, but by illustrating that the same interval can exist in more than one scale and will serve a different role in that scale. A Major 3rd from G to B is present in a G major scale and carries some important information about the key in that context, but the same interval is also present in a D Major scale, where it doesn't convey as much information about the overall key.

Another possible activity for students is for them to explore the intervallic content of the scale, and to understand what the most "important" intervals are within the scale. Students are often inclined to think that the interval of a major third between the tonic and mediant of a given diatonic major scale is the most important interval of the scale; however, pointing out that a major third also exists between $\hat{4}$ and $\hat{6}$, as well as $\hat{5}$ and $\hat{7}$, may help them to understand that the major third does not necessarily convey that much information about key. The same can be illustrated for all other intervals; while there may be a major sixth between $\hat{1}$ and $\hat{6}$, there is also a major sixth between $\hat{4}$ and $\hat{2}$ an octave higher. If students are asked to calculate intervals in this way, and to figure out what the most important interval in a scale collection is, they should eventually realize that there is only one tritone within the diatonic collection,[31] and that the tritone is therefore the "key-

[31] The tritone between $\hat{4}$ and $\hat{7}$ and $\hat{7}$ and $\hat{4}$ of a major scale involve the

defining" interval of the diatonic major scale collection. If you see a tritone between A♭ and D, for example, you know that you are in the key of E♭ Major, because that is the only major key in which that tritone occurs naturally.

A similar activity can be undertaken with the qualities of seventh chords; having students build diatonic seventh chords on each major scale degree and determine the quality of each chord will reveal that there are two types of seventh chords that only appear once within the major system, the Major-minor or dominant seventh chord (on $\hat{5}$) and the half-diminished seventh chord – and, importantly, each one of these unique chord qualities contains the tritone!

These are difficult concepts for students to understand if they are just presented as knowledge without context, but if intervals, scales and chords have all been presented as interconnected systems, this can help them build a conceptual representation of the system of tonality. It may help students understand voice-leading rules that they will encounter later, including the requirement to always resolve the chordal seventh of the dominant seventh down. It certainly helps explain why the dominant seventh chord is used so frequently, as the addition of the chordal seventh (and thus, the tritone) to the dominant triad makes the chord into the unique entity that helps determine and establish the key. It also establishes dominant-functioning chords as important members of the hierarchy of tonality, and sets the groundwork for the difficult concept of tonicization with secondary dominants.

Alternating the procedural knowledge with the conceptual knowledge in a music theory fundamentals classroom will have several beneficial results. First, it results in a constant revisiting and development of earlier information; this 'distributed practice' is a well-known strategy for skill development, extended here to cognitive tasks and the classroom setting.[32] Second, it reinforces the material and encourages students to develop their adaptive strategies. If a student is still relying on the strategy of spelling scales via the whole step–whole step–half step method, for example,

same pitches and result in an enharmonic interval (+4/°5), so there really only is one tritone per major scale. Students who have difficulty with this concept can be asked about the interval between $\hat{2}$ and $\hat{4}$ (a minor third) and $\hat{4}$ and $\hat{2}$ an octave higher (a Major 6th) – even though these two intervals involve the same pitch-classes, the intervals are not enharmonic and so are different intervals.

[32] Rittle-Johnson et al., *op. cit.*

returning to scales while working on intervals will encourage the student to adapt to using a more efficient strategy for the earlier material. Third, the iterative process helps create connections between procedures that may otherwise seem somewhat disconnected, creating a conceptual understanding of the elements of tonality.

Summary

If student performance in mathematics is a predicting factor in success in written music theory classes, there may be a cognitive link between the abstract and systematic processes in mathematics and those in music. If that is the case, music theorists should consider the vast amounts of research done in mathematics education to see if we can learn anything about best practices in that discipline.

Research in mathematics education can help us understand the cognitive processes that result in the development of skills and concepts necessary to succeed in the music theory fundamentals classroom and beyond. For example, the concept of *adaptive strategies* can help us to understand how a student progresses through cycles of knowledge development to reach the ultimate goal of immediate recall of information such as key signatures, intervals, or chord spellings, and can help us as teachers learn how to encourage a student to move beyond an inefficient strategy.

Armed with knowledge of conceptual and procedural knowledge, a music theory fundamentals instructor can be aware of the need to create connections between the critical aspects of theory fundamentals in order to work towards the goal of teaching students to develop an understanding of the abstract system of tonality. A solid procedural knowledge foundation in music theory fundamentals and conceptual knowledge of the principles of tonality will provide them with a solid foundation for their continued development, not only in the written music theory curriculum but in all aspects of their musical lives.

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