WINNING BY DEFAULT: WHY IS THERE SO LITTLE COMPETITION IN GOVERNMENT PROCUREMENT?

KARAM KANG AND ROBERT A. MILLER

ABSTRACT. Government procurement contracts generally garner a small number of bids, and it is not uncommon for only one bid to be considered. This paper quantifies multiple factors determining the extent of competition observed in the United States federal procurement data for commercially unavailable IT and telecommunications service contracts. We develop, identify, and estimate a principal-agent model of procurement where a buyer exerts costly effort to attract a more competitive field of sellers, after which she negotiates contract terms and selects a winner. Our theory predicts that negotiations lower the benefits of extra bids, and the estimated decrease in the average number of bids from negotiations is between 0.6 and 2. Although removing discretion from procurement agencies reduces the scope for corruption, we estimate expected payments would rise up to 3 percent.

1. INTRODUCTION

Procurement accounts for about 16 percent of US federal government spending.[1] In spite of its vast size, the extent of competition for a procurement contract is not very intense. For example, $241 billion (or 45 percent of the payments for procurement contracts) were paid to contracts that attracted a single bid during FY 2010, based on the Federal Procurement Data System (FPDS). This could potentially be a source of a significant inefficiency in government resource allocation, especially for commercially unavailable, customized goods that do not have readily comparable market prices.

[1]See the overview of awards by fiscal year at usaspending.gov.

In this paper, we develop, identify, and estimate a procurement model to empirically quantify the factors determining the extent of competition observed in the data.

To conduct this analysis, we incorporate two important institutional features of federal government procurement that have attracted attention from the literature but which have not yet been studied jointly. First, a procurement agency (a buyer hereafter) chooses the extent to which a contract will draw competitive bids. Federal regulations permit contracting without providing for full and open competition under broadly defined circumstances.[2] Such practices are prevalent; for instance, 1.2 million contracts were awarded without full and open competition in FY 2010, consisting of 51 percent of all contracts that were awarded during that fiscal year. Many empirical studies, such as Krasnokutskaya and Seim (2011) and Athey, Coey and Levin (2013), estimate the effects of restricting entry or providing preferential treatments to certain bidders on auction outcomes, taking such policies as given. Our paper, on the other hand, studies how competition is endogenously determined, focusing on the buyers' preferences for particular sellers and the extent of competition.[3] In this regard, a growing body of empirical literature on the demand side of public procurement (Bandiera, Prat and Valletti, 2009; Coviello, Guglielmo and Spagnolo, 2017; Rasul and Rogger, forthcoming; Best, Hjort and Szakonyi, 2017; Decarolis et al., 2017) is closely related.

Second, the final contract price can differ from, and is often much larger than, the initially agreed upon price. Our analysis distinguishes between price changes associated with the initial contract and those that result from ex-post renegotiation. The latter have been empirically studied by Gagnepain, Ivaldi and Martimort (2013), Bajari, Houghton and Tadelis (2014), and Decarolis (2014); but the former have received scant attention in the empirical literature, despite the extensive theoretical literature on optimal contracting in procurement (Laffont and Tirole, 1987; McAfee and McMillan, 1987; Riordan and Sappington, 1987). Our paper fills this gap by exploiting distinctive features in the FPDS, which specifies the stated contract type, such as firm-fixed price and cost-plus, and the reason for each price and duration change. Given this information we partition price and duration changes into those

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[2]Federal Acquisition Regulations (FAR) 6.2 and 6.3 provide details on these circumstances, including discretionary ones (e.g., limited data rights, patents, copyrights, follow-on contracts, and urgency) as well as non-discretionary ones (e.g., statutes and set-asides for small business concerns). We focus on the former type of restrictions in competition.

[3]Our paper is complementary to the empirical literature on endogenous entry in auctions when the bidders pay entry costs (Bajari and Hortacsu, 2003; Hendricks, Pinkse and Porter, 2003; Li, 2005; Li and Zheng, 2009; Athey, Levin and Seira, 2011).

that are less likely to occur for firm-fixed price contracts, and those that are not. Both types of price changes are frequent and large in size in the data; in FY 2010, the former price changes accounted for 39 percent ($209 billion) of the total procurement payments, and the latter ones for 14 percent ($75 billion). Jointly studying endogenous competition and price changes is important because competitive behavior affects initial contract terms and, hence, the final contract price.

The regulations give the buyer considerable discretion in determining contract terms, as well as the extent of competition. However the data do not contain details about how negotiations on contract terms between the buyer and the sellers proceed, many of which could be informal; we only have details on procurement outcomes. For these reasons we model the procurement process as a two-stage noncooperative game where the buyer first chooses the extent of competition among sellers, and then negotiates contract terms.[4] During negotiations, the buyer, who is less informed than the sellers about their costs, offers a menu of contracts to maximize her expected payoff from completing the procurement project.[5] We assume her preferences depend on the transfer to the winning seller, efforts she expends on soliciting additional bids if she allows competition, and her desire to award the contract to a default seller rather than opening the process to competitive bids. The latter two factors could result from corruption and capture, as well as administrative costs (Bajari and Tadelis, 2001; Bandiera, Prat and Valletti, 2009) and noncontractible quality (Manelli and Vincent, 1995).

We distinguish two types of contract outcomes, which realize after completion of a project: those informative about seller's private costs and those not. We prove that the equilibrium contract terms fully insure sellers against the latter, but separate them through their dependence on the former. The buyer prefers contracts submitted by sellers with the lowest cost; as the number of bids increases, the expected transfer falls. In this way both the contract terms and the number of bids are connected to the buyer's decisions to promote competition.

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[4]The alternative to negotiated acquisitions is a sealed bidding procedure. When only one seller is considered, sealed bidding is not possible; even when multiple sellers are considered, sealed bidding is not a dominant procedure in the data. For example, among the 1.2 million contracts that were competitively awarded during FY 2010, only 1.4 percent of them were awarded using sealed bids.
[5]Bajari and Tadelis (2001) argue that menus for contracts like those Laffont and Tirole (1993) are not used for construction contracts, and mechanisms other than contract menus such as competitive bidding, reputation, and third-party bonding companies seem to be important in addressing adverse selection problems in procurement. In the contracts that we study, competition is not intense, most contractors do not win more than one contract, and performance and payment bonds are not required (FAR 28.103).

We provide sufficient conditions under which the model is nonparametrically identified from our data. In this regard, our paper belongs to the literature on the identification of principal-agent models (Perrigne and Vuong, 2011; Gayle and Miller, 2015; An and Tang, 2016). The primitives of the model include the buyer's preferences and the distribution of seller costs. We allow continuously distributed unobserved heterogeneity to stochastically determine seller costs; unobserved heterogeneity has been emphasized in the auction literature (Krasnokutskaya, 2011). Our identification arguments rely on the equilibrium relationship between the winning contract terms and the distribution of seller costs. Having identified the seller cost distribution, we identify the buyer's preferences from her choices about how much competition to promote.

We focus on contracts for commercially unavailable IT and telecommunications services that were awarded during FY 2004-2012. We control for observed heterogeneity by focusing on a single sector because procurement contracts differ widely in the scale, scope, and competitive intensity. Our estimates of the structural model show that the buyer's cost of intensifying competition is large, and that contracting on informative outcomes correlated with sellers' private cost types is valuable. Stripping the buyer of her discretion in designing contracts would increase the average number of bids by between 0.6 and 2 from 1.5 bids under the current regime, but the price would also increase by up to $35,800 per contract (3 percent). While our empirical results are specific to IT and telecommunications services, the empirical framework itself can be readily applied to other similar types of contracts.

The rest of the paper is organized as follows. In Section 2, we delineate the institutional setting and the data, and present empirical features that motivate our model, which is described in Section 3. The equilibrium of the model is presented in Section 4, followed by the identification of the model in Section 5. Section 6 presents the estimation results, and Section 7 discusses counterfactual policies. Section 8 concludes.

2. Institutional Background and Data

The data is drawn from the Federal Procurement Data System, through usaspending.gov, which has also been used in recent studies by Warren (2014), Liebman and Mahoney (forthcoming), and MacKay (2016). For each procurement contract, we observe the solicitation procedure, the number of bids, the contract type, the history of price and duration changes, product and service code, commercial availability,

contracting agency (e.g., Department of Defense), and the location of contract performance. The purpose of this section is to describe the institutional background and the features of the data that are the most pertinent to our analysis.

We analyze contracts with specified terms and conditions for commercially unavailable services initiated in FY 2004–2012.[6] In this paper, we focus on the contracts for IT and telecommunications services, including IT strategy and architecture, programming, cyber security, data entry, backup, broadcasting, storage, and distribution, and telephone/Internet services.[7] In FY 2010, for example, they accounted for $30 billion of the total federal government spending on procurement.

We further restrict our attention to the contracts that were (i) paid for $300,000–$5 million, (ii) expected to take longer than 30 days to complete, (iii) completed before the end of FY 2014, (iv) not terminated prematurely, and (v) performed within the the continental United States.[8] There are in total 2,203 such contracts in the data, costing the government $3.2 billion (in 2010 dollars) collectively.

2.1. **Competition and Negotiation.** Table 1 presents summary statistics of the contracts in the sample by the extent of competition as specified in the data. *Full and open competition* is the default in the acquisition process, but federal regulations specify the circumstances under which a procurement agency is allowed to limit competition (FAR 6.2 and 6.3). For more than two thirds of the contracts in the sample, full and open competition was not employed. The reasons stated in the data can be categorized into three cases: (i) unavailable for competition due to domestic statutes or international agreements, (ii) set-aside for small business concerns due to statutory requirements, such as section 8(a) of the Small Business Act and the

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[6]The contracts with specified terms and conditions are definitive contracts and purchase orders, the focus of our analysis. The alternatives are indefinite delivery, indefinite quantity contracts such as federal supply schedules. They, for example, accounted for 71 percent of the 2.4 million contracts that were awarded in FY 2010, but contracts with definitive contracts and purchase orders tend to be of a larger size, accounting for 47 percent of the procurement spending of that fiscal year.

[7]Specifically, we consider the contracts with a product and service code of Category D3. The FPDS requires that a product and service code be reported for each contract, and the codes are divided into three groups: research and development (R&D), service, and products. Among the service codes, there are 48 categories, and Category D3 is for IT and telecommunications services.

[8]Each acquisition of supplies or services that has an *anticipated* dollar value between $3,500 and $150,000 is automatically reserved exclusively for small business concerns (see FAR 19.502-2). Because the anticipated payment amount does not always appear in the data, we use a threshold of $300,000 for the actual payment. We exclude the contracts performed outside of the continental United States because the cost structure could be very different from those in our sample. It is very rare for contracts to be terminated prematurely (less than 1 percent), but such termination mostly originates from the buyer. For simplicity of the analysis, we also exclude all prematurely terminated contracts.

TABLE 1. Competition for IT Service Contracts (FY 2004-2012)

Extent of competition	Obs.	Size ($M)		One Bid	Number of Bids	
		Mean	SD	Ratio	Mean	Median
No/limited competition	1,631	1.49	1.20	0.93	1.39	1
Unavailable for competition	796	1.67	1.19	0.98	1.06	1
Set-aside for small business	183	1.71	1.31	0.44	4.20	2
Not competed by discretion	652	1.20	1.12	1.00	1.00	1
Full and open competition	572	1.30	1.10	0.36	4.08	2
Sealed bids	12	2.14	1.22	0.67	1.58	1
Competitive proposals	310	1.38	1.16	0.27	4.55	3
Simplified acquisition	185	1.01	0.84	0.48	2.22	2
Other competitive procedures†	65	1.61	1.21	0.37	7.58	2
Total	2,203	1.44	1.17	0.78	2.09	1

Notes: This table provides summary statistics of all contracts with definitive terms and conditions for commercially unavailable IT and telecommunications service contracts of FY 2004-2012 with a large size ($0.3–5 million) and a long expected duration (≥ 30 days). *Size* refers to the total amount of obligated money to the government per contract as of FY 2014, in 2010 dollars. † Architect-engineer and basic research.

Historically Underutilized Business Zones Act of 1997, and (iii) discretionary. As for the third category, the detailed reasons include the existence of limited rights in data, patent rights, copyrights, secret processes, or brand (54 percent in our data), follow-on contract (19 percent), urgency (8 percent), and other/unspecified (18 percent).

Even when bids are competitively solicited, having only one bid is quite common (36 percent) and the median number of bids is 2. The number of bids can be affected by the efforts of the buyer to exchange information with potential sellers in advance, via pre-solicitation notices, draft requests for proposals, requests for information, industry conferences, public hearings, market research, and one-on-one meetings. Note that attracting and evaluating an additional bid or proposal incurs an extra administrative burden.[9]

We partition the competitive solicitation procedures into four: sealed bids, competitive proposals, simplified acquisition, and the remainders. Table 1 shows that sealed bidding was rarely used; only 12 out of the 572 fully competitively solicited ones were auctioned. FAR 6.4 delineates the conditions under which sealed bidding

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[9]Furthermore, there is anecdotal evidence that the risk of receiving a bid protest from losing sellers is not small. Federal Times reported in July 2013 on how bid protests are slowing down procurements. The article quoted Mary Davie, assistant commissioner of the Office of Integrated Technology Services at the General Services Administration: "We build time in our procurement now for protests. We know we are going to get protested."

TABLE 2. Competition and Price

	Noncompetitive (1)	One Bid (2)	Log (Total Contract Price) (3)	Log (Total Contract Price) (4)
Military-related agency	0.130** (0.055)	0.118*** (0.037)		
Competitive			0.238** (0.094)	0.038 (0.089)
Log (Numer of bids)				0.199*** (0.059)
Base duration	Yes	Yes	Yes	Yes
Agency FE	No	No	Yes	Yes
Other FEs†	Yes	Yes	Yes	Yes
N	962	962	962	962
R^2	0.171	0.168	0.317	0.327

Note: The dependent variable for Column (1) is a dummy variable that takes 1 if the solicitation was noncompetitive and 0 otherwise; the dependent variable for Column (2) is a dummy variable that indicates whether only one bid was considered (including noncompetitive and competitive); and the dependent variable for Columns (3) and (4) is the logarithm of the total contract price in 2010 dollars, defined as the total amount of obligated money to the government per contract as of FY 2014. All contracts in the final sample are included. The standard errors are clustered at the 4-digit product and service code level, and provided in parentheses; $*p < 0.10$, $**p < 0.05$, $***p < 0.01$. *Military agencies* include the Departments of Defense, State, and Homeland Security. † 4-digit product and service codes, location of the contract performance (state), year of award, and month of the award, respectively.

is required. When these conditions are not met, the most prevalent solicitation procedure is through competitive proposals, where a buyer issues a request for proposal, upon which interested sellers submit their proposals. After receiving them, the buyer determines the competitive range of the sellers and undertakes negotiations tailored to each seller, allowing the seller to revise his proposal, regarding price, schedule, technical requirements, type of contract, or other terms of a proposed contract (FAR 15.3). Although the contract type is included in solicitation, it is treated as a matter for negotiation, considering various factors such as price competition and type and complexity of the requirement (FAR 16.1). The selection of the winner is based on a comparative assessment of (revised) proposals.

We focus on contracts that for discretionary reasons are awarded without competitive solicitation (Row 4 in Table 1), and those awarded through competitive proposals (Row 7 in the table). To study the role of a buyer's discretion, we exclude contracts designated noncompetitive for statutory reasons. Our model is less suitable for analyzing sealed bidding and simplified acquisition, where there is little scope for discretion (Bajari, McMillan and Tadelis, 2008), and for studying procurement

procedures related to basic research or professional services of an architectural and engineering nature. This produces a final sample of 652 noncompetitive contracts and 310 competitive ones, worth a total of 1.2 billion.

Based on the final sample, Table 2 shows that the contracts awarded by the military-related agencies (the Departments of Defense, State, and Homeland Security) tend to be less competitive than others. This may be related to more pre-qualification requirements to work for these agencies, such as security clearance, than others.

