COGNITION AND INSTRUCTION, *23*(2), 237–309

Why Cannot Children See as the Same What Grown-Ups Cannot See as Different?– Early Numerical Thinking Revisited

Anna Sfard
The University of Haifa and Michigan State University

Irit Lavie
The University of Haifa

To think is to forget differences … . (Borges, 1964, p. 94)

Based on close observations of two 4-year-old children responding to their parents' requests for quantitative comparisons, we offer a "participationist" account of the origins and development of numerical thinking, one that portrays numbers as a product rather than a pregiven object of human communication. In parallel, we propose a participationist reinterpretation of relevant past research, most of which has been guided by the metaphor of learning-as-acquisition (of mental schemes, of concepts, etc.). The point of departure for our analyses is the assumption that thinking can be usefully conceptualized as a special case of activity of communicating and that learning arithmetic can be thought of as a development of a specialized discourse. We claim that the development of this discourse involves the ability to see as "the same" things that, so far, could only be seen as different. In the longer run, this ability will lead to the *objectification* of the discourse—that is, to the use of number words as if they signified discourse-independent entities "out there" in the world. This development commences in child's ritualized participation in arithmetical routines of grown-ups and continues in a gradual transformation of the rituals into genuine explorations. Paraphrasing Vygotsky (1978), we conclude that the numerical discourse that begins as an interpersonal affair turns in the growing mind into a matter of one's relation with human-independent world. We also claim that this kind of development

Requests for reprints should be sent to Anna Sfard, Michigan State University, DSME, Wells Hall D325, East Lansing, MI 48824. E-mail: annasfar@math.msu.edu

is a 1-way process, and the change from the interpersonal to between-person-and-the-world outlook, once accomplished, can hardly be reversed.

RECONCEPTUALIZING (NUMERICAL) THINKING: WHY DO WE NEED IT AND WHAT IS THE NATURE OF THE TASK?

> Human communication will almost always go astray unless real energy is expended. (Reddy, 1978, p. 295)

Ever since the advent of the disciplined inquiry into human cognition, numerical thinking has been a paradigmatic object of study. The tradition goes back to Jean Piaget (1952) and continues with the wide variety of developmental psychologists and misconceptions seekers, ending up, at least for now, with "participationist" writers who, like Jean Lave (1988) and Valerie Walkerdine (1988), vowed to reclaim the place of the social within the time honored trinity world–society–individual.[1] Throughout history, students of the human mind were often divided on questions of epistemology, of methodology, and as a result, of the meaning of observed phenomena, but they always agreed that mathematical thinking is a perfect setting for uncovering general truths about children's cognitive development (cf. Reed & Lave, 1979).

In this article, while showing the inherent difficulty of the endeavor, we claim that in spite of the work already done, the developmental psychologist's favorite subject requires as much attention as ever, whereas any further progress may depend on our ability to rethink traditional assumptions about human thinking. Conceptual turnaround is likely, indeed, to be the most promising way out of the controversies that have been pervading the inquiry into numerical thinking ever since it's beginning. The profusion of research data on what is regarded as an early numerical thinking notwithstanding, there is little consensus as to the way in which these data should be interpreted. Piaget's famous conservation experiments, the paradigmatic moot point in the ongoing debate on children's arithmetic (for a well-written summary of this debate, see Dehaene, 1997) may be enough to justify an attempt at reconceptualization.

Let us look at a concrete instance of children's grappling with numbers to show how the post-Piagetian conceptual system, within which we all operate as if by default, falls short of fulfilling its role of sense-making apparatus. The episode is the

---

[1]The "participationist" perspective, one that conceptualizes learning as becoming a participant in certain activity (Lave, 1988; Lave & Wenger, 1991), is contrasted here with the "acquisitionist" perspective, grounded in the metaphor of learning as acquisition (of mental scheme, concept, skill, and so forth (cf. Sfard, 1998).

beginning of the 20-min long conversation between 4-year-old Roni, her friend Eynat who is 7 months older, and Irit Lavie, who is Roni's mother (at a later stage, Roni's father, who was initially supposed to serve just as a cameraman, will spontaneously join the exchange; for the transcript of the entire conversation see the Appendix). Eynat, whom Roni knew since birth, is a daughter of Roni's parents' close friends. Both couples are well-educated professionals. The event took place in Roni's house. The aim of the conversation was to probe the two girls' arithmetical competence. The mother knew in advance that the children had already attained proficiency in counting, and she believed that they might now be able to apply this skill in comparing sets of varying cardinality. What actually happened surprised the grown-ups and left them puzzled.

Episode I

1.	Mother	I brought you two boxes. Do you know what is there in the boxes?	Puts two identical closed opaque boxed on the carpet, next to the girls.
2.	Roni	Yes, marbles.	
3a.	Mother	Right, there are marbles in the boxes.	While saying this, points to the box close to Eynat, then to the other one.
3b.	Mother	I want you to tell me in which box there are more marbles.	
3c.	Eynat		Points to the box which is closer to her.
3d.	Roni		Points to the box Eynat is pointing to.
4.	Mother	In this one? How do you know?	Points to the box the girls pointed to.
5.	Roni	Because this is the biggest than this one. It is the most.	While saying "than this one" points to the other box [the one close to her]
6.	Mother	Eynat, how do you know?	
7.	Eynat	Because… cause it is more huge than that.	Repeats Roni's pointing movement when saying "than that"
8.	Mother	Yes? This is more huge than that? Roni, what do you say?	Repeats Roni's and Eynat's pointing movement when saying "than that"
9.	Roni	That this is also more huge than this.	Repeats the above pointing movement when saying "than this"
……		……	……
10a.	Mother	Do you want to open and see? Let's open and see what there is inside. Take a look now.	
10b.	Roni		Abruptly grabs the box which is nearer to Eynat, and which was previously chosen by the girls as the one with more marbles.
10c.	Eynat		Tries to grab the same box, but gives up; turns to the other box
11.	Roni	1.. 1.. 1.. 2, 3, 4, 5, 6, 7, 8.	Opens the box and counts properly.
12.	Eynat	1, 2, 3, 4, 5, 6.	Opens the other box and counts properly.
13.	Mother	So, what do you say?	
14.	Roni	*6*	

15.	Mother	Six what? You say 6 what? What does it mean "six"? Explain.	
16.	Roni	That this is too many.	
17.	Mother	That this is too much? Eynat, what do you say?	
18.	Eynat	That this too is a little.	
19.	Mother	That it seems to you a little? Where do you think there are more marbles?	
20.	Roni	I think here.	Points to the box which is close to her (the one with 8 marbles)
21.	Mother	You think here? And what do you think, Eynat?	
22.	Eynat	Also here.	

Indeed, the conversation did not turn out as expected. Although presented with two identical opaque boxes and asked which of them contained more marbles, the girls did not bother to open the boxes and made an immediate choice ([3c], [3d]). Not only did they do this and agree in their decision, but they were also perfectly able to "justify" their action in the way that would appear adequate if not for the fact that the girls had no grounds for the comparative claims they made ([5], [7], [9]). If the startled mother hoped that her interrogation about the reasons for the choices ([4], [6], [8]) would stimulate opening the boxes and counting the marbles, she was quickly disillusioned: Nothing less than the explicit request to open the boxes ([10a]) would help. On the other hand, once encouraged to do this, the girls did not need explicit prompting to count the marbles and perform numerical comparison. It is as if the mere sight of the marbles prompted the proper routine action. As initially conjectured by the parents, the girls performed the counting and the comparing without a glitch.

The question may now rightly be asked why, if so skillful in the procedure, the children did not apply it in the first place. The explanation that in this initial task they simply were not yet aware of what was expected from them does not seem to work: The phenomenon of choosing before counting repeated itself in the next task, in spite of the fact that the immediate former experience might have already taught the girls the rules of the game.

Episode II$_a$

34a.	Mother	I am putting two boxes with marbles here. Where… in which one of them are there more marbles? Tell me.	The interviewer puts one box next to Roni and another next to Eynat. Both boxes are closed.
34b.	Roni		Exchanges the placement of the two boxes. Then changes her mind and reproduces the original arrangement of the boxes.
34c.	Eynat		Points to her box without opening it.
35.	Mother	Where are more marbles? What do you think?	

36a.	Roni		Opens her box and closes it immediately.
36b.	Roni	2.	
37a.	Mother	Eynati, do you want to open yours?	
37b.	Eynat		Opens her box and closes it immediately.
37c.	Eynat	2.	
38.	Mother	2… Indeed? Where are more marbles?	
39.	Roni	In none.	

In fact, nothing changed until the end of the session: The same scenario could be seen time and again, whenever the girls were presented with the boxes and asked "In which box are there more marbles?" (see the initial parts of the other two episodes in the Appendix).

Some readers are likely to be too familiar with the fact that "children who know how to count may not use counting to compare sets with respect to number" (Nunes & Bryant, 1996, p. 35) to be mystified by this present case. Yet, knowing what children *do not* do is not enough to account for what they *actually do*. Indeed, our young interviewees' insistence on deciding which box "has more marbles" without performing any explorations is a puzzle in itself, one that is not explained by the previously quoted time-honored observation. On the face of it, the phenomenon is close to what usually happens when "pre-operational" children are faced with number-conservation tasks. Here, too, the young interviewees tend to claim inequalities of different sets on the basis of considerations other than the results of counting. However, none of the researchers' speculations occasioned by the conservation tasks seems to apply to this case.[2] Unlike in conservation tasks, Roni and Eynat make their claims about the inequality without actually seeing the sets, so we cannot ascribe their choices to any visible difference between the objects of comparison (just to restate, the two closed boxes were indistinguishable). Neither can the children's decisions be seen as the result of the well documented interviewees' tendency to change their answer following the repetition of the interviewee's question: The girls chose one of the indistinguishable boxes already the first time round, before the parents had a chance to reiterate their request. As if all these unanswered questions about the reasons for the children's actions were not enough, the episode gives raise to additional quandaries: Why are the girls in such a perfect accord about their choices even though these choices seem arbitrary? What is it

[2]In one of the classical conservation tasks the child is shown two equivalent rows of counters arranged into a visible one-to-one correspondence. After the child decides which of the sets contains more marbles, one of the rows is "stretched" so that it becomes longer without becoming more numerous. When asked which row has more marbles, even those children who previously answered that "no row has more" usually point to the one that has been stretched. Researchers (e.g., McGarrigle & Donaldson, 1974) speculated that this new answer stems from the fact that, first, the children do not necessarily relate the words "has more" to the number of counters but rather attend to other easily visible properties of the sets, such as length; and second, in school culture, reiteration of the question is usually read as cluing about the need to change the answer (Mehan, 1979).

that evidently makes the chosen box so highly desirable? (Note that each of the girls wants this box for herself, see e.g., [10b,c].) Why after making the seemingly unexplainable decisions are the children able to answer the request for justification? On what grounds do they claim that what they chose is "the biggest" or "more huge"? None of the Piagetian claims about children's "conception of number" and, in particular, about their inability to "conserve the number" or their tendency to eschew counting suffices to answer any of these queries.

Considering the particular resilience of these and similar questions, it seems that the problem may reside in the conceptual tools we are using while trying to make sense of what we see. Although perfectly able to interpret what the grown-ups are doing, we fail when applying the same kinds of interpretation to the children's actions. Thus, for example, it does work well for us to assume that the grown-ups' questions are intended to elicit statements about the world and that the mother expects boxes to be assigned with numbers. Based on our own experiences we may also claim that these numbers appear to her, along with the boxes, as constituting an integral part of the mind-independent world. As will be argued throughout this article, this last statement is not necessarily also true for children. This, however, does not mean that children's actions are devoid of inner logic. Rather, if we are unable to see the children's reasons, it is likely to be the result of our tendency to interpret their utterances the way we would interpret our own. Yet, the children's use of words may be dramatically different from that of grown-ups.

Similarly, foundational assumptions hidden in our research language are likely to be the principal factor underlying our inability to come up with satisfactory interpretations. Such terms as *conservation* or *acquisition of [the concept of] number* are, indeed, ontologically and epistemologically laden, and those who use these terms subscribe, often unwittingly, to the "objectifying" world view according to which the notions related to number refer to externally given entities. The goal we set to ourselves in this article is to develop a terminology that would be free of these particular foundational entailments. If our conjecture about the sources of our interpretational difficulties is true, such linguistic shift may result in disappearance of at least some of the dilemmas.

The alternative approach offered on these pages is rooted in the vision of numerical thinking as an evolving form of communication. Because communication is an inherently social phenomenon, one may say that in research that grows up from this basic assumption we will be realizing the "strong programme" in the sociology (and psychology) of mathematics, in the sense of Bloor (1976): While analyzing the developing mathematical communication, we will be investigating not just "the form" in which mathematical knowledge "finds its expression," but "the very content and the nature of [this] knowledge" (p. 3). Following our basic tenet on the communicational nature of human thinking, we will sustain that numbers are discursive constructs, created for the sake of communication about the world. This, clearly, will bring about the reversal of the commonly assumed developmental order: Numbers will no longer be seen as primary to, and independent of, human discourse, as something that people merely "describe" in their talk; instead, numbers will become a by-product of the discursive growth, a result of human attempts to create better tools for communicating about the world. Rather than inquiring about children's "acquisition of the concept of number" we will thus be asking: Where does children's talk about numbers come from, how is it different from that of the grownups, and what can possibly make it change over time? Our main conjecture is that the children's numerical discourse lacks the objectification that is typical of the numerical talk of the adults. The term *objectification* will be clarified in operational terms following the introduction of the communicational framework.

Before actually engaging in the task of reconceptualization and its testing, let us make sure that its objective is well understood. Changing a discourse is not a trivial matter and for more than one reason. To begin with, trying to speak about seemingly well-known phenomena in a nonroutine way implies changes in the uses of familiar words, such as *number* or *learning*, and this kind of change requires a complex bootstrapping process that would not succeed unless the person is fully dedicated, disciplined, and always aware of her own discursive ways and of their possible alternatives. Further, changing a discourse is, clearly, a metadiscursive task, but because of the inherent blurriness of the discursive hierarchy, our efforts may be easily mistaken for an object-level attempt to establish facts about the extradiscursive reality. In other words, the effort to replace a familiar discourse with another one may be misinterpreted as a factual disagreement with the results of former research. For the same reason, the attempt "to speak differently," although potentially far-reaching in its consequences, would often be dismissed as a mere "semantic game." As we go on, we urge the reader to remain wary of all these undesirable interpretations.

THINKING AS COMMUNICATING AND MATHEMATICS AS A FORM OF DISCOURSE

The world does not speak. Only we do. (Rorty, 1989, p. 6)

As mentioned before, the long history of systematic inquiry into human numerical thinking can be roughly divided into two strands: the *acquisitionist* strand that began with the seminal work of Piaget, and the *participationist* strand, grounded in the claims about the sociocultural origins of human knowing and on the inherent situatedness of learning (Brown, Collins, & Duguid, 1989; Lave, 1988; Rogoff, 1991, 2003). Our objective in this article is to demonstrate consequences—and merits—of the participationist approach or, more specifically, to show how the assumption that "there is no physical relation for any infant which is not always and already social" (Walkerdine, 1988, p. 16) changes the vision of what it means to do mathematics and how, following this change, some long-standing controversies, such as the one surrounding Piaget's conservation tasks, seem to disappear. Our stance is, in a sense, an elaboration and radicalization of the one adopted by Valerie Walkerdine. Although Walkerdine is certainly not the only author to oppose the Platonic vision of the notion of number and to speak about mathematics in participationist terms,[3] she is one of the most outspoken, and most explicitly nonacquisitionist,[4] writers. Although Walkerdine's intellectual history is not much different from that of the majority of other investigators who dealt with mathematical thinking in participationist terms (Beach, 1995; Cole, 1996; Lave, 1988; Nunes, Schliemann, & Carraher, 1993; Saxe, 1991; Wertsch, Minick, & Arns, 1984)—like all the others, she began as a Piagetian and ended up disillusioned with the traditional acquisitionist approach—her work is made distinct by its innovative conceptual framework, inspired by the modern semiotics.

The acquisitionist and participationist schools differ in their ontological and epistemological foundations to such extent that, being participationally minded, we find it practically impossible to summarize the findings of the acquisitionist research in its own terms. We thus postpone the discussion of this latter strand until after introduction of our own language and presentation of our research. The rest of this section is devoted to the first of the two tasks—the introduction to the communicational perspective on human thinking.

Taking Walkerdine's semiotic approach one step further, we view *thinking* as a *special case of communication*, or as a type of discursive activity (cf. Ben-Yehuda, Lavy, Linchevski, & Sfard, in press; Edwards, 1997; Harré & Gillett, 1995; Sfard, 2000, 2001a, 2001b). [5] This is the basic tenet of the conceptual framework that, for obvious reasons, we call *communicational*. The word *discourse* is used here to denote any act of communication, whether verbal or not, whether with others or with

---

[3]The anti-Platonic movement in the philosophy and sociology of mathematics has a long history, and it goes back to the formalist and intuitionist schools of thought, best represented by the mathematicians David Hilbert (1862–1943) and Luitzen Egbertus Jan Brouwer (1881–1966), respectively. Even earlier, it can be found, albeit in a slightly different form, in the philosophical writings of John Stewart Mill (1843/1959). More recently, non-Platonism has been promoted by sociologists of science (e.g., Bloor, 1976), semioticians (Rotman, 1994, 2000), and in educational publications (e.g., Ernest, 1991).

[4]Walkerdine's (1988) heightened nonacquisitionist awareness did not prevent her from falling, every here and there, into acquisitionist trap, inherent in the discourse that she was occasionally using even as she was trying to leave it behind. Thus, for example, while criticizing those who talk about "the acquisitions of concepts or of meanings as if though it was a function of an object world" (p. 16), she does not point to the fact that the source of the identified weaknesses of this approach is in the very expressions "acquisition of concept or of meaning" that reify the activities under study.

[5]Walkerdine herself, although concerned in her research with what she calls "mathematical discourse," never explicitly subscribed to the equation "mathematics = a kind of discourse" that we promote in this article.

oneself, whether synchronic, like in a face-to-face conversation, or asynchronous, like in exchange of letters or in reading a book. Our focus in this article is on one particular type of thinking, which we call *mathematical.* Later, we narrow our debate down to numerical discourse, which is but a special subcategory of mathematical discourses. Four basic features of communication will be considered whenever we try to decide whether the given instance of discourse does, indeed, deserve the name "mathematical": keywords and their use, visual mediators, endorsed narratives, and routines. Let us explain in a few words what is meant by each of these terms.

A discourse counts as *mathematical* if it features mathematical words, such as those related to quantities and shapes. The conversation between Roni, Eynat, and Roni's mother, replete with such mathematical terms as number-words and comparison-words (e.g., *more*, *bigger*), can thus count as a case of mathematical discourse. This, however, is just one out of many possibilities of mathematical communication. Although many number-related words may appear in nonspecialized, colloquial discourses, mathematical discourses as practiced in schools or in the academia dictate their own more disciplined uses of these words. *Word use* is an all-important matter because, being tantamount to what others call "word meaning" (Wittgenstein, 1953), it is responsible for how the user sees the world. As will be argued in the course of our analyses, Roni and Eynat are not yet using any of the mathematical words the way these words are used by mathematically versed interlocutors. Let us emphasize that while speaking about the idiosyncrasies of children's discourse we do not mean just grammatical imperfections. Although we find some of such grammatical irregularities quite telling, our focus is on other more global distinctive properties of keyword use, such as the presence, or absence, of *objectification* (this term will be explained throughout our analyses).

Visual mediators are means with which participants of mathematical discourses identify the object of their talk and coordinate their communication. Whereas colloquial discourses are usually mediated by images of material things, that is, by concrete objects that are pointed to with the nouns or pronouns and that may be either actually seen or just imagined, mathematical discourses often involve symbolic artifacts created specially for the sake of this particular form of communication. Such symbolic mediation, however, is still absent from the incipient numerical talk of our young interviewees. Quite understandably, the only form of visual mediation that can be found in our data is concrete rather than symbolic: The mathematical task performed by the girls is described in terms of sets of marbles provided by Roni's mother and is visually mediated by these sets. In this case, the mediators are not merely seen, but also physically manipulated (e.g., the girls are touching the marbles while counting them), and this physical procedure is part and parcel of their numerical discourse. More generally, the physical operations on visual mediators—with the most elementary of them being the procedures of scanning the mediator with one's eyes in a well-defined way—would often become automated and embodied, that is, would be remembered, activated, and implemented in the direct response to certain discursive prompts, as opposed to implementation that requires deliberate decisions and the explicit recall of a verbal prescription for these operations.

