

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/231078109>

What is observable in classical and quantum gravity?

Article in *Classical and Quantum Gravity* · January 1999

DOI: 10.1088/0264-9381/8/2/011

CITATIONS

233

READS

85

1 author:



Carlo Rovelli

Aix-Marseille Université

315 PUBLICATIONS 13,956 CITATIONS

SEE PROFILE

All content following this page was uploaded by [Carlo Rovelli](#) on 08 December 2016.

The user has requested enhancement of the downloaded file. All in-text references [underlined in blue](#) are added to the original document and are linked to publications on ResearchGate, letting you access and read them immediately.

What is observable in classical and quantum gravity ?

Carlo Rovelli

*Department of Physics and Astronomy,
University of Pittsburgh, Pittsburgh, 15260 U.S.A.
Dipartimento di Fisica, Universita' di Trento, Trento, Italy.
INFN, Sezione di Padova.*

August 25, 2009

Abstract

The problem of the identification of the observable quantities in quantum gravity (or in any diffeomorphism invariant quantum theory) is considered. We recall Einstein's "hole argument" on the impossibility of a priori identifying space-time points. We argue that only by explicitly taking into account the physical nature of the bodies that form the reference system and their gravitational interactions one can get well defined gauge invariant (and "local") observables and a definition of physical space-time points. A model is considered in which general relativity is coupled to matter: the matter represents the physical reference system. The gauge invariant physical observables of this theory are displayed.

1 The problem of observability in the presence of the gravitational field.

By drawing quantum mechanics and general relativity together, we may infer that the space-time structure undergoes quantum fluctuations at small scales [1, 2]. How would we detect these fluctuations? In classical physics, space-time points are determined by material bodies like particles; but particles themselves are subject to quantum fluctuations. So how can fluctuating particles define a fluctuating space-time structure down to the Planck scale? The very concept of space-time points seems to become fuzzy in such context [3]. These and similar issues refer to the question of what precisely could be observed in a quantum theory of gravity. Indeed, one of the basic difficulties in quantum gravity is to understand which physical quantities should be predicted by the theory [4].

The answer seems simple: the observable quantities in the quantum theory should be the same as in the classical theory, or at least a subset of these. Rather remarkably, however, the problem of what precisely is observable is far from being trivial even in *classical* general relativity. Indeed, one may find in the literature substantially non-overlapping points of view on this issue [5]. At the end of the day, opposite points of view do not seem to give contradictory results in the concrete analysis of real experiments; and indeed they cohabited in the scientific community. But the problem becomes less academic and really crucial in approaching the quantum theory. Here, the precise specification of the gauge invariant observables (the “Dirac observables”) is needed both for constructing the formal apparatus of the theory and for understanding its interpretation [4]. Moreover, we believe that many discussions and disagreements on interpretational problems in the quantum domain (for instance the famous “time issue”, see for instance ref.[6]) just reflect different but unexpressed interpretations of the *classical* theory. Thus, the subtleties raised by the attempts to quantize the theory force us to reconsider the problem of observability in the classical theory.

In general, in a theory with a gauge invariance, we assume that only gauge invariant quantities can be measured and/or predicted by the theory. In general relativity, however, this assumption is rejected by many authors on the grounds that in this theory the gauge just reflects the particular reference

system in which the measures are performed [7] ¹. The disagreement can be summarized in the two following points of view, which we present here without any claim of not being strongly biased by our particular perspective:

1. *The “non-local point of view”*. According, for instance, to Dirac [8], in any gauge theory (by gauge theory we mean here [8] any theory in which the *same* set of initial data can evolve to *different* sets of (gauge related) data), determinism requires that only gauge invariant quantities are measured. This thesis has been applied to general relativity by Einstein himself in his famous “hole argument” [9], which we will review below. According to Einstein, it follows from the hole argument that points of space-time are not a priori distinguishable, so that, for instance, the Ricci scalar $R(x)$ in a certain space-time point P is not an observable quantity ².

2. *The “local point of view”*. Space-time points are physically distinguishable, because any measurement in physics is performed in the frame of a given reference system. The fact that reference systems are freely chosen in general relativity implies that the evolution is not uniquely determined. Quantities like the value $R(x)$ of a scalar (say, again, the Ricci scalar) in a given point P *are* observables quantities in gravity.

Surprisingly enough, as opposite as these two points of view may seem, nevertheless they tend to lead to the same conclusions whenever a classical experiment is analyzed, as we already remarked. Of course this fact indicates that there is a bridge between the two points of view. But the two points of view push us toward drastically divergent directions for the quantization programs. For a classical example of a discussion on this point, see for instance, the Karel Kuchar - Bryce DeWitt discussion³ at the 1988 Osgood Hill meeting on Conceptual Problems in Quantum Gravity [6].

In this paper we study the precise relation between these two points of

¹In order to appreciate the content of this issue, consider that in pure (no matter) general relativity on a compact space, essentially no reasonable gauge-invariant observable (singlet under the 4 dimensional diffeomorphism group) is known (see later for some cautionary remarks about this statement). Thus, one is forced either to give up the basic assumption that only gauge invariant quantities are observable, or to accept that no observable is available in pure compact space general relativity.

²Of course $R(x)$ is *not* gauge invariant, because $\delta_f R(x) = f^\mu \partial_\mu R(x) \neq 0$.

³Kuchar: “Quantities non invariant under the full diffeomorphism group *are* observable in gravity”; DeWitt: “Quantities non invariant under the full diffeomorphism group *are not* observable in gravity” (quoted from memory; the text of the discussion will appear in the proceedings).

view. In a companion paper ref.[10], we study the implications of these two points of view in the quantum theory. The present paper has therefore two purposes. The first is an attempt to clarify the observability issue; the second is a preliminary step necessary for the study of the observability in quantum gravity as approached in the companion paper. The questions raised at the beginning of this paper will be (tentatively) answered at the end of the companion paper.

The main concept that we shall use is the concept of material reference system. By material reference system we mean an ensemble of physical bodies, dynamically coupled to general relativity, such that these bodies can be used to define the space-time points in a sense that will be specified ⁴. By using this concept, we shall argue that the “local point of view” can be obtained from the “non-local point of view” in the context of a certain approximation. To give concreteness to these ideas (which certainly are not new, see for instance [5, 11, 12, 13, 14, 15, 16, 17]) we introduce a concrete model in which they are implemented.

The model is given by general relativity coupled to certain matter. We find in explicit form the observables of this theory that commute with the diffeomorphism constraint, and in implicit form the ones which commute with all the constraints. These, we claim, are the physical observables of gravitation, with which measuring procedures [18] are associated. In the context of the model we verify the relation between the two points of view and we discuss the approximation that recovers the second from the first one.

Armed with the machinery developed in this paper, we face the issues of the quantum theory in the next paper [10]. We will see that the possibility of defining gauge invariant observables has remarkable consequences for the quantum theory.

The central result of this paper is to show that in order to have formally well defined physical observables in general relativity, we are forced to take into account explicitly the material nature and the gravitational interactions of the bodies used as system of reference. In other words, in general relativity we are not allowed to consider reference systems as *external, non dynamical* objects, as we do in non general relativistic physics.