Table 2 also shows that more competition is associated with higher contract price, even after controlling for observed heterogeneity of each contract. This pattern is consistent with endogenous determination of the extent of competition where the buyer takes into various factors, both observed and unobserved by a researcher. Absent unobserved heterogeneity, the equilibrium of standard auction models predicts that procurement price falls as the number of bids increases.[10]

2.2. **Price and Duration Changes.** The contract price at the time of the award, *base price*, can be different from the actual price at the end of the contract, *final price*. We define the base price as the total amount of money that the government is obliged to pay at the beginning of the contract; the final price as the sum of all payments. The final price is typically higher than the base price, but not always. Similarly, the duration of a contract may change ex-post. The *base* duration is defined as the difference between the expected completion date in the contract and the starting date; and the *final* duration as the difference between the expected completion date of the last contract action and the starting date. A *delay* is then the difference between the final duration and the base duration.

The database provides the history of each record of price and duration changes. Table 3 shows the average amount and the frequency of price and duration changes by their associated reason: (i) additional work, (ii) change order and definitization of a letter contract, (iii) supplemental agreement for work within scope, (iv) exercise of an option, and (v) various administrative actions such as funding, close out and changes in the vendor information.[11]

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[10]In common value or affiliated private value auctions, a positive relationship between bids and the number of bidders may arise even in the absence of entry (Bulow and Klemperer, 2002; Pinkse and Tan, 2005).

[11]The full list of administrative actions are funding only, close out, novation agreement, re-representation of non-novated merger/acquisition, change of procurement instrument identifier, transfer action, vendor identifier change, and vendor address change. Note that we drop contracts that were terminated prematurely or canceled; hence contract modifications related to termination or cancellation are not considered here.

TABLE 3. Price and Duration Changes

	Price				Duration		Correlation‡
	All		Firm-Fixed†		All		
	Amount	Freq.	Amount	Freq.	Days	Freq.	
Base	712.2		690.9		433.4		
	(808.4)		(803.6)		(384.8)		
Final	1,256.6		1,112.7		787.1		
	(1,132.1)		(1,021.4)		(647.0)		
Change							
Any	543.6	0.69	421.5	0.64	353.7	0.57	0.41
	(860.1)	(0.46)	(702.5)	(0.48)	(588.3)	(0.50)	
Added work	23.1	0.07	16.9	0.08	10.4	0.04	0.44
	(147.7)	(0.26)	(93.2)	(0.27)	(84.1)	(0.20)	
Change order	41.0	0.13	37.1	0.11	21.5	0.09	0.35
	(199.6)	(0.34)	(196.6)	(0.32)	(138.5)	(0.28)	
Supplemental	52.0	0.19	37.0	0.18	33.2	0.16	0.31
	(233.6)	(0.39)	(178.0)	(0.38)	(139.6)	(0.37)	
Use options	211.6	0.30	169.7	0.28	141.1	0.26	0.42
	(489.2)	(0.46)	(410.6)	(0.45)	(388.2)	(0.44)	
Administrative††	215.8	0.52	160.8	0.47	147.5	0.38	0.29
	(551.0)	(0.50)	(433.5)	(0.50)	(397.8)	(0.48)	

Note: Unconditional average price (in thousand dollars, CPI-adjusted to 2010) and duration are shown and standard deviations are in parentheses. All contracts in the final sample (962 observations) are included. † Firm-fixed price contracts (653 observations). ‡ Correlation between price changes and duration changes for a given reason. †† *Administrative actions* include funding, close out, and changes in the vendor information.

There are several notable trends manifest in Table 3. First, price changes are frequent and considerable in size; 69 percent of the contracts experienced at least one price change, and the average total change in price is $543,600, 43 percent of the average final price of a contract in the sample. Duration changes are also frequent and large in size. Second, price changes and delays are positively correlated. It seems as if contracts *reward* delays, which is at odds with time incentive contracts based on moral hazard (Lewis and Bajari, 2011, 2014). Third, price changes occur regardless of the contract type as stated in the data. In particular, *firm-fixed price contracts* supposedly make the seller fully responsible for the performance costs and resulting profit or loss (FAR 16), but in our sample 64 percent of them experienced a price change. This implies that a firm-fixed price contract is not a commitment by the buyer on price changes.

However, Table 4 shows that firm-fixed price contracts have on average $181,700 less price changes associated with administrative actions than other contracts, even after controlling for observed contract attributes. Other price changes, in particular

TABLE 4. Price Changes and Contract Type

	(1)	(2)	(3)	(4)	(5)
Firm-fixed price contract	-1.649	8.685	-33.21	-52.59	-181.7***
	(11.68)	(16.05)	(20.30)	(41.52)	(47.60)
Base duration	Yes	Yes	Yes	Yes	Yes
Fixed effects†	Yes	Yes	Yes	Yes	Yes
N	962	962	962	962	962
R^2	0.404	0.385	0.281	0.314	0.289

Note: The dependent variables are the amount of price changes in thousand dollars (CPI-adjusted to 2010) for each of the five categories of reasons for modification: (1) additional work; (2) change order and definitization of letter contract or change order; (3) supplemental agreement for work within scope; (4) exercise options; and (5) various administrative actions. All contracts in the final sample are included; standard errors are provided in parentheses; *$p < 0.10$, **$p < 0.05$, ***$p < 0.01$. † 4-digit product and service codes, procurement agency, location of the contract performance (state), year of award, and month of the award, respectively.

TABLE 5. Non-repeat vs. Repeat Sellers

	Number of Sellers	Number of Contracts	Competitive Solicitation	Number of Bids	Size ($M)
Non-repeat sellers	284	284	0.33	2.38	1.16
			(0.03)	(0.54)	(0.65)
Repeat sellers (≤ 10)	282	405	0.28	1.69	1.26
			(0.02)	(0.10)	(0.54)
Repeat sellers (> 10)	52	273	0.37	2.57	1.36
			(0.03)	(0.40)	(0.74)
Total	618	962	0.32	2.14	1.26

Notes: We divide the contracts in our sample into three categories based on the seller's history of winning any of the contracts with definitive terms and conditions for IT and telecommunications services worth at least $300,000 (8,199 contracts in total): *non-repeat sellers*, *repeat sellers* with 2–10 contracts, and those with more than 10. *Size* refers to the total amount of obligated money to the government per contract as of FY 2014 (in 2010 dollars). The numbers in parentheses are standard errors.

for additional work and change order, do not exhibit a statistically significant relation with the contract type and the coefficient estimates are small. Similar patterns are also found for duration changes. Based on this observation, we divide the changes into two groups, those correlated with firm-fixed price contacts and the rest, and treat them differently in the model we present in the next section.

2.3. **Repeated Interaction.** Repeated interactions occur infrequently overall. Table 5 presents the summary statistics of our sample by the seller's history of winning contracts. To allow that the reputation of a seller may also be built from similar

contracts to those in our sample, we look at all contracts with definitive terms and conditions for IT and telecommunications services with a final price at least $300,000, initiated during the period of our study. There are in total 8,199 contracts, which were performed by 3,244 unique sellers collectively.[12] The 962 contracts in our sample were performed by 618 sellers, and 52 (8 percent) of them won more than ten of the 8,199 contracts. Table 5 shows that the contracts won by these sellers tend to be competitively solicited and have more bids. These trends do not support the hypothesis that the sellers who win multiple contracts face less competition. This implies that discretionary restrictions in competition may not necessarily be associated with dynamic incentive schemes.[13]

Furthermore, we observe the number of losing bids for each competitive contract, but we do not observe their identities. This limits our capacity to study the possibility of collusion and reputation. Most sellers win only one contract during the period of study (Table 5), and the contracts in our sample tend to appear irregularly in terms of size and requirements. These features make it difficult for sellers to maintain a collusive relationship (Porter and Zona, 1993). Although the data are not suitable to studying inter-temporal incentives, we partially accommodate long-term relationships of buyer-seller pairs through buyer preferences for no competition.

3. Model

The institutional features and data trends highlighted in the previous section are guideposts for developing our model. This section exposits the model.

3.1. **Buyer's Payoff.** Suppose a buyer is assigned to administer a procurement process for a government project. The rules described in Section 2.1 on solicitation delegate responsibility to the buyer for deciding whether she will permit competition or not. This is a choice variable in our model. We denote by η a cost to the buyer of

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[12]To identify a unique seller, we use its *parent company*'s DUNS Number. A DUNS number is a unique nine-digit identification number for each physical location of a business, and is required for all businesses to register with the federal government for contracts or grants. For example, there are 45 unique DUNS numbers that are associated with AT&T as a parent company, and we treat them as one seller in Table 5.

[13]This is consistent with the findings of the US Government Accountability Office in its report to the Congress in April 2009 (GAO-09-374). Based on their analysis of 62 contract solicitations from FY 2007 and 2008 and meetings with 121 contracting officials, the authors of the report conclude that better performance information is needed to support agency contract award decisions. The contracting officials' reluctance to rely more on past performance was found to be due, in part, to their skepticism about the reliability of information and difficulty assessing relevance to specific acquisition.

choosing a competitive process rather than simply selecting a default seller. We do not assume that η reflects a social cost, but do allow for the possibility that it might.

Should she permit competition, the second choice confronting the buyer in our model is the extent of soliciting extra bids. The various activities mentioned in Section 2.1 testify to the range of instruments available to the buyers for publicizing the request for proposals. In our model we denote the level of effort by $\lambda \in \mathcal{R}^+$, which is the arrival rate of a Poisson distribution for the number of bids exceeding one. The greater the number of bids, denoted by n, the higher the administrative costs of attracting and processing bids, denoted by $\kappa(n)$. We assume that $\kappa(n)$ is positive, increasing and convex in n.

Section 2.2 distinguishes between the base price, which we denote by p, and the final price, which we express as $p + \Delta$. The total cost of the project to the buyer is:

$$p + \Delta + \kappa(n) + \eta \quad \text{if the project is competitively solicited and draws } n \text{ bids,}$$
$$p + \Delta + \kappa(1) \quad\quad\ \text{if the project is not competitively solicited.}$$

3.2. **Payoff and Private Information of Sellers.** There are two types of sellers, low and high cost, independently distributed. The proportion of the low-cost sellers in the population, denoted by $\pi \in (0, 1)$, is common knowledge among the buyer and the sellers. The total cost of completing the project is the sum of the deterministic cost, $\alpha \in \mathcal{R}^+$ for a low-cost seller and $\alpha + \beta > \alpha$ for a high-cost seller, and stochastic cost change, denoted by $\epsilon \in \mathcal{R}$.

The seller cost type is hidden information, known to the seller only, and therefore not contractible. Contractible outcomes can be partitioned into cost changes distributed independently of seller's type, ϵ, and outcomes that are related to the seller's type, which we denote by s. In general both ϵ and s are multidimensional vectors, and the distribution of s is conditional on both seller type and the realization of ϵ. Mainly for notational purposes and data limitations, we assume both ϵ and s are real-valued and independent, and that s is an outcome but not a cost. Let $\underline{F}(s)$ denote the cumulative distribution function of s for low-cost sellers, and $\overline{F}(s)$ for high-cost sellers, with densities $\underline{f}(s)$ and $\overline{f}(s)$, respectively. We assume that s is informative but imperfect: $\underline{F}(s)$ and $\overline{F}(s)$ are defined on common support denoted by S, but $\underline{F}(s) \neq \overline{F}(s)$ for some $s \in S$.

Liquidity concerns, or the cost of working capital, lead sellers to discount (enlarge) positive (negative) values of Δ and ϵ.[14] Thus the payoff from contract (p, Δ) and

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[14]The price changes can be costly, potentially due to adaptation costs (Crocker and Reynolds, 1993; Bajari and Tadelis, 2001; Bajari, Houghton and Tadelis, 2014) and sellers' risk aversion (Baron

FIGURE 1. Timeline of the Procurement Process in the Model
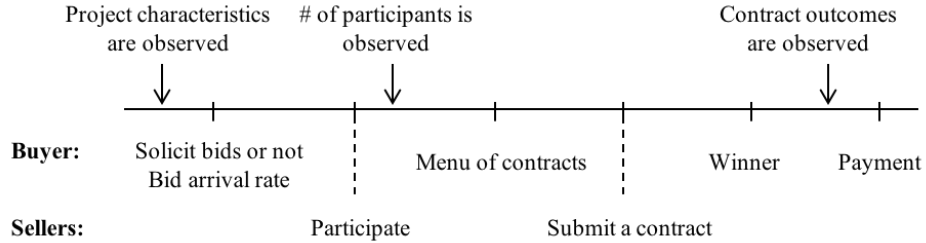
realized value of ϵ is:

$$p + \psi(\Delta - \epsilon) - \alpha \qquad \text{if the seller is low cost,}$$
$$p + \psi(\Delta - \epsilon) - \alpha - \beta \quad \text{if the seller is high cost,}$$

where $\psi(\cdot)$ is a continuous real-valued function defined on $\mathcal{R}$, with $\psi(0) = 0$, $\psi'(0) = 1$, $\psi' > 0$, and $\psi'' < 0$. We also assume there exists a maximal penalty the buyer can impose on sellers denoted by $M \in \mathcal{R}^-$ satisfying $M \leq \Delta - \epsilon$. In theory, the maximal penalty finesses situations where it might otherwise be optimal to impose an extremely steep penalty on a low-cost winner in the event of a very unlikely outcome for a high-cost seller to achieve an outcome very close to first best. In practice, M reflects limited liability and bankruptcy constraints of sellers.

3.3. **Timeline.** Figure 1 represents the timeline of the model. When a project is realized, the buyer decides whether to competitively solicit bids, and if so, she also determines the level of effort to attract and process bids, which stochastically determines the realized number of participating sellers. The buyer then offers menu of contracts, each defined by a base price plus a mapping from the contractible outcomes to the price changes. She announces her preference ordering over the items on the menu. The sellers simultaneously select a contract from the menu, and then the buyer chooses a winner following her preference ordering. Payment is made on upon completion of the project and the revelation of contractible outcomes.

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and Besanko, 1987; Laffont and Rochet, 1998; Arve and Martimort, 2016). Arve and Martimort (2016) explicitly model firms' risk aversion in a procurement context. See pages 3240–3241 for their justifications, including imperfect risk management or diversification, bankruptcy or auditing cost of issuing debt, liquidity constraints, nonlinear tax systems, and internal agency problems.

4. Equilibrium

This section characterizes and illustrates the optimal menu of contracts for given a number of bids, and then shows how the optimal extent of competition is derived.

4.1. **Contract Menu.** It is straightforward to show that the buyer, who is risk neutral, fully insures the winning seller, who has liquidity concerns characterized by $\psi(\cdot)$, against realizations of ϵ, which is revealed to both. The resulting contract is a schedule determining the base price, denoted by p, adjustments for ϵ on a cost-plus basis, and a price change net of ϵ that is a mapping of the informative outcome s, which we denote by $q(s)$. Since cost changes due to the realization of ϵ are fully compensated, $\Delta = q(s) + \epsilon$ for all (ϵ, s). We call a contract *fixed* when $q(s) = 0$ for all $s \in S$, and *variable* when $q(s) \neq 0$ for some $s \in S$.[15]

We show below that it is optimal for the buyer to offer a menu of two contracts, a fixed contract and a variable one. Presented with an optimally designed contract menu, sellers truthfully reveal their cost type through their contract selection: low-cost sellers choose the fixed contract and high-cost ones submit the variable contract. When feasible, the buyer selects a seller submitting the fixed contract. In the case of a tie, the buyer randomly selects a winner. Hence the probability that the fixed contract realizes in equilibrium when n sellers are present is $1 - (1 - \pi)^n$.