Endorsed narratives are sets of propositions that are accepted and labeled as *true* by the given community. In the case of mathematical discourse, the narratives, to be endorsed, have to be constructed and substantiated according to a set of well-defined rules, specific to this discourse. Indeed, the way mathematical statements are created and substantiated differs considerably from how one creates and endorses, say, historical, sociological, or scientific narratives. In the case of scholarly mathematical discourse, these endorsed narratives are known as *mathematical theories*, and this includes such discursive constructs as definitions, proofs, and theorems.[6] Needless to say, our young interviewees are too inexperienced to be aware of the extensive bulk of generally endorsed mathematical narratives. Their present repertoire is restricted to the basic "number facts," such as the simplest equalities (e.g., *2 and 2 is 4*, see [51]) and inequalities (e.g., *10 is more than 4*, see [76]; note that we may decide that the interlocutors are aware of such statements even if none of them is actually uttered in exactly this form). In addition to the generally endorsed "abstract" narratives such as those quoted previously, one can speak about more specific narratives that pertain to concrete objects and may be endorsed in a given situation. The aim of Roni and Eynat's activity, at least in the eyes of the grown-ups, is to create such more locally endorsable narratives: The girls are supposed to explore the boxes with marbles and to come up with endorsable statements that answer the mother's question "Which of the boxes has more marbles?"

Routines are well-defined repetitive discursive patterns characteristic of the given discourse. Specifically mathematical regularities can be noticed whether one is watching the use of mathematical words and mediators or follows the process of creating and substantiating narratives about numbers or geometrical shapes. In fact, such repetitive patterns can be seen in almost any aspect of mathematical discourses: in mathematical forms of categorizing, in mathematical modes of attending to the environment, in the ways of viewing situations as "the same" or different, which is crucial for the interlocutors' ability to apply mathematical discourse whenever appropriate—and the list is still long. Our use of the term *routine* is close to the usage proposed by Schutz and Luckmann (1973) and applied in the context of mathematics learning by Voigt (1985).

---

[6]More generally, *narrative* is any text, spoken or written, framed as a description of objects, of relations between objects, or activities with or by objects. Terms and criteria of endorsement may vary considerably from discourse to discourse, and more often than not, the issues of power relations between interlocutors, which in principle should not be relevant in this context, may in fact play a considerable role. This is certainly true about social sciences and humanistic narratives such as history or sociological theories. Mathematical discourse is supposed to be impervious to any considerations other than purely deductive relations between the narrative's different elements.

Spontaneous human tendency to reiterate previously learned discursive behaviors is what makes communication possible. Indeed, it is thanks to the regularities in our behavior that interlocutors can interpret what others are saying and are able to decide what kind of response is expected. Yet, in the majority of discourses the participants are unaware of the fact that their actions disclose structural regularities, and they certainly cannot be said to "follow the rules" of the discourse in a conscious, intentional manner. A distinctive feature of mathematical activity is that it tries to make some of its rules explicit. Indeed, such metadiscursive attempts as formulating definitions that would later control mathematical word use constitute an integral part of the mathematical discourse itself.

Although our conversations with Roni and Eynat were supposed to be "noninterventional," they were still full of opportunities for learning, if only because the counting and comparing-by-counting procedures were constantly in focus and were incessantly rehearsed. In general, one's knowledge of discursive routines is only in part a result of intentional teaching. Although the previously mentioned numerical procedures were the intended object of learning, we had also an opportunity to observe a spontaneous emergence of an *ad hoc* routine that surprised the parents and persisted in spite of the mother's efforts to counteract its recurrence. The sequence of action presented in Figure 1 repeated itself in each of our four episodes in reaction to the mother's question "Which box has more marbles?" Note that different routines may be nested one in another. In this case, the routines of comparing-by-counting are but a part of the "big" routine presented in Figure 1.

To complete these foundational preparations, let us mention the *unit of analysis* that is employed in this study. Although *number concept* seems to be an obvious candidate, we wish to argue that it is too confining to be a good choice. Vygotsky, who was one of the first researchers to devote direct attention to the issue of the unit of analysis, and who agreed, in principle, that *concept* may successfully play this role, defined the term *concept* as a word together with its meaning (Vygotsky, 1987). If combined with Wittgenstein's interpretation of the notion of meaning ("The meaning of a word is its use in language," Wittgenstein, 1953, p. 20) the term *concept* becomes tantamount to the *word together with its discursive use*. Discursive use, in turn, means the totality of proper combinations in which the word may appear. If so, the use of a word cannot be treated as a stand-alone entity and cannot be considered in separation from the discourses of which it is a part. The combined

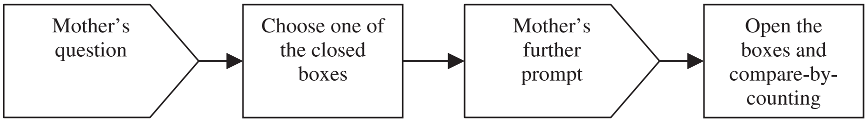

FIGURE 1 Ad hoc routine developed throughout the interaction.

Vygotskian–Wittgensteinian rendition of the term *concept* makes it clear that it is the entire discourse on numbers, rather than a single number concept, that should be considered as the proper unit to be studied by those interested in the development of numerical thinking.

The incipient numerical discourse of our young interviewees is the object of analysis in the rest of this article. Our focus will be on two characteristics of the girls' numerical talk: on their numerical keywords uses and on their routine ways of approaching numerical tasks. While investigating these properties, we will be asking ourselves what has yet to change in the children's discursive activity before they can count as fully fledged, "nonperipheral" (Lave & Wenger, 1991) participants of what is generally regarded as a proper numerical discourse. Given that the latter type of discourse is represented in our data by the contributions of the two adults who participated in the conversation, it will be called here "the grown-ups' discourse."

HOW DOES CHILDREN'S WORD USE DIFFER FROM THE WORD USE OF GROWN-UPS?

> The world for them is not a concourse of objects in space; it is a heterogeneous series of independent acts … There are no nouns … . (Jorge Luis Borges, 1964, p. 32)

A number of remarks must be made before we engage in the analyses. First, although we believe that the ways of talking within each of the two pairs, that of the children and that of the grown-ups, are uniformly distinct when analyzed on the contrasting background of the other pair, we are also aware that these two types of talk may be not as homogeneous as is perhaps implied by the generalizing names. Certain differences between individual ways of communicating can be found in each of the two groups. This is particularly true about the children. To begin with, Roni and Eynat are neither of exactly the same age nor in the same social position within the observed group. The 7-month age difference between Roni and Eynat means a different kindergarten experience and a longer history of learning for Eynat. Throughout our analyses we saw that, indeed, Eynat's numerical discourse may be considered as somewhat more advanced. Interestingly, this relative superiority is recognized by Eynat: When faced with the grown-ups' questions, Roni often lets Eynat make her decision first and then follows in her footsteps (e.g., utterances [3], [62], [71–72]; see Appendix). The fact that the conversations are coordinated by Roni's parents is also of much significance, as it introduces an element of social imbalance. The difference between the two girls' relative positions within the group may probably account, a least partially, for the fact that Roni authors more than twice as many turns as Eynat (Roni: 80 turns, Eynat: 35 turns;

more specifically, Roni took 67 vocal turns and in the remaining 13 cases she used silent pointing or head movements; for Eynat the corresponding numbers are 29 and 6). In general, Roni is much more proactive than her friend: She often volunteers her opinions even if not asked, whereas Eynat rarely speaks if not explicitly invited to do so (a deviation from this rule can be seen by the end of the conversation, in Episode IV, when there is a certain tension between the girls because of their seemingly uneven ability to deal with the task at hand). In addition, Roni seems determined to get as much of her parents' attention as possible and does not hide her impatience whenever Eynat occupies the scene for more than one turn (see the many parts in the four episodes that are marked as instances of Roni's positioning activity, in particular [106–114], [118–122], [153–160], and [223–225]).

The grown-ups' number talk is not perfectly uniform either. Moreover, it cannot always be taken as fully representative of what is generally accepted as the proper numerical discourse. Both mother and father instinctively reduce the complexity of their language and, concerned about the effectiveness of communication, adjust their ways of talking to those of the girls. The mother seems particularly eager to accommodate her talk to the needs and capacities of the children. As will be shown later, her numerical discourse is deliberately modified so as to become closer to that of Roni and Eynat. The fact that the mother and the father change their discursive ways in the presence of the children is an important thing to remember throughout our analysis, especially when we decide about what constitutes grown-ups' numerical discourse. Our claims on this latter type of talk will thus be made not just on the basis of what can be actually observed in our transcripts but also through analysis of our own numerical discourse.

Let us emphasize again that the focus of the analyses that follow is on what could be called in traditional terms "the mathematical content" of the talk, as opposed, for example, to its interactional aspects. The question "What are the discursive moves with which the adults tried to stimulate children's learning?" could become a theme of a separate study. Here, let us only remark that in concert with what was found by other researchers regarding interactions between children and their well-educated parents (Heath, 1983; Wertsch, Minick, & Arns, 1984; see summaries in Cazden, 2001), the interactional patterns induced by Roni's mother and father were reminiscent of, although not identical with, those practiced by school teachers. This structural similarity can be seen mainly in the type of questions presented to the children, in the parent's fine-tuned scaffolding actions, and in their tendency for repeating one kind of tasks several times, until the children show evidence of some mastery.

Yet another thing to remember throughout the presentation is the fact that the analyzed conversations were originally held in Hebrew and that many significant language-specific observations have been made on the basis of discursive features likely to be lost in the translation. Whenever we feel this might have happened, we add necessary linguistic explanations.

Finally, let us remark that in our analyses, we will sometimes use the method of *interpretative elaboration*. The interpretative elaboration is a text that, utterance by utterance, elaborates on the text produced by the interlocutors. This work can be compared to that of an archeologist who reconstructs an ancient artifact, say a vessel, from its remnants. Indeed, a conversation is an act of multichannel communication, only parts of which are accessible to everybody (the self-discourses that evolve along interlocutors' personal channels are private and inaudible to others). Our text completes the audible discourse to a more comprehensible, plausible whole the way the parts added by a restorer complete the ceramic pieces to a vessel. It must be stressed that neither the restorer nor the interpreter make claims to the "authenticity" or to the ultimate "correctness" of the final product, and this is true even if both of them tend to believe that the original producer, if asked, would confirm the fidelity of the reconstruction.

In the analyses that follow we focus on two types of arithmetic keywords: *number words* and *words of comparison*. Special attention is given to the uses of the comparative expression *the same,* the analysis of which, we believe, is particularly important in the task of capturing the difference between children's and grown-ups' numerical talk (enough to recall how central the use of these words is to Piagetian theory).

Number Words

The word *number* appears in the children's talk very rarely (in three utterances only: [49], [258], [260]) and the word *amount*, which is introduced by Roni's mother at one point ([97]), is never to be found in the girls' utterances. However, both girls often use concrete number words, such as *one*, *two*, *ten*, and so on. The question we will be asking in our analysis is whether the children's use of these words is objectified. The adjective *objectified*, which can be seen as a derivative of both *object* and *objective*, implies that number words are applied as if they signified self-sustained entities, likely to become objects of exploration. These invisible "somethings" are implied to be discourse-independent, just like trees or atoms or any other material object. This objectifying effect is attained and sustained through many discursive characteristics, but for now, we will limit ourselves to linguistic objectification markers, that is, to those characteristics of the number talk that make it similar to the discourse on material objects.

All the utterances containing number words have been collected and presented in Table 1. The manner of use depends on the occasion that prompts this use, and in our episodes the occasions arise mainly when the mother makes an explicit request for counting or for numerical comparison. In response to the former kind of request, the children routinely begin producing canonic number-word sequences. Often, when the sequence ends, the girls would emphatically repeat the last number word (this is especially true of Eynat; see, e.g., [24], [66]).

TABLE 1
Uses of Number Words

Episode	*Children's Uses*	*Grown-Ups' Uses*
General	Counting: 3 times [R] + 5 times [E] Single number word: 13 times [R] + 3 times [E]	
II_c	[43] Because there is [are][a] **2** in one, and in [this] one there is [are] another **2**. [R]	
		[50] There is [are][a] **2** in one box.. [F] [52] Together, there is [are][a] **4** … [F]
III_c	[71] There is [are] **4** here [E] [76] **10** is more [E]	
		[79] **10** is more than what? [M]
	[80] Than **1** and **2** [R]	
		[81a] **10** is more than **1** and **2**? [M]
	[83] **4** is also a little [R]	
		[84] And what is **10**? [M]
	[85] **10** is … [R]	
		[88a] **10** is nothing? [M]
III_d	[98] … you get **4** [R] [100] **2** and **2** is **4**. [R]	
III_e		[101] I have **2** marbles [M]
	[105] You can … put the same **4** [**4** of the same?][b] [E] [116] There is [are][a] **4** [E] [123] I make **4** I see there is [are] **4**. It is the same **4** [**4** of the same?][b] [E]	
III_f	[133] You take **4** pretty marbles You get **4** and **4**. [R] [138] There is **2** and **1** and it makes **6**. [R]	
III_g		[151a] **2** and **4** – is this the same [thing][c]? [F]
III_e		[188] **2** and **4** is this the same [thing][c]? [F]
IV_b		[231] She has **8** [M]
IV_c	[236a] there is **10**. [E]	
		[239] What is more-many,[d] **10** or **8**? [M] [241] **10** is more than **8**? [M]
IV_d	[247b] I knew that it was **10**. [R]	
		[249] **10** is more than **8**? [M]
	[250] **8** is not the end. [E]	
		[251] **8** is not the end? [M]
	[252] It [the end] is **10**. [E]	
		[254] Is there a number that is bigger than **10**? [M] [256] **1000** is bigger? [M]

Note. R = Roni; E = Eynat; M = mother; F = father.

[a]The Hebrew word *yesh* was used, which may be translated either to *there are* or to *there is*. [b]These are two possible translations of the Hebrew *arba oto davar*. [c]The Hebrew term for *the same* is composed of two words, *oto davar,* the literal translation of which is *the same thing* (*davar* means *thing*). The first word, *oto*, cannot be used without being followed by a noun. So, either one specifies what is (are) the thing(s) that is claimed to be "the same," like in "the same number" (*oto mispar*) or "the same child" (*oto yeled*) or, if one is not as specific as that and just tries to say that A and B are *the same*, one says "A and B it's the same *thing*." [d]This incorrect form stresses that the word *more* (*yoter*) is used in the sense of more numerous rather than larger.

While considering the task of comparing, it is important to make a distinction between two types of quantitative comparisons: comparison *between sets of objects* and comparison *between numbers*. This distinction is delicate, but it involves a significant difference in the use of number words. Comparison of sets involves the number words in the role of *descriptors of objects* (of sets), whereas in the other type of comparison number words appear as signifying *objects in their own right*. In grammatical terms, this means that number words are functioning either as *determiners* or as *nouns*.

The most natural linguistic indicator of the use of a number word as set descriptor is the appearance of this word in conjunction with a noun, as in *10 marbles*. Yet, one may be making this kind of use also without any explicit mention of the objects that are determined by the number words. Children's tendency for terseness may be one reason for the lack of elaboration.[7] Only too often number words would be used as stand-alone, self-contained expressions, whatever the context. Thus, the mere absence of a noun following number word cannot be taken as sufficient evidence that number words are objectified, that is, used as signifying self-sustained objects.[8] We cannot be sure of objectification even when the child's talk seems to imply abstract numerical operations ([100], [138]). We will substantiate this last claim while dealing with routines.

More reliable differentiators of objectified and nonobjectified use are the adjectives *bigger* and *smaller* and the adverbs *more* and *less*, routinely applied in the context of comparisons. When number words are used in conjunction with *more* or *less*, like in the sentence *10 is more than 8* ([241]), these words function as determiners rather than nouns, and this implies that the objects of the talk are sets and their elements. Indeed, it would be natural to complete such sentence to "10 marbles is more than 8 marbles." When number words are used as referring to self-sustained entities, the result of comparison is presented with the words *smaller* or *bigger*. This is the case with mother's question "Is there a number that is *bigger* than 10?" ([254]) and with Eynat's later utterance "When numbers don't end, then this … the number is *bigger*" ([258]). When any of the adjectives *small* or *big* is followed by a number word, it is to be understood that the latter is used as a noun. In our study, the adverbs *more* and *less* dominate the conversation, and the adjectives *big* and *bigger* appear only in the last subepisode (IV_d), which is the exchange between Roni's mother and Eynat. So, whereas there is a

---

[7]The children may be using number words as synecdoches (kind of metonym, where a part is used as signifying the whole) for sets of elements of the given numerosity; for example, if a child says "10 is more than 6" she may really mean "10 marbles is more than 6 marbles."

[8]In cases like this, singular and plural forms might help in deciding whether number words are used as descriptors of sets or as signifying self-sustained objects. Thus, saying "There is 10," in singular, can be taken as a sign of objectification, whereas the use of plural form, "There are 10," brings an implicit reference to elements of a set. Unfortunately, in Hebrew this distinction is absent, so it could not be used as an indicator of objectification in this study.

vague evidence for objectification in Eynat's numerical discourse, Roni's talk is devoid of any such features.

In accord with our expectations, we found out that the children's number talk is not objectified the way the grown-ups' talk is. In this respect, Eynat's discourse appears a little more advanced than Roni's. This statement is made on the basis of an overall analysis of the children's discourse and not just on the grounds of local sampling. No single use of a number word can be taken as sufficient evidence of the presence or absence of objectification. Moreover, the linguistic contexts of use, such as those considered previously, are only a part of the story of objectification. Other types of indicators for objectification in number talk will be discussed in the section devoted to routines.

It should be added that the claim about the lack of objectification in Roni's and Eynat's talk is corroborated by the discursive behavior of Roni's mother and father. The parents seem to anticipate the difficulty that may stem from objectified use of number words. All along, they adjust their discourse to what they expect the children to be able to understand and produce. In result, we can often identify children's uses just by studying parents' altered language, which mirrors these uses. Thus, the grown-ups eschew objectifying language. They never detach their number talk from the concrete context of boxes with marbles. They are careful to include linguistic features that indicate the context as that of comparison of sets rather than of numbers as such—see, for example, their consistent use of *more* rather than *bigger.* The mother is so determined to sustain this effect that she would not shy from bending the rules of the language. For example, to avoid linguistic form that might puzzle the children, she uses the incorrect expression *more-many* ([239]). This form stresses that the word *more* (*yoter*) is used in the sense of *more numerous* rather than *larger,* and thus makes clear that the comparison regards sets of marbles, not numbers.

For all their attempts, the grown-ups are unable to escape the objectifying language entirely. Every here and there they use expressions that refer to the *existence* of numbers, and they are evidently unaware of the disturbing ontological message such expressions may bring to the children. Thus, in [254] the mother asks "*Is there* a number that is bigger then 10?" and the father speaks about numbers as self-sustained entities every time he raises the issue of sameness (see, e.g., [151a], where he asks "2 and 4—is this the same [thing]?").

Words of Quantitative Comparisons Stating Inequality

The just asserted lack of objectification in the initial use of number words implies that one can hardly expect the children to receive such expressions as *10 is more than 8* ([249]) as referring to some imagined entities. On the face of it, their consistent use of *more* instead of *bigger* indicates that the girls refer to concrete sets of objects. However, the fact that the word *more* is often applied with reference to closed boxes hints at the absence of any numerical considerations also in this case. In fact, a close look at the children's talk leads us to doubt whether they are engaged in the activity of comparing at all. The utterances containing words such as *many*, *more*, *most*, collected in Table 2 exhibit little regularity and, in any case, do not adhere to the rules of what generally counts as a discourse of comparing. This kind of doubt seems justified in the face of two salient features of Roni's and Eynat's uses of words: (a) the obvious confusion with respect to the role of different forms of the quantitative adjectives and (b) the incompleteness of the girls' comparative clauses. Let's briefly illustrate these claims.