These ideas, we repeat, are not new, but to our knowledge they were never concretely implemented in a theory (with the possible exception, in dif-

⁴This concept is distinct from the concept of coordinate system in differential geometry.

ferent formalism, of ref.[17]). There are several consequences of this shift in perspective. We shall discuss these consequences, point out open problems, and suggest some alternative models, in the conclusions at the end of the companion paper. Finally, let us stress that the arguments we are presenting here are relevant for every theory invariant under a space-time diffeomorphism group. For instance they are relevant if one wants to give a physical interpretation to the so called “topological theories”⁵, or to any theory which include fully non-perturbative gravitational interactions. In particular, our arguments should be relevant for any non-perturbative formulation of string theory.

2 Einstein’s hole argument and the washing away of physical points

We begin by discussing the “non-local point of view”. In this section we essentially recall and discuss Einstein’s hole argument in a modern language [9, 14, 5]. In the next section we will show how, in spite of the correctness of the argument, there is still a way to recover a definition of space-time points by explicitly introducing the material reference systems in the theory. Since the argument is delicate, we introduce in this section some mathematical terminology; this will not be used in the rest of the paper.

Let us start by stating the problem in a precise way. Let M be an assigned four dimensional smooth manifold. We assume a compact space, M has a topology $\Sigma \times R$ where Σ is a compact three manifold and R is the real line. We consider the possible assignments of a pseudo-riemmanian metric tensor g on M . We may also consider other structures that represent matter, for instance a scalar field f , which is a smooth mapping from M in R , or a set of particles $X_y, y = 1, \dots, n$, which are represented by suitable (timelike with respect to g) smooth mappings from R in M . The set (M, g, f, X_y) is assumed to describe a certain classical spacetime with a certain gravitational field and a certain configuration of matter. We call it a “localized universe”. The precise relation between this structure and the physical world is the argument of this discussion.

⁵In that context, an attempt strictly related to the present one of getting space-time properties from diffeo-invariant quantities by matter couplings has been made in ref.[19].

The set of mappings $\phi : M \mapsto M$ which preserve the structure of M form the diffeomorphism group of M , which we call $Diff4$. We denote by \mathcal{C}_M the set of all localized universes for a fixed manifold M . Every $\phi \in Diff4$ induces a mapping of \mathcal{C}_M in itself. This mapping sends (M, g, f, X_y) in $(M, \phi_*g, f \circ \phi^{-1}, \phi \circ X_y)$. We call the orbits of $Diff4$ in \mathcal{C}_M “non-localized universes”. The reasons for this terminology will become clear. The question that we are asking is the following. Is a given configuration of the physical world represented by a localized universe or by a non-localized universe? More precisely: can we, by means of physical measurements, distinguish between two localized universes that are in the same orbit of $Diff4$? A negative answer corresponds to the “non-local point of view”, a positive answer to the “local” one.

On purpose, we didn’t introduce coordinates on M . Indeed, both localized universes and non-localized universes have an intrinsic geometrical meaning which is coordinate independent.

After this terminological introduction, we may come to physics. We begin by recalling Dirac’s argument [8] against the possibility of measuring non-gauge-invariant observables. Suppose the equations of motion of a classical system do not uniquely determine the evolution, so that the same initial data can evolve either to a later set of data A or to a set B, always respecting the equation of motion. Then, we say that A and B are gauge related [8]. Dirac argues that, in this situation, the only way determinism can be respected is to assume that only observables that have the same value on the set A and on the set B could be observed. Following Dirac, we call these observables gauge invariant observables, or physical observables; we will also use the expression “Dirac observables” (the three expressions have the same meaning in this paper). Since determinism is a basic principle in classical physics, Dirac postulates that measurement procedures can be attached only to gauge invariant observables.

Now suppose we have two distinct localized universes U and U’ which are in the same orbit of $Diff4$. Suppose that the matter configuration is such that there is a “hole”, namely a region without matter. Suppose also that outside the hole the matter and gravitational configurations are identical⁶. Suppose that U satisfies the Einstein field equations⁷. It follows from

⁶This is possible because the ϕ that relates U and U’ can have support inside the hole.

⁷More precisely, the components of g, f, X_y in a coordinate system O, and therefore in

the general covariance of Einstein equations that also U' satisfies the field equations. Now, let us draw a spacelike surface in U such that the hole is entirely in the past of the surface. U and U' are identical outside the hole, and therefore they have the same set of initial data on the surface. Thus, according to Dirac's postulate, U and U' are physically indistinguishable. No measurement can detect the difference. It follows that U and U' represent the same configuration of the physical world. It follows (from an immediate generalization) that the configurations of the physical world are represented by non-localized universes, and not by the single localized universes. It follows that the non-local point of view is correct.

Several comments have to be added.

a. The crucial physical input of the argument is determinism. Later we will discuss in which sense one gets around determinism in general relativity. However, it is important to stress here that by itself classical general relativity is certainly a deterministic theory: planets do not have a lot of freedom in their run around the sun !

b. The most important consequence of the discussion we presented, which was already pointed out by Einstein ⁸, is the following. Suppose we pick a point P in the manifold, and P happens to be in the hole. Suppose that U and U' differ in the following feature: in U there is a small gravitational wave around P , while in U' the geometry is flat around P (and the gravitational wave is in another region of the hole, away from P). If we could measure the local geometry in P we could distinguish U from U' . But we cannot distinguish U from U' . How is it possible ? It is possible only by assuming that the a priori specification of the point P is physically meaningless. The individuality of single points of M is washed away by general covariance. This is the reason for the terminology we introduced. The "non-localized" universes of the "non-local" point of view are differential manifolds with a metric, modulo the possibility of a priori distinguishing between points.

c. Of course the hole argument can be also expressed in coordinate language, but this tends to be confusing. Given a coordinate system O , we have that g, f, X_y can be expressed by their components $g_{\mu\nu}(x), f(x), X_y^\mu(\tau)$

any other coordinate system, satisfy the Einstein equations.

⁸Einstein spent a lot of time thinking on these issues and came back to them until the end of his life. For an enlightening reconstruction of the evolution of Einstein's ideas on the subject see ref.[5]. Recently the issue has received new consideration; the argument that we have presented in this section can be found also in refs.[21, 5, 14, [20](#), 16]

(where $\tau \in R$). The same universe is represented by different components in different coordinate systems; the transformations that send the components of a localized universe U in a coordinate system O into the components of the same universe in a different coordinate system O' is called a passive diffeomorphism. Note that it is also true that different localized universes are represented by the same components in different coordinate systems. The mapping $\phi \in Diff4$ that sends a localized universe U into a different universe U' (which have different components in *the same* coordinate system O) is called an active diffeomorphism. There is a 1-1 correspondence between localized universes and sets of components; every coordinate system defines a (different) 1-1 correspondence. The situation is illustrated in the figure

There is also a 1-1 correspondence between non-localized universes and the orbits of the passive diffeomorphisms in the space of the representatives. To understand the hole argument in coordinate language it is important to emphasize that determinism is not broken by passive transformations (which just represent the same object in different coordinate systems) but it is broken by the active transformations (which relate different objects in the same coordinate system).

d. As mentioned in note 1, essentially no reasonable gauge-invariant observable is available in pure, compact-space general relativity. In the next section we will study how it is possible to do experiments in general relativity in spite of that. Here, however, we must clarify (and partially limit) this statement. By observable we mean here a smooth function on the phase

space of the theory. By gauge-invariant we mean having vanishing Poisson brackets with the constraints. Any smooth function on the orbit space defines an observable of this kind, thus in principle observables should exist; by saying that such an observable is not available we do not mean that these objects do not exist, but simply that we do not know any of them neither explicitly (as functions of the canonical coordinates) nor in some well-defined implicit form.

