## Work and Energy

## Goals and Introduction

In this experiment, we test and apply the principle of conservation of energy and the workenergy theorem. The work-energy theorem states that if there is work, $W$, performed on an object as it moves from one location to the next, then the kinetic energy, $K E$, of the object changes. We can symbolize this relationship as
$K E_{i}+W=K E_{f}$ (Eq. 1)
where the kinetic energy, or energy of motion, of an object depends on the mass of the object, $m$, and its speed, $v$,
$K E=\frac{1}{2} m v^{2}$. (Eq. 2)
Work is performed on the object when it is subject to a force, as the object moves from one location to the next. A scenario such as that shown in Figure 9.1 is useful for defining the relationship between an applied force, $\overrightarrow{\boldsymbol{F}}$, the displacement, $\Delta \overrightarrow{\boldsymbol{r}}$, the angle between the force and the displacement, $\theta$, and the work due to the applied force. In the figure, a person is depicted pulling on a rope that is attached to a box, as the box moves from the point $P_{1}$ to the point $P_{2}$.


Figure 9.1
The work performed by the force would be
$W=F \Delta r \cos \theta .(E q .3)$

Note that we don't know if the box is initially moving or not, or whether or not it is moving at the end. The act of applying a force either delivers or removes energy from an object or system, as it moves, based on the magnitudes of the force and displacement, and the angle between them.

When the angle between the force and the displacement is greater than $90^{\circ}$, the $\cos \theta$ will be negative, meaning that the work performed by that force is negative. Note also, that forces that act perpendicular to the displacement would do 0 work! If we were to consider the work done by friction in the scenario of Figure 9.1, what would be the angle between the friction force and the displacement? Would the work done by friction be positive or negative?

The gravitational force due to the Earth is an example of a force we always experience, and one whose effect must be factored into any laboratory experiment. The work performed by gravity depends only on the vertical distance between the initial and final locations of the object, $\Delta h$. We can write the work performed by gravity in terms of a change in potential energy, $P E$. For gravity the potential energy at a height $h$ above some location we designate as $h=0$ is given by
$P E=m g h($ Eq. 4)
and the work done by gravity as an object moves from one height to another would be
$W_{g}=-\Delta P E=-m g \Delta h .($ Eq. 5)

Because conservative forces allow us to express the potential energy at specific locations, it is possible to consider both the kinetic and potential energies possessed by an object at any moment. Using the mass of the object, $m$, its speed, $v$, and its height $h$ above a location where $h=0$, we write the total mechanical energy of the system, $E$, as
$E=K E+P E=\frac{1}{2} m v^{2}+m g h .(\mathrm{Eq} 6$.

By considering total mechanical energy, including $P E$, we can then restate the work-energy theorem as follows:

1) If there is no work performed by forces besides gravity as an object moves from one position to the next, then the total mechanical energy at position 2 should equal the total mechanical energy at position 1 which we could write as
$E_{1}=E_{2}$. (Eq. 7)
2) If there is work performed by forces besides gravity as an object moves from one position to the next, then the total mechanical energy at position 2 should equal the sum of the total mechanical energy at position 1 and the work, which we could write as
$E_{1}+W=E_{2}$. (Eq. 8)

Note that we have already accounted for the work done by gravity with our potential energy terms, as part of the total energy, $E$. It is also possible that the work could be negative, as we discussed earlier, causing the final energy at position 2 to be less than the energy at position 1 .

Figure 9.2 is an illustration of the apparatus to be used in this experiment. A glider is free to move along a track that is elevated at one end, while two photogates will be placed at points $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$. The photogates will be used to measure the time it takes for the glider to pass by, once at the bottom of the air track, and once at the top. A photogate is a photocell and light source connected to an electronic timer in such a way that whenever the light hitting the photocell is interrupted, the timer is turned on. The timer is turned off as soon as the light again hits the photocell. The light hitting the photocell is interrupted by the glider with length, $L$. As the glider passes through each gate, the transit times, $\Delta t_{1}$ and $\Delta t_{2}$, are measured. From these times and measuring the length of the glider the speeds of the glider at points $P_{1}$ and $P_{2}$ are calculated, using
$v=\frac{L}{\Delta t}$. (Eq. 9)

In order to define the height, $h$, at each of the points shown, or anywhere along the track, we would need to define a reference height at which $h=0$. Typically, we choose the lowest point in the experiment to be where $h=0$, and then $h$ would always be positive for points above this height (assuming we use a positive $y$-axis pointing upward, as is common). Using this convention, the height at $\mathrm{P}_{1}$ would be 0 , and the height at $\mathrm{P}_{2}$ would be $h$.