Confusion between different forms of quantitative adjectives. Obviously, the girls do not yet have a full command of the inflection of adjectives and adverbs. To the grown-up's ear, the modifiers employed by the children would often appear misplaced. What sounds like unary superlative, describing a single object, may be used by the child as if it was in a comparative form, signifying a relation between two objects. For instance, Roni's utterances "This is *the biggest than* this one" and "This is *the most*" in [5] are both likely to be translated by an experienced listener into "This is *bigger than* this one." Similarly, her *too many* in [16] seems equivalent to the grown-ups' *more*. Finally, Eynat's *a little* ([18]) seems to be tantamount to the grown-ups' *less*. The children seem to be applying the words *more*, *the most*, *too much* interchangeably, with an apparent slight preference for superlatives, which may be the result of their being exposed to the frequent use of these in the emphatic language in which they are often addressed by grown-ups.

Incompleteness of comparative clauses. Only rarely do the girls use number words in full sentences of the type "*a* is more [bigger] than *b*" (in fact, only one such case has been observed in our transcripts: In [7] Eynat says "It is more huge than that," and Roni repeats this sentence in [9]). Utterances of this type, if they appear in the conversation, are usually "scaffolded" by the grown-ups. The following brief conversation, taken from Episode III$_c$ (see Appendix) is a good example of such scaffolding activity:

76	Eynat:	*10* is more.
77	Mother:	*10* is more?
78	Roni:	Yes, and *4* too.
79	Mother:	*10* is more than what?
80	Roni:	Than *1* and *2*
81	Mother:	*10* is more than *1* and *2*?

In this brief exchange, the mother repeatedly uses interrogative revoicing in the attempt to elicit a full comparative clause. In spite of her efforts, she only gets separate "halves" of the sentence ([76], [80]). In the end, she is compelled to do the job

TABLE 2
Uses of the Words of Comparison

Episode	*Children's Uses*	*Grown-Ups' Uses*
I_a		[3b] I want you to tell me in which box there are **more marbles**. [M]
		This question repeats itself (in this or slightly changed form): [19], [34a], [35a], [38], [62a], [89], [213a], [214a], [229], [231].
	[5] Because this is **the biggest than** this one. It is **the most**. [R]	
	[7] Because … cause it is **more huge than** that. [E]	
		[8] Yes? This is **more huge than** that? [M]
	[9] That this is also **more huge than** this. [R]	
I_b	[16] That this is **too much**. [R]	
		[17] That this is **too much**? [M]
	[18] That this too is **a little**. [E]	
		[19] That it seems to you **a little**? [M]
II_c		[44] So, this is why **there is more** in none of them? So, in both of them there is… what? [T]
		[46] And this is … **more** or **less**? [T]
	[47] **Less**. [R]	
		[48] **Less than** what? [T]
	[49] Than … than … **than big** numbers. [R]	
III_b	[69] That it is **more** here. [R]	
III_c		[75] Show me how you know that this is **more**. What tells you there is **more**? [M]
IV_c		[239] What is **more-many**, 10 or 8? [M][a]
IV_d		[249] But how do you know that 10 is **more** than 8? [M]
		[254] The end says that it is **the most**?[b] Is there a number that is **bigger than** 10? [M]
	[258] When numbers don't end, then this … the number is **bigger**. [E]	
		[259] When numbers don't end, then the number is **bigger**? [M]
	[260] … they will be **the biggest**. [E]	
		[263] What do you say about **big** numbers? [M]

Note. R = Roni; E = Eynat; M = mother; F = father.

[a]The mother used two words here, *yoter* and *harbe*, which mean *more* and *many*, respectively. The expression *more-many* is meaningful to the listener and has the sense of *more-numerous*. Using the two words in conjunction is an incorrect but common Hebrew usage. *Yoter* is enough to say *more*, but the addition of the word *harbe* (*many*) comes to stress that the comparison is between the sets of marbles, not between numbers as such (if the numbers were compared, the comparative expression would be *yoter gadol*, which means, literally, *more big*, i.e., *bigger*). [b]The Hebrew expression was *hachi harbe*, which counts as rather colloquial and not entirely

of "sewing" the two parts together all by herself ([81]). It is an open question whether the absence of more complex forms of use stems from the children's inability to build compound expressions or from their mere propensity for terseness, but in any case, we have good reasons to assume that at this early stage, the girls, not being guided by a sense of self-sustained entities called numbers, have only an approximate idea about how number words and the comparative expressions can be discursively combined.

Children's use of different forms of quantitative adjectives has been the object of inquiry for some time now. Discursive imperfections similar to those observed in our study have been documented by others, and they thus did not come to us as surprise. Although there is agreement as to the existence of the difficulty with comparative forms (*more*, *bigger*), the researchers differ in their interpretation of the sources of the difficulty. According to Matthews and Matthews (1978) the latter form is conceptually and developmentally more advanced than the simple (positive) form (*many*, *big*), and this accounts for a certain delay in its learning. Walkerdine (1988) claimed that this explanation does not tell the whole story. In her view, the child's command over different discursive forms depends on the context of use, and in school situation, where this use is usually tested, the task of comparing becomes a part of unfamiliar discursive practice within which it simply loses the meaning it has for the child in everyday activities. Walkerdine's account may be a good "first approximation" of our case as well: As already remarked, although our data collection took place in the home of one of the children and was done by this child's parents, the discourse that was practiced was probably closer to the one children knew from kindergarten than to the everyday domestic talk.

Although we agree with the spirit of Walkerdine's (1988) argument, we have just added another explanation: The girls are confused in their use of comparison words because they have difficulty deciding what is being compared. Even more strongly, we cannot be sure that the observed activity can be interpreted by the girls as one of comparing. Indeed, considering the fact that according to the children's experience, words such as *more* are used mainly in sentences of the type "*Eat (have, take) some more*" or "*I want more*" (cf. Walkerdine, 1988), these words are likely to be seen as referring to manipulations such as addition, rather than to the act of comparison.

The Comparative Expression *the Same*

As can be seen from Table 3, the occurrence of the term *the same* along our transcripts, and especially in the two girls' utterances, is so rare (cf. Walkerdine, 1988) that one can question the logic of our dealing with this expression at all. Yet, we can see a number of reasons for devoting special attention to this particular term. First, as has been remarked earlier, the issue of numerical sameness is

TABLE 3
Uses of the Expression *the Same*

Episode	*Children's Uses*	*Grown-Ups' Uses*
II_c		[54] And in each box there is **the sa …** [F] [56] And there is **the same** [**thing**][a] in each box? [F]
III_d		[97] Can it be done so that there will be **the same** [**thing**][a]? That there will be **the same marbles** in both boxes […] **the same amount of marbles** in the tw … [M]
	[98] … if you open and see that you have in **the same box** … ahm … you have **the same** [**thing**][a] then you get 4. [R]	
III_e		[102] Is it possible to change something so that there will be **the same** [**thing**][a] in both boxes? [M]
	[105] You can take from the pink box and then put **the same 4** [or "**4 of the same**"?] [b] [E]	
		[115] [Can you make it] So that there will be **the same**? [F]
	[123] If I make 4, and I count, and I see there is [are] 4, then it is **the same 4**. [E]	
III_f		[125] I want to know whether there is anything you can do … so that there will be **the same number of marbles** in the two boxes. [F] [135] Make it so that there be **the same number of marbles** in both boxes. [F] [141] So, make it so that it will be **the same** [**thing**][a] in both boxes. [F]
III_g		[145] Can you, Eynat, make it so that it will be **the same** [**thing**][a] in both boxes? [151a] 2 and 4 — is this **the same** [**thing**][a]? [F]
III_h		[152] So, make it so that it will be **the same** [**thing**][a] [F]
III_e		[185] Now, can you divide the marbles so that that there will be **the same** [**thing**][a] in the two boxes? [188] 2 and 4, is this **the same** [**thing**][a]? [F] [190] I told you to put in each box … so that there will be the same 2 and 4, is this **the same** [**thing**][a]. [F]

[a]The Hebrew term for *the same* is composed of two words, *oto davar,* the literal translation of which is *the same thing* (*davar* mans *thing*). The first word, *oto*, cannot be used without being followed by a noun. So, either one specifies what is (are) the thing(s) that is claimed to be "the same," like in "the same number" (*oto mispar*) or "the same child" (*oto yeled*) or, if one is not as specific as that and just tries to say that A and B are *the same*, one says "A and B, it's the same *thing*." [b]These are two possible translations of the Hebrew *arba oto davar.*

central to the theory developed by Piaget and to his notion of conservation. It is reasonable to assume that our interpretation of children's performance of these latter tasks depends to considerable extent on how the term *the same* is used by children and interviewers and whether it is used at all. Second, as will be argued later, the use of the expression *the same* is a sensitive indicator of the degree of objectification of one's talk. Third, before a child can incorporate a word into her own utterances, she may already be able to interpret the uses of this word made by others, that is, she may be capable of responding in a certain routine manner to utterances that include these words. Finally, in our study, children's reactions to grown-ups' uses of the words *the same* led to dramatic, puzzling reactions that we found potentially quite telling. The challenge, of course, was to find out why the children behaved the way they did.

The first of the unexpected discursive events took place after the mother presented the children with two boxes containing two marbles each (see Episode II_a). In Episode II_b, after she has answered the question "Where are there more marbles?" ([38]), with the brief "In none" ([39]), Roni is trying to do her best in response to her father's follow-up inquiry:

Episode II_b

42.	Father:	Why? Why do you say this?
43.	Roni:	Because there is [are] 2 in one, and in [this] one there is [are] another 2.
44.	Father:	So, this is why there is more in none of them? So, in both of them there is … what?
45.	Roni:	Two.
46.	Father:	And this is … more or less?
47.	Roni:	Less.
48.	Father:	Less than what?
49.	Roni:	Than … than … than big numbers.
50.	Father:	Than big numbers? That means.. If there is [are] 2 in one box and 2 also in the other, then what is there in the two boxes?
51.	Roni:	4.
52.	Father:	Aha. Together, there is [are] 4?
53.	Roni:	Yes.
54.	Father:	And in each box there is the sa …
55.	Roni:	Because it is between …
56.	Father:	I see. And there is *the same* [*thing*] in each box?
56a.	Roni:	…
57.	Father:	How many in each box?
58.	Roni:	2.
59.	Father:	Oh well …

The first thought likely to come to one's mind upon hearing this curious exchange is that Roni and her father "simply do not understand each other." To anybody who is not a little 4-year-old girl it must be quite obvious that all the father wants is to hear his daughter summarizing her findings in the sentence

"There is *the same number* of marbles in the two boxes," or even simpler, "It is *the same [thing]*." After all, for grown-ups, the words *more* and *less* that signify difference and the words that signify sameness are complementary opposites, so that whenever one does not occur, the other must be the case. Yet, although Roni clearly realizes that the word *more* does not apply to either of the two boxes (see [39]), the expression *the same* does not come to her mind as a suitable alternative. The more unexpected the girl's answers, the more determined the father's attempts. What began with a direct question [42], continues with unfinished sentences waiting for completion [44, 46, 54], and ends with the explicit statement that "there is *the same thing* in both boxes" [56], which leaves the usually eloquent Roni rather speechless [57]. In [54] the father almost puts the words into his daughter's mouth, but even this to no avail. While going through the other episodes we realized that what we observed here is not a matter of an accidental, momentary "misunderstanding," but rather of a systematic gap between the grown-ups' and children's use of the words *the same* (see in particular Episodes III_d–III_h). We speak about children's use of the expression rather than about the complete lack thereof because we have evidence showing that the words *the same* were not entirely foreign to the young interviewees. Indeed, if the father insisted on eliciting these words, it was because he had heard his daughter actually using them on other occasions. If so, the fact that the numerical context proved practically impervious to this particular expression shows that the girl could not see any connection between this present context and the situations in which the words *the same* were previously used.

Just to get a better sense of this "imperviousness," let us take a look at Table 4, presenting the interpretive elaboration of one fragment of the above conversation between Roni and her father (Episode II_b). According to the elaboration, the words *the same*, so clearly hinted at, and eventually uttered, by the father, are so far from Roni's mind that the girl would rather violate the rules of grammar and change her use of words that employ this expression (see, e.g., Roni's use of the word *less,* which, under the father's pressure [46], changes its designation several times: from being a signifier of a binary relation it becomes a unary modifier, equivalent to the expert's *small* [47]; then, under the father's further urging [48], it goes back to its role of the binary predicate [49]. The striking aspect of what we see here is that the child's use of a word may cross grammatical boundaries which are impenetrable in the eyes of the grown-up user).

The girls' helplessness with respect to the term *the same* in the numerical context becomes particularly salient in Episode III, where the parents, in full of disbelief about the children's manifest inability to respond properly, come up with the new task: They ask the girls to make the contents of two boxes "the same." This time, only one of the two boxes contains two marbles, whereas the other has four. The following sample conversation follows the girls' verdict that the box with four marbles deserves being described with the word *more:*

Episode III_d

97.	Mother:	Can it be done so that there will be the same [thing]? That there be the same marbles in both boxes [….] the same amount of marbles in the tw…
98.	Roni:	Yes, it's fine, but … but … but if you open and see that you have in the same box.. ahm.. you have the same [thing], then you get 4.
99.	Mother:	How?
100.	Roni:	2 and 2 is 4.

Evidently, the only way in which Roni is able to interpret her parents' question about sameness is to get back to the situation in Episode II_b, in which she heard the term *the same* in a similar context. One possible interpretation of this scene is that the girl, not having any reason for singling out a particular aspect of this former situation as the one that justified the use of the words *the same*, on hearing these words again reproduced the whole of the former situation indiscriminately.

The frequent appearance of the term *the same* in the father's talk is particularly effective in conveying the vision of numbers as self-sustained entities. Only such entities can be compared with each other and be found either identical or different. This conception is particularly salient in expressions such as "I want to know whether there is anything you can do … so that there be *the same number of marbles* in the two boxes" ([125]) and "2 and 4, is this *the same [thing]?*" ([188]). Much of the obvious difficulty the father encounters in his communication with Roni and Eynat stems from the lack of similar objectification in the girls' use of numbers. After all, when numbers are not things in their own right, one cannot see any sameness in the two separate boxes. The mother seems to realize all this when in [97], without actually using the word *number,* she makes unsuccessful effort to be more precise about what is supposed to be "the same" in the two boxes: "Can it be done so that there will be the same [thing]? That there be the same marbles in both boxes [….] the same amount of marbles in the tw… ."[9]

All these findings highlight the centrality of one's ability to see sameness in differently looking things in the process of objectifying one's numerical discourse. At this point, it may be appropriate to quote Henri Poincaré's[10] famous statement according to which mathematics can be seen as the *science of calling different things*

---

[9]One can claim that Roni's use of the word *two* is objectified: She recognizes this number at a glance, without counting ([36b]), and she uses existential expressions, such as "there is 2 in one [box], and in [this] one here is another 2" ([43]). Yet, if there is an objectification, it is similar to the one we witness when we use the word "pair." In [43], Roni doesn't feel a need to say "two marbles," because her attention is on the "pairness." On the other hand, just like our saying, "Here is a *pair*, and here is another *pair*" would not lead to a claim that we are watching "the same [thing]," Roni does not proceed from here to the descriptor *the same* her father is so desperately trying to elicit.

[10]French mathematician and philosopher, 1854–1912.

TABLE 4
The Segment [47]–[49] and Its Interpretive Elaboration

Speaker	*What Is Said*	*Interpretative Elaboration*
44. Father	So, this is why there is more in none of them? So, in both of them there is … what?	The father tries to elicit the use of the term *the same* with reference to the two boxes.
45. Roni	Two.	For Roni, there is no alternative to the description that says “there is more in none of them”. She thus interprets her father question “So, in both of them there is … what?” as the question about the number of marbles in both boxes.
46. Father	And this is … more or less?	Father leads Roni toward the expression *the same* by trying to make her aware that none of the only alternatives, either *more* or *less*, holds here.
47. Roni	Less.	For Roni, *the same* is not a complementary option for *more* and *less (*is not equivalent to *none has more*). Besides, according to the meta-rules of her discourse, of the two possibilities presented by the Father, one has to be true. Roni chooses *less* because she already said that *more* is not an option. The choice makes sense if the words *less* and *more* are interpreted here as *small* and *big*.
48. Father	Less than what?	Father is surprised: To his mind, Roni contradicts what she said before (because if in none of the boxes there is *more*, none can be claimed to have *less*). By the use of the incomplete comparative form “less than …” he imposes the return to the original comparative role of less.
49. Roni	Than … than … than big numbers	This statement is perfectly true both in the eyes of Roni and of her father, except that it shows that the word *less* in Roni’s former utterance [47] did not express comparison between the two boxes. The present answer corroborates our interpretation of [47].

the same name. To put it in a slightly different language, doing mathematics means speaking of many different things as if they were, in a sense, the same. Indeed, when we use, say, the word *five* with reference to objects as different as the fingers of one hand, the coin with the digit 5 engraved on one of its sides, a famous military building in Washington, DC called the Pentagon, and the equally famous Soviet Army symbol (not to speak of a certain kind of Chanel’s perfume), we are certainly applying the same name to things that, on the face of it, have nothing in common with each other. The importance of seeing samenesses becomes even more obvious if we realize that using one word as a signifier for many different signifieds is not unique to mathematics—the activity of "saming" through language is the very essence of *any* conceptualization and not just the mathematical. It is why Vygotsky (1987), to learn about human ways of building concepts, launched the experiment in which he followed children's developing ability to identify blocks that could be called a given name. Still, the way sameness is attained in mathematics is quite unique in that it does not seem to require any perceptual similarity and relies on discursive mediation instead.

Let us elaborate. The first primary kind of sameness that a young child learns to see is the sameness that pervades the discourse about the material, perceptually accessible world. It is the sameness one notices when watching continuously changing images that, although different from each other, are told to be images of *the same table* or of *the same tree* or of *the same person*. This is the type of sameness that underlies what Piaget described as the child's recognition of the *permanence of objects*. This type of "saming," or identifying, allows the child to overcome the dimension of time in her perception of reality.

Initially, therefore, the words *the same* apply only to sets of things (images) that are distant in time and are never simultaneously present. This is the kind of use we make when reacting to several different pictures with the sentence:

(a) "*The same person* appeared in court on both occasions."

At a later stage in the development of human discourses, the uses of the expression *the same* are extended, and the secondary type of sameness appears. The expression can now be employed with reference to two simultaneously present objects. This is the case with Roni's father's utterance [141]:

(b) So, make it so that there be *the same [thing]* in both boxes.

What is it that makes us use the expression *the same* in each of the two situations? The reason for the use of these words with respect to the different human images is the fact that even if we cannot retrace the continuous change that transformed one of these images into another, we are well aware that such transformation did take place over time. Moreover, we know that in reality, these different images replaced one another and never coexisted alongside each other.[11] This is certainly not the

[11]There are some other properties of this sameness that one may mention. This sameness is *direct*, in that it is perceived by us in a natural way, without any discursive mediation except, perhaps, that of the "atomic" routine of naming; it is *universal* rather than relative, that is may be stated even if the items that are being samed cannot be directly compared, that is, are not simultaneously present in our visual field.

case with "the same number" (see father's [125] and [135]) or "the same amount" (see mother's [97]) that the grown-ups are able to see in the two simultaneously present boxes, as well as in many other situations that they not necessarily perceive as being continuously transformable one into another.[12] So, let us repeat the question asked previously: What is it that remains *the same* in a directly recognizable way when we make transitions between things as different as the fingers of one hand, the coin of 5 shekels, the main building of the American Department of Defense headquarters, and the Soviet Army symbol?

At a closer look, the only thing that justifies the use of the term *the same* with reference to this set of seemingly unconnected items is *the counting procedure*, and the fact that it invariably ends with the word *five* when we move from the fingers of one hand to the pentagon and then to the pentagram. It is because of the counting procedure ending with the word *two* that Roni's father expects his daughter to apply the words *the same* to the two boxes of marbles.[13] All this is quite obvious. What is probably much less obvious is why this new kind of sameness, which is so clear to the grown-ups, is inaccessible to the girls.