There do are "global" quantities that characterize a four dimensional pseudo- Riemannian manifold. One may consider the volume of the space-like slice with the maximum volume (provided it exists). If one restricts the space of solutions to Einstein equations by certain suitable assumptions, then there are theorems which say that space-time can be foliated by trace-of-the-extrinsic-curvature=constant slices. Thus, one may consider the volume of the constant slice with $\text{tr } k=1$. These, or similar, objects are gauge- invariant. However, these quantities do not solve the problem of the observability in general relativity. First, there are strong assumptions that one has to add to the theory in order to have them defined at all. Second, and more important, these quantities are completely unrelated to anything has been measured (and, most probably, will be measured) in gravitational physics. Thus, their knowledge does not help us to solve the problem that we have posed, namely: how are *real* observations made in gravity, if we do not know any (concretely observable) gauge-invariant quantity in the theory ? Finally, our major final aim is the quantization, and we do not think a quantum field theory could be developed in terms of so non-local quantities.

Other more "local" quantities could be obtained as follows. If we take five scalar functions of the metric and we express one of them as a function of the other four, this would provide a gauge-invariant observable; alternatively, we may take the extrinsic curvature as a local time and some localized gravitational wave to define location. No one has been able to complete such kind of programs. If such programs could be implemented, the way observables are introduced in this paper will not be necessary. Still, however, we claim that "realistic" observables that are measured in real concrete gravitational experiment are of the kind that will be introduced in the next sections and not of the pure-gravity kind.

e. Suppose the hole was not really empty, but a particle was running in it. Suppose in the universe U the particle crossed the hole always through a region of flat geometry. Then, *also* in the universe U' the particle will run

through flat geometry. Thus, we may say that the fact that the geometry is flat around the particle trajectory is indeed a gauge invariant statement, which can be observed. In other words, points are not defined a priori, but matter can be used to define a localization.

3 Material reference systems. The resurrection of the physical points.

In the previous section we recalled the argument that demonstrate the correctness of the non-local point of view. In this section we show that also the local point of view is correct in a suitable sense. The idea that we shall follow is the one suggested at the end of the previous section: we can measure the local geometry without violating Dirac determinism by localizing points by means of some matter. Let us assume that among the matter included in our dynamical system, namely in our universes U , we select a set of bodies that we decide to use as reference system. These set of bodies will be used to define points. For instance we may use particles to define space points and intersections of particles trajectories to define space-time points. Another possibility (which we will follow) is to use physical variables attached to the particles (clocks) to define time instants. We stress that there are basic differences between these material reference systems and the coordinate systems of differential geometry: for instance, the coordinate systems are freely specified at every point of M , while the time evolution of our material reference systems is determined by the dynamical equations, once the initial data have been given. As noted at the end of last section, a material coordinate system defines points in a way that is independent both on passive and active diffeomorphism transformations.

Let us consider, for instance, a single particle X coupled to general relativity, and a variable T , attached to the particle, which grows monotonically along the particle trajectory. In a given coordinate system O , a localized universe U will be represented by $(M, g_{\mu\nu}(x), X^\mu(\tau), T(\tau))$. From $X^\mu(\tau)$ and $T(\tau)$ we may define the function $\tilde{X}(T)$ by

$$\tilde{X}(T(\tau)) = X(\tau). \quad (1)$$

Consider the point $\tilde{X}(0)$ in U . This can be considered a “physical point”, in the following sense. It is meaningful to say, for instance, that the geometry

in this point is flat. This is a gauge invariant statement. More precisely, consider any scalar function R of the metric (and of the other fields, if they are around), and let $R(x)$ be its components in O . Then the quantity

$$\tilde{R} \equiv R(\tilde{X}(0)) \quad (2)$$

is a physical observable quantity in the sense of Dirac. Indeed, let us consider an infinitesimal change of coordinates, induced by a vector field f . We have

$$\begin{aligned} \delta R(x) &= -f^\mu(x)\partial_\mu R(x), \\ \delta X^\mu(\tau) &= f^\mu(X(\tau)), \\ \delta T(\tau) &= 0, \end{aligned} \quad (3)$$

from which it follows that

$$\delta \tilde{R} = -f^\mu(x)\partial_\mu R(\tilde{X}(0)) + \partial_\mu R(\tilde{X}(0))\delta \tilde{X}^\mu(0) = 0. \quad (4)$$

Now, suppose that a three dimensional flow of particles, each carrying a clock, fills up the 4 manifold. By using this matter, we may define a mapping from a region of R^4 in M ⁹. This material reference system allows us to give a gauge invariant meaning to any measurement of the geometry of U .

Then, we take two steps

Step 1. We neglect in the Einstein equations the energy momentum tensor of the matter that we decided to use as a material reference system.

Step 2. We neglect in our system of dynamical equations the entire set of the equations that determine the motion of the matter of the reference system.

We claim that these two approximations define the “local point of view”. Step 2 introduces some indeterminism in the evolution of the entire system. This indeterminism, however, is not fundamental, and does not implies that Dirac determinism is violated, because it simply follows from disregarding certain equations. Step 1 implies that the system (general relativity + other matter not considered as reference system) evolves according to the Einstein equations as if the reference system were not present. This can be true only within the approximation. The degrees of freedom of the matter of the material reference system do not disappear; they enter in the definition of the physical observable quantities.

⁹We disregard the need to have more than one “chart”; global features do not play any determinant role in the present discussion.

It follows that we can give a second interpretation to pure general relativity. We can assume that the equations of motion of general relativity represent an *approximated* system in which a material reference system was defined, the geometry defined by g is the one of the space of physically observable points defined by the reference system, but step 1 and step 2 have been taken, so that the material reference system evolution is free and does not affect the rest of the physics. This is precisely the picture of the localized point of view. Once more we stress that this picture is only valid within an approximation ¹⁰. In the next section we shall verify these ideas in a specific model.

We have reached the conclusion that the *same set of equations*, vacuum Einstein equations of pure general relativity, admit two distinct interpretations which are not contradictory. According to the first one the metric tensor is a mathematical non observable gauge variable. Only diff- invariant functionals of the metric and the matter variables can be observed. The theory can be thought as a fundamental and exact theory, and the evolution is deterministic. This is the “non-local point of view” or “non-local interpretation”.

According to the second interpretation the metric tensor components $g_{\mu\nu}(x)$ are observable and express the angle and distance properties of the material bodies that form the reference system. The exact interacting matter-gravity theory is well defined. In it, the evolution of the metric is *not* given by the vacuum Einstein equations, it is given by the coupled matter-gravity equations; but the theory admits an approximation, via steps 1 and 2, in which it is reduced again to pure general relativity equations. These vacuum equations for an *observable* metric can only be true in an approximate sense because the action of the gravitational field on the matter degrees of freedom (which do participate in the definition of the observable metric) is neglected; the evolution is determined up to 4 arbitrary functions because the evolution equations of the bodies of the reference system are disregarded. This interpretation of the vacuum Einstein equations is the “local point of view” or “local interpretation”.