Denote the price of the fixed contract in the menu by $\underline{p}_n$ and the base price of the variable contract by $\overline{p}$. The buyer's expected transfer to a winning seller is:

$$[1 - (1 - \pi)^n]\,\underline{p}_n + (1 - \pi)^n \left[\overline{p} + \int q(s)\overline{f}(s)ds\right] + \mathbb{E}(\epsilon). \tag{1}$$

Appealing to the revelation principle, the buyer is limited to choosing $\underline{p}_n$ and $\{\overline{p}, q(s)\}$ subject to five constraints: two individual rationality (IR) conditions inducing each seller type to undertake the project; two incentive compatibility (IC) conditions inducing each seller to reveal his true type; and a maximal penalty condition restricting the range of $q(s)$. The IR constraints for the two types are:

$$\underline{p}_n \geq \alpha \quad \text{and} \quad \overline{p} + \int \psi[q(s)]\overline{f}(s)ds \geq \alpha + \beta.$$

To derive the IC constraint for the low-cost type, we first compute the winning probability if he chooses the fixed contract when the other sellers follow their equilibrium

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[15]Section 6 gives details on how we define *fixed* vs. *variable* contracts in our empirical analysis.

strategy, denoted by $\underline{\phi}_n$:

$$\underline{\phi}_n \equiv \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{\pi^k(1-\pi)^{n-1-k}}{k+1} = \frac{1}{n\pi} \sum_{j=1}^{n} \binom{n}{j} \pi^j(1-\pi)^{n-j} = \frac{1-(1-\pi)^n}{n\pi}. \quad (2)$$

If he chooses the variable contract instead, the probability of winning is:

$$\overline{\phi}_n \equiv \frac{(1-\pi)^{n-1}}{n}. \quad (3)$$

Thus a low-cost seller prefers $\underline{p}_n$ to $\{\overline{p}, q(s)\}$ if and only if:

$$\underline{\phi}_n\{\underline{p}_n - \alpha\} \geq \overline{\phi}_n\{\overline{p} + \int \psi[q(s)]\underline{f}(s)ds - \alpha\}.$$

The IC condition for the high-cost type can be similarly defined.

To characterize the optimal menu, four additional notations are helpful. Let $h : \mathcal{R}^+ \to \mathcal{R}$ denote the inverse of the first derivative of $\psi(q)$; that is $h[\psi'(q)] \equiv q$. Let $l(s) \equiv \underline{f}(s)/\overline{f}(s)$ denote the likelihood ratio, and define the threshold likelihood ratio associated with the maximal penalty condition by:

$$\tilde{l}(\pi) \equiv \frac{1}{\pi} - \frac{1-\pi}{\pi\psi'(M)}. \quad (4)$$

Lemma A.1 in Appendix A proves there is at most one root in $\pi \in (0,1)$ to the following expression:

$$\beta - \int_{l(s)<\tilde{l}(\pi)} \psi\left(h\left[\frac{1-\pi}{1-\pi l(s)}\right]\right)[1-l(s)]\overline{f}(s)ds - \psi(M)\int_{l(s)\geq\tilde{l}(\pi)}[1-l(s)]\overline{f}(s)ds. \quad (5)$$

We denote the root by $\tilde{\pi}$ when it exists, and otherwise set $\tilde{\pi} = 1$.

Theorem 4.1. *The minimal number of items on an optimal menu is two. All optimal menus induce a separating equilibrium amongst the sellers: low-cost sellers submit fixed contracts and high-cost sellers submit variable contracts. The optimal menu containing two items is uniquely defined by the price of the fixed contract:*

$$\underline{p}_n = \alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left(\beta - \int \psi[q(s)][1-l(s)]\overline{f}(s)ds\right), \quad (6)$$

and the variable contract:

$$\overline{p} = \alpha + \beta - \int \psi[q(s)]\overline{f}(s)ds, \quad (7)$$

$$q(s) = \begin{cases} h\left(\frac{1-\min\{\pi,\tilde{\pi}\}}{1-\min\{\pi,\tilde{\pi}\}l(s)}\right) & if \ l(s) \leq \tilde{l}(\min\{\pi,\tilde{\pi}\}), \\ M & if \ l(s) > \tilde{l}(\min\{\pi,\tilde{\pi}\}). \end{cases} \tag{8}$$

The proof is provided in Appendix A. Intuitively, two contracts suffice because there are only two seller types, and for the remainder of the paper we focus on the uniquely defined menu. In the interior solution to the buyer's problem $\underline{p}_n$ and $\bar{p}$ are found by solving two equations characterizing the IR constraint for high-cost sellers and the IC constraint for low-cost sellers, both of which hold with equality at optimum, and minimizing the resulting expression for the buyer's cost with respect to q for each s. When $\pi > \tilde{\pi}$ it is optimal for the buyer to offer a fixed contract at α; in this case the IR constraint for low-cost sellers binds at optimum. For any s such that $l(s) > \tilde{l}(\pi)$, the buyer charges the maximal penalty to sellers selecting the variable contract.

Several features of the theorem are noteworthy. First, there is no pooling equilibrium. Second, since $h(1) = 0$ and its derivative is negative, it follows from (35) that $q(s) \gtrless 0$ as $l(s) \lessgtr 1$ and $q(s) = 0$ if and only if $l(s) = 1$. In words, if a certain value of s is more (less) likely to be generated by a high-cost seller than a low-cost one, then $q(s)$ is positive (negative) so that low-cost sellers are incentivized not to mimic high-cost ones. Third, differentiating (33), the fixed contract declines with the number of bids, converging to α, manifesting the benefits of more competition. However, from (34) and (35), neither $\bar{p}$ and $q(s)$ depend on the number of bids.[16] This is because the IR condition for the high-cost type is satisfied with equality.

Appealing to (1), (33) and (34), the expected transfer to a winning seller given n bids, denoted by $T(n)$, can be written as:

$$T(n) = \alpha + (1 - \pi)^{n-1}[\beta + \Gamma] + \mathbb{E}(\epsilon), \tag{9}$$

where:

$$\Gamma \equiv \int (1 - \pi)\{q(s) - \psi[q(s)]\} - \pi\psi[q(s)][1 - l(s)]\overline{f}(s)ds. \tag{10}$$

Lemma A.3 of Appendix A proves the minimum expected transfer when s is not contractible is $\alpha + (1 - \pi)^{n-1}\beta + \mathbb{E}(\epsilon)$. Thus Γ is the difference in the expected transfer when only one bid is tendered, between the optimal menu and the constrained optimal menu containing only fixed contracts, which is negative.[17] Moreover as the

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[16]This result is similar to McAfee and McMillan (1987), Laffont and Tirole (1987), and Riordan and Sappington (1987), where the distortions due to information asymmetry are invariant to the number of bids, though expected distortions and seller profits decline with the number of bids.

[17]To prove $\Gamma < 0$, Lemma A.5 constructs a menu offering one variable contract and one fixed contract offered to low-cost sellers that induces an expected transfer which is strictly less than

number of bids increases, the absolute value of the difference, $(1-\pi)^{n-1}\Gamma$, declines at a geometric rate. The basic intuition is that in her quest to extract rent from low-cost sellers when faced with the constraint of having to accept a high-cost seller as a last resort, the buyer uses s to discriminate between the two types, and that the value of discriminating in this way declines with more bids.

4.2. **An Example.** Figure 2 displays the shape of the optimal contract menu, illustrating some of its features described above, in an example where the distribution of s for the low-cost type is $Gamma(1, 1.5)$; the counterpart for the high-cost type is $Gamma(1, 2)$; the cost parameters are $\alpha = 1000$, $\beta = 500$ and ϵ is degenerate at 0; the penalty M is high enough to be non-binding; and the costs of liquidity are modeled as

$$\psi(q) = -\psi_0 e^{-q/\psi_0} + \psi_0, \tag{11}$$

where $\psi_0 = 2500$.

The solid curve in Panel (A) of Figure 2 represents the likelihood ratio $l(s)$, while the two dotted curves in the panel show $q(s)$ for two values of π. Panel (B) depicts the expected transfers as a function of the number of bids for the same two values of π. Increasing π from $1/3$ to $1/2$ induces a steeper price adjustment schedule, evident in Panel (A), thus exposing the high-cost winner to a greater liquidity cost and increasing the amount of expected transfer to him from 1515 to 1568, as shown in Panel (B). The panel shows that the expected transfer for a variable contract is constant in the number of bids and that of a fixed contract declines and converges to 1000, almost achieved with only handful of bids.

Panel (B) also illustrates the optimal contract menu when s is not contractible and $\pi = 1/3$. Then the optimal menu reduces to two fixed contracts; the preferred contract is the downward sloping piecewise-linear line with the 1500 intercept, and the least preferred contract is the constant 1500. The value of contracting on s is always positive and declines with the number of bids. The liquidity cost associated with the optimal contract does not vary with the number of bids, but the gap between fixed contracts designed for low-cost sellers shrinks as the number of bids increases. Appealing to the definition of Γ in (10), the difference in the unconditional expected transfer with and without contracting on s is 30.69 when there is only one bid, and this difference declines to $\left(\frac{2}{3}\right)^4 \times 30.69 = 4.04$ with five bids.

$\alpha + (1-\pi)^{n-1}\beta + \mathbb{E}(\epsilon)$ and bounds the expected transfer induced by the optimal menu characterized on Theorem 4.1 from above.

FIGURE 2. Example
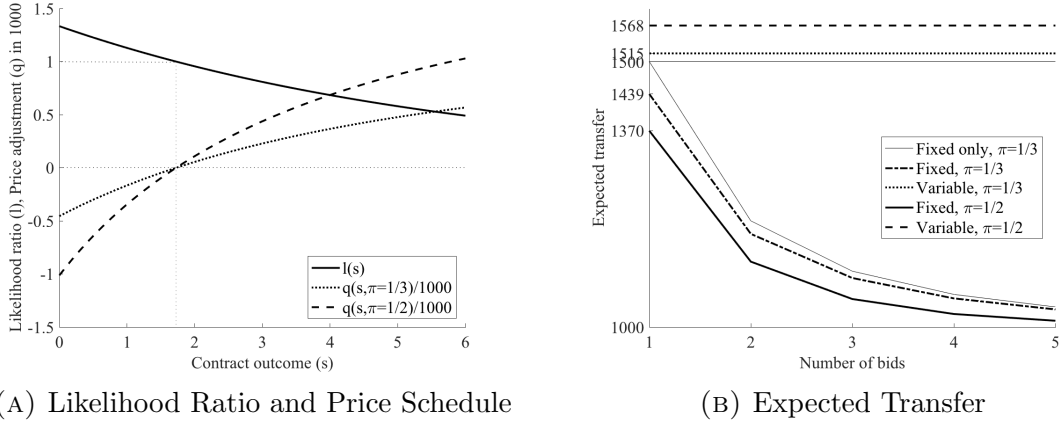
(A) Likelihood Ratio and Price Schedule   (B) Expected Transfer

Notes: The graphs above illustrate the equilibrium in an example case for two values of π where $\alpha = 1000$, $\beta = 500$, and ϵ is degenerate at 0. The distribution of s for low-cost sellers is $Gamma(1, 1.5)$ and the counterpart for high-cost sellers is $Gamma(1, 2)$.

4.3. **Extent of Competition.** The extent of competition is chosen to minimize the buyer's expected total cost. Having solved the optimal menu of contracts and the expected transfer to a winning seller for any given number of bids, we now derive the expected total cost of competitive procurement with effort λ, denoted by $U(\lambda)$. Recall that λ is the arrival rate of extra bids, denoted by j in the equation below, which follows a Poisson process.

$$
\begin{aligned}
U(\lambda) &\equiv \sum_{j=0}^{\infty} \frac{\lambda^j e^{-\lambda}}{j!} \left[T(j+1) + \kappa(j+1) \right] + \eta \\
&= \alpha + e^{-\lambda\pi}(\beta + \Gamma) + \mathbb{E}(\epsilon) + \mathbb{E}[\kappa(j+1)|\lambda] + \eta,
\end{aligned}
$$

where $T(n)$ and Γ are defined in (9) and (10), respectively. The expected total cost of noncompetitive procurement, denoted by U_0, is:

$$
U_0 = \alpha + \beta + \Gamma + \mathbb{E}(\epsilon) + \kappa(1).
$$

The buyer chooses to solicit competitive bids if and only if $U_0 \geq \min_\lambda U(\lambda)$. Because $U(\lambda)$ is convex, it attains a global minimum at its unique stationary point, denoted by λ^*. If $\lambda^* \leq 0$, then the choice reduces to the sign of η. Alternatively if $\lambda^* > 0$, then the buyer solicits competitive bids if and only if:

$$
\eta \leq (1 - e^{-\lambda^*\pi})(\beta + \Gamma) + \kappa(1) - \mathbb{E}[\kappa(j+1)|\lambda^*]. \tag{12}
$$

5. Identification

Our data comprises: whether the contract draws competitive bids, which we denote by setting $c = 1$, or not (setting $c = 0$); the number of bids, n; whether the winning contract is a variable contract, denoted by setting $v = 1$, or not (setting $v = 0$); the contract outcomes s and ϵ for both contract types; the fixed price $\underline{p}_n$ if the winning contract is a fixed contract; and the base price $\overline{p}$ and the price change net of ϵ, q, if the winning contract is variable. We assume the observations in the data are independently distributed.

Under the null hypothesis that π is constant conditional on observed project attributes, π is identified off the proportion of variable contracts $(1 - \pi)^n$ for any given $n \in \{1, 2, \ldots\}$. Thus π is over-identified from variation in n. However, this null hypothesis is rejected because Table 2 shows that conditioning on observed attributes, the number of bids is positively correlated with contract price. Consequently we treat π as an unobserved project-specific continuous variable filtering through the equilibrium and complicating identification. We also allow project and bidding costs to vary with π, expressing α, β and $\kappa(n)$ as $\alpha(\pi)$, $\beta(\pi)$ and $\kappa(n, \pi)$. Similarly $\underline{p}_n$ and $\overline{p}$ are written as $\underline{p}_n(\pi)$ and $\overline{p}(\pi)$ respectively. Since the dependence of q on s is channelled though $l(s)$, we write $q(l(s), \pi)$ for $q(s)$.

Denote the support of π by $\Pi \subset (0, 1)$ and that of s by $S \subset \mathcal{R}$. The parameter space is formed from: $\mathcal{F}_\pi$, the set of nondecreasing and continuously differentiable mappings from Π to $[0, 1]$; Ψ, twice differentiable concave functions from $\mathcal{R}$ to $\mathcal{R}$ tangent to the identity function at the origin; $\mathcal{A}$, differentiable mappings from Π to $\mathcal{R}^+$; $\mathcal{F}$, nondecreasing functions defined from S to $[0, 1]$; $\mathcal{K}$, mappings from $\{1, 2, \ldots\} \times \Pi$ to $\mathcal{R}^+$ that are continuous in the second argument for each integer; and $\mathcal{F}_\eta$, nondecreasing functions defined from $\mathcal{R}$ to $[0, 1]$. The primitives of the econometric structure comprise: the distribution of the proportion of the low-cost type, $F_\pi \in \mathcal{F}_\pi$; the liquidity cost function, $\psi \in \Psi$; project costs, $\alpha \in \mathcal{A}$ and $\beta \in \mathcal{A}$; the distribution functions of s, $\underline{F} \in \mathcal{F}$ and $\overline{F} \in \mathcal{F}$; the bidding cost function $\kappa \in \mathcal{K}$; and the distribution function of η, denoted by $F_\eta \in \mathcal{F}_\eta$.

Our empirical specification assumes the following about (s, π, η):

A1: s, π, and η are mutually independent.
A2: $F_\pi(\pi)$ is strictly increasing for all $\pi \in \Pi$.

We also restrict the parameter space so that an interior solution invariably attains, meaning neither the IR constraint for the low-cost type nor the maximal penalty constraint bind. With reference to (4) and (5):

A3: $\Pi \subset (0, \tilde{\pi})$, and $l(s) \leq \tilde{l}(\pi)$ for all $(s, \pi) \in S \times \Pi$.

Given **A3**, $q(l, \pi)$ is defined by the interior solution to the first order condition (35):

$$\pi = [1 - \psi'(q)] / [1 - \psi'(q) l]. \tag{13}$$

In addition we assume that as the proportion of the low-cost type increases, the project cost to either type declines.

A4: $\alpha(\pi)$ and $\beta(\pi)$ are nonincreasing in π.

A5: $\alpha(\pi)$ and $\beta(\pi)$ satisfy the following inequality for all π:

$$\alpha'(\pi) + \beta'(\pi) - \sup_{l \in (0,1)} \left| h'\left(\frac{1-\pi}{1-\pi l}\right) \right| \frac{1}{(1-\pi)^2} \int_{\{s:l(s)<1\}} \overline{f}(s) ds < 0.$$

These assumptions suffice to establish contracts are monotonic in π in intuitively appealing ways.