After giving more thought to the issue, we feel compelled to reverse the question: In fact, why should this kind of discursively mediated identity be readily evident to young children? The process of mathematical objectification is difficult and "unnatural" enough to require some really good reasons to be undertaken. For many people, and certainly for the children, sameness may appear to be a matter of something much more "serious" and less "human-made" than counting. We described the process of mathematical objectification as "unnatural" because it is quite unlike anything the child has been used to, so far. This is true for more reasons than those mentioned previously. Until now, all the samenesses in the child's world were primary in that they have been identified through direct, unmediated perception. With the arrival of the number talk, the child is suddenly required to renounce the direct recognition of sameness for the sake of a mediating discursive procedure, necessary to identify secondary sameness. In the case of numbers, it is the discursive process of counting that will allow her to decide whether two things can be called the same name. This means that the property of sameness that, so far, was conceived as being in the things themselves, is now to be found in what people decide *to do with words* with relation to these things. This is a difficult transition,

---

[12]As was nicely illustrated in the recent film *A Beautiful Mind,* which tells the story of the mathematician John Nash, people can vary in their ability to perceive visual transformability of things one into another. In the film, Nash is plastically shown projecting a simplified scheme of a slice of orange onto the pattern appearing on his colleague's tie.

[13]One may wonder why the Father opted for the words the *same [thing]* rather than *equal*, which might appear more natural in this context. Father's choice was probably grounded in his intuition as to the state of his daughter's vocabulary. The parent had good grounds to assume that Roni was already acquainted with the expression *the same* (which he might have heard her using in certain contexts) and equally good reasons to believe that she was still unfamiliar with the term *equal.*

the more so, that what people do with words in general, and in particular while counting, may seem quite arbitrary to a person not yet aware of the possible practical significance of such a procedure.

The gains of objectification are something to be experienced and, as such, can become a motivational factor only through the persistent use of objectified discourse. If so, paradoxically, the appreciation of the value of this type of talk seems to be both a *precondition* and the *result* of one's participation. The process of discursive growth is thus inherently circular, and if so, one cannot expect the objectification to happen overnight.

HOW DO CHILDREN'S ROUTINES DIFFER FROM THE ROUTINES OF GROWN-UPS?

> The fact is that I did not know how to understand anything. I ought to have judged by deeds, not by words. (Antoine de Saint-Exupéry, 1945, p. 30)

Routine, just to recall, is a repetitive discursive pattern. As observers, we will try to describe such patterns the way a physicist describes regularities in the motion of material bodies: by identifying the rules that define the pattern. Of course, the rules of discourse, unlike those of physics, are rarely deterministic. We call these rules *metadiscursive,* to emphasize the fact that, if explicitly formulated, they would appear as discursive constructs (propositions) the object of which is another discourse.

Any set of routine-defining metarules may be divided into three subsets. First, there is a routine *procedure*—the subset that defines, or merely constrains, the interlocutor's discursive performance. This is the *how* of the routine. The routine's *when* is given by the other two subsets, which will be called here *opening* and *closing conditions*. Opening conditions are rules that specify the circumstances in which the routine is likely to be evoked. We will look for these conditions in the *openings* of routine performances, that is, in those actions that usually precede, and may often count as what prompts, the actual implementation of the given routine procedure. Thus, for example, the mother's request to open the boxes and "see what there is inside" ([10a]) in Episode I served as the opening to the procedure of comparing-by-counting implemented by the girls (and scaffolded by the mother) in response. Considering the fact that the mere invitation to compare the boxes ([3]) did not prompt the counting, we may say that in the children's discourse, nothing less than an explicit request to "see what is inside" in conjunction with the question "In which box are there more marbles?" can count as an opening condition for the procedure of counting. The closure conditions describe circumstances that signal the completion of the routine performance. We will try to deduce these conditions from the *closing* part of routine performances. For instance, pointing to the box the contents of which gave rise to a longer counting procedure ([20] and [22] in Episode I) is regarded as the proper closing of the routine of comparing, at least in the eyes of the mother (notice how she induces this closing action in Episode I).

We are now turning our attention to the routines that were prompted by Roni's parents and performed by the children. Following, we list the routine *procedures* that we were able to observe in our study. We then assess children's proficiency in the execution of these procedures. This initial survey is thus focused on *how* the children's routines work. The *when* of the routines, that is, their opening and closing conditions, will be discussed later. Our major claim is that it is mainly this latter aspect that makes the difference between children's and grown-ups' routines. In other words, even when children's and grown-ups' routine *performances* appear indistinguishable, they may be geared toward different goals and different situations. Following this observation, we distinguish between several types of routines, depending not so much on the manner of performance as on the circumstances and goals that prompt it.

The *How* of the Routines: Routine Procedures

The majority of procedures enacted in the four episodes are *comparisons of sets* (CS). A scrutiny of Roni's and Eynat's reactions to the recurrent prompts to compare boxes with marbles leads us to the conclusion that the girls are capable of more than one type of set comparison. The possible types of CS procedures have been collected and systematized in Figure 1. Let us explain and exemplify the different components of the diagram, while also trying to decide which of the procedures were actually implemented by the participants of our study and how skillful the girls were in the performance.

The comparison which we encountered first in Episode I was *direct* (CS:D). We call it this name because the choice of one of the two boxes did not seem to be mediated by any decision procedure, for which the counting-and-number-comparing operation would be the natural candidate. It is this direct reaction to the request for comparison that puzzled Roni's parents and that became the reason for this article. As mentioned earlier, many researchers reported that young children who have a full mastery of counting may not use this skill when presented with sets of elements and asked "Which has more?" More often than not, 4-year-olds would rely in their decisions on a difference in perceptually most salient dimension, such as the length of a row of elements or the area covered by the set (Dehaene, 1997; Nunes & Bryant, 1996; Piaget, 1952; Zhou, 2002). In our summarizing diagram (see Figure 3) we included this kind of direct action under the heading of *principled set comparison* (CS:P.) The adjective *principled* comes to stress that the decision of the performer may be systematic rather than arbitrary, even if the principle behind the comparison does not meet expectations of the person who prompted the action. Our present case, however, does not seem to belong to this category, as there is no visible difference between the boxes and thus no rational reason to prefer one to another. We thus see this case as one of an *arbitrary* comparison (CS:A), with this last adjective stressing the fact that the choice is made among visually indistinguishable options.

The parents' intention was to check the children's *numerical* abilities, and thus they expected comparisons to be mediated by number talk (see CS:N in the diagram). CS:N is a special case of *mediated* set comparison (CS:M). In response to four consecutive requests for comparison, CS:N, which we also call *comparison-by-counting*, was performed by the girls four times ([10–22], [36–39], [64–69], [215–234]). As can be seen from the transcripts, each CS:N may be decomposed into subprocedures of two types: *evaluation* (E), and *comparison of numbers* (CN). Both evaluation and comparison can be performed in either of the two ways: *directly*, that is in one step, or in a *mediated* manner, with a certain composite procedure necessary to produce the closing. Let us exemplify each of these subprocedures.

The *mediated evaluation* (E:M) means counting. From the many instances of counting ([11], [12–14], [23–24], [66], [216–218], [219], [224]) we conclude that both girls have a full command of the procedure. All the "how-to-count" principles identified by Gelman and Gallistel (1978; one-to-one correspondence, constant order, cardinality) were observed, and the girls have displayed a satisfactory proficiency in performing the necessary steps. In our transcripts, the only instance of the *direct evaluation* (E:D; subitizing) occurs when the sets to be compared contain two elements ([36b], [37c], [65]).

Direct comparison of numbers (CN:D), is attained by recalling endorsed narratives, or simply *number facts*, about numerical order relations. Here is a representative example of CN:D performance:

239.	Mother:	What is more-many *[see footnote 4 in the appendix]*, 10 or 8?
240.	Roni:	10.
241.	Mother:	10 is more than 8?
242.	Eynat:	Yes.

In our study, almost all the cases of number comparisons, at least as far as we can judge, are direct and, for the most part, silent; the girls simply point to the box corresponding to the number they assessed as larger. To this there is one exception. Episode IV_d, interpretively elaborated in Table 5, is the case of mediated number comparison (CN:M). This special exchange gives us a valuable glimpse into the ways in which young children, whose numerical experience and the memory of number facts are scarce, may be deciding which of any two numbers fits with the determiner *more* or with the adjective *bigger.* Here, Eynat seems to be explaining her method. According to our interpretation, the child asserts that to decide which of a given pair of numbers deserves to be called bigger, she starts reciting number

TABLE 5
Episode IV_e and Its Interpretive Elaboration

Speaker		*What Is Said*	*Interpretive Elaboration*
249.	Mother	But how do you know that 10 is more than 8?	
250.	Eynat	Because 8 is not the end.	"Because if I counted to 10, I would arrive at 8 before the counting is completed."
251.	Mother	8 is not the end? And what *is* the end?	
252.	Eynat	It's 10.	"The last number I would have to say would be 10."
....		....	....
258.	Eynat	No, no, When numbers don't end, then this … the number is bigger.	"If I count to compare two numbers, x and y, and I arrive at x before I arrive at y, then y is the bigger."
259.	Mother	When numbers don't end, then the number is bigger? Why?	
260.	Eynat	Because many numbers … it is up to … ah … this, this, … and if they order themselves fine, then … they will be the biggest.	"When there are many numbers, from 1 to … and if they are ordered according to the regular numerical order, then the numbers in the end are the biggest."

sequence and waits to see which of the two relevant number words appears first. It is the other number word that should be used in conjunction with the determiner *more* or with the adjective *bigger*. We speculate that Eynat might have learned this method in kindergarten or from her older siblings.[14]

Let us take a closer look at our only case of non-numerical, but still mediated set-comparison, as interpretively elaborated in Table 6 (in the unshaded part of the diagram in Figure 2, which contains the non-numerical routines, this comparison was marked as CS:nN.) This episode (IV_d) preceded the conversation we have just analyzed. The utterance [247], "Because ... because ... because I saw there is a long row, and then I knew that it was 10" seems to be referring to a set comparison based on perceptual considerations. Eynat is evidently talking here about the case of sets organized in rows in such a way that the assessor is able to appreciate their relative lengths. If the procedure is performed correctly, one should assure one-to-one cor-

[14]It also seems reasonable that when the method of comparing by reciting number sequence was taught, the number 10 was prominently present as *the biggest* among those that have been compared. It might even be called "the end," as it was the last that was considered. This may also explain why Eynat, while asked by Roni's mother "What tells you that this [4] is more [than 2]" ([74]) answers, seemingly without any connection, "10 is more" ([76]). The girl might have heard questions such as the one asked by the mother in kindergarten, when the relations between numbers up to 10 were discussed.

TABLE 6
Episode IV_d and Its Interpretive Elaboration

Speaker		What Is Said	Interpretive Elaboration
246.	Mother	Eynat, how do you know [that 10 is more than 8]??	
247a.	Eynat	Because … because … because I saw there is a long row,	Eynat: "I saw that the row of 10 is longer than the row of 8"
247b.	Eynat	and then I knew that it was 10.	Eynat: "and then I knew that 10 is the bigger number"
248.	Mother	You saw a long row and then you knew it was 10?	

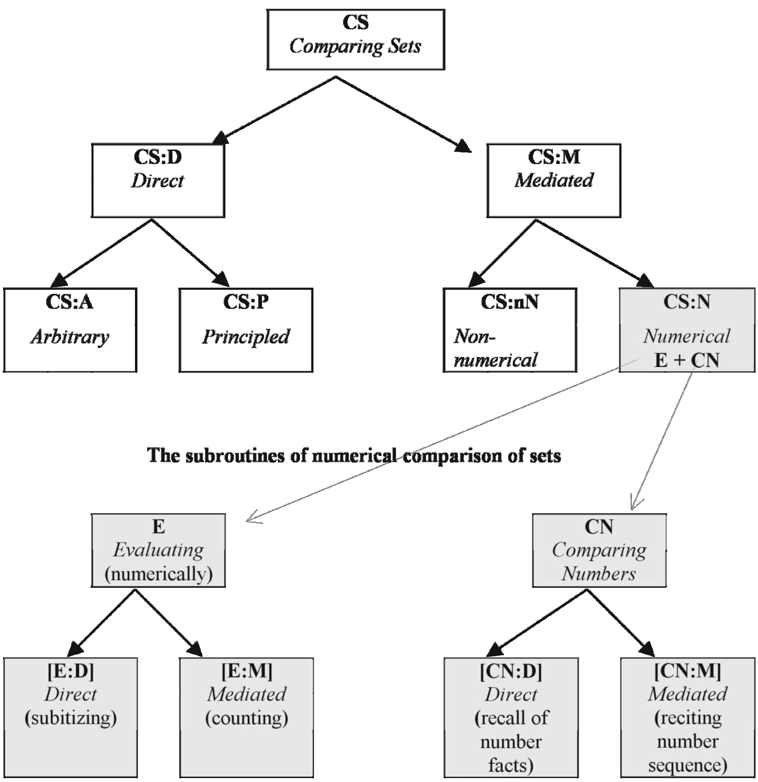

FIGURE 2 Routine procedures of quantitative comparison.

respondence (mapping) of the elements of the two rows before making any inferences. Eynat does not mention such need, so we cannot be sure whether she is aware of it.

To sum up, Roni and Eynat revealed the ability to perform a range of numerical and non-numerical procedures. In the elementary numerical procedures, such as counting or number comparison, the girls have shown satisfactory proficiency. In the composite procedures, such as numerical set comparisons (comparing-by-counting), they needed mother's prompting in making transitions between the component subprocedures. For example, in Episode I the mother spurred comparing of the results of counting by asking "So what do you say?" [13]; the situation repeated itself in a similar situation in Episode IV (see [229]). The need for this kind of scaffolding persisted from the first episode to the last, and we can only speculate whether this need was a mere result of the girls' relative reticence or was rather an outcome of their inability to perform the compound procedure in its entirety on their own. Yet, whatever the case, it was quite clear that the girls were fairly advanced in their learning of the procedures. It seems that soon there would be no difference between them and the grown-up performers, in this respect.

Thus, if the children's numerical discourse was considerably different from that of the parents, as we claim it was, the distinctive features of this discourse must have been hiding in another place. In the rest of our analysis we accept the suggestion given by Wertsch, Minick, and Arns (1984): "[I]t is often more appropriate and accurate to describe differences in subjects' performance in terms of differences in how they interpret a situation rather than of how they carry out the task," (p. 160). Following, to formulate a conjecture about the children's vision of the task, we analyze the *when* of their routines. The issue is well worth a deeper thought, as it relates this discussion to the classical debate on the transfer of learning, and to the current controversy around the inherent situatedness of cognition (Anderson, Reder, & Simon, 1996; Brown et al., 1989; Greeno, 1997; Lave, 1988; Sfard, 1998).

The *When* of the Routines: Opening and Closing Conditions

If the grown-ups who took part in our study were sometimes surprised by the children's actions, it was, indeed, not so much because of Roni's and Eyant's procedural skills as because of the girls' decisions as to when the different procedures should, or should not, be applied. Figure 3 presents the two compound routines, *direct choice* (A) and *numerically mediated comparison* (B), that were implemented one after the other four times in response to the four pairs of boxes with marbles presented by the mother. The figure specifies the opening and closing conditions of all four performances. As can be seen, form detailed analysis of the openings, the

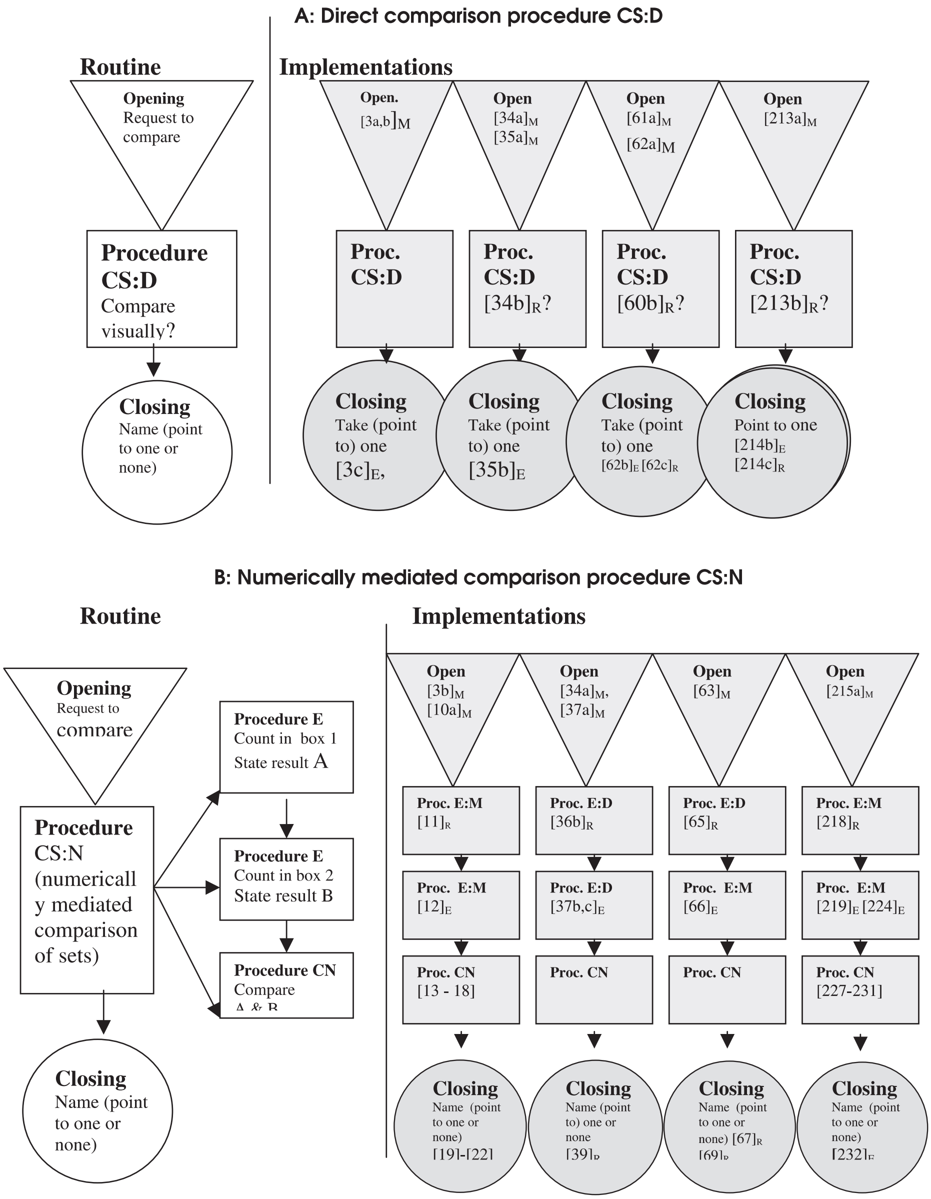

FIGURE 3 Routines of set comparison and their implementations.

mother's initial prompting, which would induce the counting action in any adult respondent, failed to do so in the case of the children.

It is quite clear that when asking their questions about the boxes with marbles, the grown-ups intend to evoke *explorations*, that is, actions that would lead to "truths about the world." In other words, their aim was to guide the children toward the production of endorsed narratives such as "This box contains more marbles." Yet, what the children were actually prompted to do was quite different. Their goals seemed to be more practical. Considering the nature of their experience and of their relations with the surrounding so far, it is not unreasonable to assume that questions intended to sound as requests for an impartial, factual comparison, were read by the girls as invitations to *deeds*, that is, to actions that produce a change in the environment and not just words. Roni's and Eynat's selection of one of the boxes may be interpreted as an act of *choosing (taking) for themselves* rather than as an attempt to make an assertion about the state of the world. This interpretation seems highly plausible considering certain particularities of the girls' behavior. Notice, for example, how in Episode I Roni grabs the box that was said to have more marbles even though it is closer to her friend [10b]. See also how in Episode II_a the girl manipulates the closed boxes, clearly concerned about having for herself the one with the label "more" [33a]. All along the four episodes the children act as if they intended to take the possession of the boxes deemed to have more marbles.