The two interpretations are not contradictory, because we may start from the non-local one, which is the fundamental one, explicitly couple the matter

¹⁰While, in non generally relativistic physics, considering the reference systems as external non dynamical objects is *not* an approximation.

that forms the reference system to general relativity and obtain a system which is well defined, and finally reobtain general relativity in the local interpretation as a well defined limit.

4 A model of a general relativity + material reference system theory, and its physical observables.

We want to implement the idea presented in the previous sections in the context of a specific model. As a preliminary exercise, let us start with the simplest possible reference system, formed by a single particle and by a single clock. Later we will fill the space with these objects. Our variables are the ones considered in the previous section,

$$g_{\mu\nu}(x), \quad X^\mu(\tau), \quad T(\tau), \quad (5)$$

where $\mu, \nu = 0, 1, 2, 3$, $\tau \in R$, $x = (t, \vec{x})$. The problem is to choose a dynamics for this system. The simplest choice is to require that the particle is free, and follows geodesics, and that the clock measures the proper time along the particle geodesic. Thus, we want the following equations of motion

$$\frac{d^2 X^\mu}{ds^2} + \Gamma_{\nu\rho}^\mu(X) \frac{dX^\nu}{ds} \frac{dX^\rho}{ds} = 0, \quad (6)$$

$$\frac{d^2 T}{ds^2} = 0, \quad (7)$$

where the proper time $s(\tau)$ is given by

$$\frac{ds(\tau)}{d\tau} = \sqrt{-\frac{dX^\mu}{d\tau} \frac{dX^\nu}{d\tau} g_{\mu\nu}(X(\tau))} \equiv r(\tau). \quad (8)$$

These dynamical equations have to be added to the gravitational equations, which have to be the Einstein equations with a right hand side given by the appropriate energy momentum tensor of the particle.¹¹

¹¹The system defined by these equations is not very physical, because of the well known difficulties of point-like masses in general relativity. In our case the difficulties are even

There are several Lagrangians that give these equations. For instance we may take the the simple action

$$S = \int \frac{1}{2M} \left(\frac{d^2 T}{ds^2} \right)^2 ds = \int \frac{1}{2M} \left(\frac{dT}{d\tau} \right)^2 \frac{1}{r(\tau)} d\tau. \quad (9)$$

This action, however, leads to an Hamiltonian with 4 powers in the momenta. A more convenient choice is

$$\begin{aligned} S &= -mc \int \sqrt{-\frac{dX^\mu}{d\tau} \frac{dX^\nu}{d\tau} g_{\mu\nu}(X(\tau)) - c^2 \left(\frac{1}{\omega} \frac{dT}{d\tau} \right)^2} d\tau \\ &\equiv -mc \int r_T(\tau) d\tau. \end{aligned} \quad (10)$$

c is the velocity of the light, m is the mass of the particle, and ω is a constant needed to adjust the dimensions of the T variable; we shall assume that T is dimensionless (for instance, it could be an angular variable), then ω has the dimensions of a frequency (the inverse of a time). The reader may check that this action gives the equation of motion that we want (in the calculation it is useful to exploit the reparametrization invariance in τ and change the parametrization to τ' defined by $\frac{d\tau'}{d\tau} = \frac{r_T}{r}$).

We are interested in the dynamical system given by the action

$$S[g, X, T] = \frac{1}{8\pi G} \int dt \int d^3x \sqrt{g} R - mc \int d\tau r_T(\tau). \quad (11)$$

where the notation for the gravitational part is the standard one [22].¹² In order to work out the Dirac observables and in view of the quantization, we need the Hamiltonian analysis of the system. As it stands, the action is not well suited for the Hamiltonian analysis, because it is the sum of two terms with different evolution parameters. In order to have the second term expressed in the same evolution parameter as the first one, it is convenient

greater, because we will consider the limit in which the matter does not affect the gravitational field and this limit is singular for a point-like particle, because the topology of the space changes from the black hole one for non zero mass to the trivial one for zero mass. (I own to J. Stachel this observation.) The next reference system that we will introduce, in which the particle is replaced by a continuum fluid will not present these difficulties.

¹²A class of related lagrangians, in which a gravity matter coupling can be used to get physical observables have recently been constructed by Karel Kuchar [23].

to fix the reparametrization invariance in τ by requiring that $X^0(\tau) = \tau$. In this way we may identify τ with t , and the particle degrees of freedom will be described by just the 3 variables $X^a(t)$, $a = 1, 2, 3$, without redundancy. Thus we may rewrite the action in the equivalent form

$$S = \int dt \left[\int d^3x \sqrt{g} R - mc \sqrt{-\dot{X}^a \dot{X}^b g_{ab}(X(t)) - 2\dot{X}^a g_{a0}(X(t)) - g_{00}(X(t)) - \frac{c^2}{\omega^2} \dot{T}^2} \right]. \quad (12)$$

The dot denotes the derivative with respect to t . We may now introduce the ADM variables $g_{ab}(x), N^\mu(x)$ [13]

$$S = \int dt \left[\int d^3x L_{ADM}[g, \dot{g}, N] - mc \sqrt{((N^0(X))^2 - (\dot{X}^a + N^a(X))^2 - \frac{c^2}{\omega^2} \dot{T}^2)} \right], \quad (13)$$

where 3 dimensional indices are raised and lowered by means of the ADM three metric. The conjugate momenta are (L is the total Lagrangian)

$$\begin{aligned} p^{ab}(\vec{x}) &\equiv \frac{\partial L}{\partial \dot{g}_{ab}(\vec{x})} = p_{ADM}^{ab}(\vec{x})[g, \dot{g}, N], \\ p_a &\equiv \frac{dL}{d\dot{X}^a} = -mc \frac{(\dot{X}^a + N^a)}{\sqrt{(N^0)^2 - (\dot{X}^a + N^a)^2 - \frac{c^2}{\omega^2} \dot{T}^2}}, \\ P &\equiv \frac{dL}{d\dot{T}} = -\frac{mc^3}{\omega^2} \frac{\dot{T}}{\sqrt{(N^0)^2 - (\dot{X}^a + N^a)^2 - \frac{c^2}{\omega^2} \dot{T}^2}}, \\ \pi_\mu(\vec{x}) &\equiv \frac{\partial L}{\partial \dot{N}^\mu(\vec{x})} = 0, \end{aligned} \quad (14)$$

where $p_{ADM}^{ab}[g, \dot{g}, N]$ are the momenta of the ADM theory. The Hamiltonian is

$$H = \frac{1}{8\pi G} \int d^3x N^\mu(\vec{x}) H_\mu^{ADM}[g, p](\vec{x}) - N^a(X) p_a - N(X) \sqrt{m^2 c^2 + \vec{p}^2 + c^{-2} \omega^2 P^2} \quad (15)$$

By computing, as usual, the commutator of the primary constraints with the Hamiltonian (or, equivalently, by considering the lapse and shift as Lagrange

multipliers from the very beginning) we obtain the first class constraints

$$\begin{cases} H_a(\vec{x}) = H_a^{ADM}(\vec{x}) - \delta^3(x, X)p_a, \\ H_0(\vec{x}) = H_0^{ADM}(\vec{x}) - \delta^3(x, X)\sqrt{m^2c^2 + \vec{p}^2 + \frac{\omega^2}{c^2}P^2} \end{cases} \quad (16)$$

The Hamiltonian, of course, vanishes weakly and the dynamics is entirely in the H_μ constraints. This concludes the Hamiltonian analysis.