Lemma 5.1. *(i) If **A3** holds then $\partial |q(l,\pi)| / \partial\pi > 0$. (ii) If **A3** and **A5** hold then $\partial\overline{p}(\pi) / \partial\pi < 0$. (iii) If **A3** and **A4** hold then $\partial\underline{p}_n(\pi) / \partial\pi < 0$ for all $n \in \{1, 2, \ldots\}$.*

The proof of the lemmas and the theorem in this section is provided in the supplementary appendix. Theorem 4.1 shows that all equilibria are separating and that low-cost sellers submit fixed contracts and high-cost sellers submit variable ones. Hence $\underline{F}(s)$ and $\overline{F}(s)$ are directly identified from the distributions for the observed s of fixed and variable contracts, as is the likelihood ratio $l(s)$. We now analyze the identification of the model primitives determining seller costs, and then those of buyer preferences.

5.1. **Sellers' Costs.** The liquidity cost function is identified from variable contracts off variation in (q, l), holding $\overline{p}$ constant. By Lemma 5.1 $\overline{p}(\pi)$ is strictly decreasing, and therefore has an inverse mapping, denoted by $\pi^*(\overline{p})$. Define the composite function $q^*(l, \overline{p}) \equiv q[l, \pi^*(\overline{p})]$: it is evident that $\partial q(l, \pi') / \partial l = \partial q^*(l, \overline{p}') / \partial l$ for all $(l, \pi', \overline{p}')$ satisfying $\overline{p}' = \overline{p}(\pi')$. Holding π constant, we totally differentiate (39) with respect to l, substitute the derivative $\partial q^*(l, \overline{p}) / \partial l$ for $\partial q(l, \pi) / \partial l$ in the resulting equation, and rearrange to obtain:

$$\psi''(q) = \left[\frac{\partial q^*(l, \overline{p})}{\partial l} \right]^{-1} \frac{1 - \psi'(q)}{1 - l} \psi'(q). \tag{14}$$

Our assumptions guarantee that $q^*(l, \overline{p})$ is uniformly Lipschitz continuous in l for any $\overline{p}$. Consequently the Picard–Lindelöf theorem applies, proving the differential equation (14) has a unique solution of $\psi'(q)$ given the normalizing constant $\psi'(0) = 1$.

Furthermore $\psi(q)$ is solved from the other normalization, $\psi(0) = 0$ for any value of $\overline{p}' = \overline{p}(\pi')$ with $\pi' \in \Pi$. The identification of $\psi(q)$ now follows from the identification of $q^*(l, \overline{p})$ directly off the variable contracts.

The remaining analysis exploits the identification of the conditional density function $f_{\pi|c,n,v}(\pi|c, n, v)$ and its cumulative distribution, $F_{\pi|c,n,v}(\pi|c, n, v)$. Identifying $f_{\pi|c,n,v}(\pi|c, n, 1)$ is immediate: since $\psi(q)$ is identified, the realizations of π associated with variable contracts are identified from (39). In equilibrium the buyer resorts to variable contracts with probability $(1 - \pi)^n$; her selection links $f_{\pi|c,n,v}(\pi|c, n, 1)$ with $f_{\pi|c,n,v}(\pi|c, n, 0)$ through Bayes' rule, identifying the latter.

Lemma 5.2. $f_{\pi|c,n,v}(\pi|c, n, v)$ *is identified.*

Turning to $\alpha(\pi)$ and $\beta(\pi)$, let $G_{\underline{p}_n|c}(p|c)$ denote the cumulative distribution function for $\underline{p}_n$ conditional on $c \in \{0, 1\}$. By Lemma 5.1, $\underline{p}_n$ is strictly decreasing in π, and by **A2** the inverse of $F_\pi(\pi)$ exists. Therefore the inverse of $G_{\underline{p}_n|c}(p|c)$ exists. Define $\underline{p}_n^*(\pi, c)$ as follows:

$$\underline{p}_n^*(\pi, c) \equiv G_{\underline{p}_n|c}^{-1}\left[1 - F_{\pi|c,n,v}(\pi|c, n, 0)|c\right]. \tag{15}$$

By construction $\underline{p}_n^*(\pi, c)$ solves for $\underline{p}_n$ given (π, c) and, by Lemma 5.2, is identified. Also since $\psi(q)$ is identified, $\overline{p}(\pi)$ is identified off the variable contracts by appealing to (39). Substituting $\underline{p}_n^*(\pi, c)$ for $\underline{p}_n$ in (33) and $\overline{p}(\pi)$ for $\overline{p}$ in (34) and manipulating the resulting equations give the expressions for $\alpha(\pi)$ and $\beta(\pi)$ in (43) below. Theorem 5.1 establishes the primitives on the seller side are identified.

Theorem 5.1. $\psi(q)$, $\alpha(\pi)$ *and* $\beta(\pi)$ *are identified, and for* $n \in \{2, 3, \ldots\}$*:*

$$\alpha(\pi) = \frac{1 - (1 - \pi)^n}{1 - (1 - \pi)^{n-1}}\underline{p}_n^*(\pi, c) - \frac{\pi(1 - \pi)^{n-1}}{1 - (1 - \pi)^{n-1}}\underline{p}_1^*(\pi, c), \tag{16}$$

$$\beta(\pi) = \overline{p}(\pi) + \int \psi\left(h\left[\frac{1 - \pi}{1 - \pi l(t)}\right]\right)\overline{f}(t)\, dt - \alpha(\pi).$$

5.2. **Buyer's Costs.** If $\kappa(n, \pi)$ is identified, then $F_\eta(\eta)$ is partially identified from the buyer's choices on competitive solicitation in response to independent variation in π, as assumed in **A1**. Recognizing its dependence on π explicitly, let $\lambda^*(\pi)$ rather than λ^* denote the optimal arrival rate of extra bids, write $\Gamma(\pi)$ for Γ in (10), and similarly express the right hand side of (12) as:

$$\Omega(\pi) \equiv (1 - e^{-\lambda^*(\pi)\pi})[\beta(\pi) + \Gamma(\pi)] + \kappa(1, \pi) - \mathbb{E}[\kappa(j + 1, \pi)|\lambda^*(\pi)].$$

We identify $F_\eta(\eta)$ for $\eta \in \{\Omega(\pi) : \lambda^*(\pi) > 0\}$ and at $\eta = 0$. From Lemma 5.2 the joint distribution of (π, c) is identified, and hence so is $\Pr(c = 1|\pi)$. Thus if $\lambda^*(\pi) > 0$ then $F_\eta[\Omega(\pi)] = \Pr(c = 1|\pi)$ is identified; otherwise $F_\eta(0) = \Pr(c = 1|\pi)$ is identified.

Finally $\kappa(n, \pi)$ is not identified without imposing parametric restrictions. Our empirical specification assumes bidding costs are:

$$\kappa(n, \pi) = (\kappa_1 + \kappa_2\pi)(n - 1) + (\kappa_3 + \kappa_4\pi)(n - 1)^2. \tag{17}$$

The first order condition for interior optimality simplifies to:

$$\kappa_1 + \pi\kappa_2 + [1 + 2\lambda^*(\pi)]\kappa_3 + \pi[1 + 2\lambda^*(\pi)]\kappa_4 = \pi\exp[-\pi\lambda^*(\pi)][\beta(\pi) + \Gamma(\pi)]. \tag{18}$$

From Lemma 5.2 and the observed distribution of (c, n), $f_{\pi,n|c}(\pi, n\,|1)$ and $f_{\pi|c}(\pi\,|1)$ are identified, which implies that $\lambda^*(\pi)$ is identified from:

$$\lambda^*(\pi) = \sum_{j=0}^{\infty} j f_{\pi,n|c}(\pi, j + 1\,|1) \big/ f_{\pi|c}(\pi\,|1).$$

Since $\beta(\pi)$ and $\Gamma(\pi)$ are also identified (by our previous arguments), the vector of coefficients $(\kappa_1, \kappa_2, \kappa_3, \kappa_4)$ in (18) solves a system of linear equations for different values of π. The rank condition sufficient to identify $(\kappa_0, \kappa_1, \kappa_2, \kappa_3)$ can be checked directly.

6. Estimation Results

We say the ith contract is competed ($c_i = 1$) if there is a competitive solicitation or more than one bid is considered.[18] The reasons for price changes specified in the data determine whether changes are categorized as informative or uninformative. It is based on the empirical findings in Table 4 that firm-fixed price contracts tend to exhibit small price changes of Columns (4) and (5): the estimated coefficients are negative and their magnitudes are larger than the counterparts in the other columns. We measure q_i by the sum of all price changes associated with exercise of an option or an administrative action; and ϵ_i is measured as the sum of all other price changes.[19] Having defined q_i, we define the contracts with $q_i \neq 0$ as variable ($v_i = 1$) and those with $q_i = 0$ as fixed ($v_i = 0$).[20] Lastly, s_i is measured by the sum of all duration

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[18]Contracts with more than one bid categorized in the data as noncompetitive, 35 contracts in our sample, are treated as competitively solicited.

[19]We consider an alternative categorization, where price changes for reasons corresponding to Columns (3) to (5) are labeled as q and remaining price changes make up ϵ. The results under this specification are displayed in Appendix B, showing that our results are robust.

[20]Note that the contract types, which we analyze in the model, do not necessarily coincide with the nomenclature of the stated contract type such as firm-fixed price or cost plus. Alternatively, we

TABLE 6. Summary Statistics

| | Competitively Solicited? | | | |
| | Yes | | No | |
	Mean	SD	Mean	SD
Number of bids	4.55	10.50	1.00	-
Fixed contract	0.44	-	0.42	-
Price of fixed contracts ($\underline{p}_n$)	1186.87	974.43	825.62	866.65
Base price of variable contracts ($\bar{p}$)	549.14	639.22	534.88	678.26
Cost change independent of seller type (ϵ)	167.34	506.07	189.08	508.11
Price change net of ϵ for variable contracts (q)	683.10	831.01	604.81	729.07
Outcome related to seller type (s) for fixed contracts	0.03	0.17	0.05	0.18
Outcome related to seller type (s) for variable contracts	1.75	2.46	1.33	2.20
Awarded by a military-related agency	0.25	-	0.38	-

Notes: This table provides summary statistics of the variables used in the estimation for the final sample of 962 contracts, consisting of 652 contracts that were not competitively solicited and 310 contracts that were competitively solicited. The contract prices are in thousand CPI-adjusted dollars to 2010. See the text for the definition of each variable.

changes associated with the same reasons that we use to define q_i, divided by the base duration of the contract. Table 6 provides summary statistics of the variables used in the estimation.

6.1. **Parameterization and Estimation.** Nonparametric estimators can be formed from the equations that establish identification, but given our modest sample size, we impose a parametric functional form, and estimate its primitives using a simulated GMM estimator. The moment conditions are, however, motivated by the identification arguments: the joint probabilities regarding competitive solicitation, the number of bids, and contract type; moments of the joint distribution of s and contract type; and the quantiles of contract prices conditional on contract type and the number of bids. In estimation we also allow for the possibility that the IR constraint binds for both seller types. While the identification arguments for $\psi(q)$ given in the previous section are robust to this extension, neither $\alpha(\pi)$ nor $\beta(\pi)$ are identified for $\pi > \tilde{\pi}$ without imposing functional form restrictions. The supplementary appendix gives further details about estimation.

The primitives of the model are parameterized as follows. We assume that the distribution of π is given by $Beta(\zeta_1, \zeta_2)$, that $\psi(q)$ takes the parametric form of (11), and that the maximal penalty is a constant at δ_0. η is assumed to follow $N(\mu_\eta, \sigma_\eta)$ for

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define the firm-fixed price contracts with $q_i = 0$ as fixed ($v_i = 0$) and the others as variable ($v_i = 1$). Table A1 in Appendix B shows that our results are not sensitive to this alternative designation of contract types.

non-military contracts and $N(\mu_\eta^m, \sigma_\eta^m)$ for military contracts. $\kappa(n, \pi)$ is parametrized as (17). The distribution of s for the low-cost sellers is:

$$\underline{F}_s(s) = \begin{cases} \underline{\rho} & \text{if } s = 0, \\ (1 - \underline{\rho})\underline{G}(s) & \text{if } s > 0, \end{cases}$$

where $\underline{G}(s)$ is the CDF of $Gamma(\underline{\alpha}_s, \underline{\beta}_s > 0)$. The counterpart for the high-cost sellers, $\overline{F}_s(s)$, is similarly defined with $\overline{\rho}$ and $\overline{G}(s)$, where $\overline{G}(s)$ is the CDF of $Gamma(\overline{\alpha}_s, \overline{\beta}_s > 0)$.

Deterministic project costs depend on whether or not the project is for military agencies, denoted by a binary variable, $m \in \{0, 1\}$. Given m, we assume the cost for low-cost sellers, α, is linear in π, and the cost differential β is a fraction of α:

$$\alpha(\pi, m) = \alpha_1 + \alpha_2 m + \alpha_3 \pi,$$

$$\beta(\pi, m) = [\beta_1(1 - m) + \beta_2 m] \, \alpha(\pi, m).$$

We assume $\alpha_3 \leq 0$, so that monotonicity results of Lemma 5.1 apply.[21] Finally we directly observe ϵ in the data and is independent of π and η by assumption. We estimate the distribution of ϵ conditional on m without parametric assumptions.

6.2. **Parameter Estimates.** Using the estimated parameters, we simulate the data and calculate key moments displayed in Table 7. The table shows the actual and predicted moments, based on 5,000 simulations of the estimated model. The overall fit of the simulated data to the actual data is good in both level and trend.

The parameter estimates and their estimated asymptotic standard errors are presented in Table 8. The 95 percent confidence intervals for our estimates of the mean direct cost of competitive solicitation (μ_η for non-military contracts and μ_η^m for military ones) are [\$11,200, \$29,900] for non-military contracts, and [\$18,800, \$48,400] for military ones. The cost of competitive solicitation is larger in the military agencies by \$13,100 with asymptotic error \$6,900, a statistically significant difference that partially explains why there is less competition for military contracts than non-military ones (Table 2).

Table 9 displays project and bidding costs, plus the expected loss to the buyer from soliciting competitive bids, for nonmilitary and military procurement projects evaluated at the median value of the ratio of the low-cost sellers (π), 0.38. Overall, 85 percent of total project costs are attributable to the predetermined components

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[21]This inequality was not imposed in estimation, but as Table 8 shows, the restriction is satisfied by our unconstrained estimate of α_3, which is statistically significant.

TABLE 7. Model Fit

	All		Non-military	
	Data	Model	Data	Model
Probability of				
No competitive solicitation	0.6778	0.7374	0.6341	0.6876
1 bid given competitive solicitation	0.2677	0.3062	0.2500	0.2913
2 bids or less given competitive solicitation	0.4258	0.5209	0.3664	0.5058
5 bids or less given competitive solicitation	0.8516	0.9162	0.8534	0.9173
Fixed contract given no competition	0.4156	0.4307	0.4254	0.4443
Fixed contract given 1 bid	0.4419	0.5788	0.4655	0.5933
Fixed contract given 2 bids or less	0.3976	0.4254	0.4310	0.4224
Fixed contract given 5 bids or less	0.4091	0.4561	0.4000	0.4620
Average transfer ($M) of fixed contracts				
Given no competitive solicitation	0.9722	0.9422	0.9027	0.8754
Given competitive solicitation	1.2551	1.2491	1.2183	1.2199
Average transfer ($M) of variable contracts				
Given no competitive solicitation	1.3590	1.2838	1.2311	1.2031
Given competitive solicitation	1.1984	1.1855	1.1193	1.1631

(α and β) and 15 percent to the stochastic component (ϵ). In expectation, low-cost sellers spend $1.02 (1.18) million to complete a project for non-military (military) agencies, and high-cost sellers an extra $0.27 (0.24) million. This cost differential represents the most a buyer can save from contracting with a low-cost seller. Relative to this differential the bidding cost of having two bids instead of one is 19 percent (22 percent), while the loss the seller incurs from competitive solicitation is lower, 8 percent (14 percent) for non-military (military) contracts. Note that the cost differential between low-cost and high-cost sellers is slightly higher for non-military contracts than for military ones.