This interpretation is also very much in tune with what was said before about the girls' use of the word *more.* If, so far, this word appeared mainly in utterances such as *Take [eat, have] some more*, it is likely to be read as an invitation to "taking for oneself" even if it now features in another context, such as the mother's question *In which box are there more marbles?* If this it true, then the words *the same* do not signify comparison either. Whereas likely to be used mainly in the reassuring sentences such as *You got the same [as your brother, as your friend]*, the term *the same* is read simply as saying that there is no reason to worry about the possibility that another person has been unduly privileged. Thus, whereas *more* stimulates the act of taking, the expression *the same* implies the justness of the performed action.

We find this interpretation useful also because it helps in accounting for the children's claims about the inequality of the two identically looking boxes. Because the aim of choosing is *to have* rather than *to know*, one feature that sets *deeds of choosing* apart from explorations is that the former type of routine action always has a definite resolution: When one chooses to have, one thing will be chosen even if the options seem indistinguishable. Indeed, nobody is likely to consider the lack of a difference as a sufficient reason to stay empty-handed! This is why Roni and Eynat do not hesitate to pick up one of the two identically looking boxes. If, in response to the mother's further interrogation ([3–5], [8]), they claim to have chosen a box that is "the biggest" and "more huge" than the other one, it is because they associate such words as *bigger*, *biggest, huge*, *more*, and *better* with the preferred option without necessarily seeing these expressions as systematically related to the images they see. This means the reversal of the "grown-up" order of things: Options are not chosen because they are better, but rather they are considered better because they are chosen.[15]

Explorations are what turn deeds into rational actions. Principled or mediated comparisons precede grown-ups' choices as a rule. Roni's and Eynat's actions will also be discursively mediated, one day. At this stage, however, when the children's numerical discourse is still unobjectified, the rationality of mediating routine may be absent even if a spotless mediating performance is present. In this situation, chanting number words while touching the marbles one by one is not unlike the incantations of meaningless rhymes that are often a part of children's play. Because the young performers show no uneasiness while making direct choices and because they cannot yet appreciate the advantages of the discursively mediated decisions, their counting and then singling out one of the number words may be not more than a *ritual*, the only importance of which is in its being practiced collectively, together with others.

The distinction between the three types of discursive routines—*deed*, *exploration,* and *ritual*—is made here according to the perceived goal of the routine performance.[16] Both deeds and explorations are geared at extradiscursive reality: They are, respectively, about changing the world and getting to know it. In contrast, rituals are socially oriented: These are acts of solidarity with those with whom they are performed. However, rituals may be more than that. Far from being just a "mean-

[15]An alternative account might be that the girls are just playing a guessing game, that is make a bet before checking. A few things weaken the status of this interpretation, at least with regard to Episode I. First, both Roni and Eynat are capable of substantiating their seemingly arbitrary choices. Of course, one may claim that what Roni really means while replying "Because this is the biggest than this one" is "Because *I think* that this is the biggest than this one." However, this latter sentence can hardly count as a response to the mother's question "How do you know?" Second, the girls do not proceed from here to counting without being prompted, and this means that they may view their first choice as the final answer. Third, they act as if the box they choose in this apparently arbitrary way was, indeed, more desirable. Above all, however, even if we decide that the girls do play the guessing game (which, indeed, may be the case in the later episodes), this interpretation does not, on its own, resolve our main dilemma: It does not bring a clear-cut answer to the question "Are the children aware of the fact that the grown-up's query 'In which box are there more marbles?' is supposed to always be answered with the numerical comparison and with such comparison only?"

[16]Compare Edwards and Mercer's (1987) definitions of ritual and principled knowledge: "*ritual* knowledge is a particular sort of procedural knowledge, knowing how to do something"; "Principled knowledge is defined as essentially explanatory, oriented toward an understanding of how procedures and processes work, or why certain conclusions are necessarily valid, rather than being arbitrary things to say because they seem to please the teacher" (p. 97). See also the distinction between *normative procedural instruction* and *truths* that was introduced by Cobb, Wood, and Yackel (1993), after Much and Shweder (1978). According to Cobb and Hodge (2002) this distinction "provides a means of capturing a crucial difference between classrooms whose mathematics is learned with what is colloquially termed understanding and those where it is not" (p. 6).

ingless game," as the word "ritual" may sometimes imply, they are likely to be a nascent form of young children's contact with the world. Indeed, if thinking is a special case of communication, and if communication begins as an interpersonal affair, this contact may only develop through other people. As Vygotsky (1978) put it, "The path from object to child and from child to object passes through another person" (p. 30). Rituals may be a part of this path. They are what initiates children's relationship with the world of things.

To sum up, our response to the question about the difference between grown-ups' and children's routines is that more often than not, a procedure that would be an exploration when performed by a grown-up becomes a ritual when implemented by a child. Roni's and Eynat's numerically mediated comparisons are certainly the case in point. In what follows, we substantiate this assertion in more detail (see Claim 1). Later, we make a case for yet another claim: Ritualized discursive performance should not be regarded as an opposite of what we use to call "meaningful activity" (see Claim 2).

Claim 1: Children's Numerical Routines Are Rituals Rather Than Explorations.

The preliminary question is whether the difference between explorations and rituals is researchable at all. Routines that differ in their goals would often produce identical performances. Moreover, the goal of an activity, as perceived by the performer, is rarely readily accessible to the observer. If so, how can we tell a "genuine exploration of quantitative relations" from the purely ritual counting-before-choosing? This, we claim, can be done by considering particular performances in a wider discursive context.

In general, the type of routine that underlies one's performance may be judged not only by the goal that guides the performer but also on the basis of such additional evidence as word-uses and mediator-uses that it involves, the range of discursive performances that would count as equivalent, the range of situations in which this performance would be regarded as adequate, the correctibility of the performance, and the list is still long. The criteria for distinguishing between deeds, rituals, and explorations have been collected in the comparative Table 7. Let us look at the different characteristics one-by-one, while trying to decide which of them can be found in Roni's and Eynat's discursive performances.

Goals. We have already observed that at this early stage, when their discourse is not yet objectified and when numbers are not much more than words of incantation, decisions made by the children cannot be interpreted as a result of their quest after "the truth about the world." At this point, children's readiness to engage into counting activities is more likely to be motivated by their concerns about their own relations with other humans than by their interest in the relations

TABLE 7
Main Characteristics of Deeds, Rituals, and Explorations

	Deed	*Ritual*	*Exploration*
Goal The routine is implemented for the sake of:	A change in environment	Relationships with others (improving one's positioning with respect to others)	Description of the world (production of endorsed narrative about the world)
The place (role) of discourse in the routine	Possibilities: The discourse may be 1. May be *absent* (*direct* deed) 2. *interwoven* into a sequence of practical actions 3. *separable* (precedes target action – premeditated deed)	May be *interwoven* or *separable*	There is only discourse
Words' and mediators' use	Possibly no active use of keywords	Template-driven use of keywords—as descriptors of extradiscursive mediators	Objectified use of keywords—as signifying objects in their own right
Flexibility			
Routine procedure	Unrestricted; may be one of bricolage as long as it brings the required closure; possibly direct—no discursive mediation	Deterministic or nearly so No equivalent process	Can be merely constraining There are other processes that count as equivalent (fit the same prompt and closure—are part of the same superroutine)
Routine opening	Any situation that evokes the need for a given state of environment would count as the same prompt	Is saturated with situational clues about the available types of mediation and to the social setting	Poor in situational clues; allows for more than one mediational mode; no clues relating to social setting
Routine closing	Any situation that can count as satisfying the prompting need	Well-defined symbolic act ("halting signal")	Specifies the required relation between opening and closing
Applicability		Restricted—the routine is highly *situated*	Broad—the routine is applicable in a wide range of situations
Correctibility	By tinkering	Cannot be locally corrected—has to be reiterated in its entirety	Parts can be locally replaced with an equivalent subroutine

(*continued*)

TABLE 7 (*Continued*)

Performers	No special requirements	Has to be performed together with (scaffolded by) others	No need for scaffolding—can be performed individually
Addressees	Possibly none (in the case of no mediating discourse)	Others (*authoritative discourse*)	Others and oneself (*internally persuasive discourse*)
Acceptability	The *result*—the change in environment—is judged as adequate; no need for human mediation of the acceptance—it depends on the environment	The activity has to be shown to strictly adhere to the rules defining the routine procedure—the acceptance depends on other people	The narrative produced through the performance must be *explicable*—the acceptance should be independent on other people

between boxes with marbles. Roni's behavior lends a strong support for this claim. In all four episodes, she watches her friend's actions and then points to the box chosen by the other girl ([3], [62b,c], [214b,c,d]). Later, she struggles to have for herself the box to which the word *more* has been applied ([10b], [34b], [35b], [60b], [64].) All this shows the contagious, self-amplifying nature of desirability. Children tend to want what others want, and whatever they want is often desired for no other reason than the fact that others want it. In addition, and as was already noted, in the child's world all the superlatives seem almost synonymous, and *more* is often exchangeable with *better.* To put it in Walkerdine's (1988) words, "In terms of consumption within our culture, it is *more* that is valued: that is what consumption is about" (p. 27).[17] It is probably because of this positive association that the mother formulates all her question about marbles in the boxes in terms of *more* rather than *less*.

Giving an answer expected by the interlocutor may be read as an act of pledging allegiance. In this context it is worth mentioning that the girls' attempts to gain the ownership of the better option clearly competes, and is successfully combined, with an equally strong need to belong with the peer. While making their choices, Roni and Eynat are careful to stress that their decisions are shared. The need for solidarity with the friend is further evidenced by Roni's repetitive use of the word *we*, through which she asserts the joint ownership of solutions ([67]: "We were right," see also [90]), of actions, ([244]: "we counted"), and even of inabilities ([191]: "We can't do it.").

[17]See similar claim made in Lakoff and Johnson (1980).

However, this is still only a part of the story. It seems that Roni is also torn between the solidarity with Eynat and the desire to have the parents to herself. This latter need often finds its expression in an open fight for attention ([106]–[114], [117]–[122], [223]). In [106]–[121], when Eynat starts outperforming her friend, Roni's struggle for domination arrives at its peak. Similar phenomenon can be seen all along the Episode III where, in the end, Roni starts to question Eynat's skills openly ([153], [154]).

To sum up, the children have different goals than those envisioned by the grown-ups. While counting and comparing, the girls are in fact preoccupied with the delicate social fabric of their little group, and the conversation on boxes with marbles is, for them, as good an occasion for interpersonal engineering as any other. While grown-ups count to get closer to the truth about the world, the children count to get closer to the grown-ups. The "exploratory" activities of the young participants are therefore a form of community-building ritual.

The role of discourse in the routine. Routine can be a combination of discourse and of practical action, that is, an action aiming at a physical change in the environment. This is certainly the case for deeds. Deeds, rituals, and explorations differ considerably in the amount of discourse involved, in the nature of the discourse, and in the way this discourse is related to the nondiscursive, practical elements of the performance. Thus, for example, explorations, at least in mathematics, would often include discourse only, and no practical actions at all. In contrast, deeds would not be called this name if they did not contain a practical action. Sometimes, the practical aim of the deed would be attained directly, without any discursive mediation. In such cases, we speak about *direct* deeds. As was already stated, Roni's and Eynat's spontaneous choices of specific boxes may probably be regarded as such. In our study, we were watching the beginning of children's transition form direct to discursively mediated choices.

In general, the discursive mediation may be incorporated into a deed in one of the two ways: It may appear in its entirety *before* the practical action, or may be *interwoven* into a sequence of practical actions.[18] The act of choosing from among

[18]Dividing candies between children using the "one for you, one for me" routine or giving the change to a buyer by successive complementing the sum to be paid to the sum obtained from the buyer are good examples of discourse-interwoven deeds. In such case, moving all of the discourse up front, so as it precedes any practical move would usually be impossible without a thorough change in the discourse. Indeed, more often than not, the mediating discourse is not just interwoven into a sequence of practical steps, but it is formed in a direct reaction to these steps. In such case there is no point in talking about a stand-alone discursive routine: The nature and the course of the discourse would be entirely dependent on the nature and course of the action. Rather than following a certain general scheme that can be presented in advance, the discourse develops step by step, with each of the steps designed in response to the result of the preceding practical action. Sometimes, the steps will be systematic and based on a previous discursive and practical experience; sometimes they will be a product of ingenious tinkering.

two boxes, preceded by counting the boxes' contents and comparing the results is an example of a premeditated action, that is, an action in which discourse comes before the practical move. In a case like this, the discursive sequence ends in an endorsed narrative that becomes the basis for the subsequent practical step. Here, the discursive part may be separated from the practical action and treated as a self-sustained exploratory routine. This is probably how many explorations come into being in the first place.

In Roni and Eynat's case, the routines of comparing-by-counting are not available as a part of the deed of choice. Rather, they seem to be separate activities, retroactively tacked to this deed. Indeed, these discursive routines are imposed by the grown-ups *after* the deed (the choice) has already been performed. As such, they do not play any practical role and can thus be only seen as rituals, performed as an act of deference to the initiative of the superiors. Their turning into explorations will go hand in hand with their integration with the deed of choosing into a single routine.

Words and mediators use. As was already noted, number words appear in Roni's and Eynat's discourses either stand-alone, as one-word utterances, or within a restricted repertoire of constant phrases, such as counting sequences or (usually incomplete) comparative clauses. We call this type of use *template-driven* and view it as a salient property of socially oriented rituals. Word use characteristic of the full-fledged mathematical explorations is quite different. Development of this more advanced use is a part and parcel of the process of objectification. In the objectified talk, number words are used in ways that imply their role of pointers to extradiscursive entities called *numbers*. The secondary role of words and mediators as mere "avatars" of the "real thing" is implicit in the term *representations*, with which they are traditionally referred to. Once the concrete mediators are relegated to the position of mere "representations," number words get life of their own, and new types of utterances can now be built around them in ways similar to those in which utterances are constructed around other, more familiar nouns. Roni's and Eynat's template-driven numerical discourse has still a long way to go before the girls arrive at this advanced use of words.

Flexibility (and situatedness). The question of *flexibility* regards the discourse as a whole rather than any specific routine. In most general terms, we are now asking how much variation is possible within the given discourse in the face of similar goals. The flexibility may thus be judged according to two criteria. First, we can keep the routine procedure constant and ask about the range of situations in which this procedure is likely to be evoked. In our study, where the focus is on the routine of counting, the relevant question is this: "What are all the situations in which a child is likely to turn to the procedure of counting spontaneously?" Here, we are thus dealing with the issue of the flexibility of the *routine opening condi-*

tions: We are asking how liberal and how encompassing these conditions are. The second criterion is that of the variability of the discursive action in the face of a given prompting situation. This time, the prompt (the opening) remains constant and we check how much variation is allowed in the procedure that is considered appropriate for this prompt. We are thus talking about the flexibility of the *routine procedure*. In our box-with-marbles comparison task, the flexibility of the procedure would mean *the child's ability to perform the task in many different ways, but always with the same result.*

Unlike deed or exploration, which are judged as successful in reaching their goal mainly on the basis of the relation between their openings and closings (or between "inputs" and "outputs"), *rituals* attain their goal through their very performance. No part of such performance is more important than any other. After all, the whole point in the ritual action is that it is strictly defined and followed with accuracy and precision so that different people can perform it in identical ways (possibly together). If so, different ritual performances cannot be seen as interchangeable just because they bring the same end product. Roni's and Eynat's numerical activity does seem to be marked by such rigidity. The girls perform numerical comparisons in a constant order and without any variation along the way. The opening likely to spur the routine performance is very specific, and thus extremely restricting. In our case, the counting will not occur unless the contents of the boxes are readily visible and the necessity to count is clearly hinted at by the grown-ups.

This makes the ritual highly situated. Thus, as long as all children's routines related to numbers are rituals, the mapping between prompting situations and the procedures is practically one-to-one, and there are no visible connections between the different discursive routines. In the child's world, partitioned into numerous disjoint sets of situations on the one hand, and disjoint sets of matching rituals on the other, two routines may not appear as related even if they feature the same words and similar procedures. At this point, all we can see while watching the children responding to grown-ups' questions is a bunch of mutually unrelated procedures rather than a consolidated numerical discourse.

Correctibility. Because of the fact that the implementer of an exploration focuses on the outcome, whereas performer of a ritual cares about the process itself, the two types of routines lead to different behaviors in the case of a performance breakdown. Explorations admit of local repairs, whereas rituals, to be considered as corrected, must be simply repeated. Once again, we did not have much opportunity to watch Roni and Eynat in the situations of routine performance breakdown, but we seem to have good reasons to assume that they have no means to correct their evaluations or numerical comparisons other than reciting number sequences again and again (as, indeed, is the case in [218–224]).

Performers and addressees. The next couple of features that set rituals and explorations apart is related to the question *who performs* the procedure and to whom this communicational activity is *addressed*. Ritual, being an inherently social action, has sense only if it is performed with others and for the sake of others. In the case at hand, the children's performance takes a form of conversation with grown-ups. The girls, at least in the first episode or two, appear unable—or maybe just unwilling—to perform the numerical comparisons on their own, without being led from one step to another by the grown-ups' questioning and without having the grown-ups' confirmation of the correctness of their actions. In contrast, in the expert performance of exploratory routines, the performer is able to play by herself all the roles—that of the interrogator and that of the respondent, that of the proposer and that of the assessor. The discourse that enfolds thorough such expert routine performance does not have to be directed at others and may be a response to the performer's self-posed question. In this latter case, from being a conversation with others it turns into a self-sustained *thought process*.

In short, Roni's and Eynat's counting is, at least at this point, an activity performed for the sake of others rather than for themselves. This observation brings to mind the Bakhtinian notion of *authoritative discourse*, a discourse that "binds us, quite independently of any power it might have to persuade us internally; we encounter it with authority already fused in it" (Bakhtin, 1981, pp. 110–111). This type of discourse is played according to somebody else's rules and is judged according to this other person's criteria of properness. In explorations, guided by what appears to be a mind-independent, superhuman reality, the discourse becomes, in Bakhtinian terms, *internally persuasive*.[19] Here, the quest after other person's approval is transformed into the quest after "the truth about the world."

Acceptability. The precise, accurate performance of a specific routine procedure is an obvious requirement in the case of ritual. This, however, is not the first focus of those who decide on the acceptability of deed or of exploration. In these latter cases, the fit between the opening and the closing—between what was expected and what was obtained—is the main, and often ultimate, concern.[20]

---

[19]"Internally persuasive discourse—as opposed to one that is authoritative—as it is affirmed through assimilation, is tightly woven with 'one's own world.' In the everyday rounds of consciousness, the internally persuasive world is half ours and half somebody's else's. It's creativity and productiveness consists precisely in the fact that such a world awakens new and independent words, that it organizes masses of our words from within, and does not remain in isolated and static condition … . The semantic structure of the internally persuasive discourse is not *finite*, it is *open*; in each of the new contexts that dialogize it, this discourse is able to reveal ever new ways to mean" (Bakhtin, 1981, pp. 110–111).

[20]As it happens in schools only too often, however, what a child considers the proper performance because it leads from a given explicit prompt to the required closure may be disqualified by the teacher, because what the teacher really wants to know is whether the student is skillful in a particular algorithm. Thus, for the student, proving a theorem by means other than mathematical induction may count as equally good as any other, and calculating a sum by hand or with the calculator may be regarded as equivalent routines. In the context of school they do not necessarily count as such. The distinction, however, must seem to the student arbitrary and, as such, is likely to decrease the effectiveness of learning.

In the case of exploration, the goal of which is to produce an endorsed narrative that answers the prompting question, an additional narrative may be required, showing that there is a satisfactory fit between the opening and closing narratives. This additional narrative may be called *substantiation* (in mathematics class, many such substantiating narratives are called *proofs*). The sufficiency of the substantiating narrative is a matter of interlocutors' judgment. It is also important to note that the activity of substantiating is recursive: It may always expand, because the substantiation itself is a narrative that may become an object of substantiation.