Now we study the gauge invariant observables. We start by looking for observables that commute with the momentum constraint H_a , which generates the fixed time diffeomorphism group $Diff_3$. Consider any scalar function R of the three metric (like the Ricci scalar). We define

$$\check{R} \equiv R(X). \quad (17)$$

\check{R} commutes with the momentum constraint. In fact, by writing $H(\vec{f}) = \int d^3x f^a(\vec{x})H_a(\vec{x})$ we have the following Poisson brackets

$$\begin{aligned} \{\check{R}, H(\vec{f})\} &= \{R(X), H^{ADM}(\vec{f})\} - \{R(X), f^a(X)p_a\} = \\ &= \delta_{\vec{f}}R(x)_{x=X} - \partial_b R(X)\{X^b, f^a(X)p_a\} = \\ &= -f^a(X)\partial_a R(X) + f^a(X)\partial_a R(X) = 0. \end{aligned} \quad (18)$$

Thus we have found a class of observables invariant under three dimensional diffeomorphisms.

Let us now consider time. Consider the two observables \check{R} and T , on the phase space $(g_{ab}(\vec{x}), p^{ab}(\vec{x}), X^a, p_a, T, P)$. Given any scalar density f , the Hamiltonian flow of the Hamiltonian constraint $H(f) = \int d^3x f(\vec{x})H_0(\vec{x})$ generates one dimensional orbits in the phase space. Along these orbits, the evolution of \check{R} and T is given by

$$\frac{d\check{R}}{dt} = \{\check{R}, H(f)\}, \quad \frac{dT}{dt} = \{T, H(f)\}. \quad (19)$$

Let $\check{R}(t), T(t)$ be a solution of these equations. For every one of such solutions we define the function $\tilde{R}(T)$ by

$$\tilde{R}(T(t)) = \check{R}(t) \quad (20)$$

or $\tilde{R} = \check{R} \circ T^{-1}$. By construction, for every assigned value of T the quantity $\tilde{R}(T)$ is t -invariant, and therefore commutes with $H(f)$.

How does \tilde{R} depend on f ? It doesn't depend at all on f . This can be shown as follows. Consider two different f which differ infinitesimally by δf . We have

$$\{\tilde{R}, H(f + \delta f)\} = \{\tilde{R}, H(f)\} + \{\tilde{R}, H(\delta f)\} = \{\tilde{R}, H(\delta f)\}; \quad (21)$$

but since

$$\{H(f), H(\vec{f})\} = H(\mathcal{L}_{\vec{f}}f) \quad (22)$$

and since we may get any variation of a scalar density like f by taking its Lie derivative $\mathcal{L}_{\vec{f}}f$ with a certain vector field \vec{f} , we have that there is always a vector field \vec{f} such that

$$\{\tilde{R}, H(f + \delta f)\} = \{\tilde{R}, \{H(f), H(\vec{f})\}\} \quad (23)$$

But since \tilde{R} commutes both with $H(\vec{f})$, because it is defined in terms of 3-diff invariant quantities, and with $H(f)$, by construction, it commutes also with $H(f + \delta f)$ by the Jacobi identity, and therefore with $H(f)$ for any f . Thus, we have shown that the definition of $\tilde{R}(T)$ does not depend on the particular f chosen.

Given any scalar function of the metric R , and any real number T , $\tilde{R}(T)$ defines an observable that commutes with *all* the constraints. Of course, in order to find the explicit form of this observable in terms of phase space variables, we have to solve the evolution equations (19); this is unavoidable in any theory in which the evolution is expressed in terms of an arbitrary unphysical parameter [20]. This fact does not implies that the observable cannot be used if we are not able to solve the evolution equations. A solution of eqs.(19) depends on a set of arbitrary initial condition. Accordingly, we can assume for, say $\tilde{R}(0)$, an arbitrary value. The arbitrariness reflects the freedom in rescaling the origin of T . In particular, we may, without loss of generality, identify $\tilde{R}(0)$ with \tilde{R} . Thus we may start from $\tilde{R}(0) = \tilde{R}$ and compute $\tilde{R}(T)$ order by order in a perturbation expansion for $T \neq 0$.

Note, in particular that the physical interpretation of the observable $\tilde{R}(T)$ (for every assigned real number T) is transparent: it expresses the value of the scalar R in the point where the particle is and at the moment in which the clock displays the value T . Indeed what we have done is just to concretize in formulas the ideas on observability presented in the previous section.

We may now easily generalize the model to a continuum of reference system particles, in order to get a complete material coordinate system and

a complete set of physical observables. We introduce a “cloud” of particles which fill the space¹³, and a clock for every particle. The particles are labelled by a three dimensional continuum index \vec{y} . Since in our notation we are disregarding the need of using more than one chart for the compact space Σ , we assume that \vec{y} runs in a single three dimensional linear space, which we denote $R_{\vec{y}}^3$. The space $R_{\vec{y}}^3$ is the space of the labels, or the “names”, of each particle of the reference system. Having matter elements distinguished by “names” is, in a sense, the peculiar property of any reference system: think, for instance, of a rod and its ticks with numbers. Our Lagrangian is the simplest one¹⁴

$$g_{\mu\nu}(\vec{x}, t), \quad X^\mu(\vec{y}, \tau), \quad T(\vec{y}, \tau). \quad (24)$$

The action is

$$S = \frac{1}{8\pi G} \int dt \int d^3x \sqrt{g} R - mc \int d\tau \int d^3y r_T(\vec{y}, \tau) \quad (25)$$

In three dimensional notation, the action becomes

$$S = \int dt \left[\int d^3x L_{ADM} - mc \int d^3y \sqrt{(N^0(X(\vec{y})))^2 - (\dot{X}^a(\vec{y}) + N^a(X(\vec{y})))^2 - \frac{c^2}{\omega^2} \dot{T}(\vec{y})^2} \right] \quad (26)$$

By repeating the Hamiltonian analysis we obtain again a theory with vanishing Hamiltonian in which, now, the constraints are the following

$$\begin{cases} H(\vec{f}) &= H^{ADM}(\vec{f}) - \int d^3y f^a(X(\vec{y})) p_a(\vec{y}) \\ H(f) &= H^{ADM}(f) - \int d^3y f(X(\vec{y})) \sqrt{c^2 m^2 + \vec{p}^2(\vec{y}) + \frac{\omega^2}{c^2} (P(\vec{y}))^2} \end{cases} \quad (27)$$

This is the theory that shall quantize in the companion paper.

¹³We prefer to keep the denomination of “particles” for this kind matter, rather than using “fluid” or similar, because this matter is not described by collective degrees of freedom like density and pressure, but by the labelling of the single infinitesimal components.