Figure 9.2

By knowing the reference height at each point and measuring the mass of the glider and speed at each point, as the glider goes up or down the incline, we will be able to calculate the total mechanical energy at each point. Because there will be some friction between the glider and the
track, we expect that the total mechanical energy will not be the same at each point. Does friction do negative or positive work? Does your answer depend on whether the glider is going up or down the incline?

Goals: (1) Measure and verify aspects of the work-energy theorem.
(2) Make appropriate measurements and calculate the kinetic and gravitational potential energies of an object.
(3) Use the work energy theorem to find the work due to friction, and the magnitude of the friction force on the glider.

## Procedure

Equipment - linear air track, glider, two photogates, balance, computer with the DataLogger interface and LoggerPro software, ruler

1) Turn on the computer, open LoggerPro and open the Two-Gate Timer link.
2) The two photogates should have their signal cables connected to DIG/SONIC 1 and DIG/SONIC 2 of the DataLogger interface. Turn on the interface with the switch.
3) Begin by leveling the track so that it is horizontal, or parallel to the table, and place the two photogates at two points along the air track, about 30 cm apart. The leveling is accomplished by turning the feet of the air track to adjust the height.

HINT: Be sure that there is enough room at either end of the air track to push the glider to ensure that you are not still pushing the glider while it is in either of the photogates. You are going to need to have space on BOTH ends. Throughout the experiment, your hand should not still be contacting the glider as it moves through either photogate. Also, be sure the glider does not bounce back into the photogate.
4) Put the glider on the air track and turn on the air. Hit "Collect" in LoggerPro to begin taking data and give the glider a brief push, so that it glides through both of the gates. Verify that both gates measure a gate time, and note which data result on the screen corresponds to the photogate plugged into port 1 (gate time 1) and which is for the one plugged into port 2 (gate time 2 ). Locate the photogate plugged into port 1 by the end of the air track with two feet and the photogate plugged into port 2 by the end of the air track with one foot. The gates will then correspond to the points $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ in Figure 9.2. Since the glider was just given an initial shove and the track is level, how should the times compare at this point?
5) Measure and record the distance between the feet of the air track. Label this value as $D$.
6) Raise the end of the air track with one foot by placing a shim with a thickness of $3-5 \mathrm{~cm}$ underneath the feet, resulting in a similar setup to that shown in Figure 9.2. Measure and record the thickness of the shim. Label this value as $H$.
7) Adjust the heights of the photogates so that the glider triggers each gate when passing through. Hit "Collect" again and let the glider slide down the full length of the incline to verify that both photogates still measure the time as the glider passes. If you have followed this setup, the photogate plugged into port 2 should measure a time first, since it should be on the right at the higher point, first passed by the glider sliding down.
8) Measure and record the distance between the photogates along the length of the air track, $d$.
9) Measure and record the mass, $m$, and length, $L$, of the glider. Use the balance to measure the mass.
10) You will now measure the time through each photogate, after the glider is given a push, so that it travels up the incline. Remember to be sure that your hand is not still in contact with the glider when it reaches the first photogate at $P_{1}$, and be sure that it gets through the second gate. Measure and record the time through each photogate for the glider traveling up the incline. Be sure to note correctly which time is associated with each photogate. Think about what should be happening to the speed of the glider as it travels up the incline. Should the time at the upper photogate be greater or less than the time measured at the lower photogate? Is this what you observed? Recall your response to the prelab question PL-3.
11) (Measure and Record) Repeat the procedure from step 10 four more times, so that you have five sets of data for the glider traveling up the incline.
12) You will now push the glider down the incline and measure the time through each photogate. Remember to be sure that your hand is not still in contact with the glider when it reaches the first photogate at $\mathrm{P}_{2}$. Measure and record the time through each photogate for the glider traveling down the incline. Be sure to note correctly which time is associated with each photogate. Think about what should be happening to the speed of the glider as it travels down the incline. Should the time at the upper photogate be greater or less than the time measured at the lower photogate? Is this what you observed? Again, recall your response to the prelab question PL-3.
13) (Measure and Record) Repeat the procedure from step 12 four more times, so that you have five sets of data for the glider traveling down the incline.

As always, be sure to organize your data records for presentation in your lab report, using tables and labels where appropriate.

## Data Analysis

Use the distance between the feet of the air track, $D$, and the thickness of the shim, $H$, to find the angle that the air track makes with the horizontal direction when the right side is raised by the shim. When the shim was inserted, the distance between the feet became the hypotenuse of a right triangle with the thickness of the shim being one of the sides. Thus,
$\sin \theta=\frac{H}{D}$.