Along the four episodes Roni's parents make numerous requirements for substantiations. By asking questions such as "How do you know?" ([4], [6], [27]) or "Why there is more here?" ([70]; see also [75], [99], [235], [243], [249]), they try to see whether the children are capable of justifying their performances. Once again, what they get in return is not necessarily what they had in mind. More often than not, the girls would respond with an utterance composed of the word *because* followed by one of the assertions made as a part of the original performance. This is the case, for example, in [5], where Roni says "Because this is the biggest than this one. It is the most," although nothing indicates that the box she points to may, indeed, contain more marbles (see also [7], [70]). Alternatively, the girls would just refer the interlocutor to the discursive actions they just performed (see [28]: "Because I already counted"). One of the striking examples is the futility of Roni's mother's attempts to elicit utterances of the type "Because x is more [bigger] than y" where x and y are number words (see, e.g., [70]). In the absence of this type of statement there is no evidence that the girls actually compare numbers. The mother's follow-up requests for substantiation of statements on the relation between numbers ([75]) are equally unsuccessful, at least initially.[21] This is yet another evidence for the ritualized nature of Roni's and Eynat's performance. In the case of ritual, which is about performing, not about knowing, there is simply no room for a substantiating narrative. In this context, request for substantiation would often lead to a story about how the task was performed.

To sum up, in the grown-ups' numerical discourse the acceptance of both deeds and explorations is supposed to be, in a sense, human-independent: The performance itself and the appropriateness of the result are seen as dictated by discourse-independent factors. The extradiscursive reality, or what counts as such, is what gives the confirmatory or refuting feedback. However, as long as numerical discourse remains unobjectified, there is nothing "in the world out there" that would impose the acceptance or rejection. Having no sense of the "reality" that is

---

[21]This latter question evokes in Eyant a free association with the number 10 that is unrelated to this activity. We conjecture that the number 10 might have been at the center of attention when the issue of comparing numbers was discussed in Eynat's kindergarten class. The fact that in Episode IV Eynat is eventually able to answer Roni's mother's question [241], "How do you know [that 10 is more than 8]?" lends support to this conjecture.

supposed to be the object of exploration, the children turn to human authority for confirmation of their performances.

Claim 2: Ritualized Does Not Mean Meaningless

The overwhelming evidence showing that Roni's and Eynat's routines are rituals rather than explorations is likely to lead to a comment that at this early stage, the children's performance is, in a sense, "meaningless." According to the popular view, ritualized performances for which the performer cannot produce a satisfactory substantiation counts as the situation of "incomprehension" or "lack of understanding" (these situations are also known in literature as "knowing *how* without knowing *why*"; see, e.g., Skemp, 1976). This would also be in tune with interpretations of children's early numerical activities that can be found in classical literature. Summarizing sentences such as "[C]hildren do not appreciate *the meaning of counting* until the end of their fourth year" (Dehaene, 1997, p. 121, italics added) are representative of the traditional discourse on early arithmetic. These formulations imply that the routine of counting has an inherent meaning, the "acquisition" or "construction" of which would usually lag behind the technical proficiency. We believe to have good reasons to object to this interpretation. First, counting does seem to be meaningful to children, although for reasons that are quite unlike those of the grown-ups. Second, having seen how discourses produce their own objects we would rather eschew the idea of meaning as somehow primary to words and their discursive use. Instead of asking about meaning we thus propose to focus on the question of how children match routines with situations. The exploration of "children's meaning" of a routine procedure is now replaced with the quest after *sets of situations that are likely to prompt the use of this procedure*, and the claim about children's growing "appreciation of the meaning of counting" gives way to the talk about the *developing awareness of the usefulness of counting routines*.

All this said, it still makes much sense to talk about *understanding*. To avoid objectification and disembodiment of this latter notion, we choose to speak about understanding in experiential terms: We talk of incomprehension when a person feels, and possibly says, that she or he "does not understand." In the light of our observations, we have good reasons to oppose the claim that Roni and Eynat, while engaged in the ritualized activity of counting, have such disturbing sense of insufficient understanding. Although the grown-ups are sometimes startled by the children's answers, the girls do not show any uneasiness about their counting and comparing performances. On the contrary, they appear fairly confident and pleased with their actions. Still, along our study we did have occasions to observe them faltering, frustrated, and desperate for help. This seemed to be the case, for example, in Episode II_b, where the father, determined to elicit the words the same in the context of two boxes with two marbles each, repeated his question time and again as Roni was running out of ideas about plausible answers.

While comparing situations of manifest incomprehension to those in which the children seem to be at peace with their own actions, we notice that the difference is not so much in the quality and appropriateness of the applied routines as in these routines' very availability. It is when the interlocutor does not recognize the task at hand as similar to those with which she has already dealt successfully in the past that a sense of incomprehension arises and is at its most acute. This claim is in tune with Wittgenstein's famous definition, according to which understanding means knowing how to go on. To put it in Wittgenstein's own words, a person is likely to say "Now I understand" when she is also able to say "Now I can do it" or "Now I can go on" (Wittgenstein, 1953, p. 59.) Sometimes, just knowing what type of result is going to count as a proper closing may be enough to give one a sense of command over situation.

The issue of when and why children may develop the sense of incomprehension is of much practical importance, because this disturbing experience may have a major impact on the process of learning: It may lead to a negative emotional reaction that would hinder any further progress, but it may also create a powerful incentive for learning. Ensuring the realization of this latter possibility may be a matter of an appropriate handling of the interaction. The ability to identify situations in which the learner experiences the sense of incomprehension is therefore of principal importance for those who wish to help children in their learning. A quick survey of our data shows that there are two types of tasks that bear the greatest risk to Roni's and Eynat's sense of understanding: First, the girls seem to have no readily available routines for dealing with questions that feature the words *the same* in the numerical context (see, e.g., Episodes III_d –III_i where the girls were repeatedly asked to make the contents of two boxes, with 2 and 4 marbles respectively, "the same"); second, as was mentioned earlier, they lack routines for substantiating their own actions. The latter situation is exemplified by the following dialogue between Roni and her mother, which takes place after the girls asserted that 4 is more than 2:

75.	Mother:	Show me how you know that this is more. What tells you there is more?
76.	Eynat:	10 is more.
77.	Mother:	10 is more?
78.	Roni:	Yes. And 4 too.
79.	Mother:	10 is more than what?
80.	Roni:	Than 1 and 2.
81a.	Mother:	10 more than 1 and 2?
81b.	Roni:	Nods "Yes"
82.	Mother:	and than 4?
83.	Roni:	4 is also a little
84.	Mother:	4 is also a little? And what is 10?
85.	Roni:	10 is…
86.	Mother:	What?

87.	Roni:	Nothing. [*Laughs, raises her shoulder in the movement that says "I don't know."*]
88a.	Mother:	10 is nothing? You don't know what to say? You can say whatever you want to say. Whatever you say is fine.
88b.	Roni:	[*Shakes her head "No."*]

The signs of incomprehension, which can be identified here, have been observed in many other situations as well. More than once that we could see at least one of the girls answering with random associations ([76], [78], [80]) or declaring surrender (as Roni does here mainly in the body language, see [87]; in other places the same was done more explicitly, in words, see [142]). In Episode II_b, where Roni was repeatedly asked the same question but was unable to change her answer, we saw her desperately looking for words ([49], [56]). In other places, after futile attempts to summon an appropriate routine, the girls were visibly losing interest ([106–114], [253], [262]).

To sum up, the request for substantiation is difficult for the children because, given their performances are rituals, they can hardly see anything that requires explanation. This means that as long as they are only required to perform the rituals, the sense of incomprehension is unlikely to emerge. Indeed, numerical rituals, far from being a meaningless, purely mechanical activity, are promising and probably the only possible departure points toward a meaningful discourse.

REFLECTION: WHAT DIFFERENCE DOES COMMUNICATIONAL APPROACH MAKE?

> "The grown-ups are certainly altogether extraordinary," he said simply, talking to himself as he continued his journey. (Antoine de Saint-Exupéry, 1945, p. 45)

> The greatest magician … would be the one who could cast over himself a spell so complete that he would take his own phantasmagorias as autonomous appearances. … We (the undivided divinity operating within us) have dreamt the world. We have dreamt it firm, mysterious, visible, ubiquitous in space and durable in time. (Jeoge Luis Borges, 1964, p. 243)

Within the conceptual framework offered on these pages, thinking has been presented as a form of communication, and learning about numbers became tantamount to the induction to a certain form of discourse, together with its particular narratives ("facts about numbers"). More generally, the term *discourse* practically replaced the term *knowledge,* and the mission of the developmental researcher became to investigate the evolution of forms of communication. This replacement brought about an important epistemological/ontological change, because in contrast to the growing mental schemes, which are the focus of the traditional research, communication is not a property of an individual, but a social activity.

According to our analyses, the incipient numerical discourse of young children, which on the face of it differs from the fully fledged numerical discourse of adults only in its extent, correctness, and the degree of fluency, is in fact qualitatively distinct. Indeed, seemingly indistinguishable discursive performances may involve different routines, prompted by different goals, and implemented through unusual uses of words. Our detailed argument presented on the former pages pointed to at least three dimensions along which children's discourse diverges from that of the grown-ups in a substantial way. The first salient property of grown-ups' discourse is its *objectification*, that is, the fact that number words are employed as if they signified externally given tangible entities. Children's discourse is devoid of this property, and this fact finds its expression in the young interlocutors' idiosyncratic use of numerical keywords and, in particular, in the fact that they seem unable to apply the term "the same" to situations that feature sets of equal cardinality. The second difference is in the *purport* of the discursive activity. While the grown-ups' goal is to produce endorsable narratives about the world (within the objectified discourse, numbers are conceived as a part and parcel of this world), children's numerical discourse is ritualized, that is, oriented toward other people rather than toward nonhuman reality. Third, grown-ups' numerical discourse is usually highly *consolidated*, that is, can be described as a rich web of routines tightly knitted together by an intricate net of partially overlapping conditions of opening and closing. In contrast, the children's numerical rituals seem to have little in common with each other, and thus appear to be but a loose collection of separate rigid procedures, with mutually disjoint fields of applicability.

The study that brought about all these findings is a part of the longitudinal research project in which we use the communicational approach to investigate development of the numerical thinking. Our admittedly ambitious undertaking is far from completed, but as we hope to have shown in this article, we can already claim some results. From what we saw so far we have learned quite a lot, and not just about the predetermined objects of our study, that is, about children and numbers, but also about other issues that, unexpectedly, began drawing our attention as we were trying to interpret our data: grownups, research methods, and human communication. The rest of this exposition can be seen as an interim stocktaking. In what follows, we check the insights that we were able to gain thanks to the communicational lens, and we formulate some questions and hypotheses that, in future, will guide our further investigations.

Numbers

Our vision of the notion of number differs considerably from the one embedded in the traditional acquisitionist discourse. Let us immediately stress: We have no disagreements with representatives of other schools about the validity of their statements. If we have any doubts, it is about the usefulness of the language in which these statements are made. Thus, for example, what is said later should not be interpreted as a controversy with Piagetian researchers over such questions as whether and when children conserve numbers or which behavior should count as indication of the absence or conservation. Rather, we differ from other schools in that we are reluctant to use the notion of conservation at all. We believe that our alternative framework, in which this term does not appear (and, therefore, the questions of conservations cannot even be formulated), will help in making sense of phenomena that, as far as we can tell, stubbornly escaped acquisitionist accounts.

Thus, although the phenomena presented in this study are in tune with the findings made by other researchers, our description and interpretation of these phenomena may be different. Let us illustrate this claim with an example. Summarizing the findings by Piaget (1952), Saxe, Guberman, and Gearhart (1987), Fuson (1988), and many others, Nunes and Bryant (1996) said: "despite the fact that other methods are not reliable and that children know how to count rather well at the age of 5 and 6, they still do not realize that counting is their best tool for construction of equivalent sets" (p. 34).

We too, in our own research, have seen children unsuccessfully grappling with the request to construct sets containing "the same number" of elements.[22] However, we would not describe this phenomenon as showing that the children "do not realize that counting is their best tool for construction of equivalent sets." We are wary of this kind of statement because it carries tacit ontological and epistemological assumptions that go directly against our vision of the developmental process. The sentence implies that "having the idea" of equivalent sets can be separated from, and primary to, activities that lead to production of such sets. Piaget (1952) made this claim explicitly: "[f]rom the psychological point of view, the need for conservation appears … to be a kind of functional *a priori* of thought" (p. 4; note that Piaget's conservation is tantamount to the ability to deal with numerical equivalence, i.e., the ability to see several sets as, in a sense, "the same"). This alleged order of things, and the very dichotomy between the idea of equivalence, on the one hand, and the relevant discursive routines, on the other hand, is unacceptable within our framework. Indeed, we claim that the children's sense of sameness can only arise through their engagement in the discursive routines of

[22]Somebody may object to the parallel between this study and those of Piaget and his followers saying that the words *the same* might have not been used by the researchers in their conversations with children. It is, however, difficult to imagine that these words could, indeed, be omitted in verbal instruction for the construction of what Nunes and Bryant (1996) called "equivalent sets." Piaget's reports on his own studies corroborate this claim in many places. For example, Piaget (1952) told us that "the child was told … to pick out of the box *the same number* of counters" (p. 66, italics added) and quotes an interviewer of a 5-year-old child as asking: "Are they *the same*?" (p. 7).

sameness production and recognition.[23] We thus prefer to eschew the talk about concepts, such as equivalence, as "essences" that preexist discourse and can be acquired by the child directly "from the world." For us, words are the kingpins of sameness: The extensive equivalence classes to which we refer with words such as *two*, *three*, or *twenty thousands*, implying that the members of each of these classes are in a certain sense "the same," would probably never become an explicit object of thought if there were no means to actually think, that is to communicate, about this sameness.

A similar position was proposed by Valerie Walkerdine (1988) who, in her seminal work on children and numbers, made a convincing case for the centrality of *symbolizing* in human numerical thinking. Semiotically minded, Walkerdine complained that "For Piaget the role of signs, both linguistic and others, such as gesture, is one of [a mere] 'representation'" (p. 160; cf. Sfard, 2000), and she deplored the fact that people tend to believe in "unmediated relationship between subject and object, between knower and known, between the subject and the physical world" (p. 159). In criticizing these common views, Walkerdine rejected the vision of human thinking as the activity of "mirroring" in which the mirrored reality is the principal player and symbols are mere auxiliary devices (cf. Rorty, 1979).

The proposed revision of the traditional perspective has a considerable impact on the overall vision of numerical development. Let us mention two changes entailed by this revision. First, the common feature of almost all known research on numerical thinking is that it takes the notion of number for granted and does not engage itself with foundational questions such as "What kind of entity is number?" or

---

[23]Needless to say Nunes and Bryant (1996) are not alone in this vision. The tacit assumption on the precedence of numbers-as-discourse-independent-entities over child's numerical thinking pervades the majority of the relevant publications. This ontological message is not necessarily a result of the researchers' conscious reflection. More likely, this view imposes itself through the objectified language deeply entrenched in our culture. The assumption about the precedence of numbers over discourse is so deeply engraved in both colloquial and scholarly parlance that it becomes practically invisible. Enough to look at the following expressions, chosen at random from representative, and highly influential, books on numbers, published in the span of the last 50 years: Piaget (1952) spoke about "child's earliest contacts with numbers" (p. 3), Nunes and Bryant (1996) seconded "Piaget's emphasis on the need to consider children's understanding of *the invariants of number* in describing their mathematical reasoning and not only their knowledge of *culturally transmitted number conventions*, such as their knowledge of counting" (p. 41, italics added), and Dehaene (1997) told us that "What distinguishes us from other animals is our ability to use arbitrary symbols for numbers" (p. 73). All these quotes carry a particular conception of number and of its place in reality. The tacit message is that numbers are self-sustained entities existing in the world along with humans and animals. The expression "child's contact with numbers" further implies that when a child is born, the numbers are already out there in the world waiting to be discovered along with stars, trees, and other material objects. This is corroborated by the idea of discourse-independent status of number suggested by Nunes and Bryant when they opposed the "understanding of the invariants of numbers" to the "culturally transmitted" ways of talking about number (e.g., counting). Dehaene's expression entailed that it is not the very idea of number that sets humans and other animals apart, but only the ability to communicate about numbers with symbols.

"What does it mean that numbers *exist*?" It is tacitly agreed that the task of psychologist is to study how people *learn* about numbers, or *acquire the concept* of number, rather than discussing the ontological status of numbers. In contrast, the communicational approach is explicit about its epistemological/ontological foundation and deliberately shifts the debate from the number as such to the *numerical discourse*.

Second, the revision of the perspective entails a change in what counts as mathematical thinking and in the answer to the question of where this thinking begins. Within the traditional developmental framework, much has been said about the starting point and the subsequent course of the process of "acquisition" or "construction" of the number concept. As remarked earlier, Piaget postulated that this process does not begin until the child is already aware of the "conservation of number." Over the last several decades this claim generated much discussion, ranging from questions about the conservation tasks and their various interpretations (see, e.g., Mehler & Bever, 1967; McGarrigle & Donaldson, 1974) and ending in the controversy over the order and timing of events in the developmental sequence. Based on his comprehensive survey of the most recent research (Simon, Hespos, & Rochat, 1995; Starkey & Cooper, 1980; Wynn, 1992, 1995; Xu & Carey, 1996), Dehaene (1997) spoke about "babies who count"—children as young as 2 months of age who show signs of awareness to changes in the cardinality of small sets. As a result, humans are said to be born with a basic *number sense.*

Once we agree that numbers are not "out there" in the world but are rather human inventions, we realize that answering the question "When does numerical thinking begin?" is not a matter of empirical discovery but of semantic decision, that is, of one's explicitly presented stance about what should count as numerical thinking. This stance and the resulting statement, in turn, are subject to our choice, whereas the motives for preferring one possible definition to another are related to what one considers as more useful. The definition adopted in this article presents numbers as certain discursive constructs and thus suggests that there is no point in talking about numerical thinking that precedes one's ability to engage, if only peripherally, in the communication on numbers. True, children may display sensitivity to differences in cardinality, and this sensitivity is crucial for their future success in the numerical discourse. These early abilities should thus be of interest to anybody who wishes to study the genesis of numerical thinking because they are what will one day make this thinking possible. Yet, the advantage of leaving these early sensitivities on the other side of the mathematical–nonmathematical divide is that this decision allows us to focus on what Vygotsky called "uniquely human," that is, on those forms of activity that are typical of humans and cannot be found in other species. Whereas discursive skills do seem to be uniquely human, the sensitivity to the cardinality, as described earlier, is probably not: Much evidence has been collected showing that some animals are visibly startled by changes in the number of elements in small sets, exactly like human babies (see research based stories about "Talented and gifted animals" summarized in Dehaene, 1997, pp. 13–40).

The question that eventually has to be asked regards our gains from the proposed change of perspective. As stated in the beginning of this article, the difficult shift in the way of looking at familiar phenomena can only appear worthwhile if it brings about new insights. We claim that the communicational approach does exactly this, and we believe to have validated this claim along the previous pages. One other example is readily available in this section: The communicational perspective provides explanation to the widely known phenomenon that the summarizing statement of Nunes and Bryant, quoted earlier, left unaccounted for—we are now on somewhat firmer grounds with respect to the phenomenon of children who are able to count but do not apply this procedure when, according to the grown-ups, such move would be most appropriate. According to the communicational perspective, the children do not use counting to build sets with "the same number" of elements because the words *the same*, already familiar to them from many everyday situations, remain unrecognized when put into the numerical context. The lack of recognition is the result of the absence of objectification in the early numerical discourse: The children simply do not know what kinds of entities should be compared in the reaction to the talk about "the same number."

Children

How does looking at mathematical thinking as a special type of discourse change our understanding of children's learning? It is now clear that the simple awareness of the constancy of counting procedures is not enough for the emergence of fully fledged, objectified, discourse on numbers. The phenomena that Piaget interpreted as showing children's unawareness of the "conservation of number" are, according to our interpretation, the result of the simple fact that in the situation of choice, the young learners have no reason to privilege the ritual of counting over other, more accessible routines, among which the direct visual comparison is the uncontested favorite. Indeed, unaware of the prospective gains of being attentive to the results of counting, why should the child prefer this exotic discursive procedure over simple visual comparison? If their preferences are to change, children have to learn about practical implications of paying attention to the last word obtained in the process of counting. These implications are not anything to be discovered on their own, and thus nothing will change without a guidance of "the initiated."