¹⁴These variables are very reminiscent of the “imbedding” variables introduced by Isham and Kuchar [24]. The relation between that approach and the present one deserves to be clarified. Note however that the Isham-Kuchar theory is a reformulation of general relativity, while we are studying a general relativity + matter theory.

By an immediate generalization of the observations done for the single particle reference system, we have that, give any scalar $R(x)$ the quantities

$$\check{R}(\vec{y}) \equiv R(X(\vec{y})) \quad (28)$$

commute with the momentum constraint. It is also possible in this context to define a *Diff3*-invariant quantity $\check{g}_{ab}(\vec{y})$ by

$$\check{g}_{ab}(\vec{y}) \equiv \partial_a X^c(\vec{y}) \partial_b X^d(\vec{y}) g_{cd}(X(\vec{y})). \quad (29)$$

$\check{g}_{ab}(\vec{y})$ satisfies

$$\{\check{g}_{ab}(\vec{y}), H(\vec{f})\} = 0. \quad (30)$$

As we did for the single particle, we define $\tilde{g}_{ab}(\vec{y}, T)$ by

$$\tilde{g}_{ab}(\vec{y}, T(\vec{y}, t)) = \check{g}_{ab}(\vec{y}, t), \quad (31)$$

where $T(\vec{y}, t)$ and $\check{g}_{ab}(\vec{y}, t)$ are defined by the evolution equations

$$\frac{d}{dt} T(\vec{y}, t) = \{T(\vec{y}, t), H(f)\}, \quad (32)$$

$$\frac{d}{dt} \check{g}_{ab}(\vec{y}, t) = \{\check{g}_{ab}(\vec{y}, t), H(f)\}. \quad (33)$$

The quantity $\tilde{g}_{ab}(\vec{y}, T)$ is, for every \vec{y} and T , a physical gauge invariant observable of the theory, which *commutes with all the constraints*:

$$\{\tilde{g}_{ab}(\vec{y}, T), H_\mu(x)\} = 0 \quad (34)$$

Note that, by redefining the integration constants of the solutions of eqs.(33), we may, without any loss of generality, identify $\tilde{g}_{ab}(\vec{y}, 0)$ with $\check{g}_{ab}(\vec{y})$, as we did in the single particle case for R ,

A simple count of degrees of freedom assures us that we have all the possible observables of the theory. We started from $6 \times \infty^3$ degrees of freedom in the metric, $3 \times \infty^3$ in the particles, $1 \times \infty^3$ in the clock. Minus $4 \times \infty^3$ gauge degrees of freedom, we have again $6 \times \infty^3$ physical degrees of freedom in our theory. These are captured by $\tilde{g}_{ab}(\vec{y}, T)$, because the evolution in T is determined by the evolution equations, the initial data of which are, indeed, the correct number. The gauge invariant quantities

$$\tilde{g}_{ab}(\vec{y}, 0), \quad \frac{\partial}{\partial T} \tilde{g}_{ab}(\vec{y}, T)|_{T=0} \quad (35)$$

characterize uniquely the classical state of the gravity+matter system.

Once more, their interpretation is completely transparent: they express the geometry of the manifold of the points defined by the particles, as seen when all the clocks display $T = 0$. In a sense, this set of observables exhausts the dynamics, since it is a complete set in a t -invariant theory. But what we are interested in, is the dynamics of the system with respect to the clocks time, namely we are interested into the full set of observables for all the T , and in the relations between the observables at different T . This new dynamics in T is reconstructed by means of the T -evolution equations which define $\tilde{g}_{ab}(\vec{y}, T)$.

$\tilde{g}_{ab}(\vec{y}, T)$ defines a metric on $R_{\vec{y}}^3 \times R$. This is the metric as measured in terms of the “rods’ ticks” and of the clocks time. This is the observable quantity of the theory.

It is important to stress the fact that the matter degrees of freedom have apparently disappeared from the theory. Indeed they do not really disappear: they have been absorbed in the new metric \tilde{g} , which, unlike g , has not 2 physical and 4 gauge degrees of freedom, but is completely physically observable.

Before concluding this section, there is an additional point that should be discussed here, since later it will be necessary. This is time reversal and the sign of the P variable.

The crucial point to note is that the constraints (and in particular the hamiltonian constraint that generates the evolution in the coordinate time) do not contain T . Therefore P is constant along each solution of the field equations. This follows also easily from the field equations and the definition of P . Thus, P is gauge-invariant and characterizes each trajectory. Now, given a solution of the field equations, consider the other solution obtained from the time reversal coordinate transformation $t \mapsto -t$. All the physical observables $\tilde{g}_{ab}(\vec{y}, T)$ are the same as in the first solution; but P becomes $-P$.

Anytime we have a discrete symmetry of this kind, the formalism allows us to choose the interpretation. We may split the system in two different systems, one with $P > 0$ and the other one with $P \leq 0$, and restrict ourselves to just one of these. In other words, we can add to our system the condition $P > 0$: this condition will be preserved in the evolution.

Physically, the two regions $P > 0$ and $P < 0$ correspond to the motions in which the value of the clock variable T increases or decreases in the coordinate

time t (or x^0). It is reasonable (but not necessary) to interpret a solution in which the clocks run backward simply as a solution in which the clocks run forward, written in time-reversed coordinates.

Note that the situation is exactly analogous to the relativistic particle one, where there are two sectors of solutions, one with $p^0 > 0$, and the other one with $p^0 < 0$, which describe the particle travelling forward or backward in Minkowsky time. Now, it is well known that as far as the particle is free we can (and will) simply disregard as unphysical the solutions in which the particle run backward in time. But in the presence of an interaction, this doubling of solutions is precisely the hint of the existence of antiparticles and of the need of a second quantization. In the case of gravity, it has been suggested several times that the quantization of general relativity may only be consistent as a "first quantization" of a "second quantized" (or "third quantized") theory. We shall not adventure along these lines: our philosophy here is more conservative, and our goal is to apply standard quantum theory to standard general relativity. The point here is that the formalism that we are presenting does allow us to do that easily, because, unlikely others "internal times", the variable T has precisely the property of growing monotonically. So that we can consistently restrict the theory to the region $P > 0$.

In conclusion, we have displayed in this section a generally covariant gravitational theory in which a complete set of gauge invariant observables can be defined. The key step was the introduction of some material degree of freedom and its gravitational interactions. The observables express the distance between the material bodies and their evolution in the clock time.

5 The "local point of view" approximation.

Let us go back to the single particle reference system. An important point to be noted here is that the evolution equations that define $\tilde{R}(T)$ simplify in the context of a certain approximation. Indeed let us take the limit of the theory in which

$$\frac{P}{\sqrt{c^2 m^2 - \vec{p}^2}} \gg \frac{c}{\omega}. \quad (36)$$

$\frac{c}{\omega}$ is a length, therefore this is the limit in which the Hamiltonian dynamical quantities with the dimensions of a length are all very big. Namely we are

considering a regime of big lengths (with respect to $\frac{c}{\omega}$; the precise meaning of $\frac{c}{\omega}$ will be determined later.).