Figure 3 shows the estimated distribution of π for noncompetitive contracts first order stochastically dominates the distribution of competitive contracts. This finding illustrates the importance of accounting for the endogeneity of the extent of competition. All else equal, prices rise when fewer sellers compete, but Table 2 shows this is not the case. Figure 3 shows buyers are less likely to promote competition when π rises. In our analysis this offsetting factor is created by bidding costs that increase with π, as both κ_2 and κ_4 are positive and significant, making competition more attractive when π is low. Furthermore, our estimates of the cost differential (as transmitted through α_3) decline with π, reducing the benefit of competitive solicitation.

Table 8. Parameter Estimates

Description	Parameter	Estimate	Parameter	Estimate
Low project cost ($M)	α_1	1.7178	α_2	0.0267
		(0.0698)		(0.0251)
	α_3	-2.1699		
		(0.1536)		
Project cost differential ($M)	β_1	0.3069	β_2	0.2583
		(0.0325)		(0.0333)
Bidding cost function ($M)	κ_1	-0.0105	κ_2	0.1530
		(0.0023)		(0.0261)
	κ_3	-0.0009	κ_4	0.0124
		(0.0004)		(0.0030)
Maximal penalty ($M)	δ_0	-0.0100		
		(0.1695)		
Liquidity cost function ($M)	ψ_0	10.4020		
		(7.5244)		
Distribution of π	ζ_1	6.7445	ζ_2	10.7797
		(0.4396)		(0.8946)
Distribution of η ($M)	μ_η	-0.0205	σ_η	0.0150
		(0.0048)		(0.0042)
	μ_η^m	-0.0336	σ_η^m	0.0237
		(0.0076)		(0.0033)
Distribution of s	$\underline{\rho}$	0.9167	$\overline{\rho}$	0.3520
		(0.0155)		(0.0221)
	$\underline{\alpha}_s$	1.7828	$\underline{\beta}_s$	0.2881
		(0.4853)		(0.0715)
	$\overline{\alpha}_s$	0.8240	$\overline{\beta}_s$	2.8523
		(0.1739)		(0.7617)

Notes: Numbers in parentheses are asymptotic standard errors.

Table 9. Estimated Cost Components for Median Contracts

(in $K)	Non-military		Military	
	Estimate	SE	Estimate	SE
Project cost for low-cost sellers, $\alpha(\pi_{med}, m)$	884.1	40.4	910.8	45.8
Project cost difference, $\beta(\pi_{med}, m)$	271.3	32.1	235.3	32.1
Expected cost change, $\mathbb{E}(\epsilon|m)$	139.1	15.8	265.2	36.7
Bidding cost with two bids, $\kappa(\pi_{med}, 2)$	52.1	8.9	52.1	8.9
Cost of competitive solicitation, $\mathbb{E}(\eta|m)$	20.5	4.8	33.6	7.6

Notes: Given that the project costs and bidding costs vary with π, the numbers in this table are evaluated at the unconditional median value of π_{med}, 0.38. Furthermore, since the bidding costs depend on the number of bids, we evaluate such costs at two bids.

FIGURE 3. Estimated π Distribution Conditional on Competition
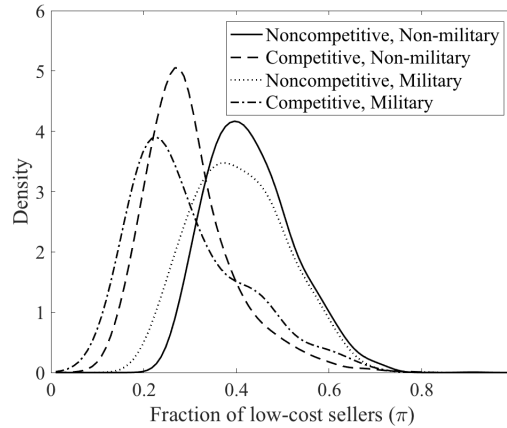
7. WHY SO LITTLE COMPETITION?

7.1. **Contract Negotiations and Informational Rent.** In our model we say the buyer conducts *contract negotiations* when she can offer contracts that are contingent upon s as well as ϵ. When contracts are allowed to depend on ϵ, but not on s, this is a special case of a *first-price sealed-bid auction*, and Lemma A.3 in the Appendix shows that in this case, it is optimal for the buyer to offer two fixed contracts with the lower-price contract taking the precedence over the higher one. Section 4.1 demonstrates how negotiating contract terms, as opposed to the latter case, helps a buyer extract informational rent from low-cost sellers, and shows that the buyer's expected savings in terms of payments to a winning seller from conducting contract negotiations are always positive and decreasing in the number of bids.

Figure 4 illustrates these cost savings from contract negotiations using our estimated parameters for a non-military contract with the unconditional median ratio of low-cost sellers ($\pi = 0.38$). Panel (A) of the figure shows the expected transfer under both cases. The expected transfer under contract negotiations with one bidder is similar to that under a first-price auction with two bids, a pattern that persists in the range of the number of bids in the graph. Consequently the marginal benefit of an extra bid is lower under negotiations. Panel (B) shows that, given competitive solicitation, the optimal number of bids is less than 2.5 for negotiations and over 3 for auctions.

To quantify the extent to which contract negotiations reduce the expected transfer from the buyer to the winning seller, we compare the current regime with a policy

FIGURE 4. The Value of Contract Negotiations

(A) Expected Transfer   (B) Marginal Benefit and Cost of Bids
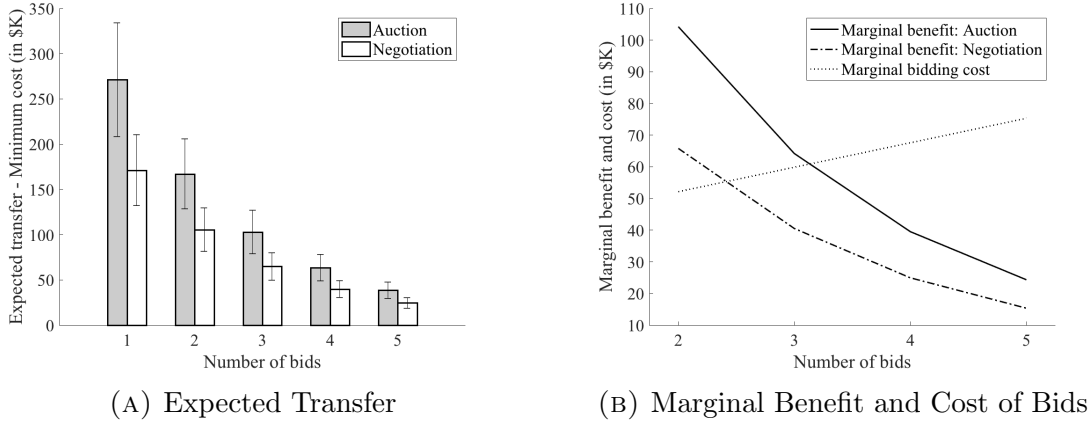
Notes: Panel (A) shows the expected transfer net of the minimum project cost for a non-military contract with the median ratio of low-cost sellers ($\pi = 0.38$) conditional on the number of bids. The error bars represent 95 percent confidence intervals. In Panel (B), we define the *marginal benefit* of a bid as the decline in the expected transfer from the marginal bid.

where a first-price sealed bid auction is used.[22] The comparison is presented in the columns labeled *Current* and (3) of Table 10. On average we find that the number of bids under the first-price auction increases by 0.7. Despite more bids, the expected transfer increase by \$35,800 per project on average, amounting to 3 percent of the expected transfer per project under the current regime (\$1.2 million). Even more striking are the results displayed in Column (4) of Table 10: if competitive solicitation was mandatory, the expected transfer would still rise, on average, by \$12,000.

7.2. **Benefits and Costs of Competition.** The expected transfer to the winning seller is just one of several dimensions to evaluate policy. Another dimension is the expected project cost, based on $\alpha(\pi, m) + \epsilon$ (for low-cost sellers) and $\alpha(\pi, m) + \beta(\pi, m) + \epsilon$ (for high-cost sellers). The liquidity cost associated with a variable contract, which arises in our model because the buyer's screening creates a distortion in risk sharing, is defined as:

$$\int \{\psi[q(l(s), \pi)] - q(l(s), \pi)\}\overline{f}(s)ds.$$

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[22]Except where indicated below, we assume that the model primitives, including the distribution of π, the project cost functions, and bidding cost functions, are invariant. To allow that the incentives for potential sellers to participate may change, one counterfactual policy (Policy (6) in Table 10) incorporates a change in bidding costs.

TABLE 10. Effects of Counterfactual Policies

(Costs in $ thousand)	Negotiation			First-Price Auction			
	Current	(1)	(2)	(3)	(4)	(5)	(6)
Number of bids	1.5	+0.3	+0.8	+0.7	+1.0	+0.6	+2.0
Transfer	1,209.5	-16.8	-45.7	+35.8	+12.0	+30.5	-36.4
Cost components							
A. Project	1,201.9	-16.9	-45.0	-30.3	-45.7	-27.2	-67.6
B. Liquidity	3.4	-0.5	-1.0	-3.4	-3.4	-3.4	-3.4
C. Bidding	16.2	+14.3	+50.1	+31.4	+49.7	+46.0	+63.2
D. Competitive solicitation	2.5	+22.5	+22.5	+6.7	+22.5	+22.5	+22.5
Aggregate costs							
A+B	1,205.3	-17.3	-46.0	-33.7	-49.0	-30.6	-70.9
A+B+C	1,221.5	-3.0	+4.1	-2.2	+0.6	+15.4	-7.7
A+B+C+D	1,223.9	+19.5	+26.6	+4.4	+23.2	+37.9	+14.8

Note: Under Policies (1) and (2), there are contract negotiations, but competitive solicitation is mandated. Policy (1) requires no minimum number of bids; Policy (2) requires at least two bids. Under Policies (3) through (6), first-price sealed bid auctions are used. Policy (3) gives the buyer the discretion to choose the extent of competition. Policies (4) through (6) mandate soliciting competitive bids. In Policy (5), $\lambda = 1.06$ for all contracts. Under Policy (6), the bidding costs (κ coefficients) are halved and $\lambda = 2.48$.

Note that because $\psi(q) > q$ for all $q \neq 0$ and $\psi(0) = 0$, the liquidity cost is always positive. Bidding costs, $\kappa(\pi, n)$, add yet another dimension to costs. Last is η, which is a mirror image of the direct benefit to the buyer from not soliciting competitive bids and awarding the project to the default seller. Table 10 summarizes how these cost components are affected by changing procurement policy. For example, Columns (3) and (4) show that reverting to a first-price auction reduces project costs (because there are more bids), eliminates liquidity costs (because there is no screening) and increase bidding costs.

To isolate the effects of greater competition, we compare how the cost components change if the seller must solicit competitive bids on all contracts. We consider the effects of two counterfactual policies; under Policy (1) in Table 10, competitive solicitation is mandatory for all projects, but buyers have discretion in choosing their effort to attract more bids. We find that the size of the expected transfer decreases by $16,900 per project, or 1 percent of the average expected transfer under the current regime.

Policy (2) in Table 10 requires at least two bids; the buyer chooses how much effort to expend to attract a greater number of bids than two. Thus if no effort is expended, there are two bids (rather than just one), so effort generates two or more bids. Under

the policy proposal, the buyer's expected total cost for a project of type π is:

$$\sum_{j=0}^{\infty} \frac{\lambda^j e^{-\lambda}}{j!} \{T(j+2,\pi) + \kappa(j+2,\pi)\} + \eta,$$

where λ is the expected number of extra bids and $T(n,\pi)$ denotes the expected transfer, as defined in (9). Buyers optimally choose λ for each project, given π. Referring to Column (2), this policy decreases the expected transfer by $45,700 per project on average, or 4 percent of the average expected transfer under the current regime.

These two alternative policies not only decrease the size of the expected transfer, but also decrease project costs and liquidity costs, on average by $17,300 and $46,000 per project, respectively. The reason is that increasing competition makes the selection of a low-cost seller more likely, thus reduce the project costs; since variable contracts are only incurred when high-cost sellers are selected, expected liquidity costs also fall. However, Table 10 shows that most of these cost savings are offset by an increase in the bidding and competitive solicitation costs.

Making welfare comparisons across policies hinges on the nature of these latter costs. Suppose the bidding cost reflects frictions in the market that are beyond the scope of a single buyer's responsibility (Kelman, 1990, 2005), and the cost of soliciting competitive bids is mostly social waste (perhaps due to unwarranted favoritism). Then we might conclude that Policy (1) is welfare-improving while Policy (2) is not because the former decreases total procurement costs (A + B + C in Table 10) by $3,000 per project while the latter increases the total procurement cost by $4,100 per project on average. However, if the cost of soliciting competitive bids represents a legitimate social cost (for example due to higher noncontractible quality of the default seller), then both counterfactual policies are suboptimal.

In measuring government inefficiency, Bandiera, Prat and Valletti (2009) propose a distinction between *active waste* and *passive waste*. The dichotomy lies in that the public decision maker (the buyer in our context) benefits from the former (e.g. corruption) but not from the latter (e.g. administrative burden). If we assume that the buyer's cost of soliciting competitive bids is associated with their own private benefits, then active waste in our model is the expected increase in transfers from not soliciting bids on every contract. Policies (1) and (2), mandating competitive solicitation, reduce the expected transfer by 1 and 4 percent respectively. On the other hand, Bandiera, Prat and Valletti (2009) estimates that the Italian governments pay for procuring standard goods up to 11 percent more than what they would

have paid absent active waste; and Di Tella and Schargrodsky (2003) estimate that procurement officers in Argentinian hospitals overprice by 10 percent. Compared to these estimates, our figures are smaller, which quite possibly reflect differences in law enforcement across countries.[23]

If we further assume that the bidding costs do not benefit the buyers, then passive waste in our model is the difference between the expected transfer under the current policy and the expected project cost of low-cost sellers. Without passive waste, the buyer can draw bids until she finds a low-cost seller, which leaves the seller no informational rent. Based on our estimates, the average passive waste is \$145,100 per contract; i.e., the expected transfer is 14 percent higher due to passive waste. This is smaller than the estimates in Bandiera, Prat and Valletti (2009), 15-43 percent, but our result is consistent with their findings that passive waste accounts for a larger portion of total estimated government waste than active waste.

7.3. **The Value of Discretion.** To assess the importance of discretion, we consider counterfactual policies in which the buyer promotes each procurement project with the same intensity and a first-price sealed auction is used. Policy (5) in Table 10 strips buyers of all their discretion; they cannot use information about the supply side, summarized by π, and cannot negotiate on contract terms either. Given that the expected transfer under a first-price auction with n bids is $\alpha(\pi) + (1 - \pi)^{n-1} \beta(\pi)$ (Lemma A.3 in Appendix A), if λ, the average number of extra bids, is chosen without regard to the value of π, then the constrained optimum level of solicitation solves:

$$\min_{\lambda \geq 0} \int \left\{ \sum_{j=0}^{\infty} \frac{\lambda^j e^{-\lambda}}{j!} \left[\alpha(\pi) + (1 - \pi)^j \beta(\pi) + \kappa(j + 1, \pi) \right] \right\} F'_\pi(v) dv.$$

Our parameter estimates yield the optimum to be 1.06, or 2.06 bids per contract on average. Column (5) shows that expected transfers increase by \$30,500, bidding costs rise by \$46,000, project costs fall by \$27,200, and liquidity costs are eliminated. The only measure of aggregate costs that declines is project and liquidity costs (A + B in the table). Moreover, referring to the $(A + B + C)$ row in Table 10, the current regime and the first four counterfactual policies all have lower costs than under Policy (5). Thus if bidding costs represent a resource cost but η is does not, then giving the buyer discretion to condition on the supply side information, including the proportion of

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[23]Transparency International's Corruption Perceptions Index annually ranks countries by the perceived levels of corruption on a scale of 100 (very clean) to 0 (highly corrupt) using expert assessments and opinion surveys. In 2016 US scored 74, Italy scored 47, and Argentina, 36.

low-cost sellers and cost differences between high and low cost sellers, confers positive value.