The more so that, as we have already noted, the process of becoming a participant of the numerical discourse is inherently circular: To become aware of this discourse's advantages one has to use it; yet, to have an incentive to use it, one has to be aware of the prospective gains of this use. Following in the footsteps of more experienced interlocutors is probably the children's only option. If so, it is now time to rehabilitate the learning that is based on ritualized action and on mimicking the grown-ups' ways with words. Trying to guess and then to meet the expert participants' expectations is sometimes the only way to learn. This type of learning is fueled by our overpowering need for communication, one of the most important elements of which is our need for acceptance. It is because of the prospective social reward that children seem truly eager to engage in mimicking and disciplined implementation of rituals.

Turning rituals into explorations is the target of further learning. This transition involves shifting the child's gaze from the discursive process to its outcome. It also means a new goal for the discursive activity: The effort to "win something" reincarnates into the action of producing "truth about the world." It is this exploratory goal that will guide children's choices of routine from now on. This change of goal will have many important entailments. Above all, it will lead to the consolidation of hitherto unrelated routines into one integrated discourse. As long as numerical routines remain rituals, they are separate, self-sustained kinds of activity; as the focus moves from the process to its product, the hitherto clear-cut divides between rigidly determined sequences of ritualized actions begin to disappear. So, for example, calculating the triple of a given quantity, on the one hand, and finding a quantity's quarter, on the other—two operations that seem to the child as two different tasks, requiring separate, irreconcilably different procedures: multiplying by 3 in the first case and dividing by 4 in the second—will eventually be recognized as calling for the same routine of multiplying the quantity by a certain factor (3 and ¼, respectively). Consolidation of different routines under one superroutine will require experience, time, and constant reflection on one's own action. However, it will have its prizes: The gradual conflation of the routines will bring about highly flexible numerical discourse and will counteract the situatedness of this kind of communication. Above all, there will be a change in the relation between the child and the discourse. From a game played according to somebody else's rules and judged according to this other person's criteria of appropriateness, the numerical discourse will turn into child's discourse-for-herself—into her internal conversation about the world. How this important change happens is the main topic of our follow-up study in which we are looking at the change in Roni and Eynat's numerical discourse as time goes by.

Grown-Ups

Our research project, originally meant as an investigation of children's mathematics, turned into an exploration of the grown-ups as well. No wonder. According to communicational approach, the thinking process we intended to study are processes of communication, and as such, they cannot be understood properly as long as only some of the participants are considered. In particular, one cannot account for children's puzzling behaviors by analyzing just their "half" of the dialogue. In our study, the cases of children's failure to act according to grown-ups' expectations are conceptualized as communication breaches rather than as the evidence of the child's as-yet-imperfect vision of the world.

Indeed, it takes at least two participants—a child and a grown-up—to produce the kind of "failures" we have seen in our study. As grown-ups, we are so successful in the project of objectification of the numerical discourse that, eventually, the number gets life of its own and starts dictating us what to think and what to do. When it happens, we become unable to stop seeing what we have learned to see, and like Roni's father, we are ignorant of the fact that in the eyes of a child we may be engaged in a virtual reality game in which the objects we are playing with remain invisible to others.

As we have just noted while reflecting on conservation tasks, all this may be true even about the most insightful and outspoken of thinkers. Indeed, also experienced researchers may not be immune to what Derrida (1976) called "the metaphysics of presence." Convinced that the child, when asked about numbers, turns her thought to this extradiscursive entity, the interviewer remained oblivious to her own decisive part in what happens next. With her role in creating the "data" unacknowledged, she is unable to see that the words and phrases that appeared in her question may return in children's response, either explicitly or implicitly, in an altered version.

The lack of sensitivity to this later possibility would often express itself in the form in which the researcher would write her report. This is the case, for example, if she chooses to tell a story *about* what children said rather than quoting children's exact words. After all, it is only through the conscious effort to minimize the activity of interpreting at the time of record-making that we can become aware of the all important difference between the sentences "None of the boxes has more" and "The two boxes have *the same*." Because to the grown-up ears these two expressions sound entirely equivalent, the mere difference "in form" can be trusted to escape the attention of the experienced interlocutor. Yet, as was shown at length in this article, for children, this difference is substantial and decisive. In our research, it was the realization of this fact that ushered us into a new vision of numerical thinking and of its development. An important opportunity would thus be lost if we did not begin with the very words that Roni and Eynat actually uttered.[24]

Thus, to perform our job as teachers and as researchers properly, we have to learn to bracket our own understanding so as to leave space for children's interpretations. To be successful, one needs to be aware of the inherent difficulty of the en-

[24]Let us remark that even Piaget, whose books are full of brief, seemingly literal quotes from his interviews with little children, may have not been sensitive enough to the subtleties of children's use of words and routines. In the absence of advanced tools for recording, Piaget had probably no choice but to use hand-written notes. We do not even know whether these notes were taken throughout the interviews or retroactively. In any case, without the kind of sensitivity to the exact wording that we have developed thanks to the unlimited recording possibilities, what Piaget has written might be his interpretation of what the child said rather then the exact words uttered by the child.

deavor. After all, bracketing is a matter of rising above one's own ways of communicating. Modifying our ways of thinking with the help of this very thinking seems as unlikely a mission as trying to change a movement of a train by pressing on its walls from inside. The task of opening our old timers' minds to those of newcomers will thus probably always remain only half done and will always invite new insights.

Communication

This study, although not intended as such, taught us also a thing or two about human communication. Rather than focusing exclusively on how children think and how this thinking is different from that of the grown-ups, we were talking about the difficulty of each of the parties with interpreting the other on this other party's own terms.

What we saw led us to modify Vygotsky's famous statement about learning as a transfer of knowledge (or of *higher mental functions*) from interpersonal to intrapersonal plane. Without contradicting Vygotsky, but using less acquisitional language and stressing slightly different aspects, we would rather say that what begins as an interpersonal affair turns in our growing minds into a matter of our relation with the human-independent world. This, it seems, is the one-way process, and the change from the interpersonal to between-person-and-the-world outlook, once accomplished, can hardly be reversed. The condition for seeing the world in mature ways is forgetting the image of the world as it was at the outset. Like with the famous picture that can be interpreted as showing either a rabbit or a duck but never the two of them simultaneously, the moment we mange to see one possibility, we cannot see the other. The inevitable closing of the developing mind to the infinity of possible ways of communicating, and thus to the infinity of ways in which the world may be seen and interpreted, is both the advantage and the price of our growing up.

There is no reason, however, to be harsh with ourselves because of our basic ignorance of the deep gap between our own and the children's ways with words. According to Grice's (1975) maxim of relevance,[25] which is corroborated and accounted for by Dennet's (1987) doctrine of intentional stance,[26] assuming that familiar words in familiar contexts are used by others the way they are used by ourselves is the basic condition of a successful conversation. Paradoxically, it may be the grown-ups' insensitivity to the difference that constitutes the most powerful incentive for the change in children's discourse. It is because of the grown-ups' naive

[25]According to Grice (1975), we are able to understand our interlocutors because we assume that what they said is relevant to the topic of the conversation, as we see it.

[26]To display "intentional stance" means to treat animated and unanimated objects as rational agents who are driven by intentions similar to our own. According to Dennet (1987), we all adopt this stance spontaneously most of the time.

insistence on communicating on their own terms that the child has no choice but to adjust her discursive ways so that they fit with those of the more experienced interlocutors. Fortunately, the young interlocutors are only too willing to do so.

The needs of grown-ups and of children complement each other and both these sets of needs stem from the underlying common need for communicating. In children, this latter need motivates learning, and in grown-ups it motivates teaching. These two complementing propensities prove themselves time and again whenever one learns to talk, and thus to think, "like a grown-up." We believe that further improvement is possible provided all the participants of this spontaneous learning and teaching activity agree to a more relaxed, less definite division of labor. We read our data as showing that the grown-ups have a lot to learn from the children, provided they open themselves to being taught.

ACKNOWLEDGMENTS

We thank the anonymous reviewers for the exciting "double-blind" conversation, for contributing generously from their thinking, and for helping us to be clearer about our own.

REFERENCES

Anderson, J. R., Reder, L. M., & Simon, H. A. (1996). Situated learning and education. *Educational Researcher, 25*(4), 5–11.

Bakhtin, M. (1981). *The dialogic imagination.* Austin: University of Texas Press.

Beach, K. (1995). Activity as a mediator of sociocultural change and individual development: The case of school-work transition in Nepal. *Mind, Culture, and Activity, 2*(4), 285–302.

Ben-Yehuda, M., Lavy, I., Linchevski, L., & Sfard, A. (in press). Doing wrong with words or what bars students' access to arithmetical discourses. *The Journal for Research in Mathematics Education.*

Bloor, D. (1976). *Knowledge and social imagery.* Chicago: The Chicago University Press.

Borges, J. L. (1964). *Labyrinths.* Bungay, Suffolk, England: Penguin.

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18*(1), 32–42.

Cazden, C. B. (2001). *Classroom discourse—The language of teaching and learning* (2nd ed.). Portsmouth, NH: Heimeann.

Cobb, P., & Hodge, L. L. (2002). *An interpretive scheme for analyzing the identities that students develop in mathematics classrooms.* Unpublished manuscript.

Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking, and classroom practice. In E. Forman, N. Minick, & A. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development* (pp. 91–119). New York: Oxford University Press.

Cole, M. (1996). *Cultural psychology.* Cambridge, MA: Belknap.

Dehaene, S. (1997). *The number sense: How the mind creates mathematics.* Oxford, England: Oxford University Press.

Dennett, D. (1987). *The intentional stance.* Cambridge, MA: Bradford Books/MIT Press.

Derrida, J. (1976). *Of grammatology* (G. C. Spivak, Trans.). Baltimore: Johns Hopkins Press.
Edwards, D. (1997). *Discourse and cognition.* London: Sage.
Edwards, D., & Mercer, N. (1987). *Common knowledge: The development of understanding in the classroom.* London: Methuen.
Ernest, P. (1991). *The Philosophy of mathematics education (studies in mathematics education).* London: Routledge Falmer.
Fuson, K. C. (1988). *Children's counting and concepts of number.* New York: Springer-Verlag.
Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number.* Cambridge, MA: Harvard University Press.
Greeno, J. G. (1997). On claims that answer the wrong question. *Educational Researcher, 26*(1), 5–17.
Grice, H. (1975). Logic and conversation. In P. Cole & J. L. Morgan (Eds.), *Syntax and semantics, Vol. 3, Speech acts* (pp. 41–58). New York: Academic Press.
Harré, R., & Gillett, G. (1995). *The discursive mind.* Thousand Oaks, CA: Sage.
Heath, S. B. (1983). *Ways with words: Language, life, and work in communities and classrooms.* Cambridge, England: Cambridge University Press.
Lave, J. (1988). *Cognition in practice*. Cambridge, England: Cambridge University Press.
Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation.* Cambridge, England: Cambridge University Press.
Matthews, G., & Matthews, J. (1978). *Early mathematical experiences.,* London: Schools Council/Addison-Wesley.
McGarrigle, J., & Donaldson, M. (1974). Conservation accidents. *Cognition, 3,* 341–350.
Mehan, H. (1979). *Learning lessons.* Cambridge, MA: Harvard University Press.
Mehler, J., & Bever, T. G. (1967). Cognitive capacity of very young children. *Science, 158*, 141–142.
Mill, J. S. (1848/1959). *System of logic: Ratiocinative and inductive.* London: Longmans.
Much, N. C., & Shweder, R. A. (1978). Speaking of rules: The analysis of culture in breach. *New directions for child development, 2,* 19–39.
Noss, R., Hoyles, C., & Pozzi, S. (2002). A study of nurses' conception of conservation. *Journal for Research in Mathematics Education, 33*(3), 204–229.
Nunes, T., & Bryant, P. (1996). *Children doing mathematics (understanding children's worlds).* Oxford, England: Blackwell.
Nunes, T., Schliemann, A. D., & Carraher, D. W. (1993). *Street mathematics and school mathematics*. Cambridge, England: Cambridge University Press.
Piaget, J. (1952). *The child's conception of number.* London: Routledge & Kegan Paul.
Reddy, M. (1978). The conduit metaphor: A case of frame conflict in our language about language. In A. Ortony (Ed.), *Metaphor and thought* (2nd ed., pp. 284–324). Cambridge, England: Cambridge University Press.
Reed, H. J., & Lave, J. (1979). Arithmetic as a tool for investigating the relations between culture and cognition. *American Ethnologist, 6,* 568–582.
Rogoff, B. (1991). *Apprenticeship in thinking: Cognitive development in social context.* New York: Oxford University Press.
Rogoff, B. (2003). *The cultural nature of human development.* New York: Oxford University Press.
Rorty, R. (1979). *Philosophy and the mirror of nature*. Princeton, NJ: Princeton University Press.
Rorty, R. (1989). *Contingency, irony, and solidarity.* Cambridge, England: Cambridge University Press.
Rotman, B. (1994). Mathematical writing, thinking, and virtual reality. In P. Ernest (Ed.), *Mathematics, education, and philosophy: An international perspective* (pp. 76–86). London: The Falmer Press.
Rotman, B. (2000). *Mathematics as sign: Writing, imagining, counting.* Stanford, CA: Stanford University Press.
Saxe, G. B. (1991). *Culture and cognitive development.* Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Saxe, G. B., Guberman, S. R., & Gearhart, M. (1987). Social processes in early number development. *Monographs of the Society for Research in Child Development,* 216, 52.

Schutz, A., & Luckmann, T. (1973). *The structures of the life world.* Evanston: Northwestern University Press.

Sfard, A. (1998). On two metaphors for learning and on the dangers of choosing just one. *Educational Researcher*, 27(2), 4–13.

Sfard, A. (2000). Symbolizing mathematical reality into being: How mathematical discourse and mathematical objects create each other. In P. Cobb, K. E. Yackel, & K. McClain (Eds.), *Symbolizing and communicating: Perspectives on mathematical discourse, tools, and instructional design* (pp. 37–98). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Sfard, A. (2001a). Learning mathematics as developing a discourse. In R. Speiser, C. Maher, & C. Walter (Eds.), *Proceedings of 21st Conference of PME-NA* (pp. 23–44). Columbus, Ohio: Clearing House for Science, Mathematics, and Environmental Education.

Sfard, A. (2001b). There is more to discourse than meets the ears: Learning from mathematical communication things that we have not known before. *Educational Studies in Mathematics*, *46*(1/3), 13–57.

Simon, T. J., Hespos, S. J., & Rochat, P. (1995). Do infants understand simple arithmetics? A replication to Wynn (1992). *Cognitive development, 10,* 253–269.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, *77*, 44–49.

Starkey, P., & Cooper, R. G., Jr. (1980). Perception of numbers by human infants. *Science, 210,* 1033–1035.

Voigt, J. (1985). Patterns and routines in classroom interaction. *Recherches en Didactique des Mathématiques, 6,* 69–118.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press.

Vygotsky, L. S. (1987). Thought and speech. In R. W. Rieber & A. S. Carton (Eds.), *The collected works of L. S. Vygotsky* (Vol. 1, pp. 19–30). New York: Plenum.

Walkerdine, V. (1988). *The mastery of reason*. London: Routledge.

Wertsch, J. V., Minick, N., & Arns, F. J. (1984). The creation of context in joint problem solving. In B. Rogoff & J. Lave (Eds.), *Everyday cognition: Its development in social contexts.* (pp. 151–171). Cambridge, MA: Harvard University Press.

Wittgenstein, L. (1953). *Philosophical investigations*. Oxford, England: Blackwell.

Wynn, K. (1992). Addition and subtraction by human infants. *Nature, 358.* 749–750.

Wynn, K. (1995). Origins of numerical knowledge. *Psychological Science, 7,* 164–169.

Xu, F., & Carey, S. (1996). Infant's metaphysics: The case of numerical identity. *Cognitive Psychology, 30,* 111–153.

Zhou, X. (2002). Preschool children's use of counting to compare two sets in cardinal situations. *Early Child Development and Care, 172*(2), 99–111.

APPENDIX
Episodes

Transcription and Coding Conventions

$OPN_{CS:N}$	Opening for the routine procedure CS:N (see the coding of the procedures below)
$PRC_{E:M(CS:N)}$	Performance of routine subprocedure E:M, within the procedure CS:N
$CLS_{Su[n]}$	Closure of the procedure Su[n]
pos	Positioning

Routine Procedures

[CS:D]	comparing sets, direct	([3], [35], [62], [214])
[CS:M]	comparing sets, mediated by mapping	([247])
[CS:N]	comparing sets, mediated numerically	([10-22], [36-39], [64-69], [215-234])
[E:D]	evaluating sets, direct, by subitizig	([36b], [37c], [65])
[E:M]	evaluating sets, mediated by counting	([11], [12-[4], [23-24], [66], [216-218], [219], [224])
[CN:D]	comparing numbers, directly (endorsed)	([16], [18], [221], [232], [234])
[CN:M]	comparing numbers, mediated, by reciting	([249-260])
Su(n)	substantiating (justifying) utterance [n]	([4-9])

Remarks

1. The person who tries to prompt a certain reaction may have a different procedure in mind than the one that is actually enacted by the person who is prompted. The same is in force for closure. The index written in the table is the routine meant by the person who is speaking, according to our interpretation.
2. Rows that do not have their own code are to be understood as being a continuation of the activity identified by the last code appearing above them. For example uncoded rows appearing after a row coded as $OPN_{CS:D}$ are to be regarded as a part of the opening.
3. Indented part of the coding zone refers to a subroutine of the last coded activity.
4. Number words are written with digits as long as there is only one possibility to read the symbol.
5. The superscript numbers throughout the episodes are footnoted at the end of the Appendix.