In this limit the Hamiltonian constraint (eq.(16)) becomes

$$H(f) = H^{ADM}(f) - f(X)\frac{\omega}{c}|P|. \quad (37)$$

The absolute value in this equation appears because of the square root of a square in eq.(16). The non differentiability around zero is of course non physical, because when P is close to zero the approximation, which is a large P approximation, breaks down. There are two separated regions in which the approximation holds; one for positive P and one for negative P . Thanks to the discussion at the end of the previous section we can restrict ourselves to the region of positive P and drop the absolute value.¹⁵

The evolution equation that define $\tilde{R}(T)$ becomes

$$\begin{aligned} \{\tilde{R}(T(t)) - \tilde{R}(t), H(f)\} &= 0, \\ \frac{d\tilde{R}}{dT}\{T, H(f)\} &= \{\tilde{R}, H(f)\}, \\ \frac{d\tilde{R}}{dT}\frac{\omega}{c}f(X) &= \{R, H^{ADM}(f)\}_{x=X}. \end{aligned}$$

By taking $f(x) = 1$ around X , we get

$$\frac{\omega}{c}\frac{d\tilde{R}}{dT} = \{R, H_0^{ADM}\}(X) + O\left(\frac{c}{\omega}\right). \quad (38)$$

Let us compare this result with the coordinate time t evolution, in pure gravity without matter, of the value of the scalar function $R(x)$ in the point $x = X$

$$\frac{1}{N}\frac{dR}{dt} = \{R, H_0^{ADM}\}(X). \quad (39)$$

The two evolutions are essentially the same: they are both driven just by the ADM Hamiltonian constraint. We have obtained the result that in the approximation considered the *physically observable* quantity $\tilde{R}(T)$ evolves in the clock time $\frac{T}{\omega}$ precisely in same way in which in the $N = 1$ gauge of pure

¹⁵A related theory, in which the constraints are linear in P not just within an approximation, but in the full theory, has been considered by Karel Kuchar [23].

gravity the *non physically observable* $R(X, t)$ evolves in t . We will come back on the interpretation of this result.

Let us now consider the full reference system. In the approximation previous considered, we have that

$$\frac{\omega}{c} \frac{\partial}{\partial T} \tilde{g}_{ab}(\vec{y}, T) = \{g_{ab}, H_0^{ADM}\}(\vec{y}, T) + O\left(\frac{c}{\omega}\right). \quad (40)$$

The observable quantity $\tilde{g}_{ab}(\vec{y}, T)$ evolves in the approximate theory exactly as the non-observable metric $g_{ab}(\vec{x}, t)$ in pure general relativity (in the gauge Lapse=0, Shift=c).

This result can be expressed as follows. We may interpret pure general relativity as the *approximate* description of a *gravitation + material reference system* theory. We may assume, in this interpretation, that the metric tensor is observable, without violating Dirac determinism. In this sense, therefore, the local interpretation of general relativity can be obtained from the non local interpretation of a coupled system in the limit in which the length $\frac{c}{\omega}$ is small.¹⁶

It is important to stress that this limit is far from being obviously well defined. Everything could go wrong in the limit, and the limit may not be continuous. We have already seen, for instance, that in the case of the point particle the limit is certainly non continuous. Thus, even with "very good reference systems" the behavior of the theory could be drastically different than free general relativity in the local interpretation. This consideration brings some doubts about the complete consistency of the local point of view. We may say that the local point of view is equivalent to the non-local one (which is the fundamental one) only up the difficulties that may emerge in taking this limit.

Let us discuss more in detail what kind of approximation is the one considered. The limit $\frac{c}{\omega} \mapsto 0$ which defines the approximation has to be taken at fixed T and P . Since the relation between P and \dot{T} involves $\frac{c}{\omega}$, the limit appears slightly different in the Lagrangian context. By substituting Lagrangian quantities in the inequality that defines the approximation we get

$$(N^0)^2 - c^2 \omega^{-2} \dot{T}^2 \ll c^2 \omega^{-2} \dot{T}^2. \quad (41)$$

¹⁶The approximation that we considered correspond to step 1 of section 3. By adding an arbitrary force acting on the reference system, one can introduce the indeterminism and obtain any other gauge.

In order that under the square root there be a positive quantity, we must have

$$(N^0)^2 > c^2\omega^{-2}\dot{T}^2. \quad (42)$$

Thus, the approximation is as good as $c\omega^{-1}|\dot{T}|$ comes close to N^0 (and $|x^a + N^a|$ goes to zero).

In order to understand this formula, consider the solution of the equation of motion of the clock. Expressed in term of the proper time s/c (in seconds), this is

$$T(s) = \alpha \frac{s}{c} + T_0. \quad (43)$$

By inserting this solution in the Lagrangian, the formulas (41) and (42) correspond to

$$\alpha < \omega, \quad \omega - \alpha \ll \omega : \quad (44)$$

the approximation is good for α very close to ω .

What we have shown is that the “clock” defined by our theory has a maximum speed. The approximation is good in the regimes in which the frequency α of the clock approaches the maximum value, which depends on the length $\frac{c}{\omega}$.

Eq. (43) provides also a precise interpretation of the quantity c/ω . In the regimes in which the approximation is good ω is equal to α and therefore $1/\omega$ is the proper time elapsed while the variable T increases by one unit, namely ω is the (proper time) frequency of the clock, and, of course, c/ω is the corresponding length.

For $\alpha \mapsto \omega$, the Lagrangian becomes imaginary, and the energy of the clock goes to infinity:

$$E = mc^2 \frac{\alpha/\omega}{\sqrt{1 - \frac{\alpha^2}{\omega^2}}} \quad (45)$$

In the limit in which ϵ is small and m too is small, in such a way that the energy E is small, we have the result that $\tilde{g}_{ab}(\vec{y}, T)$ satisfies the vacuum Einstein equation.

It is not clear to us to what extent the existence of the maximum speed of the clock is a peculiar feature of our model, or to what extent it is a consequence of some physical reason.

Before concluding this section, it is worth to make a comparison with non gravitational physics. Of course reference systems are needed anytime

there is some physics in space-time. So what is peculiar about the reference systems in gravity ?

In any non gravitational theory, it is always possible to choose a reference system which is not affected by the system under consideration. For instance, we may define space-time points in Maxwell theory by means of non charged objects that do not interact with the electromagnetic field. In the process of a measurement there will be some small interaction between reference systems objects and the Maxwell field, but (we are here in classical physics) this interaction can be made as small as one wishes. For instance, by taking the reference system heavy enough one can make the influence of the measurement on the reference system smaller than whatever given experimental error.

On the contrary, in gravity it is not possible to disregard the way the reference system is affected by the field, because an object which does not interact with the gravitational field is a non existing object. More than that, there is no way to make this interaction small. The action of the reference system over the gravitational field can be made small by taking a light reference system, but there is no way to disregard or to diminish the action of the gravitational field on the reference system. It does not help to take the reference system very heavy, or very light: the motion of everything is affected by gravity because gravity *defines* the inertial motion of objects, and there is no way to have objects that do not feel gravity: it is not even possible to write an equation of motion for a material object without the gravitational field (if the required invariances have to be respected).

Thus, while the action of the Maxwell field on the reference system can be made as small as ones want, the action of gravity on the reference system can never be neglected. This is reflected in the fact that in the Maxwell theory if we assume that we have an external reference system, given for every time, everything is fine, but if we assume that we have an external reference system given for every time in Einstein theory, we break general covariance.