An important reason for limiting discretion is that it diminishes the demand for collecting and processing information, represented by bidding costs in our model. Accordingly Policy (6) removes all discretion, as in Policy (5), but also halves bidding costs. Due to lower bidding costs, the constrained optimal number of extra bids increases to 2.48, the expected transfer decreases by $36,400 per project, and the total procurement cost ($A + B + C$ in Table 10) also decreases by $7,700 per project. Nevertheless Policy (6) cannot be justified unless there are grounds to believe that the costs of soliciting competitive bids constitute active waste, rather than more legitimate concerns.

8. Conclusion

This paper is an empirical analysis of government procurement where the procurement agencies have discretion about the extent of competition and contract terms. Our theory predicts that negotiating contract terms lowers the benefits of extra bids, and based on our estimates, decreases the average number of bids by up to 2, from 1.5 bids under the current policy. Recognizing that the objectives of the procurement agencies do not necessarily follow taxpayer priorities, we develop several distinct measures of procurement costs to evaluate counterfactual policies. Depending on the factors that comprise the bidding and the competitive solicitation costs, taxpayers might discount them as waste and prefer that all contracts to be competitively solicited. To the extent that these costs are unavoidable consequences from enlarging the competitive pool, our estimates show that allowing procurement agencies to exercise some discretion to use their knowledge of the supply side can reduce procurement costs, even as they simultaneously engage in some rent-seeking behavior.

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Appendix A. Proof of Theorem 4.1

First we prove in Lemma A.1 that the claim about a unique root of (5) is correct, and thus demonstrate the menu presented in Theorem 4.1 is well defined. Then we state and prove six additional lemmas, before proving the theorem itself.

Lemma A.1. *There is at most one root in $\pi \in (0,1)$ to (5).*

Proof. Denote (5) as a real-valued mapping from $\pi \in (0,1)$, $H(\pi)$, and rewrite it as:

$$H(\pi) = \beta - \int \tilde{H}(\pi, s)\overline{f}(s)ds,$$

where:

$$\tilde{H}(\pi, s) \equiv \begin{cases} \psi\left(h\left[\frac{1-\pi}{1-\pi l(s)}\right]\right)[1 - l(s)] & \text{if } l(s) < \tilde{l}(\pi), \\ \psi(M)[1 - l(s)] & \text{otherwise.} \end{cases}$$

If $l(s) \geq \tilde{l}(\pi)$ then $\partial\tilde{H}(\pi, s)/\partial\pi = 0$. Otherwise:

$$\frac{\partial}{\partial\pi}\tilde{H}(\pi, s) = -\psi'\left(h\left[\frac{1-\pi}{1-\pi l(s)}\right]\right)h'\left(\frac{1-\pi}{1-\pi l(s)}\right)\frac{[l(s)-1]^2}{[1-\pi l(s)]^2} > 0.$$

Taking the expectation of $\tilde{H}(\pi, s)$ with respect to s proves $H(\pi)$ is strictly decreasing in π. From (4) $\lim_{\pi\to 0}\tilde{l}(\pi) = \infty$, and $h(1) = 0$ by assumption; hence $\lim_{\pi\to 0} H(\pi) = \beta$. Therefore $H(\pi) > 0$ for all $\pi \in (0,1)$ or there exists a unique $\pi \in (0,1)$ solving $H(\pi) = 0$. $\qquad\square$

Lemma A.2. *The optimal menu includes a fixed contract.*

Proof. The proof is by a contradiction argument. Suppose to the contrary that every contract on the menu is variable. Let $\{p^{(l)}, q^{(l)}(s)\}$ denote a variable contract offered to low-cost sellers, and by choosing that contract, they obtain the expected payoff of:

$$p^{(l)} + \int \psi[q^{(l)}(s)]\underline{f}(s)ds \equiv p'.$$

Define the fixed contract of $p' + \delta$ where δ satisfies the inequalities:

$$0 < \delta < \int \left\{\psi\left[q^{(l)}(s)\right] - q^{(l)}(s)\right\}\underline{f}(s)ds \equiv \underline{R}^{(l)}.$$

A low-cost seller prefers the fixed contact to $\{p^{(l)}, q^{(l)}(s)\}$ because the former yields a higher expected payoff. The expected transfer from the buyer is lower because $\underline{R}^{(l)}$, the risk premium associated with the variable contract, exceeds δ. There are two cases to consider.

First, suppose $p' < \alpha + \beta$. We consider adding $p' + \delta$ such that $p' + \delta < \alpha + \beta$. Let this contract have the same precedence as $\{p^{(l)}, q^{(l)}(s)\}$. Then a high-cost seller would follow the same submission strategy as before because the added contract does not satisfy his IR condition. A low-cost seller, however, would switch from $\{p^{(l)}, q^{(l)}(s)\}$ to $p' + \delta$. Hence the buyer's expected transfer is reduced whenever a low-cost seller previously selecting $\{p^{(l)}, q^{(l)}(s)\}$ wins.

Second, suppose $p' \geq \alpha + \beta$. We consider replacing the whole menu with a single fixed contract of $\alpha + \beta$. Under the original menu both types are paid a risk premium and both receive a certainty equivalent of at least $\alpha + \beta$. Thus a uniform fixed contract of $\alpha + \beta$ is cheaper for the buyer, and trivially satisfies the IC and IR constraints for both types. $\qquad\qquad\square$

Lemma A.3. *It is optimal to offer more than one contract if $n > 1$. If the buyer is constrained to offer fixed contracts only, then it is optimal to offer two contracts, namely $\alpha + \beta$, and:*

$$p_n^{(l)} = \alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\beta \tag{19}$$

where the lower price takes precedence over the higher one. When $n = 1$ the menu defined by $\alpha + \beta$ and (19) collapses to the optimal one-contract menu, $\alpha + \beta$.

Proof. The last statement in the lemma is verified by setting $n = 1$ in (19). The remainder of the proof is in three parts: deriving the optimal contracts when only two fixed ones are permitted on the menu, showing that it is optimal to include more than one item if $n > 1$, and proving that offering more than two items is redundant.

First, consider a menu of two fixed contracts, $\{p^{(l)}, p^{(h)}\}$, where the former is directed towards low-cost sellers and the latter towards high-cost ones. The IR constraints require $p^{(h)} \geq \alpha + \beta$ and $p^{(l)} \geq \alpha$; the IC constraint for the low-cost seller is:

$$\underline{\phi}_n \left(p^{(l)} - \alpha\right) \geq \overline{\phi}_n \left(p^{(h)} - \alpha\right), \tag{20}$$

where $\underline{\phi}_n$ and $\overline{\phi}_n$ denote the subjective probability that a seller selecting $p^{(l)}$ wins and its counterpart for selecting $p^{(h)}$, as defined in (2) and (3). The buyer's objective is to minimize:

$$[1 - (1-\pi)^n]\, p^{(l)} + (1-\pi)^n\, p^{(h)} + \mathbb{E}(\epsilon),$$

with respect to $p^{(l)}$ and $p^{(h)}$ subject to the two IR constraints and (20). The menu characterized in the lemma solves this constrained optimization problem, and also satisfies the remaining IC constraint for the high-cost seller.

Second, if only one item is offered on the menu, then to meet IR constraints the contract price must be at least $\alpha + \beta$. The two fixed contracts defining menu referred to in the lemma satisfy both IR and IC constraints and yield a total expected transfer:

$$\begin{aligned}
T_n^F &\equiv [1 - (1-\pi)^n]\, p_n^{(l)} + (1-\pi)^n\, (\alpha + \beta) + \mathbb{E}(\epsilon) \\
&= \alpha + (1-\pi)^{n-1}\beta + \mathbb{E}(\epsilon) < \alpha + \beta + \mathbb{E}(\epsilon),
\end{aligned} \tag{21}$$

for any $n > 1$ and $\pi \in (0, 1)$.

Third, suppose multiple fixed contracts are offered to low-cost sellers, which we now denote by $\left\{ p_1^{(l)}, p_2^{(l)}, \ldots \right\}$. Also let Φ_i denote the probability of winning the contract by bidding $p_i^{(l)}$. Conditional on a low-cost seller winning the contract, the expected transfer is $\sum_i \Phi_i p_i^{(l)} \equiv p_0^{(l)}$. By construction offering $p_0^{(l)}$ instead of $\left\{ p_1^{(l)}, p_2^{(l)}, \ldots \right\}$ is equally profitable for both low-cost sellers and the buyer. Similarly, offering multiple fixed contracts to high-cost sellers does not reduce the expected transfer.   □

Lemma A.4. *It is not optimal to offer variable contracts to low-cost sellers.*

Proof. The proof is by contradiction argument. Suppose a set of multiple contracts $\mathcal{L} \equiv \left\{ p_i^{(l)}, q_i^{(l)}(s) : i = 1, 2, \ldots \right\}$ is offered to low-cost sellers in the conjectured equilibrium, where $\mathcal{L}$ includes at least one variable contract. We define the certainty equivalent of each contract $i \in \mathcal{L}$ as p_i'', defined as:

$$p_i'' \equiv p_i^{(l)} + \int \psi \left[q_i^{(l)}(s) \right] \underline{f}(s) ds \leq p_i^{(l)} + \int q_i^{(l)}(s) \underline{f}(s) ds \equiv \widetilde{p}_i'',$$

where $\widetilde{p}_i''$ is the expected transfer to a low-cost seller from submitting the ith contract. When i is a variable contract, $q_i^{(l)}(s) \neq 0$ for some $s \in S$ and appealing to the assumption that $\psi(q) < q$ for any $q \neq 0$ and $\psi(0) = 0$, it follows that $p_i'' < \widetilde{p}_i''$. Note that some contracts in $\mathcal{L}$ can also be offered to high-cost sellers.

Suppose there exists a variable contract i in $\mathcal{L}$ satisfying the inequality $p_i'' < \alpha + \beta$. If the variable contract is offered to high-cost sellers, we add a fixed contract of p_i'' to the menu; otherwise, we replacing it with the fixed contract. The new contract is designated to have the same precedence as the variable contract. High-cost sellers do not select p_i'' since $p_i'' < \alpha + \beta$ violating their IR constraint. The new fixed contract yields the same level of expected payoff as the variable contract i to low-cost sellers and therefore satisfies their IR and the IC constraints. Since low-cost sellers are indifferent, replacing or adding p_i'' does not change the submission behavior of high-cost sellers, but reduces the buyer's expected transfer (since $p_i'' < \widetilde{p}_i''$). Hence offering a variable contract with a certainty equivalent less than $\alpha + \beta$ to low-cost sellers is not optimal.

Suppose there exists at least one contract in $\mathcal{L}$ for which $p_i'' \geq \alpha + \beta$. We consider replacing all such contracts in $\mathcal{L}$ with a fixed contract of $\alpha + \beta$, while retaining the others. We also replace $\mathcal{H}$ with $\alpha + \beta$. Note that $\alpha + \beta$ is a lower bound of the expected transfer to high-cost sellers to satisfy their IR condition. Because the retained contracts in $\mathcal{L}$ are fixed contracts less than $\alpha + \beta$ by the above argument, both the IR and the IC constraints for high-cost sellers are satisfied. Note also that $\alpha + \beta$ is

less desirable to low-cost sellers than the contracts that have been withdrawn from $\mathcal{L}$. Therefore low-cost sellers would not decrease the probability of choosing the retained contracts. Thus the IR and the IC conditions for the low-cost sellers are satisfied. Therefore, the distribution of expected transfers associated with the contract for the modified contracts is first order stochastically dominated by the distribution when there are variable contracts offered to low-cost sellers with $p_i'' \geq \alpha + \beta$.   □

Lemma A.5. *The menu defined in Lemma A.3 is not optimal.*

Proof. We construct an alternative menu of contracts, comprising a preferred fixed contract $\tilde{\underline{p}}_n$ plus a variable contract $\{\tilde{\overline{p}}, \tilde{q}(s)\}$, with a lower expected transfer than T_n^F, given in (21), thus proving the Lemma. By assumption $\overline{F}(s) \neq \underline{F}(s)$ for some outcome $s \in S$, and hence for some $\epsilon > 0$, there exists $\tilde{S} \equiv \{s : \overline{f}(s) - \underline{f}(s) > \epsilon\}$ and (γ_1, γ_2) such that:

$$0 < \gamma_1 \equiv \int_{\tilde{S}} \underline{f}(s) ds < \int_{\tilde{S}} \overline{f}(s) ds \equiv \gamma_2 < 1$$

Noting that because $\psi(q)$ is monotonic its inverse exists, consider any δ satisfying the inequalities:

$$0 \leq \delta < \min\left\{\psi^{-1}\left[\frac{\beta(1-\gamma_2)}{(\gamma_2-\gamma_1)}\right], \psi^{-1}\left[\frac{(1-\gamma_2)}{\gamma_2}|\psi(M)|\right]\right\} \tag{22}$$

and define the differentiable mapping:

$$\mu(\delta) \equiv \psi^{-1}\left[-\gamma_2\psi(\delta)/(1-\gamma_2)\right].$$

We define the alternative fixed contract as:

$$\tilde{\underline{p}}_n = \alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left\{\beta + \frac{\gamma_1 - \gamma_2}{1-\gamma_2}\psi(\delta)\right\}, \tag{23}$$

and the variable contract is defined by:

$$\tilde{\overline{p}} = \alpha + \beta \text{ and } \tilde{q}(s) = \begin{cases} \delta & \text{if } s \in S, \\ \mu(\delta) & \text{if } s \notin S. \end{cases}$$

The expected payoff to low-cost sellers from choosing $\tilde{\underline{p}}_n$ is:

$$\overline{\phi}_n(\tilde{\underline{p}}_n - \alpha) = \frac{(1-\pi)^{n-1}}{n}\left[\beta + \frac{\gamma_1 - \gamma_2}{1-\gamma_2}\psi(\delta)\right], \tag{24}$$

while their expected payoff from choosing $\{\bar{\bar{p}}, q(s)\}$ is:

$$\underline{\phi}_n\left(\bar{\bar{p}} + \int \psi[q(s)]\underline{f}(s)ds - \alpha\right) = \frac{(1-\pi)^{n-1}}{n}\left\{\beta + \gamma_1\psi(\delta) + (1-\gamma_1)\psi[\mu(\delta)]\right\}, \quad (25)$$

where $\bar{\phi}_n$ and $\underline{\phi}_n$ are defined in (2) and (3). From the definition of $\mu(\delta)$:

$$\gamma_1\psi(\delta) + (1-\gamma_1)\psi[\mu(\delta)] = \frac{\gamma_1 - \gamma_2}{1 - \gamma_2}\psi(\delta). \quad (26)$$

Comparing (24) with (25) using (26) establishes the payoffs are identical, and hence demonstrates the IC constraint for low-cost sellers is satisfied with equality. By (22) and (23) their IR constraint is satisfied with strict inequality. Conditional on a high-cost seller winning the contract, his expected transfer is $\tilde{p} = \alpha + \beta$ since $\int \tilde{q}(s)\bar{f}(s)ds = 0$, implying his IR constraint is satisfied with equality. From (23) the IC constraint for high-cost sellers is satisfied with strict inequality because $\psi(\delta) > 0$ and $\gamma_1 < \gamma_2$, and by (22) δ strictly satisfies the maximal penalty constraint, $\mu(\delta) \geq M$. Therefore the proposed menu and the rules for selecting the winner constitute a direct revelation game. The expected transfer from the buyer, denoted by $\tilde{T}_n(\delta)$ to indicate its dependence on the choice of δ, is:

$$\tilde{T}_n(\delta) = \alpha + (1-\pi)^{n-1}\left\{\beta + \pi\frac{\gamma_1 - \gamma_2}{1 - \gamma_2}\psi(\delta) + (1-\pi)\left[\gamma_2\delta + (1-\gamma_2)\mu(\delta)\right]\right\} + \mathbb{E}(\epsilon).$$

Noting $\lim_{\delta\to 0}\mu(\delta) = 0$ and $\tilde{T}_n(0) = T_n^F$, we complete the proof by showing the derivative of $\tilde{T}_n(\delta)$ at $\delta = 0$ is negative:

$$\tilde{T}_n'(0) = (1-\pi)^{n-1}\pi\frac{\gamma_1 - \gamma_2}{1 - \gamma_2} < 0.$$

$\square$

Lemma A.6. *It is not optimal to offer fixed contracts to high-cost sellers.*

Proof. We prove the lemma by by contradicting the hypothesis to the contrary. First, consider a menu that includes a fixed contract priced greater than $\alpha + \beta$. Such a menu cannot offer a fixed contract of $\alpha + \beta$ to high-cost sellers as well, because their expected payoff from submitting $\alpha + \beta$ is zero, strictly less than a higher priced fixed contract. We now propose replacing all contracts that involve expected transfers of more than $\alpha + \beta$ with one fixed contract of $\alpha + \beta$. This would only leave fixed contracts, because by Lemma A.4, only fixed contracts are offered to low-cost sellers, and variable contracts with expected transfers of less than $\alpha + \beta$ do not meet the IR constraint of high-cost sellers. Under the proposal all high-cost sellers would select

$\alpha + \beta$ because no other contract satisfies their IR constraint. If all contracts offered to low-cost sellers are less than $\alpha + \beta$, then their choice probabilities would remain unchanged. Alternatively if they are offered a fixed contract of $p''' > \alpha + \beta$, they would either choose $\alpha + \beta$ or one of the remaining fixed contracts below $\alpha + \beta$. Because the $\alpha + \beta$ contract would be less desirable than the p''' contract, the probability of a low-cost seller choosing any fixed contract below $\alpha + \beta$ would increase, and the remaining mass would be concentrated at $\alpha + \beta$ (instead of at contracts with higher expected transfers). Therefore the proposal satisfies the IR conditions and every menu containing a fixed contract of more than $\alpha + \beta$ is more costly than the proposal, so not optimal.