EPISODE I: BOXES WITH 8 AND 6 MARBLES

EPISODE I_a
Direct Comparing (Choosing)

Speaker	*What Was Said*	*What Was Done*	*Type of Action*
1. Mother	I brought you two boxes. Do you know what is there in the boxes?	Puts two identical closed opaque boxed on the carpet, next to the girls.	
2. Roni	Yes, marbles.		
3a. Mother	Right, there are marbles in the boxes.	While saying this, points to the box close to Eynat, then to the other one.	
3b. Mother	I want you to tell me in which box there are more marbles.		$OPN_{CS:M}$
3c. Eynat		Points to the box which is closer to her.	$CLS_{CS:D}$, *pos*
3d. Roni		Points to the box Eynat is pointing to.	$CLS_{CS:D}$, *pos*
4. Mother	In this one? How do you know?	Points to the box the girls pointed to.	$OPN_{Su[3]}$
5. Roni	Because this is the biggest than this one. It is the most.	While saying "than this one" points to the other box [the one close to her]	$CLS_{Su[3]}$
6. Mother	Eynat, how do you know?		$OPN_{Su[3]}$
7. Eynat	Because … cause it is more huge than that.	Repeats Roni's pointing movement when saying "than that"	$CLS_{Su[3]}$
8. Mother	Yes? This is more huge than that? Roni, what do you say?	Repeats Roni's and Eynat's pointing movement when saying "than that"	$OPN_{Su[3]}$
9. Roni	That this is also more huge than this.	Repeats the above pointing movement when saying "than this"	$CLS_{Su[3]}$

EPISODE I_b
Mediated Comparing (By Counting)

10a. Mother	Do you want to open and see? Let's open and see what there is inside. Take a look now.		$OPN_{CS:N}$
10b. Roni		Abruptly grabs the box which is nearer to Eyant, and which was previously chosen by the girls as the one with *more* marbles.	*pos*

10c.			Tries to grab the same box, but gives up; turns to the other box	
11.	Roni	1.. 1.. 1.. 2, 3, 4, 5, 6, 7, 8.	Opens the box and counts properly.	$PRC_{E:M\ (CS:N)}$
12.	Eynat	1, 2, 3, 4, 5, 6.	Opens the other box and counts properly.	$PRC_{E:M(CS:N)}$
13.	Mother	So, what do you say?		$OPN_{CN(CS:N)}$
14.	Roni	6.		$CLS_{E:M(CS:N)}$
15.	Mother	Six what? You say 6 what? What does it mean "six"? Explain.		$OPN_{Su[14]}$
16.	Roni	That this is too many.		$CLS_{CN(CS:N)}$
17.	Mother	That this is too much? Eynat, what do you say?		
18.	Eynat	That this too is a little.		
19.	Mother	That it seem to you a little? Where do you think there are more marbles?		$OPN_{CS:N}$
20.	Roni	I think here.	Points on the box which is close to her (the one with 8 marbles)	$CLS_{CS:N}$
21.	Mother	You think here? And what do you think, Eynat?		$OPN_{CS:N}$
22.	Eynat	Also here.		$CLS_{CS:N}$
23.	Mother	Here? And how many marbles are here?	Points to the box which is close to Eynat	$OPN_{E(CS:N)}$
24a.	Eynat	1, 2, 3, 4, 5, 6 ….		PRC_{E}
24b.	Eynat	6!		CLS_{E}
25.	Mother	And how many here?	Shows the box next to Roni.	$OPN_{E(CS:N)}$
26.	Roni	8.		CLS_{E}
27.	Mother	Eight? How do you know?		$OPN_{Su[6]}$
28.	Roni	Because I already counted.		$CLS_{Su[26]}$

EPISODE II: BOXES WITH 2 AND 2 MARBLES

EPISODE II_a
Direct Comparing

Speaker	*What Was Said*	*What Was Done*	*Type of Action*
34a. Mother	I am putting two boxes with marbles here. Where … in which one of them are there more marbles? Tell me.	The interviewer puts one box next to Roni and another next to Eynat. Both boxes are closed.	$OPN_{CS:N}$

34b. Roni		Exchanges the placement of the two boxes. Then changes her mind and reproduces the original arrangement of the boxes.	*pos*
35a. Mother	Where are more marbles? What do you think?		$OPN_{CS:N}$
35b. Eynat		Points to her box without opening it.	$CLS_{CS:D}$

EPISODE II_b
Mediated Comparing (Direct Numerical Assessment)

36a. Roni		Opens her box and closes it immediately.	$PRC_{E:D(CS:N)}$
36b. Roni	2.		$CLS_{E:D(CS:N)}$
37a. Mother	Eynati, do you want to open yours?		$OPN_{E(CS:N)}$
37b. Eynat		Opens her box and closes it immediately.	$PRC_{E:D(CS:N)}$
37c. Eynat	2.		$CLS_{E:D(CS:N)}$
38. Mother	2 … Indeed? Where are more marbles?		$OPN_{CN(CS:N)}$
39. Roni	In none.		$CLS_{CN:D(CS:N)}$

EPISODE II_c
Explaining the Comparison ("The Same")

42. Father	Why? Why do you say this?		$OPN_{Su[39]}$
43. Roni	Because there is [are][1] 2 in one, and in [this] one there is [are] another 2.	Shows 2 with her fingers.	$CLS_{Su[39]}$
44. Father	So, this is why there is [are][1] more in none of them? So, in both of them there is [are][1] … what?		
45. Roni	2.		
46. Father	And this is … more or less?		
47. Roni	Less.		
48. Father	Less than what?		
49. Roni	Than … than … than big numbers.		
50. Father	Than big numbers? That means.. If there is [are][1] 2 in one box and 2 also in the other, than what is there in the two boxes?		
51. Roni	4.		

52. Father	Aha. Together, there is [are][1] 4?		
53. Roni	Yes.		
54. Father	And in each box there is the sa …		
55. Roni	Because it is between…		
56. Father	I see. And there is the same [thing][3] in each box?		
56a. Roni	…..		
57. Father	How many in each box?		
58. Roni	2.		
59. Father	Oh well …	Says in the tone signaling resignation.	

EPISODE III: BOXES WITH 2 AND 4 MARBLES

EPISODE III_a
Direct Comparing (Choosing)

Speaker	*What Was Said*	*What Was Done*	*Type of Action*
60a. Mother		Brings two identical opaque boxes (except that one is placed on the rug on its cover and the other on its bottom), one of which contains 2 marbles and the other 4 marbles.	
60b. Roni	I want to put [them down]	Exchanges the placement of the two boxes. Then changes her mind and reproduces the original arrangement of the boxes.	*pos*
61a. Mother	You want to do it? Fine.		
61b. Roni		Puts the boxes on the rug.	
62a. Mother	What do you say? Where are more marbles?		$OPN_{CS:N}$
62b. Eynat		Eynat points to the box on the left side.	$CLS_{CS:D}$
62c. Roni		Points to the box Eynat is pointing to.	CLSCS:D

EPISODE III_b
Mediated Comparing (Counting)

63. Mother	Both of you say it is here? How do you know? What makes you think so?		$OPN_{Su[62]=CS:N}$

64a.	Eynat	I want to open.	She pulls the box to which she pointed before as the one that contains more.	*pos*
64b.	Roni		Starts pulling the box too.	
64c.	Eynat		Takes the box to herself and opens it.	
64d.	Roni		Takes the other box and opens it.	$PRC_{CS:N}$
65.	Roni	2.		$PRC_{E:D(CS:N)}$
66a.	Eynat	1, 2, 3, 4 [… .]		$PRC_{E:M(CS:N)}$
66b.	Eynat	4!		$CLS_{E:M(CS:N)}$
67.	Roni	We were right.		$CLS_{CS:N}$
68.	Mother	You were right about what?		
69.	Roni	That it is more here.	Points to the box next to Eynat.	

EPISODE III$_c$
Explaining the Comparison

70.	Mother	Why? Why is [are][1] there more here?		$OPN_{SuCpr)}$
71.	Eynat	Because there is [are][1] 4 here.		$CLS_{SuCpr)}$
72.	Roni	4.	Speaks together with Eynat	
73.	Mother	Yes? What can yo .. […]. So 4 is … What is it?		OPN_{NCpr}
74.	Roni	It is more.		CLS_{NCpr}
75.	Mother	Show me how you know that this is more. What tells you there is more?		$OPN_{SuNCpr)}$
76.	Eynat	10 is more.		PRC_{NCpr}
77.	Mother	10 is more?		
78.	Roni	Yes. And 4 too.		
79.	Mother	10 is more than what?		
80.	Roni	Than 1 and 2.		
81a.	Mother	10 more than 1 and 2?		
81b.	Roni		Nods "Yes"	
82.	Mother	… and than 4?		
83.	Roni	4 is also a little		
84.	Mother	4 is also a little? And what is 10?		
85.	Roni	10 is …		
86.	Mother	What?		
87.	Roni	Nothing.	Laughs, raises her shoulder in the movement that says, "I don't know."	

88a. Mother	10 is nothing? You don't know what to say? You can say whatever you want to say. Whatever you say is fine.		
88b. Roni		Shakes her head for "No."	
89. Mother	In which box there is [are][1] more now?		
90. Roni	We were both right together.		*pos*
91a. Father	Indeed?		
91b. Roni		Nods "Yes"	
92. Father	And how many is [are][1] there in the other box?		
93. Roni	Si …	Opens the box	
94a. Father	How many here?		
94b. Roni		CLSes the box	
95. Roni	2.		
96. Father	2?		

EPISODE III_{d}
Making Them "The Same": Roni's Attempt 1 (Making 2 and 4, Like Before)

97. Mother	Can it be done so that there be the same [thing][3]? That there be the same marbles in both boxes [… .] the same amount of marbles in the tw …		$\text{OPN}_{\text{make the same}}$
98. Roni	Yes, it's fine, but … but … but if you open and see that you have in the same box … ahm … you have the same [thing][3], then you get 4.		$\text{CLS}_{\text{make the same}}$
99. Mother	How?		$\text{OPN}_{\text{Sumake the same)}}$
100. Roni	2 and 2 is 4.		$\text{CLS}_{\text{Sumake the same)}}$

EPISODE III_{e}
Making Them "The Same": Eynat's Attempt 1 (Making 4 and 4; Roni Interferes)

101. Mother	Aha … But I can … if I have 4 marbles here … and here I have 2 marbles …	Opens the first box, and then the other one.	$\text{OPN}_{\text{make the same}}$
102. Mother	Is it possible to change something so that there will be the same [thing][3] in both boxes?		
103. Roni & Eynat		Nod "Yes"	

104. Mother	How can you do this, Eynat?	Points to the box with 4 marbles	
105. Eynat	You can take from the pink box and then put the same 4 [or …four of the same…?] [2]	Nods “yes”	$\text{PRC}_{\text{make the same}}$
106. Mother	Shall I bring you the pink box? Do you want me to bring it?		*pos*
107. Eynat	But *I* will put!		
108. Roni	Which pink?	Shouts and interrupts Eynat.	
109. Mother	But, wait a moment … Before … The box with marbles … there are lots of marbles	*Gets up, and then sits down again*	
110. Roni	No, I don’t allow!		
111. Mother	Wait, Roni, wait	*Tries to calm her daughter, speaks along with her*	
112. Father	We won’t bring, we won’t		
113. Roni	I don’t want anybody touch the box		
114a. Mother	I am not fetching any pink box, but wait a minute, be patient!		
114b. Mother	Eynat, can you do in the boxes ’ anything with this boxes? What can you do?		$\text{OPN}_{\text{make the same}}$
115. Father	So that there is the same.		
116. Eynat	I can do so that M… eh … that there will be [are][1] 4.		
117. Mother	How can you do this? Show me.		
118. Roni	You tak … She takes from the many less!	*Shouts.*	*Pos*
119. Eynat	No, I want	*Shouts so as to be heard in spite of Roni’s shouting*	
120. Roni	I want!		
121. Mother	Ok, ok, you will do it later		
122. Roni		*Shows she is hurt, gets up and walks away.*	
123. Eynat	If I make 4, and I count, and I see there is [are][1] 4, then it is the same 4		$\text{PRC}_{\text{make the same}}$

EPISODE III_{f}
Making Them “The Same”: Roni’s Attempt 2 (Making 4 and 4)

124. Mother	Now Roni, explain what you think. Each one of you, in turn, will say what she thinks. Do you want to tell? Do you want to tell us what you think?		$\text{OPN}_{\text{make the same}}$ *pos*

125. Father	I want to know whether there is anything you can do … so that there be the same number of marbles in the two boxes.		
126. Roni	I know.		
127. Eynat	I can.		
128. Mother	You know too? Now, let's listen to Roni. We have listened to you before.		*Pos*
129. Father	Now, let's listen to Roni. Roni, make so that in both boxes… Here, go to the boxes …		
130. Roni	You take from … from … hmm … from all those pretty marbles …		
131, 132. Mother and Father	Yes? ….		
133. Roni	And … you take 1 pretty … and 4 pretty marbles … and hmmm … and then you get 4 and 4.		$\text{CLS}_{\text{make the same}}$
134. Mother	Show me.		$\text{OPN}_{\text{make the same}}$
135. Father	Show us how you do this. Show us. Make it so that there be the same number of marbles in both boxes.		
136a. Roni	But I need all the marbles.		
136b. Roni		She approaches the boxes.	
137a. Mother and Father	With these marbles only. Can you?		
137b. Roni		Nods "Yes"	
138. Roni	But here, there is 2 and 1 and it makes 6.		
139. Mother	Here, there is 2, and here there is …		
140. Roni	4.		
141. Father	So, make it so that there be the same [thing][3] in both boxes.		
142. Roni	But you can't.		$\text{CLS}_{\text{make the same}}$
143. Father	You can't?		

EPISODE III_g
Making Them "The Same": Eynat's Attempt 2 (Making 4 and 2?)

144. Mother	Eynati, what do you say?	$\text{OPN}_{\text{make the same}}$
145. Father	Can you, Eynat, make it so that there will be the same [thing][3] in both boxes?	

146. Eynat	Yes.	Nods "Yes"	
147a. Mother and Father	What will you do?		
147b. Eynat		[For illustration of the stages in Eynat's procedure see below the episode.] With her left hand takes 2 marbles from the box of 4 at her left, and with her right hand takes the 2 marbles from the other box. Transfers 1 marble from the left hand to the right hand, and throws the other 1 to the empty box on her right. Puts into the box on the left (with 2 marbles) 2 of the 3 marbles from her right hand. She organizes the 4 marbles in the left box in a line, holds 1 marble in her right hand.	$PRC_{\text{make the same}}$
148. Roni		Counts the marbles in the box left to Eynat.	*pos*
149. Eynat		While Roni is counting, gently opens her right hand and lets the 1 marble fall into the box with 1 marble on her right. The marbles and boxes are back to the initial arrangement.	$CLST_{\text{make the same}}$
150. Roni	2	Points to the box with two marbles with two fingers.	
151a. Father	2 and 4 – is this the same [thing][3]?		
151b. Roni		Shakes her head for "No."	pos

LB = left box   RB = right box   LH = left hand   RH = right hand

LB	RB	LH	RH
4	2	0	0
2	0	2	2
2	0	1	3
2	1	0	3
4	1	0	1
4	2	0	0

EPISODE III_h
Making Them "The Same":
Eynat's Attempt 3 (Making 4 and 2?)

152. Father	So, make it so that there will be the same [thing][3]. Eynat, can you do this?		$OPN_{\text{make the same}}$
153. Roni	She can't.		*pos*
154. Roni	I know that … that you can't do it.		
155. Mother	You know that it can't be done? Eynat, you can?		
156. Eynat	But I can, I can.		
157. Mother	But you can? So, let's see, perhaps she can do it, after all.		
158. Roni	Aaaa … Now … aaaa … I want to put … I want…		*pos*
159. Eynat		[For illustration of the stags in Eynat's PRCedure see below the table.] While Roni speaks, she takes 2 marbles from the box on the left, 1 marble from the box on the right, transfers 2 marbles to the right hand, puts them in the box on the right and stays with 1 marble in her hand. She strokes the marble. She transfers the marble from the left hand to the right hand and puts it in the box on the right (with 3 marbles).	
160. Eynat	I can't		
161. Eynat		She collects all the marbles in her right hand.	

LB = left box   RB = right box   LH = left hand   RH = right hand

LB	RB	LH	RH
4	2	0	0
2	1	2	1
2	3	0	1
2	3	1	0
2	4	0	0

EPISODE III$_i$
Making Them "The Same": Roni's Attempt 3 (Making 4 and 2?)

183a.	Father	I want to ask you something, girls. I have an idea, please take all the marbles from the two boxes and put them on the rug.		OPN$_{\text{divide the set}}$
183b.	Roni		Roni puts five marbles on the rug.	
183c.	Eynat		Holds one marble.	
184a.	Father	Put all the marbles together. Eynat this marble too, so that all the marbles are together	Makes clear that he is speaking about the one marble Eynat was still holding.	
184b.	Eynat		Gives the marble to Roni.	
185.	Father	Now I want to ask you. Now, can you divide the marbles so that that there will be the same [thing][3] in the two boxes?		
186.	Roni	1 … 2 … 3 … 4 …	As she names the marbles, she puts them in the box on the right. She smiles all the time	PRC$_{\text{divide the set}}$
187.	Roni	2.	Puts the other two marbles in the box on the right.	CLS$_{\text{divide the set}}$
188.	Father	2 and 4, is this the same [thing][3]?		
189.	Roni	No.		
190.	Father	So what? I told you to put in each box … so that there will be the same 2 and 4, is this the same [thing][3].		
191.	Roni	We can't do it. You have to put others.		

EPISODE IV: BOXES WITH 8 AND 10 MARBLES

EPISODE IV_a
Direct Comparing (Choosing)

212. Mother	I have a different question. Let's see… I want to ask you something else I now … Roni	Brings two closed opaque boxes with 8 marbles in one of the boxes and 10 marbles in the other.
213a. Mother	Lets see whether you are able to discover how many … no … Where is [are][1] there more marbles? I put now a lot, so let's see. One moment, who thinks … Where, where do you think there is [are][1] more … One moment!	
213b. Roni		As her mother speaks, tries to open the box next to her.
214a. Mother	Where do you think there is more, Roni?	
214b. Eynat		Points to the box next to her
214c. Roni		Points to the box next to Eynat
214d. Roni & Eynat		Try to open the box next to Eynat

EPISODE IV_b
Mediated Comparing (Counting)

215a. Mother	Do you want to check?	
215b. Eynat	I … Here …	Opens the box next to her, which each of the girls wanted for herself.
216. Roni		Opens the other box
217. Mother	Here …	
218. Roni	1, 2, 3, 4, 5, 6, 7, 8, 9, 10	
219. Eynat	1, 2, 3, 4, 5, 6, 7, 8, 9	As she counts, the marbles role and she errs without realizing
220. Mother	Let's check. Roni, what do you say?	
221. Roni	10.	
222. Mother	You say 10? And how many do you think is here? Do you want to check? Because they moved … Eynat, do you want to count again?	

223.	Roni	*I want them back!*		*pos*
224.	Eynat	1, 2, 3, 4, 5, 6, 7, 8		
225.	Roni	You promised!		*pos*
226.	Eynat	8.		
227.	Mother	8. And how many do you have, Roni?		
228.	Roni	10.		
229.	Mother	10. So were is more?		
230.	Roni	How many do you have? I haven't heard.	Turning to Eynat	
231.	Mother	She said that she has 8, so where is there more? In this box or in this one?		
232.	Eynat	In this one	Points to the box nest to Roni	
233.	Mother	Roni, what do you think?		
234.	Roni	In this one.	Points to the same box and closes it.	

EPISODE IV_c
Explaining Comparisons Between Sets

235.	Mother	Why do you think that in this one?	
236a.	Eynat	Because there is 10.	
236b.	Roni		Nodes "Yes"
237.	Mother	Because there is 10, and.. what?	
238.	Roni	Because there is 10.	
239.	Mother	What is more-many[4], 10 or 8?	
240.	Roni	10.	
241.	Mother	10 is more than 8?	
242.	Eynat	Yes.	

EPISODE IV_d
Explaining Comparisons Between Numbers: Attempt 1

243.	Mother	How do you know?	$OPN_{Su[241])}$
244.	Roni	Because … because … we counted.	PRCSu["It is 10"]
245.	Mother	You know because you counted?	
246.	Mother	Eynat, how do you know?	$OPN_{Su[241])}$
247a.	Eynat	Because … because … because I saw there is a long row,	$PRC_{Su[241])}$
247b.	Eynat	and then I knew that it was 10.	$CLS_{Su[241])}$
248.	Mother	You saw a long row and then you knew it was 10?	

EPISODE IV_e
Explaining Comparisons Between Numbers: Attempt 2

249. Mother	But how do you know that 10 is more than 8?		$OPN_{Su[241])}$
250. Eynat	Because 8 is not the end.		
251. Mother	8 is not the end? And what *is* the end?		
252. Eynat	It's 10.		*pos*
253. Roni	I want to…	Seems quite impatient.	
254. Mother	The end says that it is the most[5]? Is there a number that is bigger then 10?		
255. Roni	Yes, 1000!		
256. Mother	1000 is bigger?		
257. Roni	Yes.		
258. Eynat	No, no, When numbers don't end, then this… the number is bigger.		
259. Mother	When numbers don't end, then the number is bigger? Why?		
260. Eynat	Because many numbers… it is up to.. ah.. this, this,…and if they order themselves fine, then… they will be the biggest.		
261. Mother	I see.		
262. Roni	I want…	Seems very impatient.	*pos*
263. Mother	What do you say about the big numbers?		
264. Roni	Now, now!		*pos*

APPENDIX NOTES

[1]The Hebrew word *yesh* was used and may be translated either to *there are* or to *there is.*

[2]These are two possible translations of the Hebrew *arba oto davar.*

[3]The Hebrew term for *the same* is composed of two words, *oto davar,* the literal translation of which is *the same thing* (*davar* mans *thing*). The first word, *oto*, cannot be used without being followed by a noun. So, either one specifies what is (are) the thing(s) that is claimed to be "the same," like in "the same number" (*oto mispar*) or "the same child" (*oto yeled*) or, if one does not want to be as specific as that, and just tries to say that A and B are *the same*, one says "A and B it's the same *thing*."

[4]The mother used two words here, *yoter* and *harbe*, which mean *more* and *many*, respectively. The expression *more-many* is meaningful to the listener and has the sense of *more-numerous*. Using the two words in conjunction is an incorrect but common Hebrew usage. *Yoter* is enough to say *more*, but the addition of the word *harbe* (*many*) comes to stress that the comparison is between the sets of marbles, not between numbers as such (if the numbers were compared, the comparative expression would be *yoter gadol*, which means, literally, *more big*, that is, *bigger*).

[5]The Hebrew expression was *hachi harbe*, which counts as rather colloquial and not entirely correct, but which produces the superlative of the determiner *many* (*harbe*) in a straightforward way, preserving the connection to the source.