From a more mathematical point of view the argument goes as follows. In non general relativistic theories, there is an assumed background structure of space-time which singles out a preferred (finite dimensional) class of reference systems (the inertial systems). (The Poincare group, which relates the inertial reference systems, is not a gauge group, because once things are fixed at $t = 0$, they are fixed forever, and the resulting Poincare invariance is not a gauge invariance.) In general relativistic theories, on the contrary, the

invariance group of the space-time manifold is infinite dimensional, and nothing picks up a preferred coordinate system (or a preferred finite dimensional class of coordinate systems). The only way to have a preferred coordinate system is to let a material reference system define it.

But since, as we said, for whatever material reference system, the motions of the material reference systems objects are different in the presence of different gravitational fields, it follows that the dynamics of the reference system cannot be disentangled from the dynamics of the gravitational field itself. In principle, nothing forbids that gravitational degrees of freedom could be used to define a reference frame; in practice this has never been achieved theoretically and appears hopeless experimentally. Therefore, either we live without a definition of physical points (as in the non-local interpretation), or we explicitly introduce in the dynamical system the material reference system (as we did in the previous section), or, finally, we work in an approximate theory (as we did in this section).

6 Conclusions.

The main result of the present paper is that gravitational physics cannot be properly understood unless one takes into account the physical nature and the gravitational interactions of the bodies that form the reference system.

The reference systems are needed in order to specify the space-time points in which the measurements are performed and cannot be considered as independent from the gravitational field. In the classical theory one can always work in an approximation in which the effects and the dynamics of the material reference systems are neglected; in the quantum theory, however, the consequences of this facts are far reaching. These will be analyzed and discussed in the companion paper ref.[\[10\]](#).

A specific dynamical model of reference system has been introduced and analysed in this paper. Other models (more realistic, more fundamental) can certainly be studied.

Material reference systems provide a definition of physical space-time points. This definition allows us to introduce in general relativity a notion of locality. We recall that locality otherwise is a very obscure concept in a general covariant theory (“hole argument”). In terms of this notion of

physical locality, one can define local observables and local operators.¹⁷

The history of physics teaches us that “external” objects, represented in the theory by “absolute” concepts disappear from fundamental physics as physics develops. In this paper we have argued that general relativity *forces* us to abandon the concept of “external” reference system, in favor of an “internal” one, described by variables of the system itself. The elimination of the external-absolute reference system is automatically required by general covariance; in the theory, points are not absolute and specified a priori, but are defined by the dynamical system itself. To our perception, this conclusion reveals how, also in dealing with the concept of physical reference system, general relativity is profound and the extraordinary beautiful.

¹⁷Perhaps one can also introduce in this way a thermodynamics in general relativity, which is still an unsolved problem.

References

- [1] J.A. Wheeler, in *Battelle Rencontres 1967* ed. by C. DeWitt, J.A. Wheeler, Benjamin N.Y. 1968.
- [2] For a review of the present status of perturbative and nonperturbative methods in quantum gravity see C.J. Isham, *Quantum gravity*, in *General Relativity and Gravitation, proceedings of the 11th International conference on General Relativity and Gravitation (GR11), Stockholm 1986*, Cambridge University Press, Cambridge 1987. A. Ashtekar *Quantum Gravity, self-duality, Wilson loops and all that*, invited talk at the GR12, to appear on the proceedings, Boulder Colorado 1989. E. Alvarez, *Quantum gravity, an introduction to some recent results*, [Rev. Mod. Phys. **61**,561 \(1989\)](#).
- [3] L. De Broglie, in *International Philosophical Conference, Amsterdam 1938*, Amsterdam, 1938.
- [4] [C. Rovelli, L. Smolin, *Loop representation for quantum General Relativity*, Nucl. Phys. B133 \(1990\) 80; Phys. Rev. Lett. **61**, 1155 \(1988\)](#). C. Rovelli, in [Proceedings of the Osgood Hill meeting on *Conceptual problems of quantum gravity* Boston 1988, \(Birkhauser, Boston in press\)](#); L.Smolin, in [Proceedings of the Osgood Hill meeting](#), op.cit. .
- [5] J.Stachel, in *General Relativity and Gravitation, proceedings of the 11th International conference on General Relativity and Gravitation (GR11), Stockholm 1986*, op.cit. ; in *Proceedings of the 4th Marcel Grossman meeting on General Relativity* ed. R. Ruffini, North Holland, Amsterdam 1986.
- [6] [Proceedings of the Osgood Hill meeting on *Conceptual problems of quantum gravity* Boston 1988, op.cit. .](#)
- [7] This point of view is implicit in many textbooks on general relativity. It necessarily follows if the space-time manifold M is introduced as the manifold of the physically determined "events". For a discussion of this point see ref.[5].
- [8] P.M. Dirac, *Lectures on quantum mechanics*, Belfer Graduate School of Science, Yeshiva University, New York, 1964.

- [9] [A. Einstein, in *Albert Einstein and Michele Besso correspondence 1903-1955*, see the letter of January 3rd 1916, ed. P. Speziali, Hermann, Paris 1972; *Relativity and the problem of space* in *Relativity: the special and general theory*, Crown, New York 1961; in *Concepts of Space* by Jammer, Harvard University press, Cambridge, Mass. 1954.](#)
- [10] [C. Rovelli, *Quantum reference systems*, companion paper.](#)
- [11] [B.S. DeWitt, Phys. Rev. **160**, 1113 \(1967\).](#)
- [12] [M. Toller, Nuovo Cimento **44B**, 67 \(1978\).](#)
- [13] K. Kuchar in ‘Quantum Gravity 2’ eds. C.J. Isham, R. Penrose, D.W. Sciama. Oxford University Press, Oxford 1982.
- [14] J. Norton in *Proceedings of the 4th Marcel Grossman meeting on General Relativity*, op.cit. ; J. Earman, J. Norton, Pittsburgh University. Philosophy department. preprint 1986; J. Norton in *Measurement, realism and objectivity* ed J.Forge, Reidel 1985.
- [15] [S.W. Hawking, G.F.R. Ellis, *The large scale structure of space-time*, see pg.56, Cambridge University Press, Cambridge 1974.](#)
- [16] R.A. Coleman, H. Korte, in the GR12 abstracts, Boulder, Colorado 1989.
- [17] B.S.DeWitt, in *Gravitation: an introduction to current research*, ed. L. Witten.
- [18] [Bridgman, *The Logic of Modern Physics*, New York 1927.](#)
- [19] [S. Carlip, *Exact quantum scattering in 2+1 dimensional gravity*, Princeton preprint, IASSNS-HEP-89/4 \(1989\).](#)
- [20] [C. Rovelli, it Quantum mechanics without time: a model, in print on Phys. Rev. D \(1990\); *Time in quantum gravity: Physics beyond the Schrödinger regime*, University of Pittsburgh preprint PITT-90-02.](#)
- [21] K.Fredenhagen, R. Haag, Desy preprint 86-8066.

- [22] S. Weinberg, *Gravitation and Cosmology*, John Wiley and Sons, New York 1972.
- [23] K. Kuchar, private communication.
- [24] [C.J. Isham, K. Kuchar, Ann. Phys. **164**, 288 and 316 \(1985\).](#)