Now, use the distance between the photogates, $d$, and this angle to find the height $h$, of the point $\mathrm{P}_{2}$. Here, the distance $d$ is the hypotenuse, and $h$ is opposite the same angle $\theta$. Thus,
$\sin \theta=\frac{h}{d}$.

Using the measured times and the length of the glider, $L$, apply Eq. 9 and calculate the speed at each photogate for each trial. Organize your results in a table.

Calculate the kinetic energy, the potential energy, and the total mechanical energy at each photogate for each trial, using Eq. 2, 4, and 6. Add these results to your table.

Question 1: If there is no work performed by forces besides gravity, how should the total mechanical energy at the first gate compare to the total mechanical energy at the second gate in each trial? Is this the case in any of your trials? Is your answer different for when the glider is going up the incline versus going down the incline? Explain clearly.

During this experiment, we expected there would be friction between the glider and the track, although it may be very small. Assuming this is the only force besides gravity doing work on the glider, calculate the work done by friction using Eq. 8 for each trial. Remember that we wrote Eq. 8 for an object moving from position 1 to position 2 . This means that you will need to change Eq. 8 for the trials where the glider is going down the incline to read as $E_{2}+W=E_{1}$.

Question 2: When friction does work on the glider, is it adding energy to the glider or taking it away? How can you tell? Should the answer depend on whether the glider is going up or down the incline? Explain clearly.

Using the distance between the photogates, $d$, and the work, $W$, that you found in each trial, calculate the magnitude of the friction force, $F_{f}$, using
$W=-F_{f} d$.

Question 3: Why would there be a minus sign in this formula? Consider Eq. 3 in answering this question.

## Error Analysis

The work done by friction on the glider should be independent of the direction it is moving and the variety of initial speeds you probably measured.

Calculate the average work done by friction in the five trials when the glider was going up the incline.

Also, calculate the average work done by friction in the five trials when the glider was going down the incline.

Then, calculate the percent difference between these two averages.

Question 4: How similar are the average work up the incline and the average work down the incline? Is this what was expected? Explain clearly.

## Questions and Conclusions

Be sure to address Questions 1-4 and describe what has been verified and tested by this experiment. What are the likely sources of error? Where might the physics principles investigated in this lab manifest in everyday life, or in a job setting?

## Pre-Lab Questions

Please read through all the instructions for this Experiment to acquaint yourself with the experimental setup and procedures, and develop any questions you may want to discuss with your lab partner or TA. Then answer the Pre-Lab Questions at the end of the Experiment, and enter your answers into this Quiz. Be sure to "Submit" the Quiz before the start of your lab section on the day this Experiment is to be run.

PL-1) A glider of length 12.1 cm passes through a photogate, which registers a time of $\Delta t=$ 0.288 sec . What is the average speed of the glider, in $\mathrm{m} / \mathrm{s}$, while it is passing through the photogate?

PL-2) Bill and Ted are running the Work and Energy experiment on an air track that has feet separated by $D=50.0 \mathrm{~cm}$. After leveling the track, they shim up the feet on one end of the track using a shim they measured to be $H=3.8 \mathrm{~cm}$ tall (see Figure 9.2). What is the sine of the inclination angle of their track, $\sin \theta$ ?

PL-3) Max and Irma measure the time at the lower photogate, $\mathrm{P}_{1}$, to be
$\Delta t_{1}=0.198 \mathrm{sec}$, and the time at the upper photogate, $\mathrm{P}_{2}$, to be $\Delta t_{2}=0.250 \mathrm{sec}$. The glider was

C (A) moving faster at the upper photogate during this part of the experiment.
C (B) traveling up the incline during this part of the experiment.
C (C) moving down the incline during this part of the experiment.
C (D) not subject to a net force during this part of the experiment.
C (E) Could be moving either down the incline or up the incline

PL-4) Calculate the kinetic energy of the glider at $P_{1}$ in Max \& Irma's experiment (see PL-
3). They measured the glider to have a mass of 131 g and a length of 10.0 cm . Please answer in the SI unit of energy, Joules. [The time at the lower photogate, $P_{1}$, was $\Delta t_{1}=0.198 \mathrm{sec}$.]

PL-5) Calculate the change in potential energy of a $131-\mathrm{g}$ glider as it moves downward along the track a distance of $d=32.1 \mathrm{~cm}$, which is inclined at an angle of $\theta=6.3^{\circ}(\sin \theta=0.110)$ above the horizontal (see Figure 9.2). Please answer in Joules.