Thus, if the contradictory hypothesis is true, the optimal menu includes only one fixed contract offered to high-cost sellers, namely $\alpha + \beta$. Lemma A.4 shows low-cost sellers are not offered variable contracts. To induce a high-cost seller to select a variable contract, the buyer must offer a risk premium, so the expected transfer exceeds $\alpha + \beta$. Therefore such a menu does not include variable contracts. An optimal fixed contract menu is given in Lemma A.3. This is more costly than the menu given in Lemma A.5 which includes a variable contract for high-cost sellers, contradicting the contrary hypothesis.

<div align="right">□</div>

Lemma A.7. *The optimal menu of contracts consisting of one fixed contract and one variable contract is unique and is defined by (33), (34) and (35).*

Proof. Suppose the buyer is constrained to offer a menu of two contracts consisting of one fixed contract and one variable contract. Then by Lemma A.4, only the low-cost sellers accept the fixed contract, $\underline{p}_n$, and only the high-cost sellers accept the variable one, $\{\overline{p}, q(s)\}$ at optimum. Thus the buyer's problem is to minimize the expected transfer, (1), subject to the IR and the IC constraints and the maximal penalty constraint.

A necessary condition of the optimal menu is that the IR constraint for high-cost sellers holds with equality because otherwise the base price $\overline{p}$ could be further reduced, reducing the price and strengthening the IC constraint for low-cost sellers. We exploit this condition in two ways. First, solving for $\overline{p}$ yields (34). Substituting for $\overline{p}$ using (34) and appealing to the definitions of $\overline{\phi}_n$ and $\underline{\phi}_n$ as in (2) and (3), we rewrite the IC constraint for low-cost sellers as:

$$\underline{p}_n \geq \alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left(\beta - \int \psi[q(s)]\{1-l(s)\}\overline{f}(s)ds\right). \tag{27}$$

Second, the condition implies that the menu uniquely defined in the theorem satisfies the IC constraint for high-cost sellers. To see this note that IC constraint is:

$$\underline{\phi}_n(\underline{p}_n - \alpha - \beta) \leq \overline{\phi}_n(\overline{p} + \int \psi[q(s)]\overline{f}(s)ds - \alpha - \beta).$$

The right hand side of the above inequality is zero because the IR constraint for high-cost sellers binds, and $\underline{p}_n$ defined by (33) is bounded from above by $\alpha + \beta$ because $q(s)$ as defined in (35) satisfies $\psi[q(s)]\{1 - l(s)\} \geq 0$ for all $s \in S$.

Thus we complete the proof of the lemma by showing the menu is the solution to minimizing (1) subject to (27), the IR constraint for low-cost sellers and the maximal penalty constraint. There are two cases to consider, depending on whether or not the remaining IR constraint binds.

Suppose the IR constraint does not bind. Then the IC constraint for low-cost sellers must bind at the optimum; otherwise, the price of the fixed contract could be reduced, to the buyer's benefit. Solving for $\underline{p}_n$ from the IC constraint, and substituting the resulting expressions for $\underline{p}_n$ and $\overline{p}$, obtained from (33) and (34), into (1), we obtain the objective function of the buyer as:

$$\alpha + (1-\pi)^{n-1}\left(\beta + \int \left[(1-\pi)(q(s) - \psi[q(s)]) - \pi\psi[\overline{q}(s)]\{1 - l(s)\}\right]\overline{f}(s)ds\right) + \mathbb{E}(\epsilon).$$

The (scaled) Lagrangian for the cost minimization problem can now be expressed as:

$$L = \int \left[(1-\pi)\{q(s) - \psi[q(s)]\} - \pi\psi[\overline{q}(s)]\{1 - l(s)\} - \varkappa_1(s)(q(s) - M)\right]\overline{f}(s)ds,$$

where $\varkappa_1(s) \geq 0$ denotes the Kuhn Tucker multiplier for the maximal penalty constraint $q(s) \geq M$. The first order condition for $q(s)$ is:

$$(1-\pi)(1 - \psi'[q(s)]) - \pi\psi'[q(s)]\{1 - l(s)\} - \varkappa_1(s) = 0.$$

Rearranging terms we obtain:

$$\psi'[q(s)] = \frac{1 - \pi - \varkappa_1(s)}{1 - \pi l(s)}. \tag{28}$$

If $l(s) < \tilde{l}(\pi)$, then $q(s) = h\left[\frac{1-\pi}{1-\pi l(s)}\right] > M$ and hence $\varkappa_1(s) = 0$ solve (28). If $l(s) \geq \tilde{l}(\pi)$, then $\varkappa_1(s) > 0$ and $q(s) = M$ solve (28).

Now suppose the IR constraint for low-cost sellers binds. Then $\underline{p}_n = \alpha$. Substituting for $\underline{p}_n$ and $\overline{p}$ using (34), we simplify (1) to obtain the expected transfer of the buyer:

$$\alpha + (1-\pi)^n\left\{\beta + \int \{q(s) - \psi[q(s)]\}\overline{f}(s)ds\right\} + \mathbb{E}(\epsilon).$$

Substituting for $\underline{p}_n$ in (27) yields:

$$\beta \leq \int \psi[q(s)]\{1 - l(s)\}\overline{f}(s)ds. \tag{29}$$

Let $\varkappa_1(s) \geq 0$ denote the Kuhn Tucker multiplier for the maximal penalty constraint $q(s) \geq M$. If the IC constraint for low-cost sellers does not bind, then the first order condition for the Kuhn Tucker formulation is:

$$1 - \psi'[q(s)] = \varkappa_1(s).$$

If $q(s) > M$, then the complementary slackness condition requires $\varkappa_1(s) = 0$, and hence $1 = \psi'[q(s)]$ implying $q(s) = 0$. Therefore, either $q(s) = M$ or $q(s) = 0$. Let us define S_M as the set of contract outcomes such that $q(s) = M$ and let μ denote the probability that $s \in S_M$ for high-cost sellers. The total expected transfer can now be written as:

$$\alpha + (1 - \pi)^n \{\beta + [M - \psi(M)]\mu\} + \mathbb{E}(\epsilon). \tag{30}$$

By inspection (30) is increasing in μ, while setting $\mu = 0$ does not satisfy the IC condition for low-cost seller, or (29). This implies that when both IR constraints bind, the IC for low-cost sellers must bind as well; in other words (29) holds with equality. Now the (scaled) Lagrangian for the minimization problem can be written as:

$$\int \{(q(s) - \psi[q(s)]) - \varkappa_1(s)[q(s) - M]\}\overline{f}(s)ds + \varkappa_2\left\{\beta - \int \psi[q(s)][1 - l(s)]\overline{f}(s)ds\right\}, \tag{31}$$

where $\varkappa_2$ denotes the Kuhn Tucker multiplier for (29). The first order condition with respect to $q(s)$ is:

$$1 - \psi'[q(s)] - \varkappa_1(s) - \varkappa_2\psi'[q(s)][1 - l(s)] = 0,$$

which can be expressed:

$$\psi'[q(s)] = \frac{1 - \varkappa_1(s)}{1 + \varkappa_2[1 - l(s)]}. \tag{32}$$

Since (31) does not depend on π, neither does $q(s)$ nor $\varkappa_1(s)$ and $\varkappa_2$. Noting that $\tilde{\pi}$ solves both first order conditions (28) and (32), we equate the two and deduce $\varkappa_2 = \tilde{\pi}/(1 - \tilde{\pi})$. Substituting for $\varkappa_2$ in (32), the solution for $q(s)$ follows by appealing to the definition of $h(x)$ and setting $\pi = \tilde{\pi}$ in (35).

Summarizing, the IC for low-cost sellers always binds at the optimum, but their IR constraint may not. It immediately follows from (33) that if $H(\pi) > 0$ then $\underline{p}_n > \alpha$, but if $H(\pi) \leq 0$ then $\underline{p}_n = \alpha$. Appealing to Lemma A.1, we conclude if $\pi < \tilde{\pi}$, then

$H(\pi) > 0$ and the IR does not bind; otherwise $\pi \geq \tilde{\pi}$, then $H(\pi) \leq 0$ and the IR binds. $\qquad \square$

Proof of Theorem 4.1. By Lemma A.4 it is not optimal to offer low-cost sellers a variable contracts, and by Lemma A.6 it is not optimal to offer high-cost sellers fixed contracts. This proves the second statement in the theorem, and also implies that if an optimal menu with two contracts exists, it comprises one fixed contract directed to low-cost sellers and one variable contract for high-cost sellers. Lemma A.7 characterizes the unique optimal menu when only two contracts are permitted, proving the third statement of the theorem. Lemma A.5 shows that a menu with a variable contract is less costly than the optimal two fixed priced contract menu given in Lemma A.3, which in turn is less costly than menus containing only one contract. This proves that an optimal menu contains at least two contracts. We now show additional contracts are redundant. Noting that the low-cost seller is offered a fixed contract, exactly the same arguments used in the proof of Lemma A.3 apply here. Therefore offering several fixed contracts to the low-cost seller does not reduce the expected transfer. Also every variable contract offered to the high-cost seller must individually satisfy the IR constraint with equality, and also deter the low-cost seller from accepting it. If any two such variable contracts on the menu do not generate the same expected transfer to the high-cost seller, then offering the more expensive variable contract is suboptimal. This proves the first statement of the theorem. $\quad \square$

APPENDIX B. PROOF OF LEMMAS AND THE THEOREM IN SECTION 5

We provide the proof of the lemmas and the theorem in Section 5 on the nonparametric identification of the model.

B.1. **Proof of Lemma 5.1. Lemma 5.1 (i)**   If **A3** holds then $\partial \left| q(l, \pi) \right| / \partial \pi > 0$.

Proof. Theorem 4.1 characterizes the unique optimal menu of two contracts, defined by the price of the fixed contract:

$$\underline{p}_n = \alpha + \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left(\beta - \int \psi[q(s)] \left[1 - l(s) \right] \overline{f}(s) ds \right), \tag{33}$$

and the variable contract:

$$\overline{p} = \alpha + \beta - \int \psi[q(s)] \overline{f}(s) ds, \tag{34}$$

$$q(s) = \begin{cases} h\left(\frac{1-\min\{\pi,\tilde{\pi}\}}{1-\min\{\pi,\tilde{\pi}\}l(s)}\right) & \text{if } l(s) \leq \tilde{l}(\min\{\pi, \tilde{\pi}\}), \\ M & \text{if } l(s) > \tilde{l}(\min\{\pi, \tilde{\pi}\}). \end{cases} \tag{35}$$

By **A3** $q\,(l, \pi)$ satisfies:

$$\psi'\,[q\,(l, \pi)]\,[1 - \pi l] = 1 - \pi. \tag{36}$$

Totally differentiating (36) with respect to π and making $\partial q\,(l, \pi)\,/\partial\pi$ the subject of the resulting equation:

$$\frac{\partial q\,(l, \pi)}{\partial\pi} = \frac{l - 1}{\psi''\,[q\,(l, \pi)]\,(1 - \pi l)^2}. \tag{37}$$

By assumption $\psi''\,(q) < 0$, implying $\partial q\,(l, \pi)\,/\partial\pi \gtreqqless 0$ when $l \lesseqqgtr 1$. From (36) it follows that $q\,(l, \pi) \gtreqqless 0$ when $l \lesseqqgtr 1$. Combining both sets of inequalities $\partial q\,(l, \pi)\,/\partial\pi \gtreqqless 0$ when $q\,(l, \pi) \lesseqqgtr 0$, as claimed. $\qquad\qquad\square$

Lemma 5.1 (ii)   If **A3** and **A5** hold then $\partial\bar{p}\,(\pi)\,/\partial\pi < 0$.

Proof. Totally differentiating the base price for variable contracts, defined in (34), with respect to π, yields:

$$\bar{p}'\,(\pi) = \alpha'\,(\pi) + \beta'\,(\pi) - \int \psi'\,(q\,[l\,(s)\,, \pi])\,\frac{\partial q\,[l\,(s)\,, \pi]}{\partial\pi}\overline{f}\,(s)\,ds.$$

Define:

$$m(l, \pi) \equiv \psi'\,[q\,(l, \pi)]\,\frac{\partial q\,(l, \pi)}{\partial\pi}.$$

From (39) the first order condition for an interior solution can be rewritten as:

$$\psi'\,(q) = (1 - \pi)\,/(1 - \pi l)\ ,\ \text{or}\ q\,(l, \pi) = h\,[(1 - \pi)\,/(1 - \pi l)]\,.$$

Therefore:

$$m\,(l, \pi) = h'\left(\frac{1 - \pi}{1 - \pi l}\right)\frac{(1 - \pi)\,(1 - l)}{(1 - \pi l)^3}.$$

Note $m\,(l, \pi) \gtreqqless 0$ for $l \lesseqqgtr 1$ because $h'\,(x) < 0$ and by **A3**, $\pi l < 1$. By definition $l(s) > 0$ for all $s \in S$. Therefore:

$$\begin{aligned} \bar{p}'\,(\pi) &= \alpha'\,(\pi) + \beta'\,(\pi) - \int m\,[l\,(s)\,, \pi]\,\overline{f}\,(s)\,ds \\ &< \alpha'\,(\pi) + \beta'\,(\pi) - \int_{\{s:l(s)<1\}} m\,[l\,(s)\,, \pi]\,\overline{f}\,(s)\,ds \\ &\leq \alpha'\,(\pi) + \beta'\,(\pi) - \sup_{l\in(0,1)}|m\,(l, \pi)\,|\int_{\{s:l(s)<1\}}\overline{f}\,(s)\,ds. \end{aligned}$$

Thus $|m\left(l,\pi\right)|$ is bounded for $l \in (0,1)$ by:

$$
\begin{aligned}
|m\left(l,\pi\right)| &\leq \left|h'\left(\frac{1-\pi}{1-\pi l}\right)\right|\left|\frac{(1-\pi)(1-l)}{(1-\pi l)^3}\right| \\
&\leq \sup_{l\in(0,1)}\left|h'\left(\frac{1-\pi}{1-\pi l}\right)\right| \sup_{l\in(0,1)}(1-l) \sup_{l\in(0,1)}\left|\frac{(1-\pi)}{(1-\pi l)^3}\right| \\
&\leq (1-\pi)^{-2} \sup_{l\in(0,1)}\left|h'\left(\frac{1-\pi}{1-\pi l}\right)\right|.
\end{aligned}
$$

Therefore:

$$
\overline{p}'\left(\pi\right) < \alpha'\left(\pi\right) + \beta'\left(\pi\right) - (1-\pi)^{-2}\sup_{l\in(0,1)}\left|h'\left(\frac{1-\pi}{1-\pi l}\right)\right|\left|\int_{\{s:l(s)<1\}}\overline{f}\left(s\right)ds\right| < 0.
$$

where the second inequality holds by **A5**.   □

Lemma 5.1 (iii)   If **A3** and **A4** hold then $\partial \underline{p}_n\left(\pi\right)/\partial\pi < 0$ for all $n \in \{1, 2, \ldots\}$.

Proof. Rewriting (33) to make the dependence of $\underline{p}_n$ on π explicit:

$$
\begin{aligned}
\underline{p}_n(\pi) &= \alpha(\pi) + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left[\beta(\pi) - \int \psi\left(q\left[l\left(s\right),\pi\right]\right)\left[1-l\left(s\right)\right]\overline{f}\left(s\right)\right] \\
&\equiv \alpha(\pi) + \Psi_{0,n}\left(\pi\right)\left[\beta(\pi) - \Psi_1\left(\pi\right)\right].
\end{aligned}
$$

and hence:

$$
\underline{p}_n'\left(\pi\right) = \alpha'\left(\pi\right) + \Psi_{0,n}'\left(\pi\right)\left[\beta(\pi) - \Psi_1\left(\pi\right)\right] + \Psi_{0,n}\left(\pi\right)\left[\beta'(\pi) - \Psi_1'\left(\pi\right)\right]. \tag{38}
$$

By **A4** $\alpha'(\pi) \leq 0$. Also:

$$
\frac{\partial}{\partial\pi}\ln\left[\Psi_{0,n}\left(\pi\right)\right] = \frac{1-n\pi-(1-\pi)^n}{\pi(1-\pi)\left[1-(1-\pi)^n\right]}.
$$

The derivative is zero at $n = 1$ and $-\pi^2$ at $n = 2$. Now suppose it is negative for all $n \in \{2,\ldots,n_0\}$. For $n_0 + 1$ the denominator is positive and the numerator is:

$$
1-(n_0+1)\pi-(1-\pi)(1-\pi)^{n_0} < \pi(1-\pi)^{n_0} - \pi < 0.
$$

The first inequality follows from an induction hypothesis, and the second one from the inequalities $0 < \pi < 1$. Therefore $\Psi_{0,n}'\left(\pi\right) \leq 0$ for all (π, n). By **A3** the participation constraint is satisfied with an inequality implying $\beta(\pi) > \Psi_1\left(\pi\right)$, and hence $\Psi_{0,n}'\left(\pi\right)\left[\beta(\pi) - \Psi_1\left(\pi\right)\right] \leq 0$. The third expression in (38) is strictly negative because

$\Psi_{0,n}(\pi) > 0$, by **A4** $\beta'(\pi) \leq 0$, and:

$$\begin{aligned}
\Psi'_1(\pi) &= \int \psi'(q[l(s),\pi]) \frac{\partial q[l(s),\pi]}{\partial \pi} [1 - l(s)] \overline{f}(s) \, ds \\
&= \int (1-\pi) \frac{\partial q[l(s),\pi]}{\partial \pi} \left[\frac{1-l(s)}{1-\pi l(s)}\right] \overline{f}(s) \, ds \\
&= \int \frac{(\pi-1)[1-l(s)]^2}{\psi''(q[l(s),\pi])[1-\pi l(s)]^3} \overline{f}(s) \, ds > 0.
\end{aligned}$$

Appealing to **A3**, the second equality uses (36) to substitute out $\psi'[q(s,\pi)]$, the third equality uses (37) to substitute out $\partial q(l,\pi)/\partial \pi$, and the inequality follows from $\psi''(q) < 0$ and the assumption of an interior solution. Therefore $\underline{p}'_n(\pi) < 0$. □

B.2. **Proof of Lemma 5.2. Lemma 5.2** $f_{\pi|c,n,v}(\pi|c,n,v)$ is identified.

Proof. Rewriting (36) to solve for π, we obtain:

$$\pi = [1 - \psi'(q)]/[1 - \psi'(q)l]. \tag{39}$$

Since $\psi(q)$ is identified, the π corresponding to each variable contract $(\overline{p}, q, l)$ is identified from (39). Hence $f_{\pi|c,n,v}(\pi|c,n,1)$ is identified. Define the odds ratio related to contract types conditional on (c,n) as:

$$\varphi_{c,n} \equiv \Pr(v=1|c,n)/\Pr(v=0|c,n).$$

and note that $\varphi_{c,n}$ is identified. The joint probability that the contract type is fixed and $\pi \leq \pi^*$ can be expressed as:

$$\begin{aligned}
\Pr\{\pi \leq \pi^*, v = 0 | c, n\} &= F_{\pi|c,n,v}(\pi^*|c,n,0) \Pr(v=0|c,n) \\
&= \int_{\pi=\underline{\pi}}^{\pi^*} f_{\pi|c,n}(\pi|c,n)[1-(1-\pi)^n] \, d\pi. \tag{40}
\end{aligned}$$

Taking the derivative with respect to π^* yields:

$$f_{\pi|c,n,v}(\pi^*|c,n,0)\Pr(v=0|c,n) = f_{\pi|c,n}(\pi^*|c,n)[1-(1-\pi^*)^n]. \tag{41}$$

Similarly:

$$\begin{aligned}
\Pr\{\pi \leq \pi^*, v = 1 | c, n\} &= F_{\pi|c,n,v}(\pi^*|n,v=1)\Pr(v=1|c,n) \\
&= \int_{\pi=\underline{\pi}}^{\pi^*} f_{\pi|c,n}(\pi|c,n)(1-\pi)^n \, d\pi,
\end{aligned}$$

and taking the derivative with respect to π^* yields:

$$f_{\pi|c,n,v}(\pi^*|c,n,1)\Pr(v=1|c,n) = f_{\pi|c,n}(\pi^*|c,n)(1-\pi^*)^n. \tag{42}$$

Rearranging the quotient of (41) and (42) to make $f_{\pi|c,n,v}\left(\pi^*|c,n,v=0\right)$ the subject of the resulting equation, we obtain:

$$f_{\pi|c,n,v}\left(\pi|c,n,0\right) = \varphi_{c,n}\frac{\left[1-(1-\pi)^n\right]}{(1-\pi)^n}f_{\pi|c,n,v}\left(\pi|c,n,1\right),$$

after dropping the superscripts asterisks. Because $\varphi_{c,n}$ and $f_{\pi|c,n,v}\left(\pi|c,n,1\right)$ are identified, so is $f_{\pi|c,n,v}\left(\pi|c,n,0\right)$. $\qquad\square$

B.3. Proof of Theorem 5.1. Theorem 5.1 $\quad\psi\left(q\right), \alpha\left(\pi\right)$ and $\beta\left(\pi\right)$ are identified, and for $n \in \{2, 3, \ldots\}$:

$$\alpha\left(\pi\right) = \frac{1-(1-\pi)^n}{1-(1-\pi)^{n-1}}\underline{p}_n^*\left(\pi,c\right) - \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^{n-1}}\underline{p}_1^*\left(\pi,c\right), \qquad (43)$$

$$\beta\left(\pi\right) = \overline{p}\left(\pi\right) + \int\psi\left(h\left[\frac{1-\pi}{1-\pi l\left(t\right)}\right]\right)\overline{f}\left(t\right)dt - \alpha(\pi).$$

Proof. To prove the first equation in (43) set $n=1$ in (33) to obtain:

$$\underline{p}_1 - \alpha = \beta - \int\psi[q(s)]\left[1-l\left(s\right)\right]\overline{f}(s)ds.$$

Substitute the right hand side of this expression back into (33) yielding:

$$\underline{p}_n = \alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left(\underline{p}_1 - \alpha\right).$$

Make α the subject of the equation, and explicitly recognize the dependence of $\underline{p}_n$, $\underline{p}_1$ and α on π. To prove the second equation in (43), rearrange (39) using the definition of $h\left(\cdot\right)$ to obtain q as a function of s, specifically $q(s) = h\left((1-\pi)/[1-\pi l(s)]\right)$, substitute the expression into (34), make β the subject, and the dependence of α, β and $\overline{p}$ on π explicit.

The proof that $\psi\left(q\right)$ is identified is given in the text. Proving $\alpha\left(\pi\right)$ and $\beta\left(\pi\right)$ are identified follows from noting that their representations given in (43) are identified by Lemma 5.2 and the arguments in the surrounding text. $\qquad\square$

Appendix C. Optimal Competition under the Parametric Specification

We provide the closed-form characterization of the equilibrium contracts and competition of the estimated model. The specification on $\kappa(\pi,n)$ is

$$\kappa(n,\pi) = (\kappa_1 + \kappa_2\pi)(n-1) + (\kappa_3 + \kappa_4\pi)(n-1)^2.$$

Given this specification, the expected total cost of competed procurement with effort λ, denoted by $U(\pi, \lambda)$, is:

$$
\begin{aligned}
U(\pi, \lambda) &= \alpha(\pi) + \exp^{-\lambda\pi}[\beta(\pi) + \Gamma(\pi)] + \mathbb{E}[\kappa(\pi, j+1)|\lambda] + \eta \\
&= \alpha(\pi) + \exp^{-\lambda\pi}[\beta(\pi) + \Gamma(\pi)] + \tilde{\kappa}_1(\pi)\lambda + \tilde{\kappa}_2(\pi)\lambda(1+\lambda) + \eta,
\end{aligned}
$$

where $\tilde{\kappa}_1(\pi) \equiv \kappa_1 + \kappa_2\pi$ and $\tilde{\kappa}_2(\pi) \equiv \kappa_3 + \kappa_4\pi$.

Taking the first order condition:

$$
\pi \exp^{-\lambda\pi}[\beta(\pi) + \Gamma(\pi)] = \tilde{\kappa}_1(\pi) + \tilde{\kappa}_2(\pi)(1+2\lambda). \tag{44}
$$

Because the left hand side is decreasing in λ while the right hand side is increasing in λ, there exists a unique solution to the above equation for any given π, denoted by $\tilde{\lambda}(\pi)$. Because $\lambda \geq 0$, $\lambda^*(\pi) = \max\{\tilde{\lambda}(\pi), 0\}$. In our estimation, we numerically solve for $\lambda^*(\pi)$ for each π.

Given $\lambda^*(\pi)$, it is optimal for the buyer to hold a competitive solicitation if and only if

$$
U[\pi, \lambda^*(\pi)] \leq U_0(\pi).
$$

The above inequality can be rewritten as:

$$
\eta \leq (1 - e^{-\lambda^*(\pi)\pi}[\beta(\pi) + \Gamma(\pi)] - \tilde{\kappa}_1\lambda^*(\pi) - \tilde{\kappa}_2\lambda^*(\pi)[1 + \lambda^*(\pi)]. \tag{45}
$$

Equations (33), (34), (35), (44), and (45) characterize the equilibrium contracts and competition.

Appendix D. Simulated GMM Estimator

Let us denote the vector of the parameters of the model by θ. Our estimator minimizes a weighted sum of squared distances:

$$
g_n(\theta)'Wg_n(\theta), \text{ with } g_n(\theta) = \frac{1}{n}\sum_{t=1}^{n} g(w_t; \theta),
$$

where W is a symmetric positive-definite weighting matrix. The $g(w_t; \theta)$ vector is associated with 40 moment conditions: (i) 17 moment conditions on competition, contract type, and contract price for all projects, (ii) the same 17 moment conditions for non-military projects, and (iii) 6 moment conditions on the distribution of s, or the standardized delay.

The 17 moment conditions consist of $\Pr(c_i = 0)$, $\Pr(n_i = 1|c_i = 1)$, $\Pr(n_i \leq 2|c_i = 1)$, $\Pr(n_i \leq 5|c_i = 1)$, $\Pr(v_i = 0|c_i = 0)$, $\Pr(v_i = 0|c_i = 1)$, $\Pr(v_i = 0|c_i = 1, n_i = 1)$, $\Pr(v_i = 0|c_i = 1, n_i \leq 2)$, $\Pr(v_i = 0|c_i = 1, n_i \leq 5)$, $\mathbb{E}[n_i]$, $\mathbb{E}[p_i|c_i = c, v_i = v]$ for

$(c, v) \in \{0, 1\} \times \{0, 1\}$ where $p_i \equiv \underline{p}_i(1 - v_i) + (\overline{p}_i + q_i)v_i$, $\mathbb{E}[p_i | c_i = 1, n_i = 1]$, $\mathbb{E}[p_i | c_i = 1, n_i \leq 2]$, and $\mathbb{E}[p_i | c_i = 1, n_i \leq 5]$.The 6 moment conditions on the distribution of the standardized delay are $\Pr(s_i = 0, v_i = 0)$, $\Pr(s_i = 0, v_i = 1)$, $\Pr[s_i(1 - v_i)]$, $\Pr[s_i^2(1 - v_i)]$, $\Pr[s_i v_i]$, and $\Pr[s_i^2 v_i]$. Note that the moments as a function of θ are calculated using simulation. In our estimation, the simulation size is 5,000.

We use a two-step procedure to obtain the optimally weighted simulated GMM estimator. We start with a positive definite weighting matrix and obtain a first-step estimator, denoted by $\tilde{\theta}_n$. The asymptotic variance of $\sqrt{n}g_n(\theta_0)$, S, is estimated by:

$$\hat{S} \equiv \frac{1}{n} \sum_t g(w_t, , \tilde{\theta}_n)g(w_t; \tilde{\theta}_n)'.$$

Then we re-estimate the parameters using the optimal weighting matrix $\hat{S}^{-1}$ to obtain the optimally weighted simulated GMM estimator, which we denote by $\hat{\theta}_n$.

Under standard regularity conditions, this estimator is asymptotically normally distributed, and a consistent estimator of the asymptotic variance of $\sqrt{n}(\hat{\theta}_n - \theta_0)$ is:

$$\left(\frac{\partial g_n(\hat{\theta}_n)}{\partial \theta'}' \hat{S}^{-1} \frac{\partial g_n(\hat{\theta}_n)}{\partial \theta'} \right)^{-1}.$$

Since the moments are calculated by simulation, we use a numerical derivative of $g_n(\theta)$ to estimate the asymptotic variance of the estimator.

Appendix E. Sensitivity Analysis

We estimate the model using alternative categorizations for the price changes (q, ϵ) and the fixed versus variable contracts. In Alternative 1, price changes corresponding to Columns (3) to (5) in Table 4, as opposed to Columns (4) and (5), are labeled as q and remaining price changes make up ϵ. In Alternative 2, fixed contracts are more narrowly defined, as firm-fixed price contracts with $q = 0$. Table A1 presents the results, which are robust to these alternative specifications.

TABLE A1. Sensitivity Analysis: Alternative Categorizations

(Costs in $ thousand)	Base	Alternative 1	Alternative 2
Table 9: Cost components for median non-military contract			
Project cost for low-cost sellers	884.1 (40.4)	882.6 (42.8)	915.0 (53.9)
Project cost difference	271.3 (32.1)	257.1 (22.5)	259.4 (29.5)
Expected cost change	139.1 (15.8)	49.7 (9.0)	106.7 (13.4)
Bidding cost with two bids	52.1 (8.9)	47.7 (5.3)	46.7 (4.2)
Cost of competitive solicitation	20.5 (4.8)	20.5 (2.4)	15.5 (2.8)
Table 10: Effects of counterfactual policy (2)			
Number of bids	+0.8	+0.9	+0.9
Transfer	-45.7	-42.6	-39.6
Aggregate costs (A+B+C)	+4.1	+5.0	+7.3
Table 10: Effects of counterfactual policy (4)			
Number of bids	+1.0	+1.0	+0.9
Transfer	+12.0	+11.8	+13.4
Aggregate costs (A+B+C)	+0.6	+2.5	+0.7
Table 10: Effects of counterfactual policy (6)			
Number of bids	+2.0	+2.1	+2.2
Transfer	-36.4	-33.7	-36.8
Aggregate costs (A+B+C)	-7.7	-6.2	-8.1

Note: The estimated asymptotic standard errors are in parentheses. The unconditional median values of π are 0.38, 0.36, and 0.28, respectively. See Table 10 and the text for the counterfactual policies considered here.